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SHEAR STRENGTH OF REINFORCED CONCRETE CIRCULAR SLABS

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Few studies on the shear strength of reinforced concrete circular slabs subjected to distributed loads have been reported.

To evaluate the shear strength of such slabs, tests were conducted using test specimens in which parameters such as reinforcement ratio and diameter-to-depth ratio were varied. From the results, an evaluation method of shear strength was obtained and expressed in terms of moment-to-shear ratio, $M/V \cdot d$, and radial and circumferential reinforcement ratios. Furthermore, the effect of shear reinforcement in the slabs was also investigated.

1. INTRODUCTION

Recently, various cylindrical concrete structures, including marine structures, have been built, and it has often been necessary to design circular slabs capable of carrying a uniformly distributed load.

As the use of this type of construction spreads, the need to design circular slabs rationally has become more urgent. One of the main concerns among designers is how to determine reasonably the strength of slabs, especially shear strength under uniformly distributed loads.

In reinforced concrete structures, beam strengths[1),2)] and the punching shear of slabs[3)~5)] under concentrated loads have already been studied, and various shear strength equations have been developed.

It has been reported[1),2)] that the factors which determine the shear strength of a beam subjected to a concentrated load are : strength of concrete, reinforcement ratio, shear span-to-depth ratio, and effective depth. Where a beam is subjected to a distributed load, the span-to-depth ratio may be regarded as an additional factor.

Furthermore, in slabs which fail in punching shear, the ratio of the peripheral length of the load to the effective depth is also regarded as a factor.

Consequently, the shear strength of a circular slab under a distributed load is the result of several factors.

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However, few studies on circular slabs carrying uniformly distributed loads have been reported. Given these circumstances, tests were performed to determine the shear strength of circular slabs and were based on those factors found in the various equations for the strength of beams and slabs. The following four items were investigated.

a. Effect of shear span on the shear strength of the circular slabs.
b. Effect of radial and circumferential reinforcement ratios.
c. Effect of diameter-to-depth ratio, which corresponds to span-to-depth ratio in a beam.
d. Effect of shear reinforcement.

In order to investigate these four items, three series of shear strength tests were performed on the following test specimens.

a. Circular slabs without shear reinforcement under an annular concentrated load.
b. Circular slabs without shear reinforcement under a uniformly distributed load.
c. Circular slabs with shear reinforcement under a uniformly distributed load.

2. TEST ON CIRCULAR SLABS UNDER AN ANNULAR CONCENTRATED LOAD

(1) Test Objective

The test was performed to examine the effect of the following on the shear strength of the circular slabs (Table 1)[6].

a. Load diameter-to-depth ratio (c/d)
b. Shear span-to-depth ratio (a/d)
c. Moment-to-shear ratio $(M/V \cdot d)$

(2) Test Specimens

The diameter of all test specimens was 120 cm, while the diameter of the support (l) was 95 cm. The effective depth (d) was set at three levels : 9.5 cm (Type Ⅰ), 6.3 cm (Type Ⅱ), and 4.0 cm (Type Ⅲ). An example of the shape, dimensions, and reinforcing bar arrangement of the specimens is shown in Fig. 1, and the specifications of all 24 specimens are provided in Table 2.

The circumferential reinforcement ratio (p_θ) was fixed, while that of the radial reinforcement (p_r) was set at four levels as shown in Fig. 2.

In this test, D 6 reinforcing bars with a yield strength of 353 MPa were used. The maximum size of aggregate was 10 mm and the required average strength f'_c was 23.5 MPa. The mix proportions of the concrete are shown in Table 3 (Case Ⅰ).

(3) Test Procedure

The diameter of the annular concentrated load varied from 9.8 to 82.4 cm as shown in Table 2. The load was applied upwards to

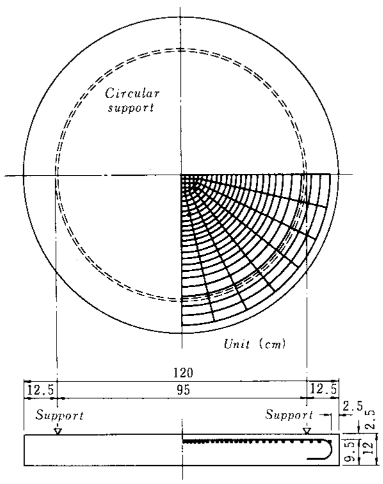


Fig. 1 Example of a circular slab test specimen.

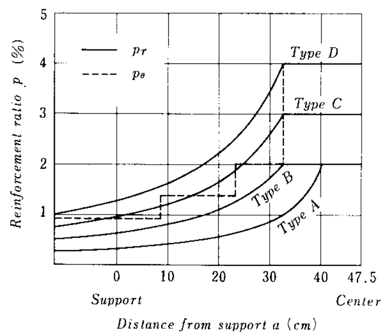


Fig. 2 Reinforcement ratio of circular slab test specimens.

Table 1 Relationship between the load point and $M/V \cdot d$.

