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3-D DYNAMIC ANALYSIS OF GROUND MOTION BY FEM WITH NON-REFLECTING BOUNDARY

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Dynamic Finite Element Method is frequently used in analyzing wave propagation problems. In the case of infinite or infinite half media, the presence of artificial boundaries introduces wave reflections from boundaries. Authors tried to solve the problem applying Smith-Cundall's method and extended the method to three dimensional problem. Smith-Cundall's method is to solve the problem by superposing two types of reflected wave from Dirichlet's and Neumann's boundaries. This method is theoretically complete. Authors made clear the weak point of the method, however, the influence of the weak point on the computed results is small, therefore, the results by this method is reliable. This method treats problems in time domain, so nonlinear problem such as liquefaction can be solved by this method in near future.

1. INTRODUCTION

Frequent applications of the Finite Difference Method and Dynamic Finite Element Method using numerical analysis are made to wave propagation problems. However, in many cases artificial boundaries are set when modeling analysis media. When ground motion propagation on infinite, or infinitehalf boundaries, is considered, the waves which should propagate to infinite distance are reflected by artificial boundaries and do not remain within an analysis model, which nullifies computed results. If there is a computer with sufficient capacity to allow setting of large numerical analysis, the influence of boundaries can be neglected. However, it cannot be applied to actual instances. In order to solve these problems, therefore, many boundary treatment methods have been proposed as follows :

(i) "Viscous Boundary" proposed by Lysmer and Kuhlemeyer is provided with a dashpot at the boundary to absorb incident wave energy at the boundary (1961) 1).

In the case of actual wave incidence, the cofficient of the dashpot becomes constant, but it cnnot be called theoretically perfect against incident angles because it is generally dependent on frequencies.

(ii) "Transmitting Boundary" proposed by Lysmer and Waas, makes, surface waves, propagating on elastic bodies on a rigid base, work on lateral boundaries by establishing a formula for the waves by FEM. 2)

The theory is perfect on assumption of modeling, and boundary conditions dependent on frequencies are provided. However, it is considered that assumption on a rigid base is not practical, and its improvement is being worked for.

(iii) "Superimposing Boundary" proposed by Smith, is a combination of the Neumann condition and

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Dirichlet condition, by which reflected waves of phase and antiphase are produced to eliminate reflected waves by taking the average of both waves 3) (1974).

This treatment method is excellent for incident angles at boundaries because of its non-dependence on frequencies, but its weak point is that reflected waves cannot be eliminated when the waves again reach the limit of the boundaries. The same treatment method as this has been proposed by Tamura and Nakamura 4) (1976).

(iv) The aforesaid weak point has been removed by Cundall and others by improving the boundary treatment method proposed by Smith 5) (1978).

The details of this method will be described in Chapter 2. However, it is considered that the method is perfect from the theoretical point of view and that there are few adverse conditions in analysis models.

Although many other methods have been proposed, only the representative examples above are introduced here.

For example, a method to obtain variables on boundaries by substitution has been proposed by Akao and Hakuno, however, it is considered that the method is lacking for general application (1980). 6)

For response calculation of the ground-structural system, the above boundary treatment methods, and a combination of these are used as required. In the famous programme, called "FLUSH", "Viscous Boundary" is used in combination with "Transmitting Boundary". 7)

On the other hand, it has been proposed by Kunar and others that the method by Cundall be applied to the lateral boundary and the models with viscous boundary be applied to the bottom boundary. (8)

Another method has been proposed by Akao and others, which is a response calculation method made by the combination of the aforesaid substitution method with the analysis method for multi-layer ground. (6)

Now, representative boundary treatment methods (i) through (iv) as aforesaid can be classified into two groups, (i), (ii) and (iii), (iv); the former is dependent on frequencies and does not allow the existence of reflected waves, while the latter is not dependent on frequencies and is based on the assumption that reflected waves exist.

