Structural Eng. / Earthquake Eng. Vol. 3. No. 1, 113 s-122 s, April 1986
Japan Society of Civil Engineers (Proc. of JSCE No. 368/ I -5)

SYMMETRY BREAKING BIFURCATION BEHAVIOR OF DOME STRUCTURES AND GROUP THEORY

By Kiyohiro IKEDA, Shogo MATSUSHITA**, and Kunio TORII****

An advanced method for a qualitative description of bifurcation buckling behavior of dome structures is proposed. While conventional finite displacement analysis technique is employed to trace the bifurcation behavior, a group theoretic method is adopted to describe the hierarchal structure of bifurcation paths.

The bifurcation behavior of polygonal-shaped truss domes is proved to be analogous to the subgroup structures of dihedral groups, which are extensively employed to describe the symmetry of polygons in mathematics. The categorization and description of the bifurcation behavior of a hexagonal-shaped truss dome structure insure the applicability and validity of the analogy in describing the bifurcation behavior. Such an analogy is able to monitor the interrelationship between the geometrical properties of the polygonal-shaped domes and their bifurcation behavior.

1. INTRODUCTION

It has been found that dome structures often exhibit bifurcation buckling behavior, along with sharp reduction of load carrying capacities. Its example can be seen in the typical load versus displacement relationships (equilibrium paths) of a spherical dome shown in Fig. 1. This figure demonstrates the existence of bifurcation path 2) branching from the path 1) at the bifurcation point A. There are apparent differences in the deformation modes of the dome between these two paths, as shown in Fig. 2.

Considerable analytical studies[1)~6)] have been focussed on the bifurcation behavior, which has a dominant influence on the performance of dome structures. Although these studies appear to be almost sufficient for the analytical tracing of complex bifurcation behavior of domes represented by a number of bifurcation points and branching paths, theoretical bases for interpreting the bifurcation behavior seem somewhat unsecure. Thompson developed a stability theory of equilibrium paths and performed a categorization of

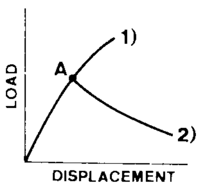
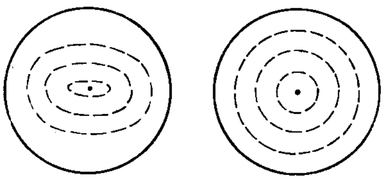


Fig. 1 Typical Equilibrium Paths of a Spherical Dome.



Path 1) Path 2)

Fig. 2 Contours of Typical Deformation Modes of a Spherical Dome.

* Member of JSCE, Ph. D., Research Fellow, The Technological Univ. of Nagaoka, Niigata, Japan, 949-54.
** Graduate Student, The Technological Univ. of Nagaoka.
*** Member of JSCE, D. of Eng., Associate Professor, The Technological Univ. of Nagaoka.

bifurcation points[7]. However, his theory was not compatible with conventional bifurcation tracing strategies in that it necessitated the higher-ordered derivatives of total potential energy to be computed.

Extensive mathematical studies on bifurcation behavior have been conducted by applied mathematicians[8),9)]. They interpreted the behavior as a process of losing symmetry and called such behavior the 'symmetry breaking bifurcation'. The behavior has been found to be analogous to the subgroup structure of a symmetry group and a simple qualitative explanation of the behavior has been performed with the use of group theory. However, most of these studies are concerned with the bifurcation behavior of the solution of ordinary differential equations and only a few investigations have been made regarding the bifurcation behavior of structures.

The objective of this paper is to combine these two completely different approaches developed in different fields and arrive at better conceptual understanding of bifurcation buckling behavior of dome structures. While finite displacement analysis and other methods developed in the field of structural engineering are utilized to trace the bifurcation behavior, a group theoretic method is employed for the qualitative description of the behavior. An emphasis is placed on identifying the geometrical symmetry of bifurcation modes.

2. GROUP THEORY FOR DESCRIBING SYMMETRY

This chapter introduces several mathematical concepts and relevant terms used to express the geometrical symmetry of figures[10]. These will serve as convenient tools for describing the geometric symmetry of bifurcation modes of structures.