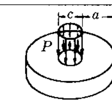
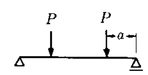
	Circular slab	Beam
Load point	 d; Effective depth	 d ; Effective depth
$M/V \cdot d$	$M=\frac{P\left(a+\frac{c}{2}\right)}{4}\beta[(1-\nu)(1-\beta^2) -2(1+\nu)\ln\beta]$ $V=P\left[\nu=1/6, \beta=\frac{c}{2a+c}\right]$	$\left.\begin{matrix}M=Pa\\V=P\end{matrix}\right\} M/V \cdot d=\frac{a}{d}$

Table 2 Specifications for test specimens and test results.

No	Specimen	f_c' (MPa)	d (cm)	a (cm)	a / d	c (cm)	c / d	M/Vd	Vtest (N)	τ_u (MPa)	Reinforcement ratio p_r (%)	Reinforcement ratio p_θ (%)	Vcal ① (N)	Vtest / Vcal ①	Vcal ② (N)	Vtest / Vcal ②
1	SIA 1	24.0	9.5	9.5	1.00	76.0	8.0	0.82	5.403	2.12	0.31	1.5	4.923	1.10	4.918	1.10
2	SIA 2	24.6	9.5	12.4	1.31	70.2	7.4	1.00	4.859	2.04	0.33	1.5	4.668	1.04	4.096	1.19
3	SIA 3	24.5	9.5	16.5	1.74	62.0	6.5	1.20	3.677	1.72	0.37	1.5	4.236	0.87	3.338	1.10
4	SIA 4	24.3	9.5	27.1	2.85	40.8	4.3	1.42	2.721	1.81	0.50	2.0	3.334	0.82	2.269	1.20
5	SIA 5	24.6	9.5	40.1	4.22	14.8	1.56	1.00	1.520	2.10	0.88	2.0	1.981	0.77	1.573	0.97
6	SIA 6	24.6	9.5	42.6	4.48	9.8	1.03	0.79	1.476	2.56	1.03	2.0	1.785	0.83	1.535	0.96
7	SIB 1	23.3	9.5	9.5	1.00	76.0	8.0	0.82	6.816	2.67	0.60	1.5	5.090	1.34	5.650	1.21
8	SIB 2	24.5	9.5	12.4	1.31	70.2	7.4	1.00	5.492	2.31	0.64	1.5	4.864	1.13	4.757	1.15
9	SIB 3	24.6	9.5	16.5	1.74	62.0	6.5	1.20	4.217	1.98	0.71	1.5	4.442	0.95	3.903	1.08
10	SIC 1	23.5	9.5	9.5	1.00	76.0	8.0	0.82	8.581	3.35	1.01	1.5	5.325	1.61	6.498	1.32
11	SIC 2	24.6	9.5	12.4	1.31	70.2	7.4	1.00	6.129	2.58	1.07	1.5	5.139	1.19	5.455	1.12
12	SIC 4	24.1	9.5	27.1	2.85	40.8	4.3	1.42	2.859	1.90	1.60	2.0	3.717	0.77	3.105	0.92
13	SID 1	24.0	9.5	9.5	1.00	76.0	8.0	0.82	9.821	3.85	1.25	1.5	5.551	1.77	6.939	1.42
14	SID 2	24.4	9.5	12.4	1.31	70.2	7.4	1.00	6.659	2.80	1.33	1.5	5.276	1.26	5.794	1.15
15	SID 3	24.5	9.5	16.5	1.74	62.0	6.5	1.20	5.266	2.47	1.47	1.5	4.844	1.09	4.774	1.10
16	SID 4	24.2	9.5	27.1	2.85	40.8	4.3	1.42	3.040	2.03	1.99	2.0	3.844	0.79	3.328	0.91
17	SⅡA 1	23.3	6.3	6.3	1.00	82.4	13.08	0.88	4.889	2.79	0.34	1.0	3.403	1.44	3.699	1.32
18	SⅡA 2	23.5	6.3	9.1	1.44	76.8	12.19	1.20	3.383	2.06	0.36	1.5	3.579	0.95	2.885	1.17
19	SⅡA 3	22.3	6.3	14.9	2.37	65.2	10.34	1.71	2.108	1.49	0.41	1.5	2.962	0.71	2.046	1.03
20	SⅡA 4	23.6	6.3	27.1	4.30	40.8	6.48	2.15	1.442	1.55	0.60	2.0	1.922	0.75	1.327	1.09
21	SⅢA 1	22.0	4.0	4.0	1.00	87.0	21.75	0.92	3.324	2.91	0.32	1.0	2.412	1.38	2.612	1.27
22	SⅢA 2	23.6	4.0	8.0	2.00	79.0	19.75	1.70	1.662	1.59	0.35	1.5	2.432	0.68	1.720	0.97
23	SⅢA 3	21.7	4.0	13.4	3.35	68.2	17.05	2.51	1.142	1.26	0.40	1.5	2.079	0.55	1.234	0.93
24	SⅢA 4	23.1	4.0	27.1	6.78	40.8	10.20	3.38	0.628	1.12	0.62	2.0	1.442	0.44	0.755	0.83

Setting point of the parameters

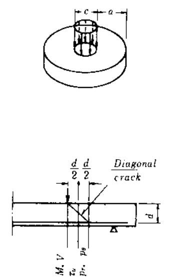


Table 3 Mix proportion of concrete.

