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# OPTIMUM DISTRIBUTION OF INPUT ENERGY AND STIFFNESS IN EARTHQUAKE RESISTANT DESIGN FOR SHEAR MULTI-MASS SYSTEMS

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Based on the control of input energy distributed to each story in multi-mass systems, the relation between input energy and structural characteristics is examined fundamentally to establish the rational earthquake resistant design. As the results of numerical analyses, the plastic response of shear multi-mass systems can be treated as the linearized systems in apparent, in the case of equal stiffness distribution between upper and lower story. The theoretical formulation based on the modal analysis is developed to the elasto-plastic systems and the method of input energy control is discussed. The optimum distribution of input energy and stiffness in the earthquake resistant design are also presented.

## 1. INTRODUCTION

In the earthquake resistant design of structures, it is important to balance aseismic safety with economy. Many research works in the past have focused mainly on the aseismic safety, and the methodology for securing safety in earthquakes were developed by various analytical and experimental studies. In the earthquake resistant design, however, it is required to use the rational and simple method taken account of both economy and aseismic safety.

As structures behave inelastic under severe earthquake motion, the methods based on the energy concept are studied recently to reduce the structural damage and to secure the aseismic safety of structures. While the restoring force of structures and earthquake motion show very complicated features, the method mentioned above is very applicable in the earthquake resistant design because the energy quantity combining the structural strength and the plastic deformation capacity can be easily obtained. Akiyama[1)~3)] and Matsushima[4),5)] proposed the optimum distribution of yield-shear coefficient to estimate the structural damage by the cumulative ductility factor of structures. Suzuki et al.[6),7)] proposed to use the structural strength and the maximum plastic deformation as an index for earthquake resistant design.

On the other hand, the authors suggested that the inelastic response of structures in elasto-plastic multi-mass systems subjected to seismic motion could be estimated easily by the input energy, and presented how to control the input energy of structures[9)]. The purpose of this study is to describe the applicability of these results to the earthquake resistant design, and the relation between the input energy and the characteristics of shear multi-mass structures, taking account of the restrictive conditions for both aseismic safety and economy. The method how to decide the optimum distribution of input energy and

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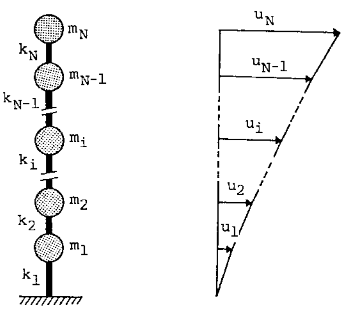
structural stiffness are also presented. The medium and low rise structures were chosen in this study. It has been already made clear by the authors that the structural safety and the extent of damage of structures in severe earthquake motion can be examined by the quantity of input energy, and that the quantity of input energy depends on such structural constants as mass and stiffness[9]. In this study, the structural safety is evaluated by the relation between the input energy and the energy absorption capacity of structures, and the economy, another important design factors, is evaluated by the total stiffness of structures.

## 2. FUNDAMENTAL CHARACTERISTICS OF SHEAR MULTI-MASS SYSTEMS ANALYZED BY MODAL ANALYSIS

If a multi-mass system having non-linear restoring force could be replaced properly to an equivalent linearized system, it would be possible to evaluate analytically the response of inelastic structures with modal analysis. In a multi-mass system having several masses, the fundamental mode of vibration is predominant in structural response, compared with higher order of vibration mode. This indicates that the total input energy and the input energy distributed to each story can be controlled by handling the fundamental mode of vibration in multi-mass systems[9].

### (1) Basic formulation of input energy and stiffness distribution in multi-mass systems

Fig. 1 (b) shows the fundamental mode of vibration in the given multi-mass system of Fig. 1 (a). When the response of multi-mass system can be evaluated properly by the fundamental mode of vibration, the input energy distributed to each story $E_i (i=1-N)$ due to sinusoidal excitation is defined by the mass distribution of the system and the fundamental mode of vibration, and the input energy to the system (the total input energy $E$) is given as $E=\sum_i^N E_i$. The ratio of the input energy distributed to the i-th story $E_i$ to the total input energy $E$ (the input energy distribution) can be written as[9]




(a) Model of a multi-mass system. (b) First mode.

Fig. 1

$$
\begin{aligned}
E_1/E &= m_1 u_1 / \{m_1 u_1 + m_2(u_2-u_1) + \cdots\cdots \\
&\quad + m_N(u_N - u_{N-1})\} \\
E_i/E &= m_i(u_i - u_{i-1}) / \{m_1 u_1 + m_2(u_2-u_1) + \cdots\cdots \\
&\quad + m_N(u_N - u_{N-1})\} : i \geqq 2
\end{aligned}
\qquad \cdots\cdots (1)
$$

where $u_i$=the $i$-th element of fundamental mode of vibration.