A group denotes a non-empty set S satisfying the following three properties : (1) there exists an identity element in S; (2) for every choice of the elements a, b and $c \in S$ the relationship $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ is satisfied; (3) every element in S has an inverse in S. A subgroup of S indicates a non-empty subset of S which satisfies all these three properties for defining a group. Isometries of a plane can be defined as one-to-one mappings on the Euclidian plane $R^2 = R \times R$ that preserve the distance between any two points in the plane, where R is the set of all real numbers. Isometries are the products of reflections, translations, and rotations. Given a figure in the Euclidian plane, the figure is stated more symmetric if its configuration can be preserved by more isometries. In describing the symmetry of the figure, it is mandatory to utilize its symmetry group, which is made up of a series of isometries that preserve the configuration of the figure[8)~10)].

The symmetry group of a regular n-gon ($n=3, 4, 5, \cdots$) is called the dihedral group of degree n (D_n). Dihedral groups have been extensively employed in describing the bifurcation behavior in the field of applied mathematics[8),9)]. The elements of the group consist of the following isometric transformations :

$$\sigma_j \text{ and } \tau\sigma_j \quad j=1, 2, \cdots, n \qquad (1)$$

where σ_j is the $360(j-1)/n$ degree clockwise rotation about the origin and τ is the reflection in the y-axis; the multiple of the two transformations denotes that the transformations are achieved in sequence from the right to left. The number of the elements of this group, equal to $2n$, is called its order. The geometrical meanings of these transformations are schematically illustrated in Fig. 3. The existence of an element σ_j in a symmetry group denotes that relevant figure is point symmetric about the origin regarding a $360(j-1)/n$ degree rotation, while the presence of $\tau\sigma_j$ expresses the line symmetry in the straight line intersecting with the y-axis at the origin at an angle of $360(j-1)/n$ degrees.

In order to demonstrate the usefulness of dihedral groups in expressing geometrical symmetry, a dihedral group of degree three, D_3 is employed here to describe the displaced states of a regular triangular rigid plate shown in Fig. 4. This plate is supported with vertical strings at the nodes 1, 2, and 3 and initially placed in the $x-y$ plane. The group D_3 consists of the six elements σ_1, σ_2, σ_3, $\tau\sigma_1$, $\tau\sigma_2$, and $\tau\sigma_3$, where σ_j ($j=1, 2$, or 3) expresses the $120 \times (j-1)$ degree rotation about the z-axis. This group has the following five subgroups :

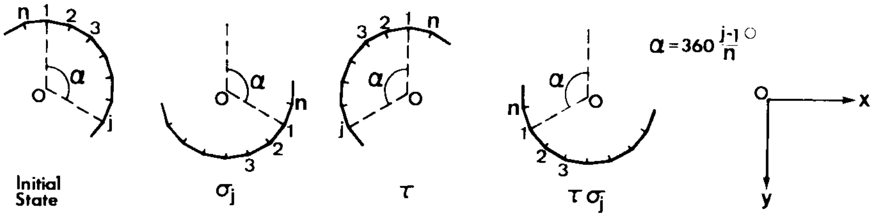


Fig. 3 Geometrical Transformations Caused by the Elements of Dihedral Group D_n.

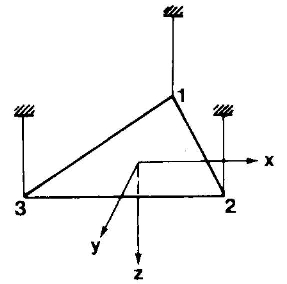
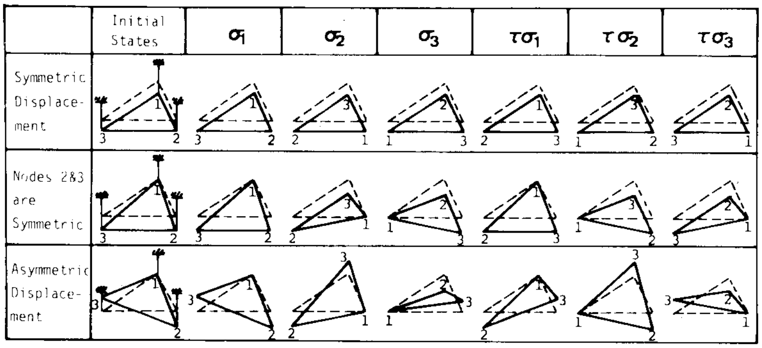


Fig. 4 Regular Triangular Rigid Plate Supported with Vertical Strings.

Table 1 Effects of the Transformations Caused by the Elements of D_n on the Geometric Configurations of the Triangular Plate.