In this paper, it is considered that wave propagation problems in elastic media be solved in the time domain. Application of the boundary treatment method by Cundall to three dimensional problems indicates that an equivalent analysis to infinite domain can be made by finite models.

2. THEORETICAL BACKGROUND OF NON-REFLECTIVE BOUNDARY

(1) Methods by Smith and Cundall

It was proposed by Smith that the reflected waves produced at boundaries could be removed by neatly superimposing Neumann's condition on that of Dirichlet. It was with Cundall that boundary conditions were contrived further based on the idea proposed by Smith. Now, the difference between the two can be examined using a simple model.

Wave Equation on u:

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}\ ;\ -\infty<0,t>0 \qquad (1)$$

The solution of equation (1) for the free boundary has a reflected wave in the same phase as the incident wave and a reflected wave in the opposite phase for the fixed boundary, therefore, by taking an average of these two solutions, the term of reflected wave is erased. This is the superimposing boundary proposed by Smith. On the other hand, the method proposed by Cundall is based on the assumption that Equation (2) is satisfied.

$$\frac{\partial u}{\partial t}(0,0)=-c\cdot f'(0),\ \frac{\partial u}{\partial x}=f'(0) \qquad (2)$$

If solved by a boundary condition as expressed in Equation (3), solution as expressed in Equation (4) is given.

i) $\frac{\partial u}{\partial t}(t,0)=\frac{\partial u}{\partial t}(0,0)=-cf'(0)$ ······ (3)

(Solution is expressed in terms of u_1)

$u_1(t,x)=f(x-ct)+A_1 f(-x-ct)$ ······ (4)

When Equation (4) is substituted in Equation (3), the following solution is obtained [Equation (5)].

$$\frac{\partial u_1}{\partial t}(t,0)=-c\{f'(-ct)+A_1 f'(-ct)\}=-cf'(0)$$

$\therefore\ A_1=-1+\frac{f'(0)}{f'(-ct)}$ ······ (5)

As shown in Equation (5), A_1 becomes a function of t, therefore, u_1 does not satisfy the original wave Equation (1). As stated hereinafter, however, the time of reflected wave within the boundary region is only (3-4) $x\Delta t$. Therefore, A_1 within microtime, seems to be constant, which does not make any difference. Eventually, a larger factor seemed to prove that the adequacy of this of this method is that wave Equation (1) has symmetrical expression to t and x. Boundary conditions in i) and ii) have also expressions symmetrical to the first differential of t and the first differential of x. It seems that if u_1 of Equation (6) has an error, it has symmetrical expression against the error in analysis u_1 of Equation (8). Therefore, they will completely compensate each other in the process of adding u_1 and u_2.

Therefore, the following Equation is given.

$u_1(t,x)=f(x-ct)+\left\{-1+\frac{f'(0)}{f(-ct)}\right\}f(-x-ct)$ ······ (6)

If solved by a boundary condition as expressed in Equation (7), solution (as expressed in terms of u_2) is given in Equation (8), the same as in i).

ii) $\frac{\partial u}{\partial x}(t,0)=\frac{\partial u}{\partial x}(0,0)=f'(0)$ ······ (7)

$u_2(t,x)=f(x-ct)+\left\{1-\frac{f'(0)}{f'(ct)}\right\}f(-x-ct)$ ······ (8)

The reflected waves can be erased by calculation of $(u_1/u_2)/2$.

Initial condition of waves moving to the positive direction of x is satisfied in u_1 and u_2 while the initial condition is satisfied under the condition of $f(x>0)=0$. The differerence in limitation of this initial condition of $f(x)$ is of importance, and will be the decisive factor in the difference between the two. In the method by Cundal, initial condition in any time, $t=t_0(>0)$, can be set :

$u(t_0,x)=f(x-ct_0),\ \frac{\partial u}{\partial t}(t_0,0)=-cf'(-ct_0),\ \frac{\partial u}{\partial x}(t_0,0)=f'(-ct_0)$ ······ (9)

However, it cannot be set by the method by Smith.

