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A NEW APPROACH TO PREDICT THE STRENGTH OF COMPRESSED STEEL STIFFENED PLATES

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This paper provides a simplified approach to the strength of compressed steel stiffened plates from a knowledge of the catastrophe theory. The strength prediction for both global and local buckling modes is presented.

The elasto-plastic buckling stress is firstly obtained with consideration of the elasto-plastic behavior of the material and the residual stresses of both stiffeners and plate panels. Then, the reduction of the ultimate strength due to the initial out-of-flatness can be explicitly determined by the imperfection sensitivity curve based on the concept of the bifurcation set in the catastrophe theory.

1. INTRODUCTION

The stability and the strength of steel plates and stiffened plates in the elasto-plastic range have been one of the subjects of the greatest concern of civil engineers. They are being used for such members as plate and box girders, chords of trusses and arches, bridge piers and towers.

Several theoretical and numerical procedures have been proposed so far on problems of the initial buckling, the postbuckling and the ultimate strength of stiffened plates. They can be classified with respect to the method of approach[4] : firstly, the orthotropic plate approach ; secondly, the beam-column analysis using the concept of the "effective width" ; thirdly, descretization methods such as finite strip method, finite difference method, finite element method adopting the incremental energy-approach ; and finally, the nonlinear bifurcation theories based on the hypoelasticity and the topological considerations.

The first orthotropic approach for stiffened plates was initiated by Huffington et al. [1]. They determined four orthogonal rigidities of "equivalent" elastic homogeneous orthotropic plates. Improvements of the procedure for flat plates in the inelastic region beyond elastic limit were made by Stowell[2] and Bleich[3] using some reduction factors of orthogonal rigidities. Attempts have been made to evaluate the elasto-plastic buckling stress of stiffened plates. Mikami et al. [4] studied the inelastic buckling stress of continuous stiffened plates through the Bleich's factors.

The second beam-column approach have been developed by Faulkner[5], Little[6], Carlsen[7], Horne et al. [8),9] and Rhodes[10]. They derived foundamental relationships between the average axial stress and the corresponding strain in the theoretical and numerical form, and compared the results with available

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experimental results. Also, Moolani et al.[11] discussed on the parametric study of the behavior of eccentrically stiffened plates.

The third approach of discretization is now being widely accepted as reasonably accurate. The early researches in Japanese civil engineering field on the numerical elasto-plastic buckling stress of stiffened plates were accomplished by Usami[12], Hasegawa et al.[13], and Yoshida et al.[14] using finite strip methods. Furthermore, the large-deflection elasto-plastic analyses of compressed stiffened plates have been developed by many researchers in the world such as Crisfield[15], Komatsu et al.[16], Marchesi[17] and Webb et al.[18] through finite element methods and finite difference methods in order to solve the relevant nonlinear simultaneous equations.

The final group of approach is based on the concept of nonlinear bifurcation. Tvergaard et al.[19] investigated the elasto-plastic bifurcation behavior, the initial postbuckling behavior and the imperfection sensitivity of eccentrically stiffened plates. They employed an incremental linearized Rayleigh-Ritz method for the stiffened plates regarding as hypoelastic plates neglecting the effect of elastic unloading. They also discussed the stability and the imperfection sensitivity of the elastic simultaneous interaction among the global buckling of the panels as a wide Euler column and the local buckling of the plates between the stiffeners[20),21]. Some powerful contributions on such interaction problems of stiffened plates have been also provided by Koiter[22] and van der Neut[23].

These theoretical and numerical analyses allow the maximum ultimate strength of stiffened plate models to be determined in an isolated form for a selected set of geometrical and material parameters.

The authors have proposed a new simplified approach to evaluate the ultimate strength of steel slender structures such as columns, beams and unstiffened plates in the elasto-plastic range[24)~27]. The approach does not require a nonlinear process of simultaneous equations concerned, and means a simplified prediction of the imperfection sensitivity of the structures in view of the singular bifurcation set through the catastrophe theory. This paper reports an application of the procedure to the elasto-plastic strength of compressed rectangular stiffened plates with axial longitudinal stiffeners. The strength prediction for both global and local bucklings of stiffened plates is presented.