$E=\langle\sigma_1\rangle \quad C_3=\langle\sigma_1, \sigma_2, \sigma_3\rangle \quad D_1^j=\langle\sigma_1, \tau\sigma_j\rangle \quad j=1,2 \text{ or } 3 \cdots\cdots (2)$

where the parenthesis $\langle \ \rangle$ is used to express the elements of a group.

Table 1 schematically illustrates the effects of these six transformations on the three different displaced states of the plate. In the case where the plate displaces uniformly in the vertical direction, its configuration can be preserved by each of the six transformations since all the nodes are symmetric. Such a case is henceforth stated that the displaced plate has D_3 (with an order six) as its symmetry group. In the case where the nodes 2 and 3 displace identically, the configuration of the plate can be preserved only by the transformations σ_1 and $\tau\sigma_1$. The plate deformed in this fashion has the group D_1^1 with an order two as its symmetry group. In the case where the three nodes displace in an asymmetric fashion, the configuration of the plate can be preserved only by the identity transformation σ_1 and has a group E with an order one. As can be demonstrated by these examples, the more symmetric a structure, the greater the order of its symmetry group. The order will be an appropriate parameter for expressing the level of symmetry of a structure.

3. BIFURCATION BEHAVIOR AND SYMMETRY GROUPS

A theoretical method for describing and interpreting the bifurcation buckling behavior of structures is advanced. This method is based on the findings by applied mathematicians that bifurcation behavior, in general, is a process of losing symmetry and is analogous to the subgroup structure of a symmetry group[8),9)]. Fujii[8)] found that the bifurcation behavior of certain systems is 'G-covariant' (covariant with a symmetry group G) and applied symmetry groups to the description of bifurcation behavior of structures. In explaining the term G-covariant, let us consider a problem of finding the equilibrium paths of a discretized structural system. Such a problem can be interpreted as the problem of finding the set of variables (f, x) satisfying the equations of equilibrium :

$H(f, x)=0 \cdots\cdots (3)$

where f denotes the loading parameter and x is the unknown nodal coordinate vector. This problem can be stated G-covariant if the following relationship is satisfied[8)].

$H(f, T_g x)=T_g H(f, x)$ for all g, f and x ·· (4)
where g is an element of the group G and T_g is the transformation caused by the element.

His theoretical findings with respect to a G-covariant system are summarized as : (1) The paths of a G-covariant system are characterized by their symmetry group G', which is a subgroup of G ; (2) the path having G as its symmetry group is called a fundamental path ; (3) a path will preserve its symmetry group until it reaches at a bifurcation point ; (4) when bifurcation paths branch from an equilibrium path at a bifurcation point with a single root, the symmetry groups of the bifurcated paths are the subgroups of the symmetry group of the equilibrium path ; (5) all the single critical points on the path with a trivial symmetry group E, in general, are the stationary points of the loading parameter f.

These findings are applicable to the description of bifurcation buckling behavior of structures simply by replacing the term 'path' used above by the 'equilibrium path'. Of course, an appropriate symmetry group must be selected and the covariancy of the behavior with the symmetry group must be verified in applying the findings to the description. These theoretical findings will greatly contribute to the qualitative description of the bifurcation behavior of polygonal domes.

4. VERIFICATION OF D_n-COVARIANCY OF SIMPLE TRUSS DOMES

It was demonstrated in the previous chapter that the bifurcation behavior of a system which is covariant with a symmetry group is characterized by the properties of the group. In applying group theory to the description of bifurcation behavior of structures, it is required to verify that the behavior is covariant with a symmetry group. In this chapter, as an attempt of such a verification, the behavior of an n-gonal reticulated truss dome (see Fig. 5) under symmetric vertical loading is proved to be covariant with the dihedral group of degree n, which is a symmetry group of a regular n-gon. This dome is made up of a series of elastic truss members with identical sectional and material properties.

As we have seen, the verification of covariancy can be achieved by showing that Eq. 4 is satisfied. Using the equilibrium equations for the finite-displacement problems of elastic truss members developed in Ref. 6, one can write the equilibrium equations for this dome as :

$H_i(f, x)=K_{ij}(x)x_j-F_{0i}(x)-f\cdot f_i=0 \quad i, j=0,1,2\cdots,n$ ················ (5)

where the subscripts i and j denote that the corresponding vector or matrix is related to the i-th or j-th node, K_{ij} is the three by three sub-matrix of the nonlinear stiffness matrix, F_{0i} is the three-dimensional nonlinear vector, and f_i is the three-dimensional normalized nodal-load vector. The summation convention applies to the dummy variable j.

