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BOUNDARY TYPE FINITE ELEMENT METHOD FOR SURFACE WAVE PROBLEMS

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A new finite element method for the analysis of surface wave problems is presented in this paper. The characteristic of this method is that the interpolation equation is selected to satisfy the Helmholtz equation in each element. This follows that the variational functional to be minimized can be formulated as in the form that all integrations are limited just on the boundary of the element. The numerical solutions obtained are compared with the analytical and observed results. From these comparative studies, it is concluded that the present method provides a useful and valuable tool for the analysis of surface wave problems.

1. INTRODUCTION

In general it is wellknown that the surface wave problems can be classified into two groups, namely, inner and outer problems. The classification depends on the configuration of the area to be analyzed. The problem of oscillation inside the closed region is called as the inner problem, for example, the oscillation in lake and water tank etc. On the contrary, wave propagation over the open region is referred to the outer problem, which includes, for example, waves on the ocean etc. Recently, to solve these problems, there have been presented a number of numerical methods. For inner problems, the finite difference method and the finite element method are usually used. Whereas, in the case of outer problems, the finite difference method[1], the finite element method[2], the boundary element method[3,4] and the combinational method of those[5~8] are employed. Using the conventional methods presented previously, it has been found that the extremely fine grids and a large number of element must be employed to get the satisfactory accuracy. Therefore, a plenty of computational time is necessary and a large size of computer core storage are required. To avoid this, this paper presents a method to analyze surface wave problems based on the finite element method borrowing the idea of the boundary element method.

The basic procedure of the method presented in this paper is based on the following idea. It is quite common that the mild-slope equation can be solved by using the corresponding variational equation. In case that the interpolation equation in the approximate method satisfies the Helmholtz equation in each elment, the variational equation can be reformulated in the form that consists only of the boundary integral. In the

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conventional boundary element method, the analytical solution is successfully used for the interpolation. However, the singularity property, i. e., the solution tends to infinity at the origin, must be used in the solution procedures. Sometimes, the singularity considerably decreases the accuracy of the numerical solution. To avoid this, the present method employs the interpolation equation based on the trigonometric function series based on the triangular and quadrilateral finite elements. Introducing the interpolation equation into the variational equation, the boundary type finite element equation can be derived. In recent paper[9], the authors have shown that the boundary type finite element method is useful for the analysis of surface waves on constant water depth. In this paper, the present method is extended to be adaptable for the slowly varying water depth problems. An oral representation was made in the reference[10].

Several numerical tests have been performed to show the validity of the present method. This method has been applied to the oscillation problem of lake and to the problem of wave diffraction and refraction due to an island. The computed results obtained by the present method are compared with the analytical solution and observed results. It has been clearly shown that the present method is extremely powerful for the analysis of surface wave problems.

2. BASIC EQUATION

A steady state surface waves with infinitesimal amplitude on the slowly varying water depte can be generally described by the mild-slope equation, assuming irrotational flow of incompressible fluid. Denoting the water elevation η, the governing equation can be written in the form[6),11)].

$$\nabla\cdot(CC_g\nabla\eta)+\omega^2\frac{C_g}{C}\eta=0 \qquad \text{in } \Omega \qquad (1)$$

where Ω is an arbitrary domain. C and C_g express phase velocity and group velocity, and ω is angular frequency. The frequhncy ω is derived from the dispersion relation.

$$\omega^2=gk\tanh kh \qquad (2)$$

where k, g and h are wavenumber, gravity acceleration and water depth respectively.

In case of inner problem, the following boundary conditions are introduced as shown in Fig. 1 (a)

$$\eta=\hat{\eta} \qquad \text{on } \Gamma_1 \qquad (3)$$

$$\eta_{,n}=\frac{\partial\eta}{\partial n}=\hat{\eta}_{,n} \qquad \text{on } \Gamma_2 \qquad (4)$$

where n means the unit normals to the boundary, superscripted ^ denotes the value which is specified on the boundary Γ_1 and Γ_2 and there is no overlap between Γ_1 and Γ_2.

