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A STUDY OF RIGIDITY AND STRENGTH IN TORSION OF H-BEAM STIFFENED WITH TRANSVERSE STIFFENERS

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The torsional rigidity of H-beam with transverse stiffeners will increase due to its stiffeners, and the average angle of rotation per unit length will be smaller than that of H-beam without stiffener.

In this study, torsional experiments of H-beams with transverse stiffeners were performed. From these results, the torsional rigidity of the H-beam with stiffeners was evaluated as the imperfectly restricted warping torsion caused by transverse stiffeners, and the torsional strength was discussed.

1. INTRODUCTION

The behaviour and strength in torsion of stiffened H-beams have not sufficiently been investigated 1), though these of unstiffened beams have already been studied and can be predicted acculately.

Torsional rigidity of H or I-type beams is generally negleced in designing of bridges or frames, since it is relatively small. But torsional rigidity of these beams should be acculately evaluated in exact prediction of torsional response or lateral bucking load of the beam, and sometimes could not be neglected in the design of curved girders. In torsion of H-beam having transverse stiffeners, the stiffener is twisted by the warping of flange of the beam, reversely the warping of H-beam is restricted by the resistance of the stiffeners. Consequently, the torsional rigidity and strength in practice is greater than those of unstiffened H-beam itself. However, because this effect of warping restraint due to a transverse stiffener is not generally so large, the effect will be evaluated by adding in torsional rigidity of the beam.

In this study, torsional tests of stiffened H-beams are performed. And the estimation of rigidity and strength in torsion of stiffened H-beams is described by parameters which indicate the effect of warping restraint due to transverse stiffeners.

2. TORSIONAL TEST

Test beams are made of SS-41, 250×125×6−9 rolled H-beams, and transverse stiffeners are located at uniform intervals. Dimensions of test beams are shown in Table 1. The test No. 1 which has no stiffener presents St. Venant's pure torsion. For No. 2∼No. 5, the thickness of the stiffener t_s varies but the interval l is constant, and for No. 6∼No. 9, l varies but t_s is constant.

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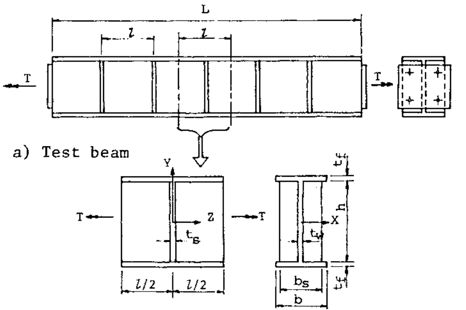

a) Test beam

b) Element with one transverse stiffener

Fig. 1 Test beam and element of analytical model.

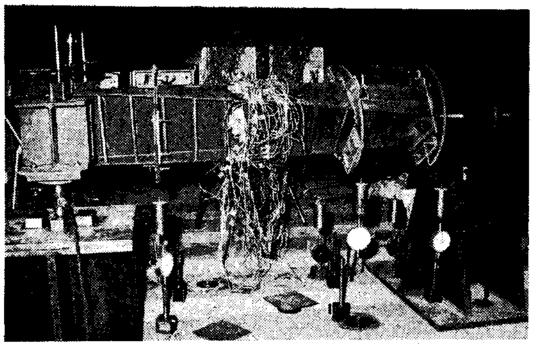

Fig. 2 Test arrangements.

Table1 Dimensions of test beam.

No.	L (cm)	l (cm)	b_s (cm)	t_s (cm)	k $\times 10^7$ (kg·cm^3)	κ	R
1	128.0						
2	128.0	16.0	12.5	0.5	2.53	0.135	28.91
3	128.0	16.0	12.5	0.8	7.08	0.135	10.33
4	128.0	16.0	12.5	1.0	11.67	0.135	6.26
5	128.0	16.0	12.5	1.3	22.02	0.135	3.32
6	127.2	5.3	12.5	1.0	15.62	0.045	4.68
7	127.2	10.6	12.5	1.0	12.91	0.089	5.66
8	127.2	21.2	12.5	1.0	10.99	0.179	6.65
9	127.2	31.8	12.5	1.0	10.19	0.268	7.17

Modulus of elasticity $E=2.00\times10^5$ MN/m^2 (2.04×10^6 kg/cm^2), Poisson's ratio $\nu=0.3$, and yielding stress $\sigma_y=277$ MN/m^2 (2830 kg/cm^2) are obtained by tensile tests of specimens and used for the analysis.

The beams are tested as St. Venant's pure torsion, subjected to a concentrated torque at the loading end. The loading end is construted of round rod which is supported by bearing, and circular plate which are wound with the wire. The round rod is bolted to the end plate of the test beam as shown in Fig. 1 a). The torque is induced by streching down the wire, and measured by load cell at the other end at which the rotation of the test beam is restrained by the H-beam bolted to the end plate of the test beam, as shown in Fig. 2.