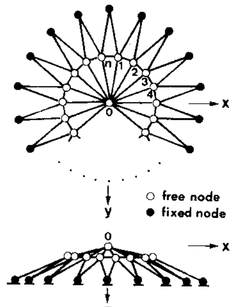


Fig. 5 n-gonal Truss Dome.

For the cases of symmetric vertical loading, all the normalized load vectors for the nodes 1 through n must be equal, i. e.,

$f_i=f_j \quad i, j=1,2,\cdots n$ ·· (6)

Under this symmetric loading, the nodes 1 through n displace symmetrically until a bifurcation point is reached. For such symmetrically displaced state, the transformation T_g associated with the element of the dihedral group causes merely a permutation among the nodes 1 through n. Hence the transformation T_g reperesents the following permutation of the node numbers :

$i\rightarrow l_i \quad i=1,2,\cdots n$ ·· (7)

where the index l_i takes a value either $1, 2\cdots$, or n. With the use of this permutation in Eq. 5, the left hand side of Eq. 4 for this dome structure becomes :

$H_i(f, T_g x)=K_{ij}(T_g x)T_g x_j-F_{0i}(T_g x)-f\cdot f_i=K_{l_i l_j}x_{l_j}-F_{0l_i}-f\cdot f_i$ ···································· (8)

In this equation, the dummy variable l_j can be replaced by a variable j and the vector f_i is equal to f_{l_i} as indicated in Eq. 6. With the use of these relationships, Eq. 8 will lead to :

$H_i(f, T_g x)=K_{l_i j}x_j-F_{0l_i}-f\cdot f_{l_i}=T_g H_i(f, x)$ ·· (9)

Thus, Eq. 4 is satisfied for all the transformations initiated by the elements of a group D_n. Hence the bifurcation behavior of this n-gonal dome under symmetric vertical loading is D_n-covariant and is analogous to the subgroup structure of D_n. Such an analogy is expected to hold for other types of domes and different kinds of structures as well. The verification of the applicability of the analogy to more general cases will be a topic requiring future studies.

5. DESCRIPTION OF BIFURCATION BEHAVIOR OF DOME BY GROUP THEORY

The Fujii's findings regarding symmetry groups, of great assistance in describing bifurcation buckling behavior, were proved to be applicable to the n-gonal truss dome in the previous chapter. His findings are employed here for qualitative description of the bifurcation behavior of a hexagonal truss dome (see Fig. 6) under the symmetric vertical loading pattern listed in Table 2. This dome is a special case (n=6) of that n-gonal dome so that its bifurcation behavior is covariant with the dihedral group D_6, which is a symmetry group of a hexagon. The bifurcation behavior of the dome is investigated here by means of the group D_6.

Table 2 Vertical Loading Pattern.

NODE NO.	PATTERN
0	0.5
1	1
2	1
3	1
4	1
5	1
6	1

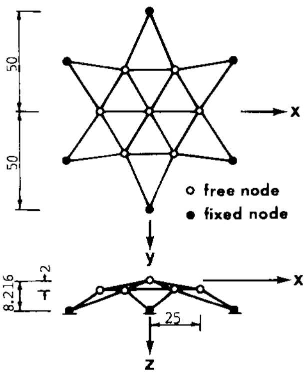


Fig. 6 Hexagonal Truss Dome (unit in cm).

This group consists of the 12 elements σ_j and $\tau\sigma_j (j=1, 2, \cdots 6)$, where σ_j denotes the $60\times(j-1)$ degree rotation about the z-axis. The authors have obtained all the subgroups of this group by investigating all the possible subsets of the group on the basis of the aforementioned three basic properties for defining a group. Consequently, this dihedral group had the following 15 subgroups :

$$C_6=\langle\sigma_1,\sigma_2,\sigma_3,\sigma_4,\sigma_5,\sigma_6\rangle \quad C_3=\langle\sigma_1,\sigma_3,\sigma_5\rangle \quad C_2=\langle\sigma_1,\sigma_4\rangle \quad E=\langle\sigma_1\rangle$$
$$D_3^j=\langle\sigma_1,\sigma_3,\sigma_5,\tau\sigma_j,\tau\sigma_{j+2},\tau\sigma_{j+4}\rangle \quad j=1,2$$
$$D_2^j=\langle\sigma_1,\sigma_4,\tau\sigma_j,\tau\sigma_{j+3}\rangle \quad j=1,2,3$$
$$D_1^j=\langle\sigma_1,\tau\sigma_j\rangle \quad j=1,2,\cdots 6 \qquad (10)$$

Figure 7 illustrates the deformed configurations of the dome associated with these subgroups. As can be seen, the three subgroups D_6, C_6 and D_3^1 represented the symmetrically displaced configuration. Such a configuration should be represented by the group D_6, which has the greatest order among them.