In case of outer problem, water elevation is assumed to be the sum of incident wave η_{in} and scattered wave η_{sc} as

$$\eta=\eta_{in}+\eta_{sc} \qquad (5)$$

where η_{in} is assumed as follows.

$$\eta_{in}=A\exp\{ik(x\cos\theta+y\sin\theta)\} \qquad (6)$$

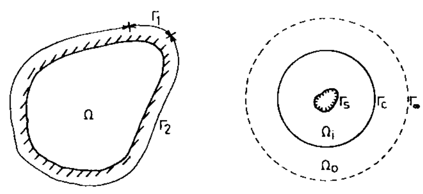
in which A denotes the incident wave amplitude, θ is the incident wave angle and i is the imaginary unit respectively.

As the boundary conditions, the following conditions are introduced on the boundary as shown in Fig. 1 (b).

$$\eta_{,n}=\hat{\eta}_{,n} \qquad \text{on } \Gamma_S \qquad (7)$$

$$\lim_{r\to\infty}\sqrt{r}\left(\frac{\partial\eta_{sc}}{\partial r}-ik\eta_{sc}\right)=0 \qquad \text{on } \Gamma_\infty \qquad (8)$$

where Γ_S is the boundary of structures, Γ_∞ is the infinite boundary and r is the distance from the pole. The following continuity condition should be satisfied on Γ_C, which is the boundary artificially located between Ω_i and Ω_0.



(a) inner problems (b) outer problems

Fig. 1 Definition sketch.

$$\left.\begin{array}{l}\eta^{\Omega_i}=\eta^{\Omega_0}\\ \eta_{,n}^{\Omega_i}+\eta_{,n}^{\Omega_0}=0\end{array}\right\} \quad \text{on } \Gamma_c \qquad (9)$$

where superscripts Ω_i and Ω_0 mean the values on the side of Ω_i and Ω_0 on the boundary Γ_c, respectively.

3. BOUNDARY TYPE FINITE ELEMENT METHOD

For the discretization of the basic differential equation, the variational principle can be usefully introduced. Generally, the variational functional to be minimized for the mild-slope equation (1) is expressed as follows.

$$\Pi=\frac{1}{2}\int_{\Omega}\left[CC_g(\nabla\eta)^2-\omega^2\frac{C_g}{C}\eta^2\right]d\Omega-\int_{\Gamma_2}CC_g\eta\hat{\eta}_{,n}d\Gamma \qquad (10)$$

The functional is transformed into the following form after integrating the first term by parts.

$$\Pi=\frac{1}{2}\int_{\Gamma}CC_g\eta\eta_{,n}d\Gamma-\frac{1}{2}\int_{\Omega}CC_g\eta(\nabla^2\eta)d\Omega-\frac{1}{2}\int_{\Omega}\omega^2\frac{C_g}{C}\eta^2d\Omega-\int_{\Gamma_2}CC_g\eta\hat{\eta}_{,n}d\Gamma \qquad (11)$$

Introducing the relation $\omega=Ck$ into equation (11), the variational functional can be written in the following form.

$$\Pi=\frac{1}{2}\int_{\Gamma}CC_g\eta\eta_{,n}d\Gamma-\frac{1}{2}\int_{\Omega}CC_g\eta(\nabla^2\eta+k^2\eta)d\Omega-\int_{\Gamma_2}CC_g\eta\hat{\eta}_{,n}d\Gamma \qquad (12)$$

Assuming that the interpolation equation for water elevation η satisfies the Helmholtz equation in Ω, equation (12) can be simplified as :

$$\Pi=\frac{1}{2}\int_{\Gamma}CC_g\eta\eta_{,n}d\Gamma-\int_{\Gamma_2}CC_g\eta\hat{\eta}_{,n}d\Gamma \qquad (13)$$

This functional is the basis for deriving the boundary type finite element equation. The wave field to be analyzed is divided into a finite number of elements, and then the variational functional can be rewritten as follows.

$$\begin{aligned}\Pi&=\sum_{e=1}^{m}\Pi_e\\ &=\sum_{e=1}^{m}\left[\frac{1}{2}\int_{\Gamma e}CC_g\eta\eta_{,n}d\Gamma-\int_{\Gamma_{2e}}CC_g\eta\hat{\eta}_{,n}d\Gamma\right] \qquad (14)\end{aligned}$$

where e denotes the e-th element, m is the total number of elment. Γ_e expresses the boundary of e-th finite element and Γ_{2e} is a part of Γ_e on which the boundary corresponds to Γ_2.