Defining the mass ratio of each story to the first as $a_i = m_i/m_1$ and the ratio of the input energy distributed to each story as $b_i = E_i/E$, Eq. (1) is expressed by the following simultaneous equations as the function of vibration mode $\{u_1\}$.

$$
\begin{bmatrix}
a_1(1-1/b_1)-a_2 & a_2-a_3 & a_3-a_4 & \cdots\cdots & a_N \\
a_1-a_2+a_2/b_2 & a_2-a_3-a_2/b_2 & a_3-a_4 & \cdots\cdots & a_N \\
\vdots & & & & \vdots \\
a_1-a_2 & a_2-a_3 & a_3-a_4 & \cdots\cdots & a_N - a_N/b_N
\end{bmatrix}
\begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{Bmatrix} = 0 \cdots\cdots\cdots\cdots (2)
$$

When the value of $a_i$ and $b_i$ are given, the eigenvector $\{u_1\}$ of the fundamental vibration can be obtained from Eq. (2), and then if the fundamental natural frequency $\omega$ and the mass $m_i$ are given, the stiffness of each story $k_i$ can be determined for the given modes of vibration $\{u_1\}$.

$$
\begin{aligned}
k_1 &= \omega^2 \sum_{i=1}^{N} m_i u_i / u_1 \\
k_i &= \omega^2 \sum_{i=2}^{N} \left\{ \sum_{j=i}^{N} m_j u_j / (u_i - u_{i-1}) \right\} \qquad (N > i \geqq 2) \cdots\cdots\cdots (3) \\
k_N &= \omega^2 m_N u_N / (u_N - u_{N-1})
\end{aligned}
$$

The total stiffness of the system is given by $K=\sum_{i=1}^{N} k_i$. The ratio of the stiffness in each story to the total stiffness (the stiffness distribution) $k_i/K$ is determined from the total stiffness $K$ and Eq. (3).

(2) Relation between the stiffness of each story and potential energy

When the system in Fig. 1 (a) is vibrating with the fundamental mode of vibration shown in Fig. 1 (b), the potential energy $V$ of its system is given as follows.

$$V=\{k_1 u_1^2+k_2(u_2-u_1)^2+\cdots\cdots+k_N(u_N-u_{N-1})^2\}\psi_1^2/2 \quad (4)$$

where $\psi_1$ is the generalized co-ordinate. In determining the stiffness of each story, one of the important problems is how to distribute the stiffness properly. By using the Lagrange's multiplier, let us obtain a condition which minimize the potential energy under the condition that the total stiffness $K$ is constant.

Introducing a parameter $\lambda$, the functional $F$ of $V$, $\lambda$ and $k_i$ is formed as follows

$$F=V+\lambda(k_1+k_2+\cdots\cdots+k_N-K)$$

When the functional $F$ is minimized, the condition that minimize $V$ can be obtained from following equations.

$$\frac{\partial F}{\partial k_i}=\frac{\partial F}{\partial \lambda}=0 \quad (i=1,2,\cdots\cdots,N)$$

The general relation among elements in the fundamental mode $u_i$ is then given as follows.

$$u_i=i\cdot u_1 \quad (i=1,2,\cdots\cdots,N) \quad (5)$$

The shape of fundamental mode which satisfies Eq. (5) has an inverted triangle. On the other hand, when the mass and the natural frequency are given, the fundamental mode of vibration which minimize the total stiffness K can be obtained from the following equations.

$$\frac{\partial K}{\partial u_1}=\frac{\partial K}{\partial u_2}=\cdots\cdots\frac{\partial K}{\partial u_N}=0$$

As a result, the same relation as Eq. (5) is obtained. That is, the fundamental mode of vibration which minimizes the total stiffness also minimizes the potential energy $V$ for the given mass distribution. In other word, if the fundamental mode of vibration is chosen as an inverted triangular shape ($u_i=i\cdot u_1$), both total stiffness and potential energy of the system become minimum. The energy distribution in this condition is given by the following equation.

$$\frac{E_i}{E}=\frac{m_i}{m_1+m_2+m_3+\cdots\cdots+m_N} \quad (i=1,2,\cdots\cdots,N) \quad (6)$$

The stiffness of each story is introduced by Eq. (3) and (5) as follows.

$$k_i=\omega^2\sum_{j=i}^{N} j\cdot m_j \quad (i=1,2,\cdots\cdots,N) \quad (7)$$

(3) Relation between input energy distribution and stiffness distribution

In the case of a multi-mass system having constant mass distribution, the input energy distribution to each story is given by Eq. (1) as follows.

$$\begin{aligned} &E_1/E=u_1/\{u_1+(u_2-u_1)+(u_3-u_2)+\cdots\cdots+(u_N-u_{N-1})\}=u_1/u_N \\ &E_i/E=(u_i-u_{i-1})/u_N : i\geqq 2 \end{aligned} \quad (8)$$

As the element in the fundamental mode of vibration is given by the formula of $u_i=i\cdot u_1$ in Eq. (5), the input energy distribution to each story is given by

$$\begin{aligned} &E_1/E=u_1/N\cdot u_1=1/N=b_1 \\ &E_i/E=\{i-(i-1)\}u_1/N\cdot u_1=1/N=b_i : i\geqq 2 \end{aligned} \quad (9)$$

The input energy distribution becomes equal ($E_1/E=\cdots=E_i/E=\cdots=E_N\cdots E=1/N$).

The natural frequency $\omega$ is varied with the difference of stiffness distribution. In the case of the system having constant mass and constant stiffness distribution, its fundamental mode of vibration is determined by using the modal analysis as follows.

$$\begin{aligned} &u_2=(2-m\omega^2/k)u_1 \\ &u_3=\{(2-m\omega^2/k)^2-1\}u_1 \\ &u_4=\{(2-m\omega^2/k)^3-(2-m\omega^2/k)\}u_1 \\ &\vdots \end{aligned} \quad (10)$$

It is clear that the above equations give the different relationship of $u_i = i \cdot u_1$. Accordingly, the input energy to each story is not equally distributed.