2. BASIC CONCEPTS

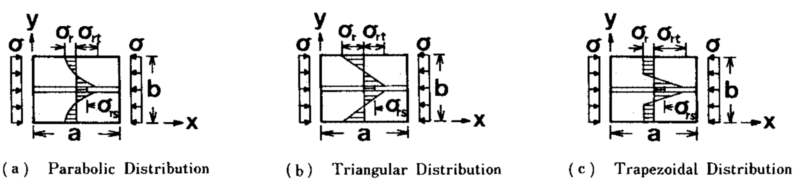
(1) Residual stress distribution

A rectangular stiffened plate with four edges simply supported under uniaxial compression as shown in Fig. 1 is analyzed as a typical basic model. The stiffened plate has only several longitudinal stiffeners of equal area and equal flexural rigidity arranged in certain equal distance. The residual stress is assumed to be distributed in an appropriate form in the local plate panel with the magnitude σ_r and σ_{rt} in compression and tension, respectively, and to be uniform in the stiffener section with the magnitude σ_{rs}.

In view of the self-equilibrium of the residual stress distribution over the entire stiffened plate, the equivalent tensile residual stress $\overline{\sigma}_{rs}$ may be assumed to be distributed uniformly in the global orthotropic plate in addition to the residual stress distribution of the plate panel alone. Hence, the magnitudes of σ_r and σ_{rt} can be replaced by the corresponding "prime" value, respectively :

$$\sigma'_r=\sigma_r-\overline{\sigma}_{rs} \text{ and } \sigma'_{rt}=\sigma_{rt}+\overline{\sigma}_{rs} \quad \cdots\cdots (1)$$

where



(a) Parabolic Distribution (b) Triangular Distribution (c) Trapezoidal Distribution

Fig. 1 Distributions of residual stresses.

$$\bar{\sigma}_{rs}=\frac{N\delta}{1+N\delta}\sigma_{rs}$$

and δ refers to the ratio of a stiffener area A_s to the plate panel area bt, and N denotes the number of stiffeners. The relationship between σ'_r and σ'_{rt} can be determined from the initial self-equilibrium condition of each distribution of residual stress.

The distribution of residual stress with the maximum compressive stress σ_r is assumed to be in either parabolic, triangular or trapezoidal form as shown in Fig. 1 (a) ~ (c). The relationships among the tangent modulus E_t, the secant modulus E_s, the average axial strain $\bar{\varepsilon}$ and the average axial stress $\bar{\sigma}$ can be obtained as :

$$E_t\equiv\frac{d\bar{\sigma}}{d\bar{\varepsilon}}=kE \text{ and } E_s\equiv\frac{\bar{\sigma}}{\bar{\varepsilon}} \qquad (2\cdot a)$$

and

$$\bar{\sigma}=\sigma_Y-(3-2k)k^2\sigma'_r$$

$$\bar{\varepsilon}=\frac{1}{E}[\sigma_Y+3(1-k)^2\sigma'_r-\sigma'_r]$$

$$\text{for parabola } 0\leqq k\leqq 1 \qquad (2\cdot b)$$

$$\bar{\sigma}=\sigma_Y-k^2\sigma'_r$$

$$\bar{\varepsilon}=\frac{1}{E}[\sigma_Y-(2k-1)\sigma'_r]$$

$$\text{for triangle } 0\leqq k\leqq 1 \qquad (2\cdot c)$$

$$\bar{\sigma}=\sigma_Y-k^2\frac{(\sigma'_r+\sigma_Y)^2}{4\sigma'_r}$$

$$\bar{\varepsilon}=\frac{1}{E}\left[2\sigma_Y-k\frac{(\sigma'_r+\sigma_Y)^2}{2\sigma'_r}\right]$$

$$\text{for trapezoid } 0\leqq k\leqq\frac{2\sigma'_r}{\sigma'_r+\sigma_Y} \qquad (2\cdot d)$$

where E and σ_Y refer to the Young's modulus and the yielding stress of the material, respectively. Furthermore, k denotes the ratio of the elastic portion of the cross section to the total section of the plate, namely, it indicates the global tangent modulus factor of the orthotropic plate section.