Case	Maximum aggregate size (mm)	Slump (cm)	air (%)	W/C (%)	S/a (%)	Unit amount (N/m³) W	C	S	G	Admixture No. 70
I	10	15	4	61.5	50	1726	2815	8973	9091	7.0
II	10	15	4	57	51	1902	3344	8895	8238	2.5
III	5	15	3	72	—	2687	3727	14906	—	9.3

the specimen with a hydraulic ram (with a capacity of 1 960 kN) installed beneath the specimen. The periphery of the specimen was simply supported using Teflon seats coated with silicon oil to minimize horizontal restraint.

(4) Test Results and Discussion

a) Failure mode and shear strength

Radial cracking was observed first, followed by circumferential cracking. After the tests, the specimens were cut, and the cracking patterns of the sections were observed. The diagonal crack (A), bending crack (B) connecting to crack (A), and crack (C) along the reinforcing bar were found (Fig. 3), and the conical punching shear failure mode was observed.

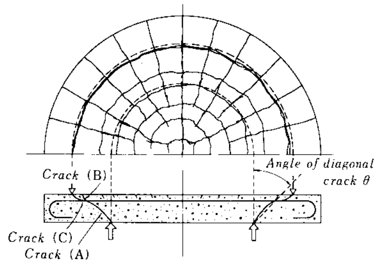


Fig. 3 Crack pattern.

When investigating the role of the various factors in shear strength, nominal shear strength τ_u is often used, and this varies according to which part of the test specimen is examined. The section examined may be at the load point itself (as suggested by Moe) or at a point $d/2$ outside the load point (ACI 318-83 Building Code)[4),7)].

In this test, since the diagonal cracks on all the specimens were at about 45° to the surface, sections at $d/2$ outside the load point were examined. The positions used for the calculation of the other parameters (reinforcement ratios, $M/V \cdot d$) are shown in the figure attached to Table 2.

b) Effect of load diameter-to-depth ratio (c/d)

It has been reported[3)-5)] that the punching shear strength of a slab decreases as the ratio of the peripheral length of the load to the effective depth of the slab increases. For the purposes of analysis, load diameter c was used instead of peripheral length, and the relationship between c/d and shear strength was examined as shown in Fig. 4. It was found that shear strength decreased slightly as c/d increased when $a/d>2$. However, when $a/d\leqq 2$, shear strength increased as the position of the section examined approached the support. The relationship between τ_u and c/d depended on d or l/d as shown in Fig. 4.

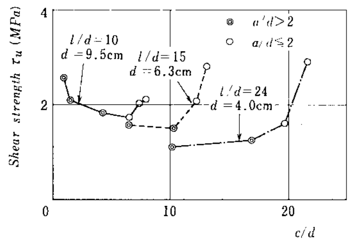


Fig. 4 Relationship between τ_u and c/d.

It is therefore inappropriate to express shear strength τ_u in terms of c/d alone.

c) Effect of shear span-to-depth ratio (a/d)

It is widely known that the shear strength of a beam is approximately inversely proportional to the shear span-to-depth ratio. In the test, when $a/d\leqq 2$, the shear strength τ_u of the circular slabs, as in beams, decreased as a/d increased, as shown in Fig. 5. On the other hand, when $a/d>2$, the shear strength depended on the effective depth d or the diameter-to-depth ratio l/d. In particular, the shear strength of a slab specimen with $d=9.5$ cm increased in the range where $a/d>2$, while that of a beam with the same effective depth was generally thought to decrease. It seems therefore to be difficult to determine the shear strength of a circular slab using a/d, when the load is applied at the center portion of the slab.

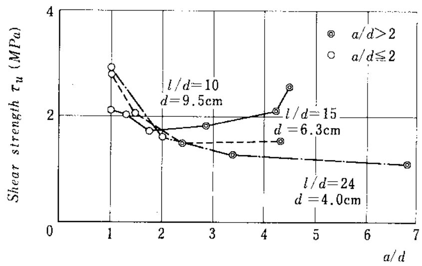


Fig. 5 Relationship between τ_u and a/d.

d) Effect of moment-to-shear ratio $(M/V\cdot d)$

The authors have put forward the hypothesis that a/d in a beam can be substituted by moment-to-shear ratio $(M/V\cdot d)$ in a circular slab, where M and V values were calculated using the equations shown in Table 1. The equations used are obtained from the theory of elasticity for thin circular plate.

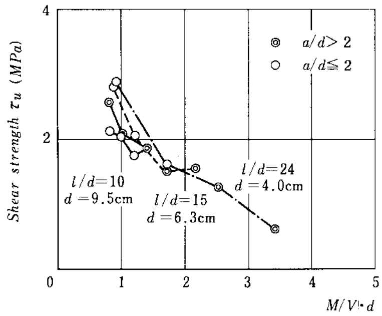


Fig. 6 Relationship between τ_u and $M/V\cdot d$.