The initial condition is renewed and reflected waves are erased in Δt of uniform time interval in the method proposed by Cundall, therefore, the domain in which reflected waves exist can be limited to a small domain near the boundaries $(-c\Delta t<x<0)$.

In the method proposed by Smith, reflected waves proceed to all domains, and when reflected waves from the point, $x=0$, are reflected again to reach the point of $x=0$, if other boundaries exist, it has the weak point that it can erase the waves no longer. However, the method by Cundall solves this problem. Analysis of assumptively infinite domains can be made in finite domains for as much time as desired.

(2) Application of Cundall's method to Three Dimensional Problems

a) When the followwing condition of non-reflective boundaries is considered,

$\frac{1}{c^2}\cdot\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\Delta^2 u$ ······ (10)

the waves proceeding to the x direction are generally expressed as follows :

$u_t(t,x,y,z)=\exp(i\xi x+iny+i\xi z-i\omega t),\ (\xi^2+\eta^2+\zeta^2=\omega^2/c^2,\ \xi>0)$ ······ (11)

As is the same case with the first dimension, trial calculation of the amplitude of reflected waves is made under Neumann and Dirichlet conditions as follows : (They are expressed in terms of A^N and A^D resectively.)

$$\frac{\partial u}{\partial t}(t,0,y,z)=-i\omega(1+A^D)\exp(i\eta y+i\xi z-i\omega t)=-i\omega\exp(i\eta y+i\xi z-i\omega t_0)$$

$$\therefore\ A^D=-1+\exp(i\omega(t-t_0)) \qquad (12)$$

$$\frac{\partial u}{\partial x}(t,0,y,z)=i\xi(1-A^N)\exp(i\eta y+i\zeta z-i\omega t)=i\xi\exp(i\eta y+i\zeta z-i\omega t_0)$$

$$\therefore\ A^N=1-\exp(i\omega(t-t_0)) \qquad (13)$$

Therefore, if the average of the two solutions obtained from i) and ii) is taken, the terms of reflected waves in the equations are erased.

Now, in the case of condition i), the following equation is given.

$$\frac{\partial u}{\partial y}(t,0,y,z)=iu\exp(i\eta y+i\zeta z-i\omega t_0)=\frac{\partial u_i}{\partial y}(t_0,0,y,z)$$

$$\frac{\partial u}{\partial z}(t,0,y,z)=i\zeta\exp(i\eta y+i\xi z-i\omega t_0)=\frac{\partial u_i}{\partial y}(t_0,0,y,z) \qquad (14)$$

b) Combination of Neumann's Condition and Dirichlet's Condition in 3 D Elastic Media

Equation of 3 D elastic media is expressed as follows :

$$\rho\frac{\partial^2 u}{\partial t^2}=(\lambda+2\mu)\,\mathrm{grad}\,(div\,\boldsymbol{u})-\mu\,\mathrm{rotrot}\,\boldsymbol{u}\ ;\ \boldsymbol{u}=(u,v,w) \qquad (15)$$

where, ρ : density, λ,μ : Lame constant

When the above equation is expressed using potentials, the following equation is given.

$$\frac{\partial^2\phi}{\partial t^2}=\alpha^2\nabla^2\phi\ ;\ \alpha^2=\frac{\lambda+2\mu}{\rho} \qquad (16)$$

$$\frac{\partial^2 \boldsymbol{A}}{\partial t^2}=\beta^2\nabla^2\boldsymbol{A}\ ;\beta^2=\frac{\mu}{\rho},\ \ \boldsymbol{A}=(A_x,A_y,A_z),\ \ \boldsymbol{u}=\mathrm{grad}\,\phi+\mathrm{rot}\,\boldsymbol{A} \qquad (17)$$

Now, in the case of incidence of waves into the boundary of $x=0$, like a), it may be solved by boundary condition as to ϕ and A as described in a). As ϕ and A are obtained independently, combinations of boundary conditions as shown below are considered.