It was noted that some of the groups denote basically the same configuration. For example, the deformed states corresponding to the subgroups D_2^1, D_2^2 and D_2^3 are identical since the configurations for D_2^2 and D_2^3 can be obtained by rotating the configuration for D_2^1 about the z-axis through an angle of 120 or 240 degrees. The notation D_2^j is employed here for representing these three subgroups for simplicity. Likewise, the notation D_1^{2j-1} can be utilized for representing the three subgroups D_1^1, D_1^3 and D_1^5 ; and D_1^{2j} for D_1^2, D_1^4 and D_1^6. Consequently, the configuration of the dome can be represented by the seven subgroups ; D_6, D_3^2, D_2^j, D_1^{2j-1}, D_1^{2j}, C_2 and E. The subgroups defined in this manner, named the 'effective subgroups' of D_6 by the authors, are henceforth employed in place of the subgroups of D_6 so as to simplify

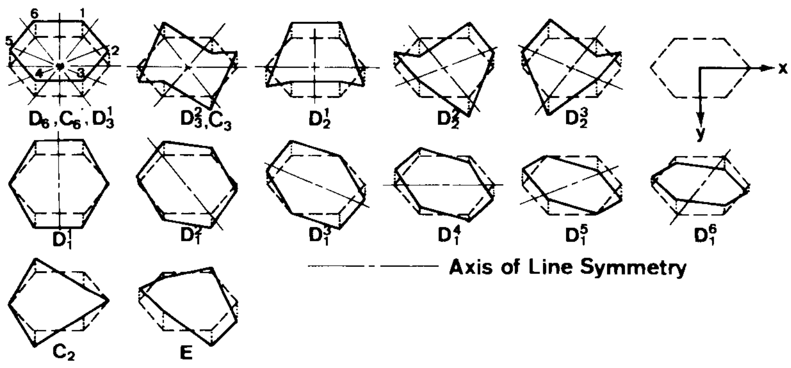


Fig. 7 Deformed Configurations of the Dome Associated with the Subgroups of D_6.

the discussion.

Every other node displaces identically for the mode having a group D_3^2 as a symmetry group. The group D_2^j represents a bifurcation mode which is line symmetric in two axes and point symmetric regarding a 180 degree rotation about the origin. The groups D_1^{2j-1} and D_1^{2j} express the modes which are line symmetric in an axis, while the group C_2 does what is point symmetric regarding a 180 degree rotation. The group E denotes a completely asymmetric mode with no axis or line symmetry.

The following inter-group relationships exist among these seven subgroups :

$$D_6 \supseteq D_3^2(\text{or } D_2^j) \supseteq D_1^{2j} \supseteq E \quad D_6 \supseteq D_2^j \supseteq D_1^{2j-1}(\text{or } C_2) \supseteq E \cdots\cdots \quad (11)$$

where the symbol $A \supseteq B$ denotes that the group B is a subgroup of a group A.

As a numerical example, the analytical bifurcation behavior of this dome (see Fig. 8) was obtained with the use of the analysis strategy and the computer program developed by Nishino et al[6]. At the same time, the deformation modes of the dome were investigated. This investigation made clear that each bifurcation path can be characterized by one of the effective subgroups. The equilibrium paths of the dome, therefore, were categorized on the basis of these effective subgroups and illustrated in Fig. 8. In this and the subsequent figures, the bold-solid lines express the equilibrum paths which have the group D_6 as their symmetry group, the long-dash lines denote those having D_3^2, the short-dash lines do D_2^j, etc. The symbol) denotes the equilibrium path, the symbol ● expresses the bifurcation point with a single root, and ○