To confirm the above hypothesis, shear strengths τ_u of the specimens obtained in the test were plotted against $M/V\cdot d$ in Fig. 6. Shear strength varied only slightly even when a/d, c/d, or l/d were changed, provided $M/V\cdot d$ was constant.

Therefore $M/V\cdot d$ is considered to be an important factor in determining the shear strength of a circular slab.

e) Application of an equation for punching shear strength

"Commentary Equation 7.7.2" (Eq. (1)) described in JSCE's "Limit State Design for Concrete Structures (Proposal)"[8)] was used as an equation for punching shear strength of slabs under concentrated loads. The correspondence of this equation to the results of the test was examined as shown in Table 2.

$$V_{cp}=0.425(0.85+0.4\,d/r)(1+\beta_p+\beta_d)f'^{\frac{1}{3}}_c u_p d \quad\cdots\cdots (1)$$

Where,

$\beta_p=\sqrt{p_w}-1\leqq 0.73$

(p_w : Average tensile reinforcement ratio in two directions, %)

$\beta_d=\sqrt[4]{100/d}-1\geqq 0$

(d : Effective depth, cm)

u_p : Effective peripheral length, cm

r : Side length of the square load, cm

f'_c : Compressive strength of concrete (MPa)

When applying the equation, the load diameter c was substituted for r.

Fig. 7 shows the ratio of shear strength V_{test} of the circular slabs (obtained from the test) to $V_{cal①}$ obtained using Eq. (1). In the figure the ratio is compared with c/d.

The values of $V_{test}/V_{cal①}$ are widely scattered for both $a/d>2$ and $a/d \leqq 2$, which means that Eq. (1) is not applicable for the calculation of punching shear strength.

f) Examination of flexural shear strength

In the same JSCE reference, the equation (Eq. (2)) for the shear strength of a beam ("Commentary Equation 7.1.1")[8] is also described. The results of the test were examined with this equation by using $M/V \cdot d$ instead of a/d. It was assumed that some effects of circular configuration was taken into account by using $M/V \cdot d$ of the circular slab.

$$V_u=0.20 f_c'^{\frac{1}{3}}(0.75+1.40\ V \cdot d/M)(1+\beta_p+\beta_d) b_w \cdot d \cdots (2)$$

Where,

b_w : Width of beam (peripheral length of examined section of a circular slab)

β_p : Obtained from average tensile reinforcement ratio in two directions as in Eq. (1).

As shown in Fig. 8, the average value of the ratios of shear strength V_{test} of the circular slabs to $V_{cal②}$ obtained using Eq. (2) is 1.10 and the coefficient of variation is 13.0 %. This means that Eq. (2) shows good agreement with the values obtained experimentally.

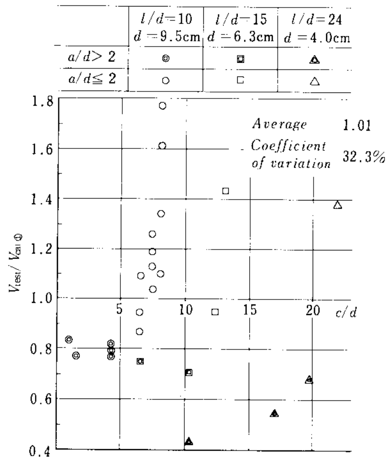


Fig. 7 Experimental value vs calculated value (Eq. (1)).

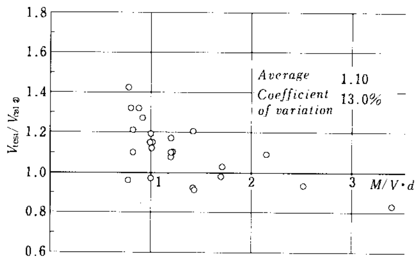


Fig. 8 Experimental value vs calculated value (Eq. (2)).

As described above, if an annular concentrated load is applied, Eq. (2) is effective for predicting the shear strength of a circular slab.

3. TEST ON CIRCULAR SLABS UNDER A UNIFORMLY DISTRIBUTED LOAD

(1) Test Objective

To determine the shear strength of circular slabs under a uniformly distributed load, the test was performed with regard to the following items[9),10)].

a. Effect of radial and circumferential reinforcement ratios (p_r and p_θ).

b. Effect of diameter-to-depth ratio (l/d).

(2) Test Specimens

In the test, two types of test specimens were used. The diameter of one was 120 cm, while that of its peripheral support was 95 cm. The diameter of the other was 240 cm, that of its support 200 cm.

The parameters used for the 120 cm diameter specimens were : a) effective depth d, 6.3–18.2 cm ; b) radial reinforcement ratio p_r, 0.36–2.0 % (at 1 d from support) ; and c) circumferential reinforcement ratio p_θ, 0.59–3.05 % (at support).

The parameters used for the 240 cm diameter specimens were : a) effective depth d, 20 cm (constant) ; and b) reinforcement ratios p_r, 0.6–1.9 % ; and p_θ, 1.1–3.2 %.

The basic test specimen, Specimen A 0, had a diameter of 120 cm and an effective depth of 9.5 cm. The radial reinforcement ratio p_r was about 0.39 % (at 1 d from support), while the circumferential p_θ was

Table 4 Conditions and test results.