$$\text{i-a)}\quad \left\{\frac{\partial\phi}{\partial x},\ \frac{\partial A_x}{\partial x},\ \frac{\partial A_y}{\partial t},\ \frac{\partial A_z}{\partial t}\ ;\mathrm{const}\right\}_{x=0} \qquad (18)$$

$$\text{ii-a)}\quad \left\{\frac{\partial\phi}{\partial t},\ \frac{\partial A_x}{\partial t},\ \frac{\partial A_y}{\partial x},\ \frac{\partial A_z}{\partial x}\ ;\mathrm{const}\right\}_{x=0} \qquad (19)$$

Note "Const" means constant regardless of time. (only y and z become functions)

In the case of combination of i-a), the following equation is given.

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial x}+\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)=\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial A_z}{\partial t}\right)-\frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial t}\right) \qquad (20)$$

Therefore, the following is obtained.

$$\left.\frac{\partial u}{\partial t}\right|_{x=0}=\mathrm{const} \qquad (21)$$

Now, explanation is given as to why A_z at left of equation (20) becomes constant. By equation (18), the following equation is obtained.

$$\left.\frac{\partial^2 u}{\partial t^2}\right|_{x=0}=0$$

Where : $\left.\frac{\partial^2 A}{\partial t^2}\right|_{x=0}=0$

and the following equation can also be establised.

$$\nabla^2 A|_{x=0}=0$$

Therefore, it must be as follows ;

$$\left.\frac{\partial^2 A_z}{\partial x^2}\right|_{x=0}=\text{const}$$

From equation (22), equation (23) is obtained.

$$\frac{\partial v}{\partial x}=\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\partial A_x}{\partial x}\right)-\frac{\partial^2 A_z}{\partial x^2} \qquad (22)$$

$$\left.\frac{\partial v}{\partial x}\right|_{x=0}=\text{const} \qquad (23)$$

Therefore, the following combinations are obtained.

$$\text{i -b)}\quad \left\{\frac{\partial u}{\partial t},\ \frac{\partial v}{\partial x},\ \frac{\partial w}{\partial x}\ ;\text{const}\right\}_{x=0} \qquad (24)$$

Conversely, the following equation is obtained from equation (20) and (21) through calculation.

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial t}\right)-\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x}\right)=-\frac{\partial}{\partial t}\left(\frac{\partial^2 A_x}{\partial x^2}+\frac{\partial^2 A_z}{\partial y^2}+\frac{\partial^2 A_z}{\partial x^2}\right)=-\frac{1}{\beta^2}\frac{\partial^3 A_z}{\partial t^3} \qquad (25)$$

When boundary condition at $x=0$ is considered, it becomes as follows :

$$\left.\frac{\partial^3 A_z}{\partial t^3}\right|_{x=0}=\text{const} \qquad (26)$$

However, it is equivalent to equation (27), and it is the condition of i -a). It can be verified likewise on other conditions. Therefore, the boundary condition of i -a) is equivalent to that of i -b).

$$\left.\frac{\partial A_z}{\partial t}\right|_{x=0}=\text{const} \qquad (27)$$

Likewise, the following can be obtained from ii -a).

$$\text{ii -b)}\quad \left\{\frac{\partial u}{\partial x},\ \frac{\partial v}{\partial t},\ \frac{\partial w}{\partial t}\ ;\text{const}\right\}_{x=0} \qquad (28)$$

Taking an average of i -b) and ii -b), reflected waves are erased.