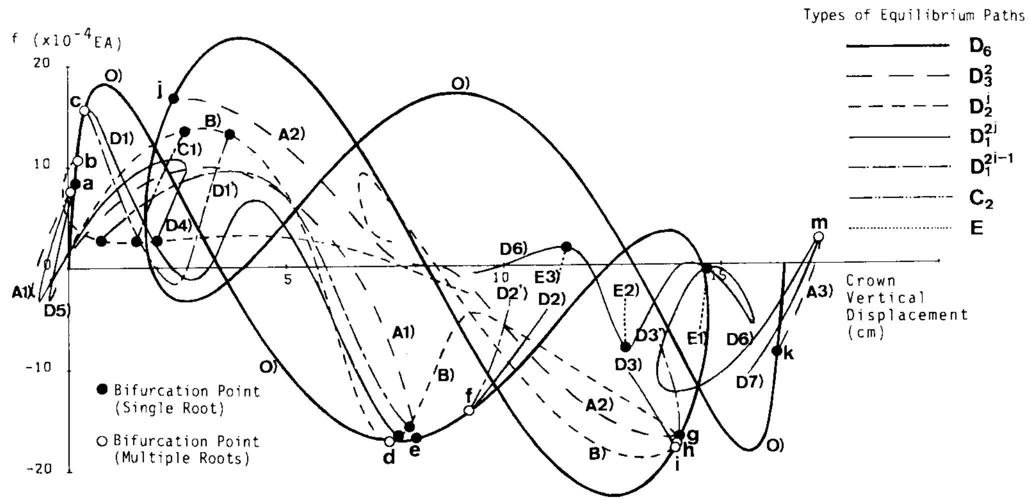


Fig. 8 Equilbrium Paths for the 6–gonal Truss Dome.

denotes that with a double root.

This analytical bifurcation behavior displayed the existence of seven different types of paths corresponding to the seven effective subgroups defined above. Thus, the effective subgroups are able to categorize the bifurcated paths and the number of effective subgroups appear to express the number of different types of bifurcation paths.

There existed an apparent hierarchy among the equilibrium paths. When two or more equilibrium paths intersect at a bifurcation point, the path with the higher-ordered symmetry group can be defined as the main equilibrium path and the other paths the bifurcation paths. For example, the path A 3) is the bifurcation path of the path O) at the bifurcation point k. However, at the same time, the path A 3) is the main path of the branching path D 6) at the point m. Thus, the terms 'main' path and 'bifurcation' path have only relative meanings.

The equilibrium path O) had D_6 as its symmetry group so that this path can be stated the fundamental equilibrium path from the Fujii's theorem (2). A series of bifurcation paths branched from the fundamental path, at the bifurcation points a, b, $\cdots$, and k. These consisted of (1) the paths A 1), A 2) and A 3) with a group D_3^2 as a symmetry group; (2) the path B) with D_2^j; (3) the paths D 1), D 2), and D 3) with D_1^{2j}; (4) the paths D 1′), D 2′), and D 3′) with D_1^{2j-1}. The paths D 4), D 5), D 6), and D 7) with a group D_1^{2j} further bifurcated from the paths A 1), A 2), and A 3); the path D 4) then intersected with the path B) with a group D_2^j. Such a feature is associated with the fact that a group D_1^{2j} is a subgroup of a group D_3^2 as well as of D_2^j. The paths E 1), E 2), and E 3) with a trivial subgroup E further branched from the path D 6). One consequence of these is that the bifurcation process among these sets of equilibrium paths followed the inter-group relationship (11). Moreover, the other bifurcation paths satisfied this relationship as well. It can be stated that the hierarchal structure of equilibrium paths is analogous to the subgroup structure of the (effective) subgroups. In addition, it was noted that the Fujii's theorem (4) holds for the cases of double roots as well, that is, the symmetry groups of the bifurcation paths are the subgroups of the symmetry group of the main path for both single and double roots.

The reduction of the order of symmetry group was observed in association with the progress of bifurcation buckling of the dome. For example, in the course of a branching process represented by a series of equilibrium paths O), A 3), D 6), and E 1), the order of symmetry group was constantly reduced from 12 to six, two, and finally one. Similar reduction processes were observed for the other bifurcation paths as well. Such a feature is based on the Lagrange's theorem[10], which states that "the order of a group is divided by the order of its subgroup". The bifurcation buckling behavior of the dome can be characterized by the reduction of the symmetry of the dome associated with the decrease of the order of symmetry group.

No bifurcation path branched from the bifurcation paths E 1), E 2), and E 3) with a trivial symmetry group E, as can be expected from the Fujii's theorem (5). The deformed states of the dome represented by the trivial group E, which have completely asymmetric geometrical configurations, cannot lose any more symmetry. Hence they should not have any further bifurcation paths from the standpoint of 'symmetry breaking bifurcation' advanced by applied mathematicians[8),9)].