As stated above, when the fundamental mode of vibration is to satisfy the relation of $u_i = i \cdot u_1$, the total stiffness of the system having constant mass distribution becomes minimum. There is a relation $k_i > k_{i+1}$ in the stiffness of each story. The stiffness of each story in the system having constant mass is given by the relation $u_{i+1} - u_i = u_1$ as follows.

$$k_i = \{N(N+1) - i(i-1)\} \cdot m\omega^2/2 \quad \cdots\cdots (11)$$

where $i = 1, 2, \cdots, N$. That is, in the case of the system in which the mass and the natural frequency are given, the potential energy of the system reduce to the minimum, if the input energy is to be distributed in proportion to the mass distribution. In this case, the stiffness of each story is given by Eq. (11) in the case of the constant mass distribution.

## 3. OPTIMUM DISTRIBUTION OF INPUT ENERGY

When the fundamental vibration is predominant, the eigenvector of fundamental vibration mode can be determined by handling the input energy distribution to multi-mass system. Once the eigenvector $\{u_i\}$ is determined, the eigenvalue problem can be applied to find the stiffness of each story.

Considering the structural safety from the relation between the input energy and the energy absorption capacity of structures, an idea is to maximize the energy absorption capacity and at the same time to minimize the amount of the input energy. For example, it is attained easily to make the stiffness size in the lowest story relatively weak[2], but the concentration of the input energy at a specific story makes the stiffness size of other stories increase and cannot reduce the total stiffness.

### ( 1 ) The most suitable distribution of input energy

The elementary stage of earthquake resistant design is to obtain the stiffness distribution, and the most suitable design at this stage is to define the stiffness distribution so as to satisfy both safety and economy. As described in chapter 2, the stiffness of each story is obtained by distributing the input energy to each story. The optimum design based on the energy concept is to plan the most suitable distribution of input energy. In the following, the way of determining the most suitable distribution of input energy is discussed to reduce the total stiffness and to make use of the energy absorption capacity effectively.

According to the experiments by the authors[8] on the energy absorption capacity of reinforced concrete columns, the energy absorption capacity up to the ultimate state was mainly evaluated by the behaviour of axial tensile reinforcement. The experiments also showed that the energy absorption capacity was proportional to the stiffness of structural members. In the elastic behaviour of structural members, the energy absorption capacity was not exactly proportional to the stiffness, but in the plastic behaviour the energy absorption capacity up to the ultimate state of members was proportional to the size of elastic stiffness. The energy absorption capacity of the $i$-th story $W_{ci}$ was then assumed in proportion to the elastic stiffness of each member $k_i$ by $W_{ci} = \alpha \cdot k_i$ ($\alpha$=the proportional constant).

As an example, let us take a 3 DOF system having three equal masses. Using the six types of the fundamental mode of vibration and taking the natural frequencies as the same constant value $\omega = 2\pi/0.6 = 10.47$ (1/s), the stiffness of each story and the total stiffness were obtained as shown in Table 1. Case 1 in the table is the one that the stiffness of each story has constant value, and Case 6 is the one that the fundamental mode of vibration has an inverted triangular shape. The total stiffness becomes minimum naturally in

Table 1 Stiffness distributtion for Case 1-6.

| Case | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $k_1$ | 2825 | 3491 | 5057 | 8184 | 6804 | 3327 |
| $k_2$ | 2825 | 2618 | 3035 | 4464 | 2268 | 2767 |
| $k_3$ | 2825 | 1746 | 1011 | 744 | 1134 | 1648 |
| $\sum k_i$ | 8475 | 7855 | 9103 | 13392 | 10206 | 7742 |
| First mode | 2.25<br>1.80<br>1 | 3.12<br>2.12<br>1 | 5.56<br>2.48<br>1 | 10.92<br>2.71<br>1 | 7.41<br>3.75<br>1 | 3<br>2<br>1 |

$m_1 = m_2 = m_3 = 50t (= 5.102 t \cdot s^2/m)$
$k_i$ : Unit; t/m=98N/cm
$\omega = 2\pi/T_0 = 2\pi/0.6sec = 10.47/s$

Table 2 Rate of stiffness distribution and energy distribution for Casb 1-6.

| Case | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| i | $k_i/K$ | $E_i/E$ | $k_i/K$ | $E_i/E$ | $k_i/K$ | $E_i/E$ | $k_i/K$ | $E_i/E$ | $k_i/K$ | $E_i/E$ | $k_i/K$ | $E_i/E$ |
| 1 | 0.333 | 0.444 | 0.444 | 0.321 | 0.556 | 0.180 | 0.611 | 0.092 | 0.667 | 0.135 | 0.430 | 0.333 |
| 2 | 0.333 | 0.356 | 0.333 | 0.359 | 0.333 | 0.266 | 0.333 | 0.157 | 0.222 | 0.371 | 0.357 | 0.333 |
| 3 | 0.333 | 0.200 | 0.222 | 0.320 | 0.111 | 0.554 | 0.056 | 0.751 | 0.111 | 0.494 | 0.213 | 0.333 |

Table 3 Values of $(k_i/K-E_i/E)\cdot 100$.