When Dirichlet condition in i -b) and ii -b) is considered, the following can be obtained in i -b) like equation (14).

$$\left.\frac{\partial u}{\partial y}\right|_{x=0}=\text{const},\ \left.\frac{\partial u}{\partial x}\right|_{x=0}=\text{const} \qquad (29)$$

As to ii -b), the following can also be obtained.

$$\left.\frac{\partial v(w)}{\partial y}\right|_{x=0}=\text{const},\ \left.\frac{\partial v(w)}{\partial z}\right|_{x=0}=\text{const} \qquad (30)$$

Now, stress working on plane of $x=0$ is considered. In the case of i -b), the following are given.

$$\sigma_{xy}=\mu\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\right),\ \sigma_{xz}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) \qquad (31)$$

Therefore, the following equation are obtained.

$$\sigma_{xy}|_{x=0}=\text{const},\ \sigma_{wz}|_{x=0}=\text{const} \qquad (32)$$

Although reverse verification is omitted, i -b) and ii -b) can be expressed from the above as follows :

$$\text{i -c)}\quad \left\{\frac{\partial u}{\partial t},\ \sigma_{xy},\ \sigma_{xz}\ ;\text{const}\right\}_{x=0} \qquad (33)$$

$$\text{ii -c)}\quad \left\{\sigma_{xx},\ \frac{\partial v}{\partial t},\ \frac{\partial w}{\partial t}\ ;\text{const}\right\}_{x=0} \qquad (34)$$

Boundary conditions of i -c) and ii -c) are of simple form for handling in numerical analysis such as the Finite Element Method.

(3) problems in multi-reflection

In Section 2-2, boundaries at $x=0$ were reviewed. However, the problem arose that the incidence of reflected waves from plane at $x=0$ into plane at $y=0$ occurs when boundary at $y=0$ is added. For such multi-reflection problems, the matter can be settled by obtaining the following four solutions. (As to equation in a) of 2-2)

$$\left.\begin{array}{l} \text{i)}\ \left.\dfrac{\partial u}{\partial x}\right|_{x=0}=\text{const},\ \left.\dfrac{\partial u}{\partial y}\right|_{y=0}=\text{const};\ \text{ii)}\ \left.\dfrac{\partial u}{\partial x}\right|_{x=0}=\text{const},\ \left.\dfrac{\partial u}{\partial t}\right|_{y=0}=\text{const} \\ \text{iii)}\ \left.\dfrac{\partial u}{\partial t}\right|_{x=0}=\text{const},\ \left.\dfrac{\partial u}{\partial y}\right|_{y=0}=\text{const};\ \text{vi)}\ \left.\dfrac{\partial u}{\partial t}\right|_{x=0}=\text{const},\ \left.\dfrac{\partial u}{\partial t}\right|_{y=0}=\text{const} \end{array}\right\} \cdots\cdots\cdots (35)$$

Although verification of the above is omitted, it is self-explanatory that it can erase even rereflected waves of reflected waves. (See Fig. 1)

Now, if renewal of the initial condition is made every Δt, the above reflected waves against any $\xi,\eta>0$) exist in the following domain.

$$\{-c\Delta t<x<0,\ -c\Delta t<y<0\} \cdots\cdots (36)$$

Therefore, four solutions, i) through iv), may be obtained only in this domain.

Further, in the case that boundary at $z=0$ is added, eight solutions are required, which are shown in Fig. 2, like Fig. 1.

Now, the method by Smith is compared with that by Cundall on the assumption of a rectangular parallelopiped. In the method by Smith, it is necessary to obtain sixty-four types of solutions ($2^6=64$), because six boundary regions exist. On the other hand, eight solutions may be required in the method by Cundall by setting adequate time intervals for renewal of the initial condition. This is because only the influence by three boundaries may be considered due to the existence of reflected waves in the only domain definite distance apart from the boundaries.

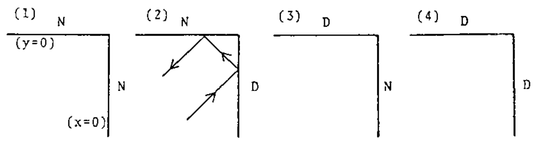
Of course, this method has some defects. This is that re-reflected waves from boundary corners are produced because grid points are treated as belonging to either side of a boundary surface. Re-reflected waves from those corners are so small as to enable ignoring them in calculation. However, this may adversely effect the result in an extreme case.