For the interpolation equation, trigonometric function series is employed based on a three node triangular element and a four node quadrilateral element. The configuration and the coordinate system are shown in Fig. 2. The origin of the local coordinate system is placed at centroid of each element. The interpolation equations are :

for a three node triangular element,

$$\eta=\left[\cos\left(\frac{k}{\sqrt{2}}x\right)\cos\left(\frac{k}{\sqrt{2}}y\right)\quad \cos\left(\frac{k}{\sqrt{2}}x\right)\sin\left(\frac{k}{\sqrt{2}}y\right)\quad \sin\left(\frac{k}{\sqrt{2}}x\right)\cos\left(\frac{k}{\sqrt{2}}y\right)\right]\begin{Bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{Bmatrix} \qquad (15)$$

and for a four node quadrilateral element,

$$\eta=\left[\cos\left(\frac{k}{\sqrt{2}}x\right)\cos\left(\frac{k}{\sqrt{2}}y\right)\quad \cos\left(\frac{k}{\sqrt{2}}x\right)\sin\left(\frac{k}{\sqrt{2}}y\right)\quad \sin\left(\frac{k}{\sqrt{2}}x\right)\cos\left(\frac{k}{\sqrt{2}}y\right)\right.$$
$$\left.\sin\left(\frac{k}{\sqrt{2}}x\right)\sin\left(\frac{k}{\sqrt{2}}y\right)\right]\begin{Bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_4\end{Bmatrix} \qquad (16)$$

where $\alpha_1 \sim \alpha_4$ are constants which are determined by the fact that the water elevation expressed in equation

(15), (16) must be coincide with the nodal values at each nodal point. These above interpolation equations are to be the eigensolution to the Helmholtz equation in a closed area.

Introducing the nodal coordinates into the interpolation equation and determining the constants α_1, α_2, α_3, α_4, the following equation can be obtained.

$$\{\tilde{\eta}\}=[G]\{\alpha\} \cdots\cdots (17)$$

where $\{\tilde{\eta}\}$ denotes the water elevation at each nodal point. For the wavenumber k, the value at the centroid of each element is used.

The interpolation equations (15) and (16) can be rewritten in the matrix form as :

$$\eta=[P]\{\alpha\} \cdots\cdots (18)$$

Eliminating $\{\alpha\}$ from equation (18) using equation (17), the water elevation for any point inside the element can be derived in the following form.

$$\eta=[P][G]^{-1}\{\tilde{\eta}\}=[N]\{\tilde{\eta}\} \cdots\cdots (19)$$

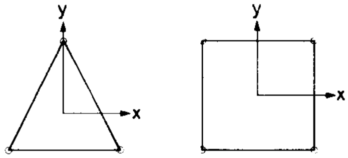
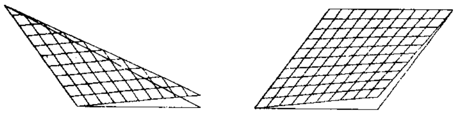


Fig. 2 Elements and coordinate system.


(a) s/L = 0.1

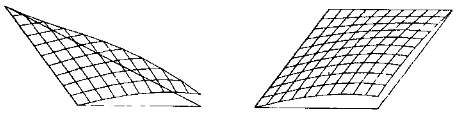

(b) s/L = 0.33

Fig. 3 Interpolation function.

where $[N]$ denotes the interpolation function for the water elevation. Fig. 3 shows a typical interpolation function for a three node triangular element and four node quadrilateral element in case that the ratio between element length s and wave length L are 0.1 and 0.33 respectively.

On the other hand, the normal derivative $\eta_{,n}$ can be described as follows using equation (18).

$$\eta_{,n}=\frac{\partial}{\partial n}[P]\{\alpha\} \cdots\cdots (20)$$

Moreovers, from equation (17), it is obtained that

$$\eta_{,n}=[Q][G]^{-1}\{\tilde{\eta}\}=[M]\{\tilde{\eta}\} \cdots\cdots (21)$$

where $[M]$ is the interpolation function for the derivative of water elevation.