| Case | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | -11.1 | +12.3 | +37.6 | +51.9 | +53.2 | + 9.7 |
| 2 | - 2.3 | - 2.6 | + 6.7 | +17.6 | -14.9 | + 2.4 |
| 3 | +13.3 | - 9.8 | -44.3 | -69.5 | -38.3 | -12.0 |

Case 6. Table 2 shows the stiffness distribution and the input energy distribution defined by the fundamental mode of vibration. For convenience, if the proportional constant $\alpha$ is chosen as $\alpha=1$, the stiffness distribution in Table 1 can be regarded as the energy absorption capacity of each story. In other words, the stiffness distribution for each case shown in Table 1 is regarded as the ratio of the energy absorption capacity of each story. Since the input energy is distributed to each story as shown in Table 2, the propriety of the input energy distribution is examined by comparing with the stiffness distribution. Table 3 represents the relative relation between the stiffness distribution and the input energy distribution given in Table 2. The numerical values express $(k_i/K-E_i/E)\cdot 100$. Both in Case 6, which has the minimum total stiffness, and in other cases, the story which has enough energy absorption capacity and the story which has not enough are mixed, and the energy absorption capacity is not utilized effectively. From both standpoints of aseismic safety and economy, the stiffness and input energy distributions in all cases shown in Table 2 are not suitable. The most suitable distribution of input energy is to be planned so as to balance the energy absorption capacity of each story with the input energy distributed to each story. It brings the results that the input energy in each story should be made equal to the stiffness distribution.

(2) The method to equalize stiffness distribution with input energy distribution

The simplest case that the stiffness distribution is equal to the input energy distribution is the one that the stiffness of each story has the same size and that the fundamental mode of vibration has an inverted triangular shape. However, the fundamental mode of vibration when the stiffness of each story has the same size is not coincident with that when the total stiffness becomes minimum. It is clear in chapter 3 that the multi-mass system having the characteristics stated above does not exist. Since the input energy can be controlled by handling the fundamental mode of vibration, both two systems, the system having equal stiffness and the system having equal input energy distribution are first defined in fundamental mode of vibration. The latter system has an inverted triangular shape in the mode. The average values of the elements of the modes in two systems are calculated. For example, a mode in a 3 DOF system having equal mass and stiffness in every stories is assumed as $\{u_1\}=\{1.0, 1.802, 2.247\}^T$, and an another mode having equal input energy distribution is assumed as $\{u_1\}=\{1, 2, 3\}^T$. Averaging these values, the third new mode shape is obtained as $\{u_1\}=\{1.0, 1.901, 2.624\}^T$. Next, the stiffness distribution and the input energy distribution are calculated by Eq. (3) and by Eq. (1) on the basis of the new mode shape, respectively.

$k_1/K=0.3896$, $E_1/E=0.3811$, $k_3/K=0.2561$, $E_3/E=0.2755$

$k_2/K=0.3543$, $E_2/E=0.3434$,

Comparing two values, some differences exist in the results. Therefore, averaging again the above values, the new input energy distribution is defined as follows,

$E_1/E=(0.3896+0.3811)/2=0.3854$, $E_3/E=(0.2561+0.2755)/2=0.2651$

$E_2/E=(0.3543+0.3434)/2=0.3489$,

Substituting these values into Eq. (2), the fundamental mode of vibration is obtained as $\{u_1\}=\{1.0, 1.905, 2.595\}^T$. The same process should be repeated until the stiffness distribution is equal to the input energy distribution. In this case, after repeating two times, the following result is obtained.

$k_1/K=0.3860$, $E_1/E=0.3859$, $k_3/K=0.2649$, $E_3/E=0.2651$

$k_2/K=0.3491$, $E_2/E=0.3491$,

Finally, the fundamental mode of vibration is calculated as $\{u_1\}=\{1.0, 1.905, 2.592\}^T$. The total stiffnesses $K$ are $K=15.147\ m\omega^2$ in the case of equal stiffness distribution, $K=14.000\ m\omega^2$ in the case of the equal input energy distribution and $K=14.239\ m\omega^2$ in the case that both distributions are equal. It means that the final system is regarded as the one having the average characteristics of the two.

In this way, averaging two values of $k_i/K$ and $E_i/E$, an unique system which has same distribution in stiffness and input energy can be obtained. It may be due to the facts that the equations for the stiffness distribution and input energy distribution, given in Eq. ( 1 ) and ( 3 ), respectively, are expressed as the function of the mass and the elements $u_i$ of fundamental mode of vibration, and moreover, that the sum of two distributions takes a certain value.

As a simple example, consider a 2 DOF system having equal two masses. Assuming the relation of $u_2=\beta\cdot u_1(\beta>1)$ between elements of the fundamental mode of vibration, the sum of the stiffness distribution and the input energy distribution is given as follows.

$k_1/K+E_1/E=1+(\beta-1)/(\beta^2+\beta-1)\beta=F(\beta)$, $k_2/K+E_2/E=2-F(\beta)$

Fig. 2 illustrates the above relations for $\beta$. It is obvious from the figure that each value of the sums approaches a certain value with $\beta$ increased ($\lim_{\beta\to\infty} F(\beta)=1$). Though the average values obtained from the above equations show some variation within a range of small $\beta$, it can be regarded almost constant.