(2) Ideal elasto-plastic buckling load

From the boundary condition, the buckling and the initial imperfection modes of the equivalent global orthotropic plate are assumed in both elastic and elasto-plastic ranges as follows :

$$W=wY(y)\sin\frac{m\pi x}{a},\quad W_0=w_0Y(y)\sin\frac{m\pi x}{a},\quad Y(y)=\sin\frac{n\pi y}{b} \qquad (3)$$

for all edges simply supported

in the coordinate system as shown in Fig. 1. In which, w, w_0, $Y(y)$, m and n designate the magnitude of the total out-of-plane deflection, that of initial out-of-plane deflection, mode of deflection in y-direction, numbers of half waves in x-and y-direction, respectively.

Consider the compressed rectangular stiffened plate with N longitudinal stiffeners with stiffener parameters : [28]

$$\delta\equiv\frac{A_s}{bt} \text{ and } \gamma\equiv\frac{EI_s}{bD_e} \qquad (4)$$

where b, t and D_e refer to the total width, the net thickness and the elastic flexural rigidity of the plate panel, respectively. Also, A_s and I_s denote the cross-sectional area and the moment of inertia of a stiffener, respectively. The torsional rigidity of stiffener itself is not considered in the paper. Through the classical orthotropic approach using partial differetial equations, the basic equation of equilibrium of the stiffened plate can be written as : [3),28]

$$D_e \nabla_p^4 W + \bar{\sigma} t_0 \frac{\partial^2 W}{\partial x^2} = 0 \qquad (5)$$

where

$$D_e \equiv \frac{E t^3}{12(1-\nu^2)}, \quad t_0 \equiv t[1+(N+1)\delta],$$

$$\nabla_p^4 \equiv k_1 \frac{\partial^4}{\partial x^4} + 2(k_2 + 2\,k_4) \frac{\partial^4}{\partial x^2 \partial y^2} + k_3 \frac{\partial^4}{\partial y^4}$$

in which, ν, t_0 and $k_j (j=1, 2, 3, 4)$ refer to the Poisson's ratio, the equivalent thickness and the constants to designate flexural and torsional rigidities of the orthotropic plate in the elasto-plastic range, rspectively.

Upon substitution of W in Eq. (3) into Eq. (5) and through the Galerkin's method, the critical stress $\bar{\sigma}_{cr}$ can be defined in the following form :[25)]

$$\bar{\sigma}_{cr} = \frac{D_e \int_0^b Y Y_1 dy}{t_0 \int_0^b \left(\frac{m\pi}{a}\right)^2 Y^2 dy} \qquad (6)$$

where

$$Y_1(y) \equiv k_1 \left(\frac{m\pi}{a}\right)^4 Y(y) - 2(k_2 + 2\,k_4) \left(\frac{m\pi}{a}\right)^2 \frac{d^2 Y}{dy^2} + k_3 \frac{d^4 Y}{dy^4}$$

The buckling coefficient K_S can be given by

$$K_S = \frac{\bar{\sigma}_{cr}}{\sigma_0} \qquad (7)$$

where

$$\sigma_0 \equiv \frac{\pi^2 D_e}{b^2 t}$$

thus, K_S can be obtained as

$$K_S \equiv \frac{\left(\frac{n\phi}{m}\right)^2}{1+(N+1)\delta} \left[k_1 \left(\frac{m}{n\phi}\right)^4 + 2(k_2 + 2\,k_4) \left(\frac{m}{n\phi}\right)^2 + k_3 \right] \qquad (8)$$

where

$\phi \equiv \frac{a}{b}$: aspect ratio

Let us define a factor f by :