The end plate of the beam is not welded to the flanges as shown inFig. 1 a), otherwise the warping of flange is restricted by the end plate and is not free at the end.

3. RESULTS OF TESTS AND THEORY

(1) Rate of twist

Fig. 3 and Fig. 4 show torqus vs. average rate of twist curves. These figures show that the larger t_s is or the smaller l is, the smaller the rate of twist appears. Since t_s of No. 2 is small, the rate of twist is nearly equal to that of No. 1, but the rate of twist of No. 4 is 50 % less than that of No. 1. And it is also noticed that the rate of twist of No. 1 increases as well as in elastic region when the magnitude of the torque is beyond the fully-plastic torque for pure torsion T_{pst}, which is immediately derived as 0.231 tm from "Nadai roof". This steady increase is caused by "large torsion" due to spiral tension of flange fiber, which is certified from the measured axial strains of the flange tip.

Moreover, it should be remarked that curves for stiffened beams become paralle to that of pure torsion when the torque reaches a certain value, for example, the inclination of the curve for No. 7 is similar to that for No. 1 when the torque is larger than $T=0.4$ tm.

(2) Evaluation of torsional rigidity for stiffened H-beam

Considering the effect of stiffener in H-beam torsion as the warping restraint of stiffener, the evaluation of torsional rigidity for stiffened H-beams will described herein.

The H-beam stiffened with transverse stiffeners are constructed of the beam elements shown in Fig. 1 b). The torsional rigidity for stiffened H-beam is derived from solving the differential equation for torsion about the element of the beam. Now, the boundary conditions of the beam element are given as following.

$$\theta=0 \text{ at } z=0, \cdots\cdots \quad (1\cdot\text{a})$$

$$EC_w\theta''=k\theta' \text{ at } z=0, \cdots\cdots \quad (1\cdot\text{b})$$

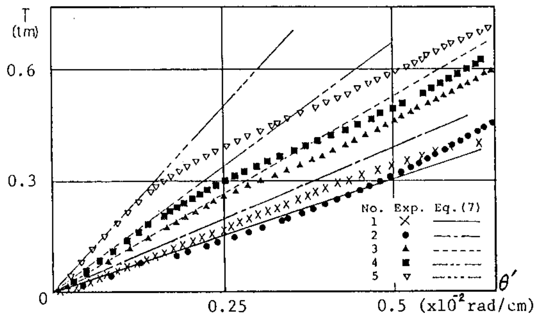

Fig. 3 Torque vs. average rate of twist (k=const.).

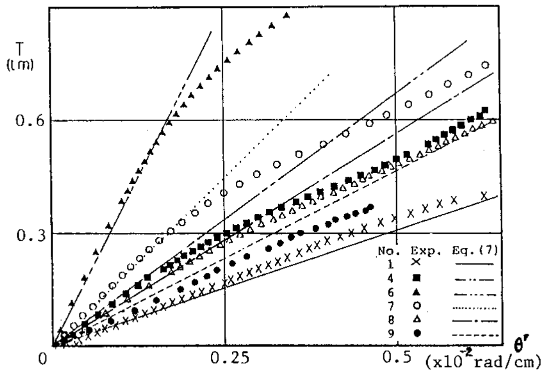

Fig. 4 Torque vs. average rate of twist (t_s=const.).

$\theta''=0$ at $z=l/2$ and $z=-l/2$, ………………………………………………… (1 · c)

where θ is rotation angle, $\theta'=d\theta/dz$, and k shows the spring constant due to a transverse stiffener between warping moment $EC_w\,\theta''$ and rate of twist θ'. Eq. (1 · c) is exactly sufficient for the H-beam in which the stiffener locates at $l/2$ from the end of H-beam in uniform torsion. However, it is checked by finite element analysis considered spring constant k, that eq. (1 · c) is approximately sufficient for usual H-beam having end stiffeners, though the warping moment adjacent to beam end, indeed, is not correct in practice.

Solving the following equation under the conditions (1 · a~c),

$-EC_w\,\theta'''+GJ\theta'=T,$ ………………………………………………… (2)

torsion angle between ends of the beam element leads to

$$\Delta\theta=\frac{Tl}{GJ}\left(1-\frac{2}{2\,\varkappa R+\varkappa\coth(\varkappa/2)}\right). \quad\cdots\cdots\cdots (3)$$

In eq. (3), parameter $\varkappa$ and R is

$\varkappa=l\sqrt{GJ/EC_w},$ ………………………………………………… (4)

$R=\sqrt{GJEC_w}/k.$ ………………………………………………… (5)

Parameter R shows the effect of warping restraint due to a stiffener, and perfect warping restraint is obtained when $R=0$.

The values of these parameters are shown in **Table1**.

From eq(3), the torsion angle between stiffened H-beam ends θ^* are approximately expressed as

$\theta^*=n\Delta\theta,\ (n=L/l).$ ……………………… (6)

Fig. 5 shows the relationship of the average rate of twist $GJ\Delta\theta/Tl$ and $1/R$ for the each value of $\varkappa$. And experimental value for $T=0.1$ tm are also shown in this figure. As shown in **Fig. 5**, the smaller $\varkappa$ or R is the smaller the rate of twist becomes.