3. APPLICATION OF AFORESAID METHODS TO THREE DIMENSIONAL ELASTICITY PROBLEMS

The results of application of the aforesaid methods to Three Dimensional Elasticity problems is introduced here. In the case of Three Dimensional Elasticity, analysis models incur a large amount of limitation due to the degree of freedom. However, a model not required for external memory of the computer was set.

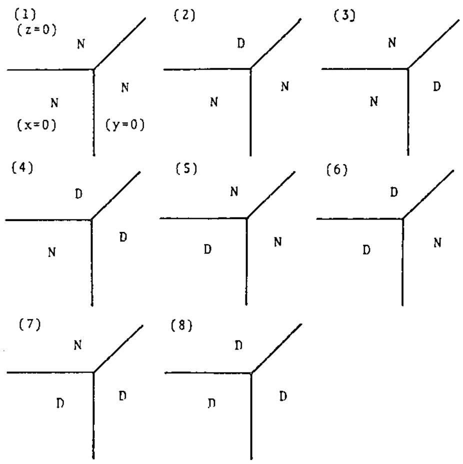
a) Half infinite elastic surface excitation

An analysis model is shown in Fig. 3. It is assumed to be a rectangular parallelopiped consisting of 2 400 units ($15\times16\times10=2400$) of solid elements. It is also assumed that one surface is of free surface and the others are non-reflective boundaries. All regions are only divided into an inner region and a boundary region, but the boundary region is not divided further to fine



(N: Neumann condition, D: Dirichlet condition)

Fig. 1 Boundary conditions at two boundaries ($x=0$, $y=0$).



(N: Neumann condition, D: Dirichlet condition)

Fig. 2 Boundary conditions at three boundaries ($x=0$, $y=0$, $z=0$).

regions. The inner region is inside the dotted lines shown in Fig. 3. Jointing of both regions is realized by having a common element along the boundary. For the degree of freedom it is required to set one kind in the inner region and eight kinds in the boundary region. Physical characteristics of the finite element models is set as below when one side length of a solid is expressed in terms of h.

$\frac{V_S}{h}=50^{(s^{-1})}$ $\sigma=0.3$ Where, V_S : Shearing wave velocity, σ : Poisson's ratio

When uniformly distributed sinusoidal force of one wave length is applied to the area shown in the oblique lines, trial output of response wave from observed points S_1 to S_5 is provided.

Fig. 4 shows response waves when one wave of sine type external force (cycle $T=40\,\Delta t$) is applied to the z direction under the following conditions; $0<t$ After primary portion of wave movement has passed, no remarkable reflected waves occur though there still remains some undulation. When running time from the first peak of displacement wave shown in Fig. 4 is obtained and converted in terms of phase velocity, the following are obtained.

0.92 V_S in $S_3 \to S_1$, 1.78 V_S in $S_3 \to S_5$ (0.95 V_p, provided V_p is velocity of P wave).

It is considered that wave like Rayleigh waves are propagated from observed points S_3 to S_1 and waves close to p waves are propagated from S_3 to S_5. These calculations were made by Hitachi super Computer S 810, and time to obtain results shown in Fig. 4 was 150 seconds by CPU.

b) Response of lens shaped soft surface layers

As shown in Fig. 5 finite element models are so set that lens shaped deposited layers considered as analysis models are included in inner regions. Input in x direction. By symmetry in construction, modeling is made of half of a region to obtaine finite models consisting of 3 456 elements (24×12×12=3 456). Physical characteristics of top and bottom layers are shown in Fig. 5. Boundary surfaces of both layers are close to approximation in step shape because of their simplicity. For shorter waves, five times of element size, influence of approximation in step shape is considered smaller. The area inside of dotted lines in Fig. 5 is the inner region.