$$f = \frac{K_S}{K_{SE}} \qquad (9)$$

where K_{SE} refers to the minimum elastic buckling coefficient. Thus,

$$f = \frac{\bar{\sigma}_{cr}}{\sigma_0} \frac{1}{K_{SE}} = \frac{\bar{\sigma}_{cr}}{\bar{\sigma}_E} \qquad (10)$$

where

$$\bar{\sigma}_E \equiv K_{SE} \sigma_0 = K_{SE} \frac{\pi^2 D_e}{b^2 t}$$

$$K_{SE} \equiv \frac{2[1+\sqrt{1+(N+1)\gamma}\,]}{1+(N+1)\delta}$$

that is, $\bar{\sigma}_E$ refers to the Euler buckling stress for the global buckling of the stiffened plate.

Then, the non-dimensionalized equation of equilibrium in the elasto-plastic range can be rewritten as :[25)]

$$f \tilde{\sigma}_E \tilde{w} - \tilde{\sigma} \tilde{w} = 0 \qquad (11)$$

where

$$\tilde{\sigma}_E \equiv \frac{\bar{\sigma}_E}{\sigma_Y} = \frac{1}{R_{SE}^2}, \quad \tilde{w} \equiv \frac{w}{t}, \quad \tilde{\sigma} \equiv \frac{\bar{\sigma}}{\sigma_Y}, \quad R_{SE} \equiv \frac{b}{t}\sqrt{\frac{12(1-\nu^2)}{\pi^2 K_{SE}}\frac{\sigma_Y}{E}}$$

in which, the symbol "$\sim$" designates the non-dimensionalization in terms of the yielding strength, σ_Y, and the thickness, t, for the stress and the displacement, respectively. R_{SE} and K_{SE} refer to the generalized width-thickness ratio for the buckling of the global stiffened plate and the corresponding elastic buckling coefficient, respectively.

In this paper, however, numerical examples are demonstrated using only the Bleich's approach to evaluate the elasto-plastic buckling stress. Now, using the Bleich's factor τ, the coefficients k_j in Eq. (5) are defined in the elasto-plastic range as follows : [3)-5)]

$$k_1 \equiv \tau[1+(N+1)\gamma], \quad k_2 \equiv \nu\sqrt{\tau}, \quad k_3 \equiv 1, \quad k_4 \equiv \frac{1-\nu}{2}\sqrt{\tau} \quad \cdots\cdots (12)$$

In order to take into account the effects of residual stress on the elasto-plastic buckling stress, the factor τ is set to be equal to the tangent modulus k in Eq. (2). Upon substitution of Eq. (12) into Eq. (9), the factor f can be obtained by :

$$f = \frac{1}{[1+(N+1)\delta]n^2 K_{SE}}\left\{\left[\sqrt{k_c}\sqrt{1+(N+1)\gamma}\left(\frac{m}{\phi}\right) - n^2\left(\frac{\phi}{m}\right)\right]^2 + 2n^2\sqrt{k_c}\,[1+\sqrt{1+(N+1)\gamma}]\right\} \cdots (13)$$

where k_c refers to the value of the factor k in Eq. (2) at the elasto-plastic buckling point.

Now, by evaluating the minimum values of K_S, the value of f can be obtained simply as

$$f = f^c = \sqrt{k_c} \quad \cdots\cdots (14)$$

since, for the elasto-plastic buckling,

$$(K_S)_{\min} \equiv \frac{2\sqrt{k_c}\,[1+\sqrt{1+(N+1)\gamma}\,]}{1+(N+1)\delta} \quad \cdots\cdots (15)$$

at

$$n = 1 \text{ and } \phi = m\sqrt[4]{k_c[1+(N+1)\gamma]}$$

Thus, using Eqs. (7), (8), (11), (14) and (15), the elatso-plastic buckling stress $\bar{\sigma}_{cr}$ can be given by

$$\tilde{\sigma}_{cr} = f^c \tilde{\sigma}_E \quad \cdots\cdots (16)$$

where

$$\tilde{\sigma}_{cr} \equiv \frac{\bar{\sigma}_{cr}}{\sigma_Y}$$

It implies that the elasto-plastic buckling stress of the stiffened plates can be expressed in the unified form similar to that of columns, beams or compressed plates.