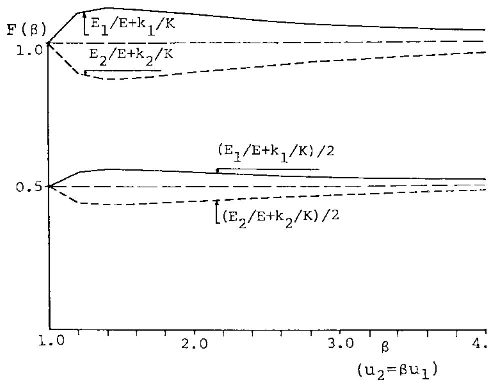



Fig. 2 Relation between $\beta$ and $F(\beta)$.

Based on these facts that the sum of the stiffness and input energy distribution, especially the average value of the two, is almost equal, the more effective method can be introduced in averaging. It means that the same result can be obtained also by using either of the two modes, the mode of the system having equal stiffness distribution and the mode of the system having equal input energy distribution. The simplest method is to use the system having equal input energy distribution. In a N-DOF system, the elements $u_i$ of the fundamental mode of vibration in the system having equal input energy with constant masses is given by the relation $u_i=i\cdot u_1$ $(i=1, 2, \cdots, N)$. The input energy distribution is given as $E_i/E=1/N$, and the stiffness of each story is given by Eq. (11). If both two values, $k_i/K$ and $E_i/E$, are not equal in comparison between the stiffness and input energy distribution, these values are averaged next as the new input energy distribution. This method requires slightly more repetitions than the method stated before, but is very effective because of its simplicity.

( 3 ) Numerical examination

Above discussion has been confined to the elastic system. The problem how the system having the elastic characteristics defined by the method of section ( 2 ) behaves in elasto-plastic response, and the problem if the relation between the stiffness distribution and the input energy distribution may be satisfied or not in the case of elasto-plastic behaviour are examined next numerically.

The system used here was three 6 DOF systems having equal mass ($m_i=50$ t ; $i=1$-$6$) in each story. The stiffness of each story was defined so that the elastic natural period $T_{01}$ was equal to 0.8 s. Table 4 shows the defined stiffness of each story. In the table, CASE A, B and C represent the stiffness distributions in the cases of (A) the equal stiffness distribution, (B) the equal input energy distribution and (C) the stiffness distribution equal to the input energy distribution, respectively. Fig. 3 illustrates the stiffness distribution and the input energy distribution to compare the difference between them. Fig. 4 shows the comparison between the planned energy distribution by theoretical formula (illustrated by the solid lines) and the numerical results obtained by the elastic and elasto-plastic analyses. The sinusoidal excitation having 300 gal in amplitude and 10 sec in duration time was chosen to know how the input energy $E$ was distributed to each story. The elastic analyses of the system were made by the input sinusoidal excitations in the periods of 0.5–1.5 sec. When the structural condition changes plastic, the period of plastic response (the apparent natural period) becomes longer because of the stiffness degradation[9]. Chosen the elasto-plastic stiffness ratio $\eta$ of each story as constant, the apparent natural period $T_e^*$ of the system is given by $T_e^* = T_{01}/\sqrt{\eta}\,(\eta<1.0)$ [11]. Therefore, the input sinusoidal excitations in the periods of 1.5–2.5 sec were used for the elasto-plastic analyses. The elasto-plastic stiffness ratio $\eta$ corresponds to the range between 0.1–0.3 in this case. As shown in Fig. 4, the elastic analyses plotted by the marks ○ coincide with the planned energy distribution in all cases, and the elasto-plastic analyses by the marks ● agrees closely with the planned energy distribution.

The reason why the elasto-plastic analysis developed by the theoretical formula of the elastic system is in good agreement with that of the elastic analysis is given as follows. The behaviour of a given system may be regarded as the linearized system in which all stories change plastic at the same time. If the elasto-plastic stiffness ratio $\eta$ is equal in each story, the plastic stiffness is the products of $\eta$ and elastic stiffness $k_i$, and the mode shape of multi-mass system having this plastic stiffness distribution is equivalent to that of the elastic system. In other words, when the stiffness distribution is not different between upper and lower story, the plastic response of shear multi-mass system can be treated as the linearized system[9]. In the figure, the stiffness distribution is illustrated by broken lines. As described in section (1), the stiffness distribution can be regarded

Table 4 Stiffness of each story in CASE A, B, C.

| CASE | | A | B | C |
|---|---|---|---|---|
| STIFFNESS OF EACH STORY $k_i$ | 1 | 5415 | 6610 | 5964 |
| | 2 | 5415 | 6292 | 5802 |
| | 3 | 5415 | 5665 | 5475 |
| | 4 | 5415 | 4723 | 4957 |
| | 5 | 5415 | 3462 | 4192 |
| | 6 | 5415 | 1890 | 3044 |
| $\sum k_i$ | | 32490 | 28642 | 29434 |

[Unit : (t/m)=98(N/cm)]

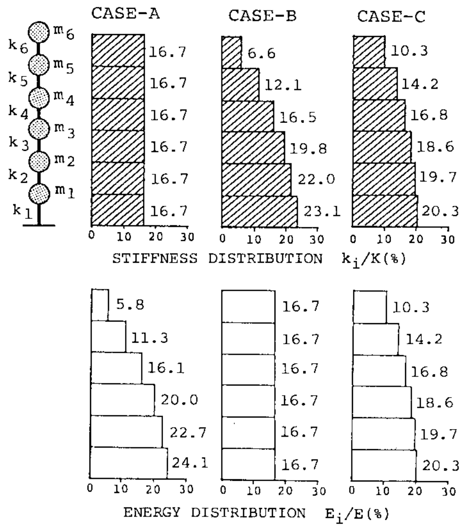



Fig. 3 Comparison of stiffness distribution and energy distribution in CASE A, B, C.