Introducing equation (19) and (21) into equation (14), the variational functional can be rearranged in the form.

$$\Pi_e=\frac{1}{2}\{\tilde{\eta}\}^T[G]^{-1T}\left[\frac{1}{2}\left(\int_{\Gamma e} CC_g[P]^T[Q]d\Gamma+\int_{\Gamma e} CC_g[Q]^T[P]d\Gamma\right)\right][G]^{-1}\{\tilde{\eta}\}$$
$$-\{\tilde{\eta}\}^T[G]^{-1T}\int_{\Gamma_{2e}} CC_g[P]^T d\Gamma\hat{\eta}_{,n} \cdots\cdots (22)$$

Minimizing the functional equation (14), it is obtained that

$$\delta\Pi=\sum_{e=1}^{m}\delta\Pi_e=0 \cdots\cdots (23)$$

From equation (23), the boundary type finite element equation can be obtained. For each element,

$$\delta\Pi_e=[k]\{\tilde{\eta}\}-\{f\} \cdots\cdots (24)$$

where

$$[k]=[G]^{-1T}[D][G]^{-1}$$

$$[D]=\frac{1}{2}\left(\int_{\Gamma e} CC_g[P]^T[Q]d\Gamma+\int_{\Gamma e} CC_g[Q]^T[P]d\Gamma\right)$$

$$\{f\}=[G]^{-1T}\int_{\Gamma_{2e}} CC_g[P]^T d\Gamma\hat{\eta}_{,n}$$

Superposing the finite element equation (24), the global finite element equation can be obtained.

$$[K]\{\tilde{\eta}\}=\{F\} \cdots\cdots (25)$$

where $[K]$ and $\{F\}$ are constructed by superposing the element coefficients $[k]$ and $\{f\}$ for all elements in the wave field.

4. INTEGRATION METHOD FOR COEFFICIENTS

In the calculation of line integrals of coefficients, the term CC_g is approximated by the value taken at the centroid of each element. The evaluation of the integral which is concerned on the boundary can be carried out by integrating from one side of the element to another, for example, in case of four node element

$$\int_{\Gamma e}\cdots d\Gamma=\int_1^2\cdots d\Gamma+\int_2^3\cdots d\Gamma+\int_3^4\cdots d\Gamma+\int_4^1\cdots d\Gamma \qquad (26)$$

where the numerical numbers indicate the corresponding nodal points for the element.

In equation (25) while calculating the matrix $[K]$ and $\{F\}$, it is necessary to transform the coordinate system from cartesian coordinate (x, y) to (s, t) coordinate system. For example, during integration from node 1 to node 2, the transforming equation can be obtained as follows (see Fig. 4).

$$\left.\begin{aligned} x&=s\cos\theta_1-t\sin\theta_1+x_1\\ y&=s\sin\theta_1+t\cos\theta_1+y_1\end{aligned}\right\} \qquad (27)$$

While integration on Γ_{12}, t vanishes and hence the transforming equations can be obtained as follows.

$$\left.\begin{aligned} x&=s\cos\theta_1+x_1\\ y&=s\sin\theta_1+y_1\end{aligned}\right\} \qquad (28)$$

In this paper, the concept of this relation is used throughout in the calculation of the line integrals.

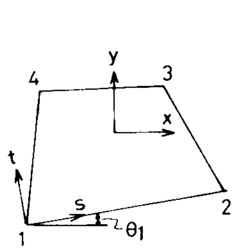


Fig. 4 Integration method.

In case of four node quadrilateral element, the various coefficient matrices can be written as :

$$[P]=\Big[\cos\left(\frac{k}{\sqrt2}x\right)\cos\left(\frac{k}{\sqrt2}y\right)\quad \cos\left(\frac{k}{\sqrt2}x\right)\sin\left(\frac{k}{\sqrt2}y\right)\quad \sin\left(\frac{k}{\sqrt2}x\right)\cos\left(\frac{k}{\sqrt2}y\right)\quad \sin\left(\frac{k}{\sqrt2}x\right)\sin\left(\frac{k}{\sqrt2}y\right)\Big]$$