(3) Postbuckling path

The rigorous prediction of the significant postbuckling behavior of stiffened plate in a closed form is entirely difficult even in the elastic range. Thus, for simplicity, a modification on the von Kármán's equations is made in order to evaluate such postbuckling reservation using the analogous orthotropic plate approach. The modified von Kármán's equations for such orthotropic plates in the elasto-plastic range lead to the following postbucking path : [25)]

$$\tilde{\sigma} = \tilde{\sigma}_{cr} + \tilde{C}_p \tilde{w}^2 \quad \cdots\cdots (17)$$

at

$$n = 1 \text{ and } \phi = m\sqrt[4]{k_c[1+(N+1)\gamma]}$$

where

$$\tilde{C}_p \equiv \frac{3(1-\nu^2)}{4K_{SE}}\frac{1}{R_{SE}^2}\frac{E_S}{E}\frac{1+k_c[1+(N+1)\gamma]}{\sqrt{k_c[1+(N+1)\gamma]}}, \quad R_{SE} = R\sqrt{\frac{K}{K_{SE}}}, \quad R \equiv \frac{b}{t}\sqrt{\frac{12(1-\nu^2)}{\pi^2 K}\frac{\sigma_Y}{E}}, \quad K \equiv 4(N+1)^2$$

and R denotes the generalized width-thickness ratio for the local buckling of the plate panel. It is clearly seen that Eq. (16) provides the elastic postbuckling path if $E_S = E$ and $k_c = 1$.

(4) Ultimate strength

Ultimate strength of the actual stiffened plates can not be determined by evaluating only the elasto-plastic buckling strength. It is further affected by the initial lateral deflection and the plastic unloading curve as well as by both the residual stress and the elasto-plastic postbuckling path. The plastic unloading curve is obtained from the failure mechanism corresponding to the ultimate state of the stiffened plate. The failure mechanism of the plate under uniaxial pure compression is assumed to consist of fold lines for the global buckling mode as shown in **Fig. 2**. Many investigations have been performed on the failure mechanism curve in the last two decades. In the present paper, the following simple interaction formula between the in-plane axial stress and the associated bending moment is applied :[25)]

$$\tilde{\sigma}^2+\tilde{m}=1 \qquad (18)$$

where

$$\tilde{m}\equiv\frac{M}{M_p}$$

in which, M and M_p refer to the bending moment perpendicular to teh corresponding fold line and the full plastic moment, respectively. Then, for the global buckling mode of the stiffened plate, the plastic unloading curve can be approximately predicted by :

$$\tilde{w}=\tilde{w}_p\equiv A\frac{1-\tilde{\sigma}^2}{\tilde{\sigma}} \qquad (19)$$

where, for $\phi^*\equiv\phi/m\geqq\cot\theta$,

$$A=\frac{1}{2}\frac{1+\phi^*\cot\theta+N\delta\dfrac{h_s}{t}}{1+\dfrac{N(N+2)}{N+1}\delta} \quad (N\ \text{: even})$$

$$A=\frac{1}{2}\frac{1+\phi^*\cot\theta+N\delta\dfrac{h_s}{t}}{1+(N+1)\delta} \quad (N\ \text{: odd})$$

for $\phi^*<\cot\theta\leqq 1$,

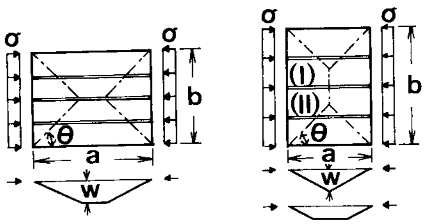
$$A=\frac{1}{2}\frac{1+\phi^*\cot\theta+N\delta\dfrac{h_s}{t}}{2-\phi^*\tan\theta+2N_{\rm I}\delta+\dfrac{N_{\rm II}(N_{\rm II}+2)}{N+1}\delta\dfrac{\cot\theta}{\phi^*}} \quad (N=N_{\rm I}+N_{\rm II})$$