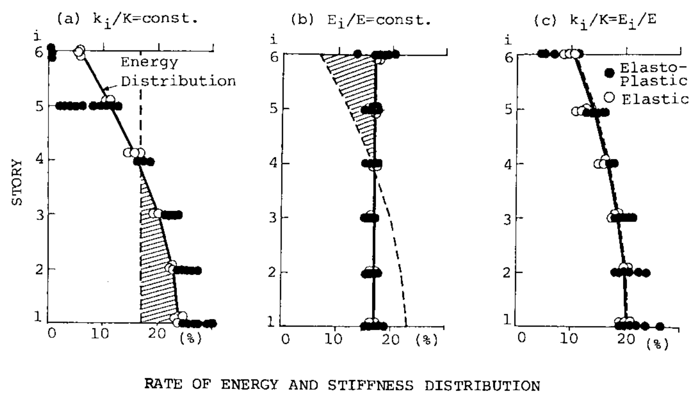



Fig. 4 Comparison between planned energy distribution and results of numerical analyses.

equivalent to the distribution of energy absorption capacity. Fig. 4 (a) and (b) show that the input energy is more than the energy absorption capacity in the shaped portions of the stories. These values are not the amount of the energy quantity, but the energy proportion of each story to the total energy. Accordingly, in Fig. 4, the aseismic safety is not directly examined from the quantitative standpoint of both input energy and energy absorption capacity, but the balance of the stiffness and input energy distribution in the system can be known, and the most suitable stiffness distribution can be determined in reference to the safety and economy. Fig. 4 (c) shows the results of numerical analyses in the system in which the stiffness distribution is equal to the input energy distribution. It is clear from this figure that the numerical results are somewhat scattered by the difference of excitation periods, but are well coincident with the lines defined by the theoretical formula. In this case (CASE C) the input energy and the energy absorption capacity are well coincident in each story, and this case may be regarded as the most appropriate plan of distribution that satisfy both aseismic safety and economy.

## 5. NUMERICAL EXAMPLES TO DECIDE STIFFNESS

Based on the results obtained in the previous chapter, the simple numerical examples to decide the stiffness of structures were given as follows. The input energy distribution were taken such three cases as (1) the equal input energy distribution in each story (Model A), (2) the equal distribution between the stiffness and input energy (Model B) and (3) the case shown in Fig. 5 (Model C) which was presented by Yamada et al[10]. Yamada et al. have reported on the optimum aseismic design of shear multi-mass system. It is based on the criterion that the optimum design is attained when the ductility factor becomes equal to the target ductility requirements. The third example (Model C) is given as the optimum result in the case of the target ductility requirements of 2.0. The natural period $T_{01}$ and the mass $m_i$ in Model A and B are taken equal to this example.

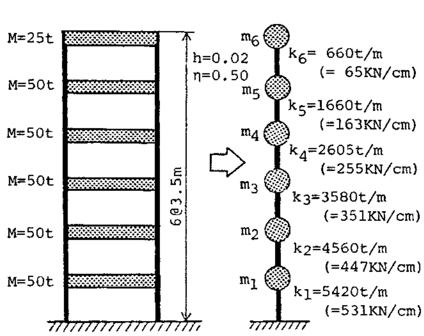


Fig. 5 An example model of six mass structure.

(1) In the case of equal input energy distribution (Model A)

Considering only the fundamental vibration, the input energy distribution is given by the following expression.

$$\frac{E_i}{E}=\begin{cases} m_1u_1\Big/\Big\{m_1u_1+\sum_{r=2}^{N} m_r(u_r-u_{r-1})\Big\} : i=1 \\ m_i(u_i-u_{i-1})\Big/\Big\{m_1u_1+\sum_{r=2}^{N} m_r(u_r-u_{r-1})\Big\} : i\geqq 2 \end{cases} \qquad (12)$$

The masses of each story are $m_1=m_2=m_3=m_4=m_5$ and $m_6=m_1/2$ by the given conditions. Substituting these values into Eq. (12), the input energy distribution is given as follows.

$$\frac{E_i}{E}=\begin{cases} u_1/0.5(u_5+u_6) : i=1 \\ (u_i-u_{i-1})/0.5(u_5+u_6) : i\geqq 2 \end{cases} \qquad (13)$$

Defining the input energy distribution to each story as $b_i=E_i/E$ and arranging each equation in Eq. (13) by the elements of fundamental mode $\{u_1\}$, the following expressions are obtained.

$$u_1=b_1\times 0.5(u_5+u_6)$$
$$u_i=b_i\times 0.5(u_5+u_6)+u_{i-1}=u_1\times\Big(\sum_{r=1}^{i} b_r\Big)/b_1 \quad : i=2\sim 5$$
$$u_6=b_6\times(u_5+u_6)+u_5=u_1\times(b_1+b_2+b_3+b_4+b_5+2b_6)/b_1 \qquad (14)$$

In Eq. (14), defining the value of input energy distribution as $b_i=1/6=0.1667$ ($i=1-6$), the fundamental mode of vibration $\{u_1\}$ in the case of the equal input energy distribution is obtained as follows.