$$\begin{aligned}[Q]=\Big[&\left\{-\frac{k}{\sqrt2}\sin\left(\frac{k}{\sqrt2}x\right)\cos\left(\frac{k}{\sqrt2}y\right)\right\}\cdot l+\left\{-\frac{k}{\sqrt2}\cos\left(\frac{k}{\sqrt2}x\right)\sin\left(\frac{k}{\sqrt2}y\right)\right\}\cdot m\\ &\left\{-\frac{k}{\sqrt2}\sin\left(\frac{k}{\sqrt2}x\right)\sin\left(\frac{k}{\sqrt2}y\right)\right\}\cdot l+\left\{\frac{k}{\sqrt2}\cos\left(\frac{k}{\sqrt2}x\right)\cos\left(\frac{k}{\sqrt2}y\right)\right\}\cdot m\\ &\left\{\frac{k}{\sqrt2}\cos\left(\frac{k}{\sqrt2}x\right)\cos\left(\frac{k}{\sqrt2}y\right)\right\}\cdot l+\left\{\frac{k}{\sqrt2}\sin\left(\frac{k}{\sqrt2}x\right)\sin\left(\frac{k}{\sqrt2}y\right)\right\}\cdot m\\ &\left\{\frac{k}{\sqrt2}\cos\left(\frac{k}{\sqrt2}x\right)\sin\left(\frac{k}{\sqrt2}y\right)\right\}\cdot l+\left\{-\frac{k}{\sqrt2}\sin\left(\frac{k}{\sqrt2}x\right)\cos\left(\frac{k}{\sqrt2}y\right)\right\}\cdot m\Big]\end{aligned}$$

$$[G]=\begin{bmatrix} \cos\frac{k}{\sqrt2}x_1\cos\frac{k}{\sqrt2}y_1 & \cos\frac{k}{\sqrt2}x_1\sin\frac{k}{\sqrt2}y_1 & \sin\frac{k}{\sqrt2}x_1\cos\frac{k}{\sqrt2}y_1 & \sin\frac{k}{\sqrt2}x_1\sin\frac{k}{\sqrt2}y_1\\ \cos\frac{k}{\sqrt2}x_2\cos\frac{k}{\sqrt2}y_2 & \cos\frac{k}{\sqrt2}x_2\sin\frac{k}{\sqrt2}y_2 & \sin\frac{k}{\sqrt2}x_2\cos\frac{k}{\sqrt2}y_2 & \sin\frac{k}{\sqrt2}x_2\sin\frac{k}{\sqrt2}y_2\\ \cos\frac{k}{\sqrt2}x_3\cos\frac{k}{\sqrt2}y_3 & \cos\frac{k}{\sqrt2}x_3\sin\frac{k}{\sqrt2}y_3 & \sin\frac{k}{\sqrt2}x_3\cos\frac{k}{\sqrt2}y_3 & \sin\frac{k}{\sqrt2}x_3\sin\frac{k}{\sqrt2}y_3\\ \cos\frac{k}{\sqrt2}x_4\cos\frac{k}{\sqrt2}y_4 & \cos\frac{k}{\sqrt2}x_4\sin\frac{k}{\sqrt2}y_4 & \sin\frac{k}{\sqrt2}x_4\cos\frac{k}{\sqrt2}y_4 & \sin\frac{k}{\sqrt2}x_4\sin\frac{k}{\sqrt2}y_4 \end{bmatrix}$$

where l and m are the direction cosines of the outward normal to the boundary.

5. COUPLING WITH BOUNDARY ELEMENT METHOD

For the analysis of outer problem, the radiation condition must be considered as the boundary condition at infinity. Thus, for the purpose of efficient numerical computation, wave field is divided into two domains, one of which is the inner domain Ω_i with arbitrary water depth and the other is the outer domain

Ω_0 with constant water depth as shown in Fig. 1 (b). On the domain Ω_i, the present method is applied and the boundary element method is introduced on the domain Ω_0.