As can be seen from these investigations of the analytical bifurcation behavior of the truss dome under symmetric vertical loading, the behavior is analogous to the subgroup structure of effective subgroups and the Fujii's theorems are capable of qualitatively explaining the behavior. The effective subgroups, which have much simpler subgroup structures than the original dihedral group does, appear to be efficiently applicable to the qualitative description of bifurcation buckling behavior of dome structures. The theoretical bases of effective subgroups are currently somewhat unsecure so that more studies should be required prior to arriving at general conclusions.

6. BIFURCATION BEHAVIOR OF POLYGONAL—SHAPED DOME STRUCTURES

Conventional dome structures are often constructed to be polygonal shaped, or analogous to it. However, the effects of geometric configurations of such domes on the bifurcation behavior have not been made clear. As an attempt to address this problem, interrelationships between the bifurcation behavior of polygonal-shaped domes and the degrees of polygon are investigated.

The hierarchal structure of the subgroups of a dihedral group is highly dependent on the algebraic property of its degree since the order of a group is divided by the order of its subgroup [10]. Therefore, the number of the subgroups of dihedral group is expected to be directly proportional to the number of factors of the order. In the case where the degree of a dihedral group n is equal to a prime number, the order of its subgroups is equal to either 1, 2, or, n, which is a factor of the order of the dihedral group, $2n$. By contrast, the subgroup structure of a dihedral group is more complex for the case where the degree is not a prime number associated with the increased number of factors of the order of a group. Such a fact can be monitored from an interrelationship between the number of effective subgroups for a series of dihedral groups and their degrees (see Fig. 9). This interrelationship was obtained by following the same procedure employed for finding the effective subgroups for the group D_6. As can be seen, the dihedral groups of prime degrees D_3, D_5, and D_7 had only three effective subgroups, while those of non-prime degrees D_4, D_6, and D_8 had significantly increased number of effective subgroups. Hence the polygonal domes represented by the dihedral groups of prime degrees are expected to possess much simpler bifurcation path structures than those of non-prime degrees.

These discussions can be confirmed from the comparison of the bifurcation behavior of six-gonal and seven-gonal domes. While Figs. 6 and 10 illustrate their geometrical configurations, Figs. 8 and 11 demonstrate their bifurcation behavior. The seven-gonal dome (with a prime degree) exhibited the simpler hierarchal structure of the bifurcation paths, expressed by the decrease of the types of paths and the reduction of the order of symmetry groups of branching paths, despite the fact that the dome had a more complex geometrical configuration than the six-gonal dome (with a non-prime degree) did.

These conclusions were derived with the aids of simple example truss-dome structures so that it will be a natural course of the future research to treat more general cases and to investigate the quantitative influence of the geometrical symmetry on the strength of polygonal domes. Group theory will be of great assistance in such an investigation and provide us deeper insights into bifurcation behavior.

7. CONCLUDING REMARKS

This paper has advanced a group theoretic method for describing the bifurcation buckling behavior of dome structures. While the theoretical findings by applied mathematicians, especially by Fujii, formed the

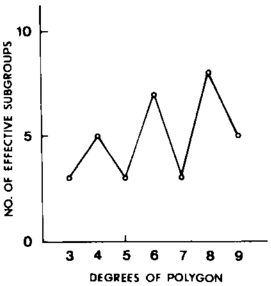


Fig. 9 Correlation between the No. of Effective Subgroups and the Degrees of Polygons.

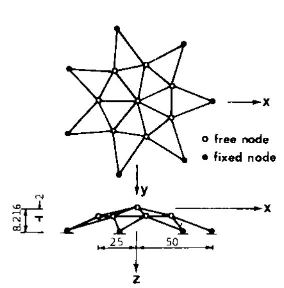


Fig. 10 Seven-gonal Truss Dome (unit in cm).

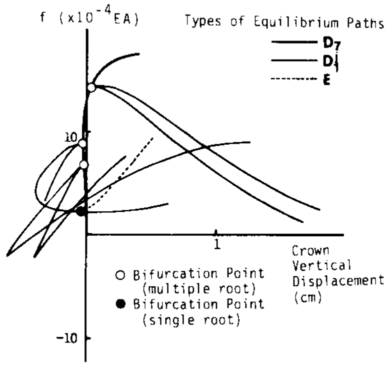


Fig. 11 Equilibrium Paths of the 7–gonal Dome.

basis of this method, these findings were extended and revised herein so as to make them applicable to structural engineering problems.