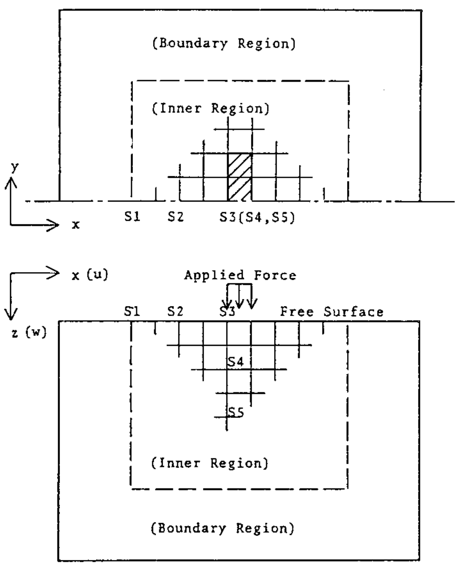


Fig. 3 FEM model used for an impulsive excitation on free surface of three dimensional elastic half space.

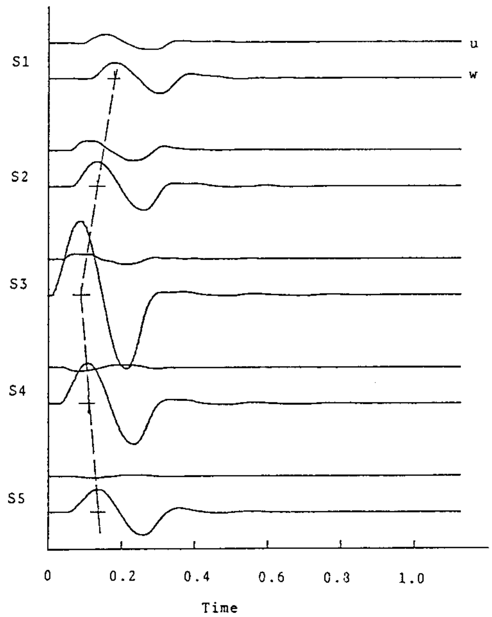


Fig. 4 Computed displacement responses due to impulsive loading in z direction with the conditions as shown in Fig. 3.

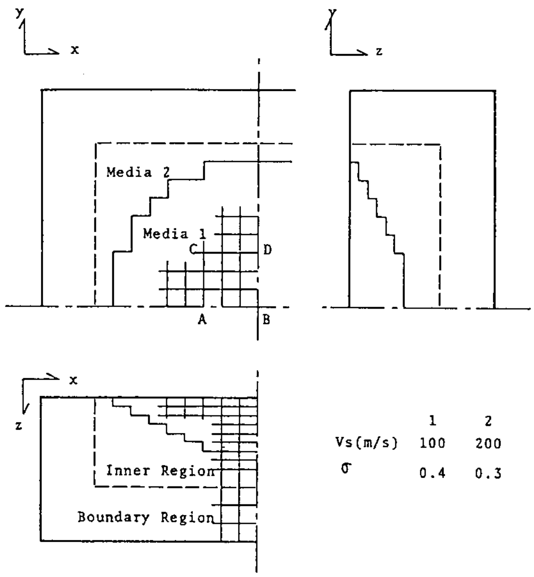


Fig. 5 Three dimensional FEM model of ground surface.

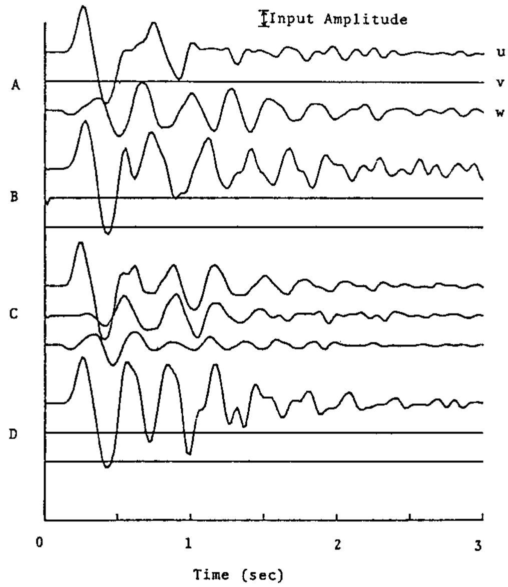


Fig. 6 Layer with soft deposit like a lens. Computed velocity response of soft surface layer shown in Fig. 5 with impulsive input at the bottom (3-D).