Generally, the variational functional to be minimized for the outer domain Π_0 is expressed as follows.

$$\Pi_0=\frac{1}{2}\int_{\Gamma_0} CC_g\eta\eta_{,n}d\Gamma \qquad (29)$$

Therefore, the variational functional equation for the present combinational method of boundary type finite elements is:

$$\Pi=\Pi_i+\Pi_0$$

$$=\frac{1}{2}\int_{\Gamma_i} CC_g\eta\eta_{,n}d\Gamma+\frac{1}{2}\int_{\Gamma_0} CC_g\eta\eta_{,n}d\Gamma-\int_{\Gamma_S} CC_g\eta\hat{\eta}_{,n}d\Gamma \qquad (30)$$

where Π_i denotes the variational functional for the inner domain. Using the Hankel function of the first kind zeroth order $H_0^1(kr)$ for the fundamental solution for outgoing scattering wave η_{sc}, the boundary integral equation can be written as:

$$\eta_{sc}(p)\left(1-\frac{\alpha}{2\pi}\right)=\frac{i}{4}\int_{\Gamma_C}\{\eta_{sc}(H_0^1(kr))_{,n}-H_0^1(kr)\eta_{sc,n}\}d\Gamma \qquad (31)$$

where p is the pole and r is the distance from it and α denotes the angle of rotation of the tangent at point p. Linear interpolation function $[L]$ is introduced for both water elevation and derivatives, i.e.,

$$\eta=[L]\{\bar{\eta}\} \qquad (32)$$

$$\eta_{,n}=[L]\{\bar{\eta}_{,n}\} \qquad (33)$$

Introducing equation (32) and (33) into (31) and rearranging the terms, the following matrix form is derived.

$$[A]\{\bar{\eta}_{sc}\}=[B]\{\bar{\eta}_{sc,n}\} \qquad (34)$$

The above boundary integral equation (34) is derived for the scattering wave. Since the variational equation is formulated for the total wave phenomena, the integral equation has to be transformed for the total wave. Introducing equation (5) and (9) into equation (34), the following equation is obtained.

$$\{\bar{\eta}_{,n}\}=-[B]^{-1}[A]\{\bar{\eta}\}+[B]^{-1}[A]\{\bar{\eta}_{in}\}-\{\bar{\eta}_{in,n}\} \qquad (35)$$

Substituting equation (19), (21), (32) and (33) into equation (30) and eliminating the normal derivative using equation (35), and minimizing the functional, the final form of the computational equation can be derived in the following form.

$$[K+H]\{\bar{\eta}\}=\{F\} \qquad (36)$$

where

$$[K]=[G]^{-1T}\left[\frac{1}{2}\left(\int_{\Gamma_i} CC_g[P]^T[Q]d\Gamma+\int_{\Gamma_i} CC_g[Q]^T[P]d\Gamma\right)\right][G]^{-1}$$

$$[H]=-([B]^{-1}[A])^T\int_{\Gamma_0} CC_g[L]^T[L]d\Gamma$$

$$\{F\}=-([B]^{-1}[A])^T\int_{\Gamma_0} CC_g[L]^T d\Gamma\bar{\eta}_{in}+\int_{\Gamma_0} CC_g[L]^T d\Gamma\bar{\eta}_{in,n}+[G]^{-1T}\int_{\Gamma_S} CC_g[P]^T d\Gamma\hat{\eta}_{,n}$$

In order to compute equation (36), the frontal solution technlque is used. Therefore, it can be possible to solve the problem which consists of a large number of unknown varibles using the limited computational time.

6. NUMERICAL TESTS

In order to show the varidity of the present method, several numerical computations have been carried out, comparing with the analytical solutions and computational results obtained by the conventional finite element method using linear interpolation function.

Consider a rectangular basin with slowly varying water depth $h=h_0(1-x/a)$ as shown in Fig. 5. The water depth is assumed to be zero along the line $x=+a$ and uniformly increases to the opposite end $x=-a$ where it is $2h_0$, h_0 being the depth along the line $x=0$ and equal to the mean depth. Hidaka[12] had

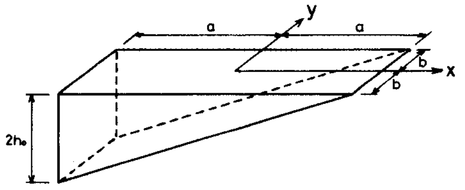


Fig. 5 Rectangular basin.

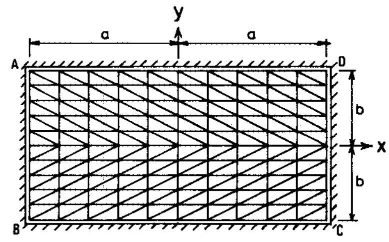


Fig. 6 Finite element idealization.

obtained the exact value for the eigen-frequency and water elevation only when the ratio b/a equals to 0.5. Fig. 6 is the finite element idealization based on the three node triangular element for the rectangular basin. In this computation, the values for a, b and h_0 are assumed to be 50 m, 25 m and 5 m, respectively.