Test on circular slabs under a uniformly distributed load

No	Specimen	f_c' (MPa)	Reinforcement ratio (%) p_r 1d from support	Reinforcement ratio (%) p_θ On support	Reinforcement ratio (%) p_θ 1d from support	Reinforcement ratio (%) Center	d (cm)	l/d	M/Vd	$\frac{M_{max}}{V_{max}d}$	q_{test} (MPa)	τ_u (MPa)	Failured section (from support)	Comparison of experiment and Calculated value q_{cal1} (MPa)	$\frac{q_{test}}{q_{cal1}}$	q_{cal2} (MPa)	$\frac{q_{test}}{q_{cal2}}$	Yield condition ○:Yield, ×:Not yield Radial 1d from support	Circumferential 1d from support
1	A 02	23.5	0.39	0.96	1.42	2.1	9.5	10	2.11	1.98	0.721	1.262	1.5d	0.66	1.10	0.66	1.10	○	○
2	A 03	23.9	〃	〃	〃	〃	〃	〃	〃	〃	0.696	1.218	〃	〃	1.06	〃	1.05	○	○
3	A 1	23.5	〃	1.48	2.09	〃	〃	〃	〃	〃	0.755	1.321	〃	〃	1.15	0.71	1.07	○	×
4	A 3	23.5	〃	2.48	3.73	〃	〃	〃	〃	〃	0.824	1.442	〃	〃	1.25	0.81	1.01	○	×
5	A 4	23.5	〃	3.04	4.47	〃	〃	〃	〃	〃	0.873	1.528	〃	〃	1.33	0.87	1.00	○	×
6	5 A	24.6	0.51	1.14	1.64	〃	18.2	5	1.54	1.03	2.599	2.352	0.8d	1.21	2.15	1.23	2.12		−
7	12 A	24.2	0.38	1.10	1.10	〃	8.0	12	2.25	2.35	0.565	1.197	1.7d	0.57	0.99	0.57	0.99		−
8	15 A	24.4	0.39	1.11	1.11	〃	6.3	15	2.47	2.99	0.392	1.086	2.0d	0.45	0.87	0.46	0.85	−	−
9	B 1	23.5	0.75	1.52	2.00	〃	9.5	10	2.11	1.98	0.922	1.614	1.5d	0.77	1.19	0.88	1.04	○	○
10	B 2	23.5	〃	2.05	3.00	〃	〃	〃	〃	〃	0.922	1.614	〃	〃	〃	0.93	0.99	○	×
11	5 B	24.7	1.05	1.15	1.66	2.0	18.2	5	1.54	03	3.138	2.840	0.8d	1.48	2.12	1.63	1.93	−	−
12	12 B	24.0	0.72	1.13	1.13	〃	8.0	12	2.25	2.35	0.735	1.557	1.7d	0.65	1.14	0.71	1.04	−	−
13	15 B	24.3	0.70	1.15	1.15	〃	6.3	15	2.47	2.99	0.527	1.460	2.0d	0.51	1.03	0.56	0.94	−	−
14	C 1	20.4	1.13	1.52	2.05	2.4	9.5	10	2.11	1.98	0.971	1.699	1.5d	0.82	1.18	1.01	0.96	○	×
15	E 0	23.5	2.00	1.00	1.37	5.3	〃	〃	〃	〃	1.402	2.454	〃	1.03	1.36	1.40	1.00	○	○
16	E 4	23.5	〃	3.05	4.39	〃	18.2	〃	〃	〃	1.716	3.003	〃	〃	1.67	1.62	1.06	○	×
17	UD-8 0.5-1.0	25.2	0.63	1.08	1.08	2.0	20.0	8	1.77	1.58	1.461	2.082	1.15d	0.76	1.84	0.82	1.77	○	○
18	UD-8 1.0-2.0	23.1	1.26	2.16	2.16	4.0	〃	〃	〃	〃	1.569	2.236	〃	0.96	1.63	1.28	1.22	○	×
19	UD-8 1.0-3.0	21.6	〃	3.24	3.24	〃	〃	〃	〃	〃	1.765	2.515	〃	0.94	1.88	1.39	1.27	○	×
20	UD-8 1.5-3.0	18.9	1.90	3.24	3.24	6.0	〃	〃	〃	〃	1.844	2.628	〃	1.04	1.77	1.67	1.11	×	×
21	C-2 1	25.3	0.36	0.59	0.59	1.9	10.6	9	1.90	1.77	0.716	1.139	1.3d	0.74	0.97	0.68	1.09	○	○

Test on the effect of shear reinforcement

No	Specimen	f_c' (MPa)	p_r 1d from support	p_θ On support	p_θ 1d from support	Center	d (cm)	l/d	M/Vd	$\frac{M_{max}}{V_{max}d}$	q_{test} (MPa)	τ_u (MPa)	Failured section (from support)	Shear reinforcement ratio (at 1.5d from support) (%)	Reinforcing range	Radial 1d from support	Circumferential 1d from support
22	SG2D3	24.3	0.39	0.96	1.42	2.1	9.5	10	2.11	1.98	1.344	2.352	1.5d	0.65	support ~ 3.5d	○	○
23	SG2D4	25.1	〃	〃	〃	〃	〃	〃	〃	〃	1.294	2.265	〃	0.4	support ~ 3.4d	○	○
24	SG2D5	24.9	〃	〃	〃	〃	〃	〃	〃〃	〃	1.285	2.249	〃	0.2	support ~ 2.6d	○	○
25	SG2D6	24.8	〃	〃	〃	〃	〃	〃	〃	〃	1.069	1.871	〃	0.1	support ~ 2.7d	○	○

0.96 % (at support). In this test 21 specimens were used, and the parameters are summarized in Table 4.