in which, θ denotes the angle of the yielding fold line as shown in **Fig. 2**. In the second type in **Fig. 2**(b), $N_{\rm I}$ and $N_{\rm II}$ refer to the numbers of longitudinal stiffeners on the fold lines of (I) and (II), respectively. Moreover, h_s designates the height of a stiffener from the surface of the plate panel, and the factor h_s/t can be given as a function of the stiffener parameters.

Now, let us consider the "equivalent bifurcation point" as the intersection of the elasto-plastic postbuckling path in Eq. (17) with the plastic unloading curve in Eq. (19). The point can be obtained by solving the following simple quartic polynomial equation :

$$\tilde{C}_pA^2\tilde{\sigma}^4-\tilde{\sigma}^3-(2\tilde{C}_pA^2-\tilde{\sigma}_{cr})\tilde{\sigma}^2+\tilde{C}_pA^2=0 \qquad (20)$$

Let $\tilde{\sigma}^*$ and $\tilde{w}^*$ designate a proper real root of the equation and the corresponding deflection calculated by Eq. (17) or (19), respectively. Hence, the equivalent bifurcation point can be given by the point $C(\tilde{\sigma}^*, \tilde{w}^*)$ in **Fig. 3**.



(a) $\phi^*\geqq\cot\theta$ (b) $\phi^*<\cot\theta$

Fig. 2 Plastic failure mechanisms.

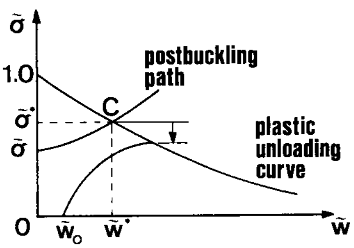


Fig. 3 Equivalent bifurcation point.

In order to evaluate the ultimate strength of the imperfect plate, a pseudo-potential may also be defined near the equivalent bifurcation point C. Then, the ultimate strength $\tilde{\sigma}_m$ of the imperfect stiffened plate can be predicted in terms of the bifurcation set through the catastrophe theory. It can be defined by a set of singular points :[27]

$$\frac{\tilde{\sigma}_m}{\tilde{\sigma}^*}=1+\alpha^*\tilde{w}_0-\sqrt{2\alpha^*\tilde{w}_0\left(1+\frac{1}{2}\alpha^*\tilde{w}_0\right)} \quad \cdots (21)$$

where

$$\tilde{\sigma}_m\equiv\frac{\sigma_m}{\sigma_Y}$$

and α^* can be approximated by the slope of the plastic unloading curve at the equivalent bifurcation point C, that is,

$$\alpha^*=-\frac{1}{\tilde{\sigma}^*}\frac{1}{d\tilde{w}_p/d\tilde{\sigma}}\bigg|_{\tilde{\sigma}=\tilde{\sigma}^*}=\frac{\tilde{\sigma}^*}{A(1+\tilde{\sigma}^{*2})} \quad \cdots (22)$$

Furthermore, the concept of the "equivalent initial imperfection" is adopted herein in order to describe the actual strength behavior of the stiffened plate :

$$\tilde{w}_0^*\equiv\mu(R)\tilde{w}_0 \quad \cdots (23)$$

where

$$\mu(R)\equiv\mu_c\left(\frac{R}{R_p}\right)^\beta$$

μ_c is a constant specified below, and R_p refers to the value of the generalized width-thickness ratio R for the local plate buckling between stiffeners, at which the buckling point changes from the elasto-plastic to purely elastic, and is alternatively used the generalized width-thickness ratio R_{SE} for the global stiffened plate buckling.