In this case, it is presumed that there is no internal attenuation, because adjacent areas of irregular region become one dimensional problems against incident seismic waves. Earthquake motion U_e is calculated independently by the conventional multi-reflective theory. On the other hand, wave displacement U_b at lateral boundary is the sum of earthquake motion ascending over boundary and wave composition x diffusing to the outside region passing through the boundary. As U_e is calculated independently, diffused wave U_f can be identified by retracting U_b from U_e. Therefore, wave eliminating treatment may be provided to U_b at the boundary. It is possible to input any seismic motion, but one sine wave of 3 Hz is input in x direction in the bottom, and response velocity of the free surface is checked. Observed points are A, B, C, and D shown in Fig. 5, velocity waves of which are shown in Fig. 6. Response was obtained in 3 seconds, provided time interval is set in $\Delta t/=1/200$ (s).

From Fig. 6, it is clear that a complex wave movement phenomenon is formed by repetition of reflecqion and reflection in the top layer. Because of much difference in physical characteristics of the top and bottom layers, wave motion energy is retained in the soft top layer, by which large amplitude of vibration continues for a longer time. It is remarkable, especially at observed points on asymmetrical surface, and four waves 3-4 times of input amplitude continue at D point.

4. CONCLUSION

The study was successful by creation of non-reflective boundaries using Cundall's method and application of it to the three dimensional finite element method. It is expected that seismic motion characteristics in the soil of a complex structure, dynamic interaction of soil and structures, liquefaction of soil, and forecast of earthquake motion using hypocenter fault models that have been difficult to clarify, will be made clear by application of this finite element method in the near future.

In closing, we wish to express our appreciation to the Ministry of Education for their cooperation extended to us in carrying out this study, subsidising our research and study on natural disasters.

REFERENCE

1) Lysmer, J. and Kuhlemeyer, R. L. : Finite dynamic model for infinite media, J, EM, Div., ASCE 95, 859-877, 1969.
2) Lysmer, J. and Waas, G. : Shear waves in plane infinite structures, J. EM, Div., ASCE 98, 85-105, 1972.
3) Smith, W. : A non-reflecting plane boundary for wave propagation problems, J. Comp. Phys. 15, 492-503, 1974.
4) C. Tamura, and Y. Nakamura : Analysis method for soil vibration taking dissipation of dynamic energy of waves underground into consideration. /Reports of Institute of Industrial Science, Universitv of Tokyo, Vol. 28, No. 8, 1976.
5) Cundall, P. A., Kunar, R. R., Carpenter and Marti, J. : Solution of infinite dynamic problems by finite modelling in the time domain, Proc. 2 nd, Int. Conf. Appl. Num. Modelling, Madrid, Spain, 1978.
6) K. Akao, M. Hakuno : Wave Treatment at Infinite Boundaries in Dynamic Analysis/proc. of Japan Society of Civil Engineers No. 336, 21-29, 1983.
7) Lysmer, J., Udaka, T., Tsai C. and Seed, H. B. : FLUSH-a computer program for approximate 3-D analysis of soil structure interaction problems, Report No. EERC, 75-30, Univ. of California, Berkeley, 1975.
8) Kunar, R. R. and Rodriguez-Ovejero, L. : A model with non-reflecting boundaries for use in explicit soil-structure interaction analysis, Int. J. EESD, Vol. 8, 361-374, 1980.

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