Reinforcement consisted of four types of deformed bars, i.e., D 6, D 10, D 13, and D 16. Their yield strengths f_{sy} were 350–370 MPa. The mix proportion of the concrete and mortar used in the test is shown in Table 3 (Cases Ⅱ and Ⅲ for the 120 cm and the 240 cm specimens, respectively).

(3) Test Procedure

The test specimens were supported in the same way as in the annular concentrated load test, and a uniformly distributed load was applied using a hydraulic rubber bag. Reinforcing bar strains and surface strains of the concrete on the compression side were measured with wire strain gages.

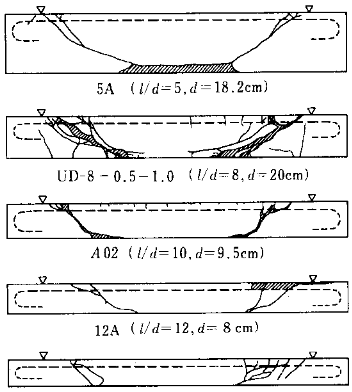


Fig. 9 Crack pattern.

(4) Test Results and Discussion

a) Crack patterns and reinforcing bar strain

After the test, the specimens were cut into two parts to observe crack patterns. These are shown in Fig. 9. All of the specimens had cracked diagonally at a point around $l/4$ from the support. In Specimen 5 A, with an l/d of 5, a clear arch appeared. In one specimen with an l/d of 8, an arch was observed, while the other three specimens with the same l/d had sheared conically before arching occurred. Specimens with an l/d larger than 10 sheared conically.

From these results, it seems that arches appear when l/d is less than 8.

For those specimens with an l/d of 10, the strains of the radial reinforcing bars near the diagonal crack (at 1 d from support) had reached the yield strain, as shown in Table 3, when the maximum load was applied. However, it can be seen that for some specimens the circumferential reinforcing bars did not yield.

b) Critical section and reinforcement ratios

The applicability of Eq. (2) was examined first.

Since the effects of the reinforcement ratios in the two directions were not clear in the case of the circular slabs under a uniformly distributed load, only the radial reinforcement ratio p_r (instead of p_w) was used, and the location of the critical section was determined by the following procedure.

If it is assumed that the shear strength ($V_u/b_w d$) of a section can be calculated by using $M/V \cdot d$ and p_r in Eq. (2), the shear strength curve shown in Fig. 10 can be obtained by considering resultant forces M and V produced when the uniformly distributed load is applied. Here, M and V values were calculated using the equations obtained from the theory of elasticity for thin circular plate, as in the case of the circular slabs under annular concentrated load.

When the shearing stress caused by the uniformly distributed load q (0.68 MPa) contacts at Point A shown in Fig. 10, it is assumed that Point A corresponds to the critical section.

The location of the critical section, Section A, determined in this way, agrees closely with that obtained experimentally as shown in Fig. 10. Considering the above and the presence of diagonal cracks at 45°, it is possible that the radial reinforcement ratio to be used in the calculation is the value at the position $d/2$ outside Point A shown in Fig. 10.

c) Effect of reinforcement ratios (p_r and p_θ)

Using reinforcement ratios p_r and p_θ determined by the above method, the relationship between the ultimate load q_{test} and p_r and p_θ of the specimen with an l/d of 10 was obtained. It was proved that the ultimate load q_{test} was linearly related to the reinforcement ratios p_r and p_θ, and that the effect of p_r was about four times that of p_θ within the range used in this test (Fig. 11).

Since the effect of reinforcement ratios differs from β_p in Eq. (2), the following linear equation is used as β_p according to the results of the test.

$$\beta_p = p_r + \frac{p_\theta}{4.35} - 1 \quad \cdots\cdots (3)$$

The equation in which $\beta_p = \sqrt{p_w} - 1$ (see Eq. (2)) is to be replaced by Eq. (3) given that p_r and p_θ express the shear strength of the circular slab under a uniformly distributed load.

