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ON THE FORM DRAG CAUSED BY SAND WAVES

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Using field and laboratory data, the form drag component of the total friction factor of an alluvial stream is plotted varsus the dimensionless complex reflecting the geometry of sand waves. It is assumed that the flow, which approximates two-dimensional condition, is tranquil, and thus that sand waves are ripples and/or dunes. A particular attention is given to the case when ripples are superimposed on dunes. It has been found that, in general, the relationship between the form drag component and the dimensionless complex mentioned, appears to be linear when this complex is large and it becomes logarithmic when it is small. However, it has also been found, that this general trend tends to deteriorate when the relative sand wave height increases beyond a certain (critical) value.

1. GENERAL

The purpose of this technical note is to finalise the previous technical note of the senior author (Ref. 17)). At the end of Ref. 17) it has been mentioned that a series of experimental measurements are being conducted at Queen's University in order to reveal how the friction factor of an open channel flow is to be determined when the flow bed is covered by two modes of sand waves : dunes with ripples superimposed on them. These measurements (which were carried out by simulating sand waves with the aid of rigid triangular elements) are completed now, and their results are presented herein. A further objective of this note is to discuss the form of the function

$$f''=\phi(\Xi) \qquad (1)$$

in the light of the recent data. Here f'' is the "form drag component" of the total friction factor $f=f'+f''$ (where f' is the "pure friction component") and Ξ is the dimensionless complex

$$\Xi=\frac{\Delta^2}{\Lambda h} \qquad (2)$$

where Λ and Δ are sand wave length and sand wave height respectively, and h is the flow depth.

A number of papers concerning the resistance to flow in alluvial streams have been published recently, and among them Refs. 2), 3), 5), 6), 7), 11), and 12), are noteworthy. However, to the authors' knowledge, none of the papers produced to date deal with the form drag component f'' as an explict function of the "geometry" presented by two modes of sand waves : dunes and ripples superimposed on them (which is the case when the grain size Reynolds number X happens to be within the interval $\simeq 5<X<\simeq 30$ (Refs. 14), 15), 16)). This is why at the end of the present text the function (1) is discussed on the basis

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of the single variable Ξ (determined by a single mode of sand waves).

It is assumed that the steady state turbulent open channel flow is in equilibrium and that it can be treated as two dimensional. The flow depth h is measured from the average bed level.

2. EXPERIMENTS

The present measurements were carried out in the glass walled adjustable-slope flume : length×width×height=20 m×0.76 m×0.6m. The triangular bed features simulating dunes and ripples had skin roughness $k_s \simeq 2.5$ mm. Two types of "dunes" were used :

$\Lambda = 1$ m ; $\Delta = 0.03$ m and $\Lambda = 1$ m ; $\Delta = 0.06$ m

The length λ and the height δ of "ripples" (superimposed on "dunes") varied within the intervals :

$5\ \text{cm} \leq \lambda < 18\ \text{cm}$ and $0 \leq \delta \leq 1.5$ cm

The actual values of λ and δ are given in Table 1. Thus a total of 135 runs were conducted. The slope of the flume bed was fixed at $S=0.001$. Flow depths h (and flow rates Q) were varied within

$0.044\ \text{m} \leq h \leq 0.182$ m and $0.0046\ \text{m}^3/\text{s} \leq Q \leq 0.0524\ \text{m}^3/\text{s}$.

This technical note should not be burdened by further details related to the present experiments; and any additional information, if desired, can be obtained by writing to the authors.

Table 1

Symbol	λ (cm)	δ (cm)	Δ/Λ	No. of Runs
⦶	–	0	0.03	10
▲	15	0.7	0.03	10
▲	11	0.7	0.03	15
●	18	1.3	0.03	9
○	15	1.3	0.03	11
◑	10	1.3	0.03	11
⊖	9	1.3	0.03	11
⊠	9	1	0.06	8
⊟	18	1.5	0.06	16
◧	15	1.5	0.06	14
◪	6	1	0.06	11
◆	9	1.5	0.06	9