Several mathematical concepts used to express the geometrical symmetry of figures were introduced. Especially, the usefulness of dihedral groups in describing the symmetry of displaced state of figures was demonstrated with the use of a simple example.

The Fujii's findings regarding the bifurcation behavior of systems covariant with a symmetry group were introduced, along with the definition of a term, covariancy. The bifurcation behavior of simple polygonal-shaped truss domes under symmetric vertical loading was proved to be covariant with the dihedral groups and analogous to the subgroup structures of these groups.

A dihedral group of degree six was applied to the qualitative description of analytical bifurcation behavior of a hexagonal truss-dome structure, which was proved to be D_6-covariant. The 'effective subgroups' of D_6, representing potential bifurcation modes, were advanced through an investigation of the deformation modes of the dome. As can be expected from Fujii's theorems, the hierarchal structure of bifurcation paths was analogous to the subgroup structure of effective subgroups. The effective subgroups could serve as a convenient tool for describing the bifurcation behavior of dome structures. In addition, a method for distinguishing the difference between the main and the bifurcation paths at a bifurcation point was advanced.

The effects of geometrical configurations of domes on their bifurcation behavior were investigated for a series of polygonal-shaped truss domes. The algebraic properties of the degrees of the polygons exerted great influences on the bifurcation behavior of the domes. For example, the polygonal domes with prime degrees exhibited much simpler hierarchal bifurcation path structures than those with non-prime degrees, represented by the decrease of the types of paths and the reduction of the orders of symmetry groups of branching paths.

These conclusions were derived on the basis of many hypotheses and their verification was performed with the use of simple axis-symmetric, truss-dome structures. Future studies should be required to verify these for more general cases, such as actual dome structures or non-axis-symmetric structures. When fully developed and verified, group theory could be valuable in describing the bifurcation behavior of dome structures, without necessitating costly computations.

8. ACKNOWLEDGEMENT

The authors are grateful to Prof. Fumio Nishino of the University of Tokyo for his valuable academic suggestions, especially in the early stage of this research.

REFFRENCES

1) Endou, A., Hangai, Y. and Kawamata, S. : Post-buckling analysis of elastic shells of revolution by the finite element method, Report of the Institute of Industrial Science, 26, The University of Tokyo, July 1976.
2) Yoshida, Y., Masuda, N. and Matsuda, T. : A discrete element approach to elasto-plastic large displacement analysis of thin shell structures, Proceedings of the Japan Society of Civil Engineers, No. 288, pp. 41~55, August 1979 (in Japanese).
3) Fukumoto, Y. and Mizuno, E : Static and quasi-static stability and imperfection sensitivity of reticulated shells, Proceeding of the Japan Society of Civil Engineers, No. 288, pp. 29~40, August 1979 (in Japanese).
4) Hosono, T. : Analysis of elastic buckling problem by arc length method, Part 1. The nature of incremental solution at the buckling point, Transactions of the Architectural Institute of Japan, AIJ, No. 242, pp. 41~48, April 1976 (in Japanese).
5) Hosono, T. : Analysis of elastic buckling problem by arc length method, Part 2. The arc length method for actual numerical analysis, Transactions of the Architectural Institute of Japan, AIJ, No. 243, pp. 21~30, May 1976 (in Japanese).
6) Nishino, F., Ikeda, K., Sakurai, T. and Hasegawa, A. : A total lagrangian nonlinear analysis of elastic trusses, Proceedings of the Japan Society of Civil Engineers, No. 344, I-1, pp. 39~53, April 1984.
7) Thompson, J. M. T. and Hunt, G. W. : A general theory of elastic stability, John Wiley and Sons, London, 1973.
8) Fujii, H. : Striped patterns in the nature, Nonlinear phenomena and analyses, Introduction to modern mathematics [1], a Special Issue of Mathematics Seminar, pp. 81~108, October 1979 (in Japanese).

9) Fujii, H. : Tracing global bifurcation phenomena, Numerical analyses and nonlinear phenomena, Introduction to modern mathematics [2], a Special Issue of Mathematics Seminar, pp. 75~122, November 1981 (in Japanese).

10) Baumslag, B. and Chandler, B. : Theory and problems of group theory, Outline Series in Mathematics, McGrow-Hill Book Company, June 1968.

(Received July 15 1985)