As a result, the form of $\mu(R)$ may be approximated by

$$\mu_c\equiv\frac{1}{8},\quad \beta\equiv2\left(1-\frac{R}{R_p}\right) \quad \cdots (24)$$

similarly to the case of compressed plate panels so as to formulate a unified strength prediction for both global and local bucklings of stiffened plates. Finally, the imperfection sensitivity or the load-carrying capacity can be determined by Eq. (21); however, with the slope α^* of Eq. (22) and the equivalent imperfection of Eqs. (23) and (24)[24)~27)].

On the other hand, the ultimate strength for the local buckling of plate panel of stiffened plate may be evaluated as four edges simply supported rectangular plate in the same manner.

3. NUMERICAL EXAMPLES AND DISCUSSIONS

Several numerical illustrations are provided on the strength of the simply supported stiffened plates under in-plane uniaxial compression. A single longitudinal stiffener is assumed to be spliced with equal interval and can be characterized by the geometrical and material parameters such as N, δ, γ and E/σ_Y. Moreover, torsional rigidities of the stiffener are assumed to be neglected. The type of residual stress distribution of the local plate panel is assumed to be either of a parabola, a triangle or a trapezoid as shown in **Fig.** 1 (a) ~ (c), and that of the stiffener to be tensile uniformly distributing in its cross section. For all the distribution types, the magnitude of the maximum compressive residual stress σ_r in the local plate panel and that of the uniform tensile residual stress σ_{rs} in the stiffener are restricted to $0.4\,\sigma_Y$ and $0.2\,\sigma_Y$, respectively. Moreover, the magnitude of initial deflection of the stiffened plates are assumed to be $a/1\,000$ and $b/300$ with its global and local modes, respectively. The values are prescribed on the basis of the tolerances allowed by the JRA Specifications for Highway Bridges, which "a" denotes the half-wave length of the stiffened plate for the global buckling and "$b/2$" indicates the width of the loaded edge of the plate panel.

For given parameters such as N, δ, γ and E/σ_Y, all the bifurcation sets or the ultimate strength curves can be calculated under the following condition : the "equivalent" orthotropic plates have such aspect ratio ϕ to take the least buckling strengths in the elasto-plastic ranges. That is,

$$n=1 \text{ and } \phi=m\sqrt[4]{k_c[1+(N+1)\gamma]} \quad \cdots\cdots (25)$$

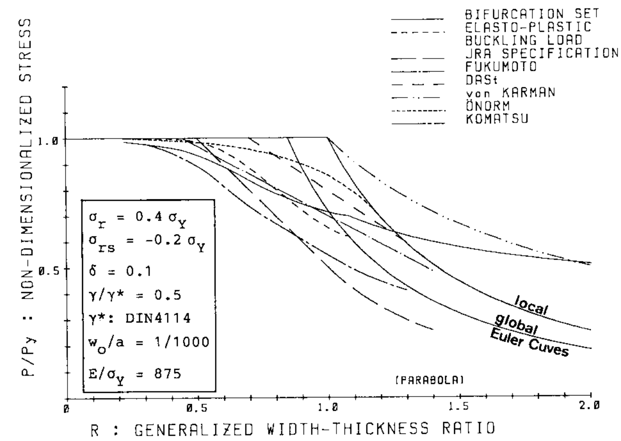
The ultimate strength curves in the elastic range for the slender stiffened plates are entirely the same, regardless of the residual stress types of distribution from definition of f in Eq. (9). In the elasto-plastic range for intermediate values of R, the ultimate strengths in the case of the trapezoidal distribution are shown to be the lowest, and those of the triangular distribution are the highest. However, the effect of the types of residual stress distributions on the strengths may be found to be insignificant quantitatively and qualitatively. Therefore, in this paper, the numerical results are presented with only the parabolic distribution for the types of global and local bucklings.

Fig. 4 (a) and (b) illustrate the proposed bifurcation sets for the global and the local bucklings of stiffened plates, respectively. In these figures, the generalized width-thickness ratio R of the local plate panel is chosen as the abscissa ; whereas the ordinate designates the non-dimensionalized ultimate strength in terms of the yielding stress σ_Y.