3. RESULTS AND DISCUSSION

In Ref. 8) it has been shown that the energy loss due to the gradual contraction along the upstream surface ab of sand waves, such as ripples or dunes, is much smaller than that due to the sudden expansion at their abrupt downstream surface bc (Fig. 1). Considering this, it has been found (in Refs. 4) and 13)) that the form drag loss f'' corresponding to dunes or ripples can be determined as follows :

$$\left.\begin{aligned} &\text{Dunes (only)} : (f''_\Delta/8) = \frac{1}{2}\Xi \\ &\text{Ripples (only)} : (f''_\delta/8) = \frac{1}{2}\xi \quad \left(\text{with } \xi = \frac{\delta^2}{\lambda h}\right) \end{aligned}\right\} \quad \cdots\cdots (3)$$

Since energy is additive, one would expect that, in the case of dunes having ripples superimposed on them, the total form drag loss f'' should be given by the sum

$$f'' = f''_\Delta + f''_\delta \quad \cdots\cdots (4)$$

which implies

$$(f''/8) = \frac{1}{2}(\Xi + \xi). \quad \cdots\cdots (5)$$

The method of simple addition embodied by (5) appears to be justified by the measurements conducted: the overall pattern of experimental points shown in Fig. 2 exhibits a satisfactory agreement with the 45°-straight line representing (5). Note, however, that the right hand ends of the patterns tend to diverge from the 45°-line and bend upwards-indicating thereby higher than predicted energy losses. From the data-analysis it has been found that this "divergence" begins to take place when the relative flow depth

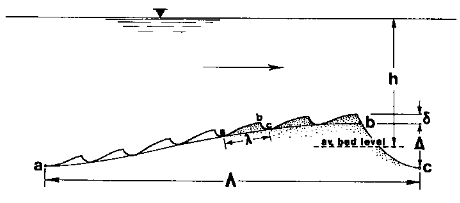

Fig. 1

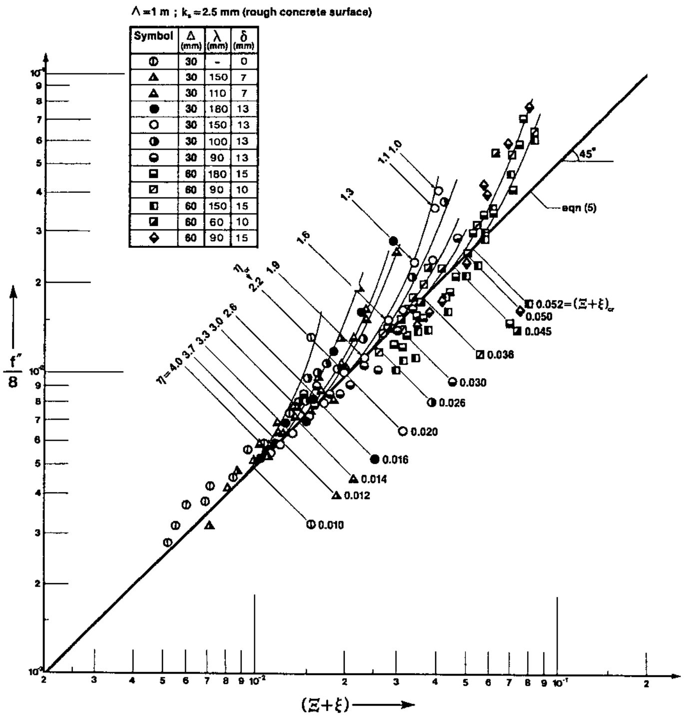

Fig. 2

$$\eta = \frac{h}{\Delta + \delta} \cdots\cdots\cdots\cdots\cdots\cdots (6)$$

begins to become smaller than a certain (critical) value η_{cr}. When η is small ($\eta < \eta_{cr}$) then the "near by" free surface impedes the "free" deformation of the stream lines due to bed features and the flow must spend an additional energy to overcome this "impeding action".

It is intended now to consider the form of the function (1).