$\{u_1\}=\{1, 2, 3, 4, 5, 7\}^T$ ··· (15)

Since the fundamental mode of vibration $\{u_1\}$ and the mass $m_i$ are already given, the stiffness $k_i$ is obtained from the following equation.

$[K]\{u_1\}=\omega^2[M]\{u_1\}^T$ ··· (16)

Developing Eq. (16), the following expressions are obtained.

$$\begin{aligned}
(k_1+k_2)u_1-k_2u_2&=\omega^2 m_1 u_1\\
-k_2u_1+(k_2+k_3)u_2-k_3u_3&=\omega^2 m_2 u_2\\
-k_3u_2+(k_3+k_4)u_3-k_4u_4&=\omega^2 m_3 u_3\\
-k_4u_3+(k_4+k_5)u_4-k_5u_5&=\omega^2 m_4 u_4\\
-k_5u_4+(k_5+k_6)u_5-k_6u_6&=\omega^2 m_5 u_5\\
-k_6u_5+k_6u_6&=\omega^2 m_6 u_6
\end{aligned}$$

Rearranging by the stiffness of each story $k_i$,

$$k_1=\omega^2(m_1u_1+m_2u_2+m_3u_3+m_4u_4+m_5u_5+m_6u_6)/u_1$$

$$k_i=\omega^2\left(\sum_{r=i}^{6} m_r u_r\right)/(u_i-u_{i-1}) \quad : i=2\sim6 \quad \cdots (17)$$

Substituting the elements of fundamental mode, $u_1=1$, $u_2=2$, $u_3=3$, $u_4=4$, $u_5=5$, $u_6=7$ and the masses $m_1=m_2=m_3=m_4=m_5=5.102$, $m_6=2.551$ $(\mathrm{t\cdot cm^{-1}\cdot sec^2})$ into Eq. (17), the stiffness and stiffness distribution of each story are obtaind as follows.

$k_1=94.39\ \omega^2$, $k_2=89.29\ \omega^2$, $k_3=79.08\ \omega^2$, $k_4=63.78\ \omega^2$, $k_5=43.37\ \omega^2$, $k_6=8.93\ \omega^2$,

$k_1/K=0.2492$, $k_2/K=0.2357$, $k_3/K=0.2087$, $k_4/K=0.1684$, $k_5/K=0.1145$, $k_6/K=0.0236$

$K=\sum_{i=1}^{N} k_i=378.84\ \omega^2$ ··· (18)

(2) In the case of equal distribution of stiffness and input energy (Model B)

The method of averaging, described in chapter 4. (2), is used here. Since the stiffness disribution for the case that the input energy of each story is equal [$b_i=0.1667$ $(i=1-6)$] is given by Eq. (18), these average are calculated at first.

$i=1$ : $(0.1667+0.2492)/2=0.2081\rightarrow b_1$, $i=4$ : $(0.1667+0.1684)/2=0.1677\rightarrow b_4$

$i=2$ : $(0.1667+0.2357)/2=0.2014\rightarrow b_2$, $i=5$ : $(0.1667+0.1145)/2=0.1408\rightarrow b_5$

$i=3$ : $(0.1667+0.2087)/2=0.1879\rightarrow b_3$, $i=6$ : $(0.1667+0.0236)/2=0.0953\rightarrow b_6$ ··· (19)

By using the average values given in Eq. (19) as the new input energy distribution, the fundamental mode of vibration is calculated by Eq. (14).

$\{u_1\}=\{1.0, 1.967, 2.870, 3.676, 4.353, 4.812\}^T$

Substituting the masses and the above fundamental mode of vibration into Eq. (17), the stiffness of each story and their distribution are obtaind as follows.

$k_1=83.02\ \omega^2$, $k_2=80.58\ \omega^2$, $k_3=75.17\ \omega^2$, $k_4=66.05\ \omega^2$, $k_5=50.94\ \omega^2$, $k_6=26.74\ \omega^2$,

$k_1/K=0.2170$, $k_2/K=0.2107$, $k_3/K=0.1965$, $k_4/K=0.1727$, $k_5/K=0.1332$, $k_6/K=0.0699$

$K=\sum_{i=1}^{N} k_i=382.50\ \omega^2$ ··· (20)

Since there is the difference between the input energy distribution given by Eq. (19) and the stiffness distribution given by Eq. (20), the average values are calculated again and the same process is repeated several times. In this case, the input energy distribution and the stiffness distribution coincided after repeating five times.

$\{u_1\}=\{1.0, 1.969, 2.872, 3.671, 4.313, 4.693\}^T$

$k_1=82.50\ \omega^2$, $k_2=79.91\ \omega^2$, $k_3=74.57\ \omega^2$, $k_4=65.95\ \omega^2$, $k_5=52.96\ \omega^2$, $k_6=31.44\ \omega^2$,

$k_1/K=0.2130$, $k_2/K=0.2063$, $k_3/K=0.1925$, $k_4/K=0.1703$, $k_5/K=0.1367$, $k_6/K=0.0812$

$K=\sum_{i=1}^{N} k_i=387.33\ \omega^2$ ··· (21)

The total stiffness $K$ is increased by 2.24 % in comparison with the case of the equal input energy distribution.

(3) In the case of Model C

The natural period $T_{01}$ and the fundamental mode of vibration in Model C are given as follows.