Since parameter $\varkappa$ is usually less than 0.4, and eq. (3) will be led to the following expression because of $\varkappa\coth(\varkappa/2)\fallingdotseq 2$

$\Delta\theta/l=T/(GJ+k/l).$ ……………………… (7)

Eq. (7) shows that k/l increases in torsional rigidity due to warping restraint of a stiifener.

Spring constant k will be derived, in the following.

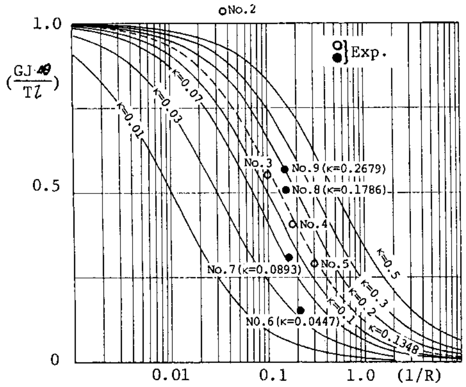

Fig. 5 Average rate of twist by restrained warping of torsion (where experimental values are when T =0.1 t · m).

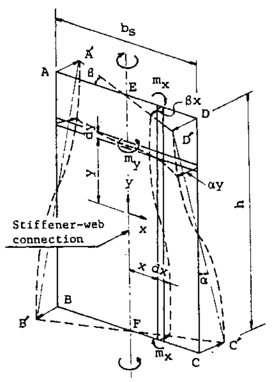

Fig. 6 Stiffener deformation by warping of H-beam.

Fig. 6 shows a vertical stiffener, in which $\overline{AD}$, $\overline{BC}$ or $\overline{EF}$ are connections between flange or web respectively, and which will deforms as broken line. Spring constant k will be able to expressed as,

$$k = G^* J^* h + \frac{1}{\theta'_{(z=0)}} \left(\int_{flg} m_x \cdot x dx + \int_{web} m_y \cdot y dy \right)$$
$$= \frac{E t_s^3}{12 b_s h} \left(\frac{2 b_s^2 h^2}{1+\nu} + \xi b_s^4 + \eta h^4 \right) \cdots\cdots\cdots\cdots\cdots\cdots (8)$$

in which $G^* J^* h$ is the quantity due to pure torsion of a stiffener, m_x, m_y in the second terms shows reactions of bending moment per dx or dy strip corresponding with torsional displacements $\beta \cdot x$, $\alpha \cdot y$ as shown in Fig. 6, and ξ, η show relative stiffeness between the stiffener and flange or web respectively. The values of k are also shown in Table—1. Curves based on eq. (7) and eq. (8) are shown in Fig. 3, Fig. 4. Thus eq. (7) gives excellent correlation with test results.

(3) A proposal for evaluation of strength in pure torsion

Since the steady increase of the torsion angle is obtained due to the effect of "large torsion", the collapse torque could not be obtained obviously. The strength in pure torsion without the effect of "large torsion" can be given as follows:

Generally, stiffeners will yield earler depending on the warping of the flange, the beam will be in the state of St. Venant's torsion beyond the torque in this state, because the effect of warping restraint of stiffeners can not be expected. Therefore the strength of stiffened H-beam can be given as

$$T_p = T_{pst} + M_{sfp} \cdot h / l \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (9)$$

where T_{pst} is fully-plastic torque of H-beam itself due to "Nadai roof", and M_{sfp} shows fully-plastic torque of a stiifener which neglects the effect of m_x or m_y in eq. (8) for simplicity. The second term in eq. (9) shows the torque for warping torsion.

If the yielding of flange occurs earlier, M_{sfp} is replaced by in-plane fully-plastic bending moment of the flange plate.

4. SUMMARY AND CONCLUSIONS

The torsional rigidity and strength of stiffened H-beam in torsion has been evaluated and formulated by regarding it as imperfect warping restraint torsion. Owing to transverse stiffeners, the rate of twist can be smaller as shown in Fig. 3, 4 or Fig. 5. This effect of warping restraint can be included in the torsional rigidity of H-beam, which is given by $GJ + k/l$. Spring constant k is obtained from eq. (8). The average rate of twist can be evaluated by using parameters of eq. (4) or (5). Finally the torsional strength of stiffened H-beams can be calculated by eq. (9).

REFERENCE

1) Heins, C. P. and Potocko, R. A. : Torsional stiffening of I-girder web, Proc. of ASCE, Vol. 105, No. ST 8, Aug., 1979.
2) Shimada, S. : Shearing strength of steel plates under torsion of box-girders, Proc. of JSCE, No. 114, Feb., 1965.
3) Fujii, K. and Ohmura, H. : Torsional tests of H-beams (Imperfect restraint of warping), Bridge and Foundation, Vol. 16, No. 11, Nov., 1982.

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