The recent f'' data of other sources is plotted versus Ξ in the (extended) diagram in Fig. 3. The laboratory data of Ref. 10), which was obtained in the same flume as the present data, covers the comparable range and it exhibits also a satisfactory agreement with the straight line $f''/8 = (1/2)\Xi$ implying

$$\phi(\Xi) = 4\,\Xi. \cdots\cdots\cdots\cdots\cdots\cdots (7)$$

In Ref. 17) it has been pointed out that, in general, the relative sand wave height Δ/h cannot be regarded as "small" and therefore, in general, "...the energy loss due to sand waves is mainly because of the sudden enlargement and the separation of flow at their abrupt downstream surfaces (the hypothesis leading to

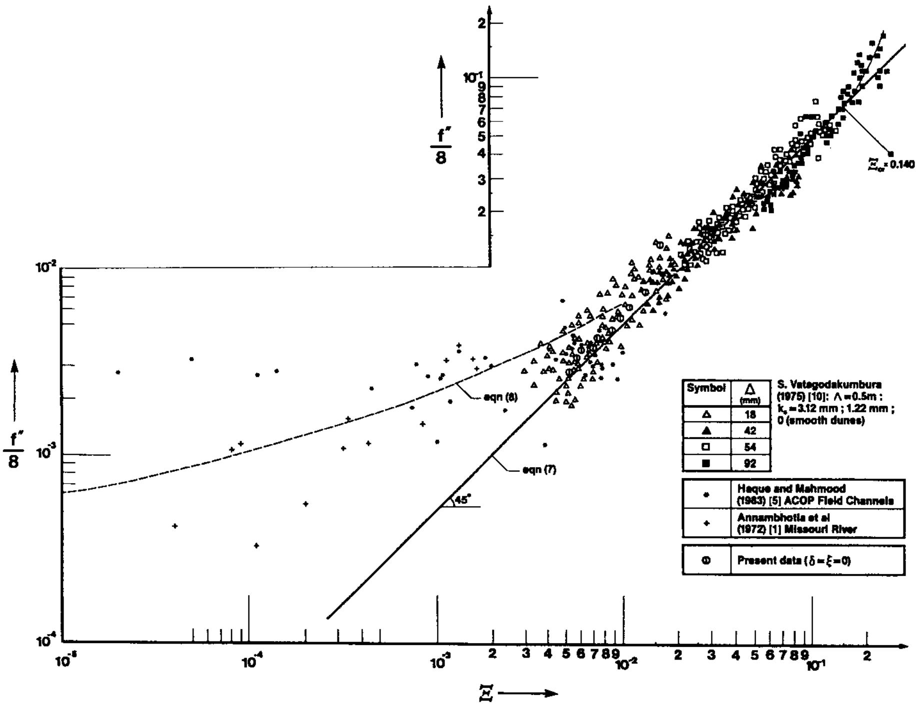

Fig. 3

(7)) rather than because of the manifestation of sand waves as a "generalised roughness"-motivation leading to the logarithmic form

$$\phi(\Xi)=[3.3\log(\Xi^{-1})-2.3]^{-2} \quad (8)$$

(suggested in Ref. 9)". But this means that if Δ/Λ is "small" then the manifestation of the logarithmic law would be only natural. The behaviour of the field data (of Refs. 1) and 5)) plotted in **Fig. 3** appears to be consistent with this statement. Indeed, in spite of the substantial scatter, one can realise from **Fig. 3**, that the point patterns tend to "transfer" themselves from the straight line implying (7) to the curve of the logarithmic law (8). This happens when Ξ is of the order of $\simeq 10^{-2}$, and therefore, for the present, the function f'' can be evaluated by adopting

the linear form (7) : if $\Xi > \simeq 10^{-2}$, and the logarithmic form (8) : if $\Xi < \simeq 10^{-2}$ ……………… (9)

APPENDIX—LIST OF SYMBOLS

f : Darcy-Weisbach friction factor
f' : "pure friction" component of f
f'' : "form drag" component of f
h : flow depth (measured from the average bed level)
k_s : height of the effective "skin roughness" (of sand waves)
Q : flow rate
S : slope of the equilibrium flow
X : grain size Reynolds number (grain size) (shear velocity) / (kinematic viscosity)
Δ, Λ : height and length of the basic sand waves (dunes)
δ, λ : height and length of the superimposed sand waves (ripples)
$\Xi=\Delta^2/(\Lambda h)$: dimensionless variable reflection the influence of the basic sand waves on f''
$\xi=\delta^2/(\lambda h)$: dimensionless variable reflecting the influence of the superimposed sand waves on f''
η : relative flow depth (as defined by Eq. (6))
ϕ : dimensionless function determining f''

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