To examine the validity of this equation, the values of q_{test}/q_{cal2} were obtained for 10 specimens with an l/d of 10. As shown in Fig. 12, the average was 1.03 while the coefficient of variation was 4.2 %. The calculated values well approximated the experimental values.

d) Effect of diameter-to-depth ratio (l/d)

The shear strength of a beam which carries a uniformly distributed load is affected by the span-to-depth ratio (l/d). It has been reported[2] that for values of l/d below 12 shear

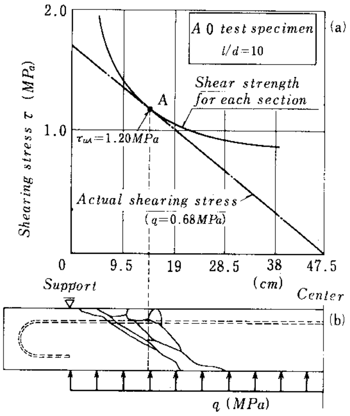


Fig. 10 Shear strength curve and actual shearing stress.

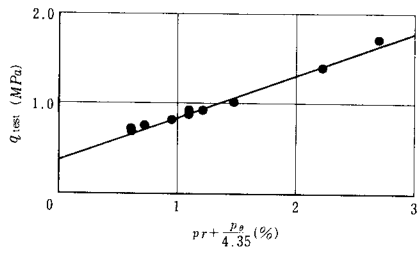


Fig. 11 Relationship between the maximum load and the converted reinforcement ratio.

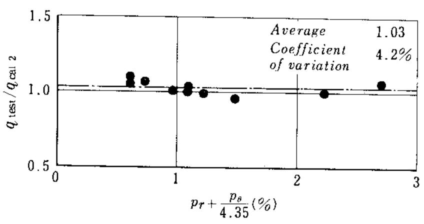


Fig. 12 Comparison of measured and calculated loads.

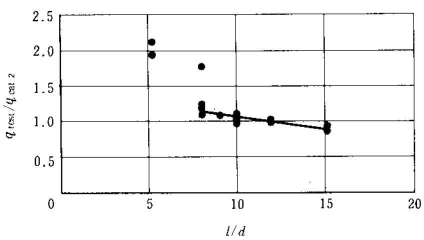


Fig. 13 Effect of diameter-to-depth ratio.

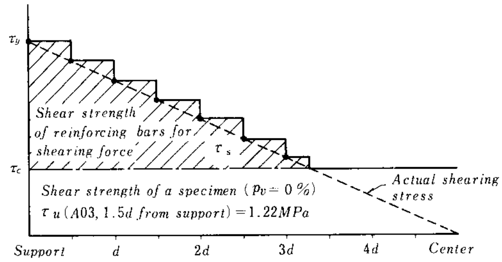


Fig. 14 Arrangement of shear reinforcement.

strength rapidly increases as l/d decreases ; this is thought to result from the arch action of the concrete.

To confirm the effects of l/d in a circular slab, values of q_{test}/q_{cal2} for specimens with l/d of 5–15 were calculated using the method stated in Sec. 3. (4). c).

As shown in Fig. 13, when $l/d \geqq 8$ and the specimen had sheared conically, the values of q_{test}/q_{cal2} were between 0.85 and 1.27. The relationship between q_{test}/q_{cal2} and l/d can be regarded as a linear one. However, when $l/d \leqq 8$ and arching occurred, the values of q_{test}/q_{cal2} were between 1.77 and 2.12, and actual strength increased rapidly.

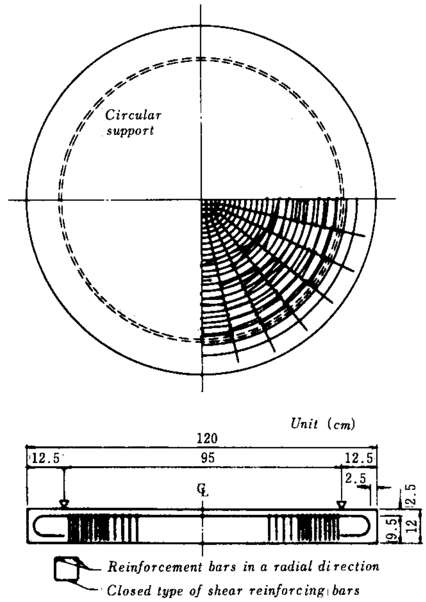


Fig. 15 Arrangement of reinforcing bars in Specimen SG 2 D 5 (p_v=0.2 %).

4. TEST ON THE EFFECT OF SHEAR REINFORCEMENT

(1) Test Objective

The test was performed to examine the effect of shear reinforcement.

(2) Test Specimens

Specimens with the same specification as Specimen A 0 utilized in the test for the uniformly distributed load were used. The shear reinforcement was arranged as shown in Fig. 14 according to "Equation 7. 3. 1" described in the "Limit State Design for Concrete Structures (Proposal)"[8]. Eq. (4) shows the equation for the calculation of shear strength with shear reinforcement.

$$\tau_y = \tau_c + \frac{A_w f_{wd} z (\sin\alpha + \cos\alpha)}{b_w d \cdot s} \quad (4)$$

Where,

A_w : Sectional area of shear reinforcing bars

f_{wd} : Yield strength of shear reinforcing bars

z : $d/1.15$

α : Angle between shear reinforcing bars and the axis of member

s : Spacing of shear reinforcing bars

b_w : Width of member (periferal length at the section examined)

The test was performed with the following conditions :

a. The shear stress resisted by concrete τ_c was assumed to be the same as the shear strength of Specimen A 0, τ_u=1.22 MPa.

b. Four shear reinforcement ratios p_v at 1.5 d from the support were used (0.1, 0.2, 0.4 and 0.65 %), and the shear strength τ_s for each ratio was assumed to be "$\tau_y - \tau_c$".