Table 1 Comparison of eigen-frequency.

exact	present method	F.E.M.(linear)
25.73	26.03(error 1.2%)	24.58(error -4.5%)

Table 1 shows the comparison between analytical and computed eigen-frequency when the mode of this basin is $(m, n)=(0, 1)$. Here m and n denote the total number of wave nodes in the x-direction and y-direction, respectively. In this table, F. E. M. denotes the finite element method based on the linear interpolation function. Here it can be seen that the solution which is obtained by the present method is well enough comparing to the one obtained by the conventional finite element method.

Fig. 7 illustrates the computed water elevation along the boundary A–D and C–D which are compared with the conventional finite element and analytical results. It can be seen that the water elevation computed by the present method is closer to the analytical solution than that by the conventional method.

The convergence criterion can be checked numerically as shown in the following figures. Fig. 8 and Fig. 9 represent the relation between the error percentage and the total number of nodes when the mode of this basin $(m, n)=(0, 1)$. The ordinate represents the error percentage and the abscissa is the total number of nodes. It can be seen that the present method is good in accuracy compared with the conventional finite element method.

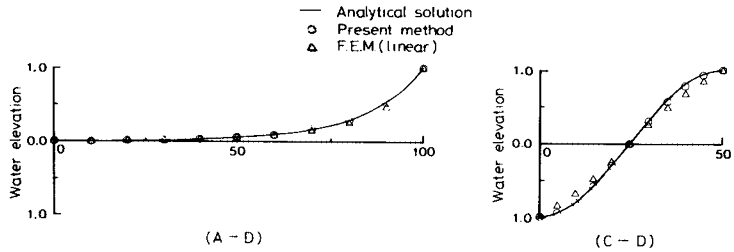


Fig. 7 Computed water elevation.

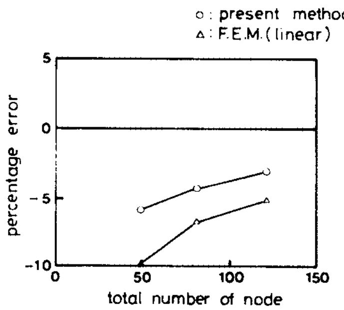


Fig. 8 Percentage error (three node element).

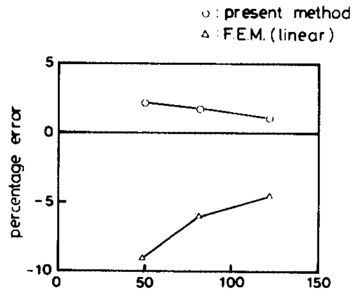


Fig. 9 Percentage error (four node element).

7. APPLICATIONS

(1) Applications to inner problem

As the first application problem, the analysis of natural oscillation problem in lake is analyzed. For the numerical study, Lake Yamanaka which is located at the northeastern side of Mt. Fuji in Japan is chosen.

Fig. 10 is the finite element idealization and water depth for Lake Yamanaka. The total number of finite elemnts and nodal points are 185 and 115, respectively. The eigen-frequency of this lake is calculated and shown in Table 2. The table shows the comparison between computed and observed eigen-frequency[13]. In this table T_1, T_2 and T_3 denote the first, second and third mode of this lake respectively. The computed results are well agreement with the observed results.

(2) Application to outer problem

The present combinational method is applied to the analysis of wave amplitude distribution around an island on parabolic shoal as shown in Fig. 11.

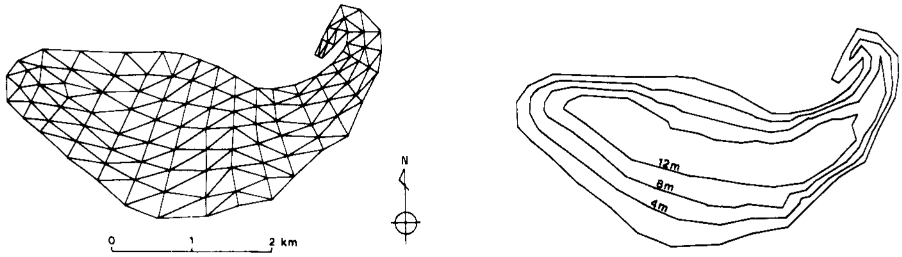


Fig. 10 Finite element idealization and water depth for Lake Yamanaka.