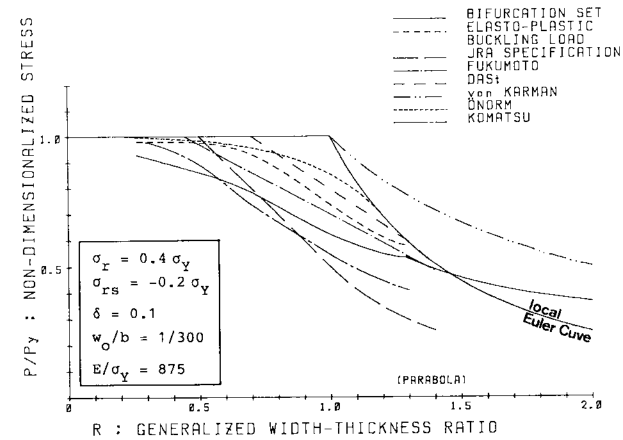
Here, the results of the strength prediction by the present analysis is shown together with those by von Kármán, Fukumoto's data-base approach[31] for high strength steel stiffened plates, Komatsu's large-deflection elasto-plastic finite element analysis[16], and furthermore by the practical design formula such as JRA, DASt and ÖNORM[30].

Especially, in the case of the latter local buckling, Fig. 4 (c) compares the same results with the numerical calculations by Crisfield, Little, Harding, Dawson, Horne and the JRA specifications for compressed plate panels[25].

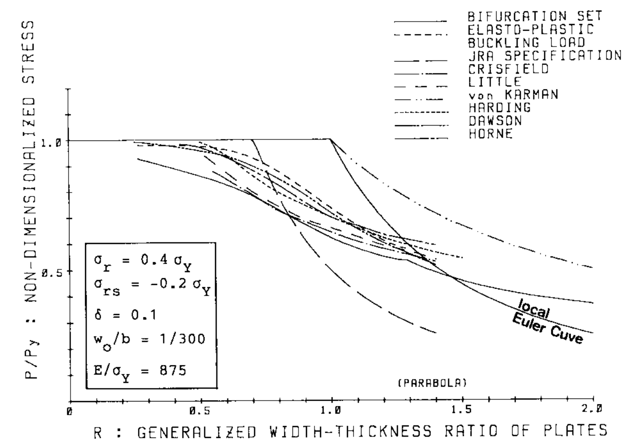
For both the global buckling of the



(a) global buckling



(b) local buckling compared with the results of stiffened plates



(c) local buckling compared with the results of local plate panels

Fig. 4 Ultimate strength curves of stiffened plates.

stiffened plate as a wide Euler column and the local buckling of the local plate, it may be found that the present imperfection sensitivity curve in Eq. (21) using Eqs. (22), (23), and (24) gives a unified strength prediction of stiffened plate. Moreover, this unified method of approach has been consistently applicable to columns, beams and compressed unstiffened plates as well[27].

In this paper, all the calculations on the ultimate strength of stiffened plates are made for only tipical values of parameters; N, δ, γ and E/σ_Y as shown in these figures. For various values of such parameters, the similar strength prediction may be explicitly determined by Eq. (21) through Eqs. (22), (23) and (24).

4. CONCLUSIONS

A simple unified approach to the ultimate strength of compressed steel stiffened plates is presented in the light of the catastrophe theory. The main conclusions are summarized as follows:

(1) The inelastic strength prediction of the stiffened plates may be explicitly determined in the form of the bifurcation sets or the imperfection sesitivity curves characterized by the 1/2-power rule for both global and local bucklings.

(2) The bifurcation sets can be explicitly defined near the "equivalent bifurcation point" being the intersection point of the elasto-plastic postbuckling path with the plastic mechanism curve.

(3) The initial imperfections are modified and replaced by the "equivalent imperfections" proposed herein.

(4) All the calculations herein can be made using only a microcomputer with small memory storage.

(5) The general philosophy adopted in this paper may also be applicable to other type of engineering structures such as arches, trusses and shells as well as columns, beams and compressed plate panels.

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