For the shear reinforcement, D 3 reinforcing bars were used as shown in Fig. 15. Details of the specimens are shown in Table 4.

(3) Test Procedure

A uniformly distributed load was applied. The test procedure was the same as that used on the specimens without shear reinforcement.

(4) Test Results and Discussion

a) Crack patterns and deflection

Fig. 16 shows the crack patterns on the cut surfaces of the test specimens. When the shear reinforcement ratio p_v was more than 0.2 %, the circumferential cracking zone was wider than when p_v was 0.1 % (at 1.5 d from the support).

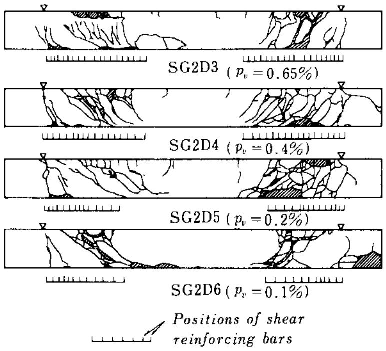


Fig. 16 Crack pattern.

Fig. 17 shows the relationship between load and deflection at the center of specimen. When shear reinforcement was not provided, the load decreased rapidly (shown by the broken line) after the "inflection point" was reached, while the deflection increased. On the other hand, when shear reinforcement was provided, the load still increased after the "inflection point" as deflection increased. From the test, it was confirmed that the use of shear reinforcement in a circular slab increased its ductility as it does in a beam.

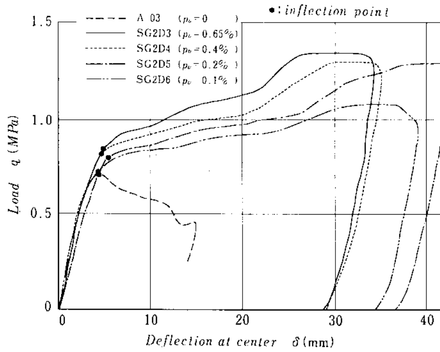


Fig. 17 Change in deflection at center.

It was also confirmed that the radial and circumferential reinforcing bars yielded at the above-mentioned "inflection point" whether or not shear reinforcement was used.

b) Effect of shear reinforcement ratio p_v

The relationship between the shear reinforcement ratio p_v at 1.5 d from the support and the ultimate shear strength of the slab is shown in Fig. 18. The straight line obtained using Eq. (4) is also shown. When $p_v \leqq 0.2$ %, the slope of the curve of strength (τ_y) vs shear reinforcement ratio (p_v) actually obtained in the test was larger than that calculated with Eq. (4). However, when $p_v > 0.2$ %, the actual ultimate strength seemed to have reached an upper limit; whereas the calculated strength continued to increase linearly.

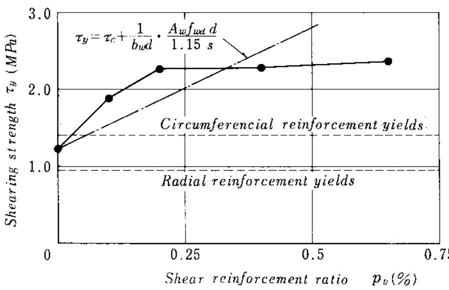


Fig. 18 Relationship between τ_y and p_v.

5. CONCLUSIONS

In order to provide a rational design for circular slabs, evaluation of shear strength is of great importance. In this paper, the various factors influencing the shear strength of the slabs have been studied, and the equations for shear strength examined experimentally.

The results obtained from this study are summarized below.

a. After examining the relationships between shear strength and the load diameter-to-depth ratio c/d, shear span-to-depth ratio a/d and moment-to-shear ratio $M/V \cdot d$, it was proved that the shear strength of a circular slab was most closely related to $M/V \cdot d$.

b. When a uniformly distributed load was applied, the effect of the circumferential reinforcement ratio p_θ on shear strength was about one-fourth that of the radial reinforcement ratio p_r.

c. The failure mode of the circular slabs under uniformly distributed load was dependent on the diameter-to-depth ratio (l/d). When $l/d \leqq 8$, arching occurred before the specimen failed. As a result, a rapid increase in shear strength was observed when $l/d \leqq 8$. When $l/d \geqq 8$, the specimens sheared conically. In this study, an evaluation method of the shear strength of the slabs where $8 \leqq l/d \leqq 15$ was investigated.

d. It was proved that the ductility of a circular slab with shear reinforcement increased as it does in a beam. It was also proved that "Equation 7.3.1" described in the "Limit State Design for Concrete Structures (Proposal)"[8] can be utilized for the evaluation of shear strength of circular slabs when the shear reinforcement ratio p_v at the critical section (at 1.5 d from the support) is below 0.2 %. When $p_v > 0.2$ %, the strengths of the slab specimens were almost constant.

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