Table 2 Comparison of eigen-frequency (in minutes).

	observed	present method
T_1	15.61	16.29
T_2	10.57	10.66
T_3	5.46	5.56

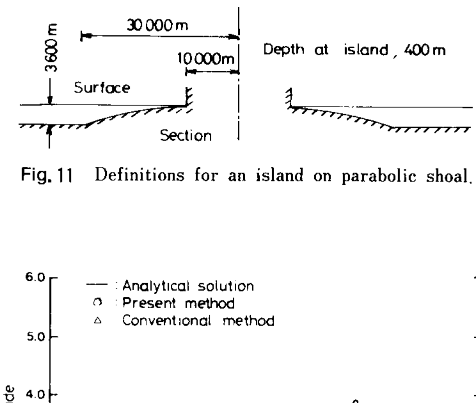


Fig. 11 Definitions for an island on parabolic shoal.

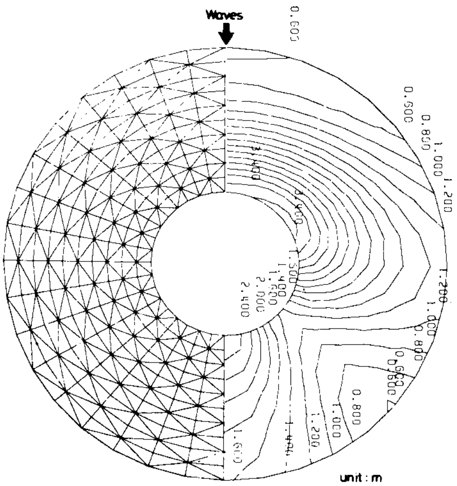


Fig. 12 Finite element idealization and computed wave amplitude distribution.

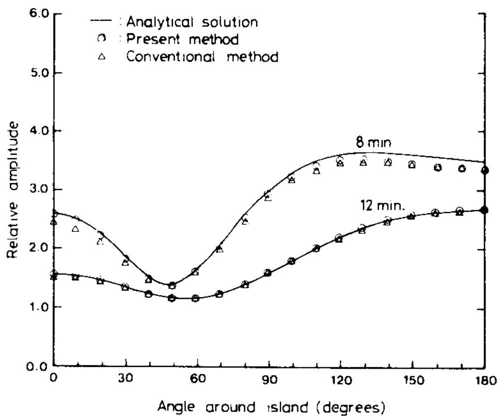


Fig. 13 Computed wave amplitude at island on parabolic shoal.

Fig. 12 represents the finite element idealization and computed wave amplitude distribution around an island. The total number of finite elements and nodal points are 720 and 396, respectively. In this case, the incident wave period is assumed to be 8 minutes and the incident wave amplitude assumed to be 1.0 m, respectively.

Fig. 13 illustrates the computed wave amplitude on an island which is compared with the conventional finite element method and analytical results. The analytical solution was obtained by Homma[14]. From this figure, it can be seen that the wave amplitude computed by the present method is well in agreement with the analytical solution compared with the conventional finite element solution. This becomes conspicuous in the rear side of the island.

8. CONCLUSION

The boundary type finite element method has been presented in this paper for the analysis of surface wave problems. The characteristic points of this method is as follows. The interpolation equation has been chosen so as to satisfy the Helmholtz equation in each element using the trigonometric function series. This enables that the variational functional to be minimized can be formulated as in the form that the integration is limited just on the boundary of the element. This follows that the final equation can be formulated by the calculation of line integral.

From the numerical examples, it can be seen that the solutions obtained by the present method is assured to be satisfactory close to the analytical solution. The CPU time by the present method to achieve better accuracy compared to the conventional finite element method requires the same amount as in the case of conventional finite element method. Moreover, the coarse finite element mesh idealization can be adaptable compared with those employed in the conventional numerical methods. Therefore it is possible to reduce the computer core storage as well as CPU time to a considerable extent. The analysis in this paper was done only for the long wave problems, whereas it is also possible to solve short wave problems.

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