Table 5 Relation between stiffness distribution and input energy distribution in each Model.

| | | Model A | | | Model B | | Model C | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $k_i$ | $k_i/K$ | $E_i/E$ | $k_i$ | $k_i/K{=}E_i/E$ | $k_i$ | $k_i/K$ | $E_i/E$ |
| STORY | 1 | 4587 | 0.249 | 0.167 | 4009 | 0.213 | 5420 | 0.293 | 0.139 |
| | 2 | 4339 | 0.236 | 0.167 | 3883 | 0.206 | 4560 | 0.247 | 0.158 |
| | 3 | 3843 | 0.209 | 0.167 | 3624 | 0.193 | 3580 | 0.194 | 0.181 |
| | 4 | 3099 | 0.168 | 0.167 | 3205 | 0.170 | 2605 | 0.141 | 0.203 |
| | 5 | 2108 | 0.115 | 0.167 | 2574 | 0.137 | 1660 | 0.090 | 0.216 |
| | 6 | 434 | 0.024 | 0.167 | 1528 | 0.081 | 660 | 0.036 | 0.104 |
| K | | 18410 | | | 18823 | | 18485 | | |

[Unit : $k_i$(t/m)]

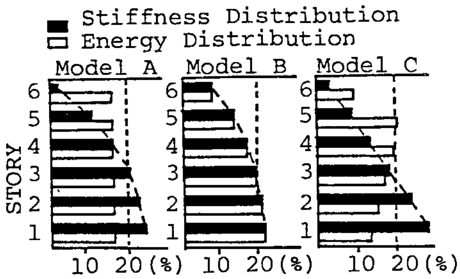



Fig. 6 Relation between stiffness distribution and energy distribution.

$T_{01}=0.901$ sec ($\omega=6.971$/sec)

$\{u_1\}=\{1.0, 2.134, 3.431, 4.887, 6.441, 7.931\}^T$

The input energy distribution $b_i (i=1-6)$ is given by Eq. (12).

$b_1=1/0.5(6.441+7.931)=0.1392$ $\quad$ $b_4=(4.877-3.431)/7.186=0.2027$

$b_2=(2.134-1)/7.186=0.1579$ $\quad$ $b_5=(6.441-4.887)/7.186=0.2163$

$b_3=(3.431-2.134)/7.186=0.1805$ $\quad$ $b_6=(7.931-6.441)/14.372=0.1037$

(4) Comparison of stiffness and input energy distribution among Model A, B, C

The stiffnesses of each story in Model A, B were calculated by using the first natural period of Model C, $T_{01}=0.901$ sec. Table 5 shows the comparison of the stiffness and the input energy distribution in each model. It is clear in Table 5 that the total stiffnesses in Model A, B and C are almost equal each other, but that the total stiffness in Model A, which has the equal input energy distribution, is the minimum of the three. Fig. 6 illustrates the relation between the stiffness distribution and the input energy distribution in each model. As shown in the figure, the stiffness distribution in Model C shows a triangular shape, and the input energy distribution of the fourth and fifth stories is somewhat large in volume. This fact is coincident with the result of the author's previous paper[9] which was obtained from the elasto-plastic response analysis with the earthquake records and sinusoidal waves.

In Fig. 6, comparing the relation between the stiffness distribution and the input energy distribution, the input energy distribution exceeds the stiffness distribution at the fifth and sixth stories in Model A, and at the fourth and fifth stories in Model C. Assumed that the stiffness size of each story is in proportion to the size of energy absorption capacity, the failure may be occurred at the story in which the input energy distribution exceeds the stiffness distribution. Model C is the case designed by the large iterative calculation so as to equalize the ductility factor of each story, but the almost same result was obtained in Model A and B by the simple calculation with figures mentioned above. Considering the balance of energy absorption capacity to input energy distribution, Model B, which makes the stiffness distribution equal to the input energy distribution, can be regarded the most rational.

## 6. CONCLUSIONS

The purpose of this study is to eastablish the rational earthquake resistant design based on aseismic safety and economy, and to study the relation between the input energy imparted by earthquake motion and the structural characteristics. According to this purpose, the mothod how to determine the most suitable distribution of input energy in shear multi-mass systems and how to determine the stiffness size of each story are examined fundamentally, The concluding remarks are summarized as follows.

(1) When the fundamental mode of vibration is predominant in structural response, the input energy to the system can be controlled by handling the fundamental mode of vibration. By examining the relation between the stiffness distribution and the input energy distribution, and by using the method of controlling

the input energy, the system which satisfy the given conditions can be determined.

(2) The sum of the stiffness distribution and input energy distribution can be regarded as almost constant. The system which satisfy the optimum distribution can be obtained easily by applying the method of averaging.

(3) The theoretical formulations developed in this study are based on the modal analysis in the elastic system. When the stiffness is distributed to the system so as not to differ between upper and lower story, the plastic response can be treated apparently as the response of elastic system.

(4) Defining the total stiffness of system as an index for the economy and defining the relation between the input energy distributed to the system and the energy absorption capacity as an index for the safety, the optimum input energy distribution is obtained by handling so as to equalize the stiffness distribution with the input energy distribution under the restrictions of two indices.

Based on the method presented in this study, the stiffness size of each story can be obtained easily by calculating the input energy distribution, when the mass and the first natural period of system are given. The optimum stiffness distribution to each story can be obtained by the simple calculation with figures without using the dynamic design procedure.

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