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FORMULATION OF DISTORTIONAL BEHAVIOR OF THIN-WALLED CURVED BEAM WITH OPEN CROSS SECTION

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An investigation of the effects of cross sectional distortion on the elastic behavior of a thin-walled open section curved beam is presented. In particular, the influence of the distortion on the longitudinal stress and on the transverse bending moment is studied.

The formulation of the problem is derived from the principle of virtual work by assuming the strain field of distortion, and a practical modification of the conventional curved beam theory is established.

The theoretical solutions are compared with the experimental results, and the effects of curvature, longitudinal stringers and transverse stiffening frames are discussed for design purposes.

1. INTRODUCTION

The theory for a thin-walled curved beam with arbitrary cross section has been established by taking into account the wall thickness not extremely small compared with other dimensions[1] as well as the different radius of curvature of arc fibers[2)~4)]. In those analyses, however, it is assumed that the cross section of any beam is stiffened against distortion. This implies the provision of an adequate number of stiffening diaphragms, transverse frames and diagonal bracings. Where such stiffeners are spaced widely apart, the cross section will distort and the analysis must be modified to account for this.

The distortion may result in considerable reduction in overall stiffness of the structure and may set up unfavourable stress conditions. The present study arose from an investigation of the guideway beam for the suspended monorail system[5)~7)], in which it is impossible to insert cross-bracings and lateral members for obtaining increased stiffness and for preventing the distortion.

The theory for distortion in the past has been focused chiefly on thin-walled straight beams of closed section unstiffened or stiffened by diaphragms[8], and the open section beam has received a little attention[9),10)]. Curved beams are, in particular, subject to distortion of larger magnitude than straight ones, owing to the existence of radial components in axial stresses[11]. Takahashi et al[12]. proposed a method of analysis for the distortion of curved thin-walled open section members, with singly symmetrical cross section, but their theory is limited in that it can only be applied to the distortion of single degree of freedom and to curved members without stiffening frames.

The present paper describes a practical method for estimation of the cross sectional distortion of thin-walled curved beams with open section stiffened by transverse frames. The theoretical solutions are

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compared with the experimental results and the accuracy of the analytical method is also discussed.

2. COORDINATE SYSTEMS

A thin-walled and circular-curved beam of open cross section is shown in Fig. 1 (a). We define the geometry of the beam by a fixed cylindrical coordinate system (ρ, α, ζ), in which ρ is a radial coordinate, α is an angular coordinate and ζ is directed normal to the plane of the beam. We denote the radius of curvature at the neutral point and at the shear center of the section by R_0 and R_S, respectively, as shown in Fig. 1 (b). Note that the neutral point does not coincide with the centroid in curved beams. To define the geometry of the cross section, we establish in addition to the usual x, y coordinates an orthogonal curvilinear coordinate system (n, s), with its origin D. The tagential coordinate s is measured counterclockwise along the center line of the wall, and n is in the direction of outward normal to s. The wall thickness t is a function of s. Since the orthogonal axes x and y pass through the neutral point of the section, we note that

$$\int_A \frac{x}{\rho}\, dA=0, \quad \int_A \frac{y}{\rho}\, dA=0 \cdots\cdots (1)$$

where A is the cross sectional area of the beam.

3. DISTORTION OF CROSS SECTION AND ITS DEGREES OF FREEDOM

Let us consider a thin-walled polygonal cross section consisting of N plate elements whose junctions are numbered 0, 1, 2, $\cdots$, $N-2$ as shown in Fig. 2(a). The cross sectional distortion is defined by the deflection of each plate in its own plane accompanied with its transverse bending. It is assumed that the effect of both the local bending of each plate without nodal displacements, as illustrated by dotted lines in Fig. 2(b), and its transverse stretching on the distortion is negligible.

As a result of these assumptions, an open section beam consisting of N plates has N degrees of freedom for movements in the x-y plane. Since those degrees include three rigid-body displacements, the degree of freedom for the cross sectional distortion itself is $N-3$. The $N-3$ components of the distortion θ_1, θ_2,

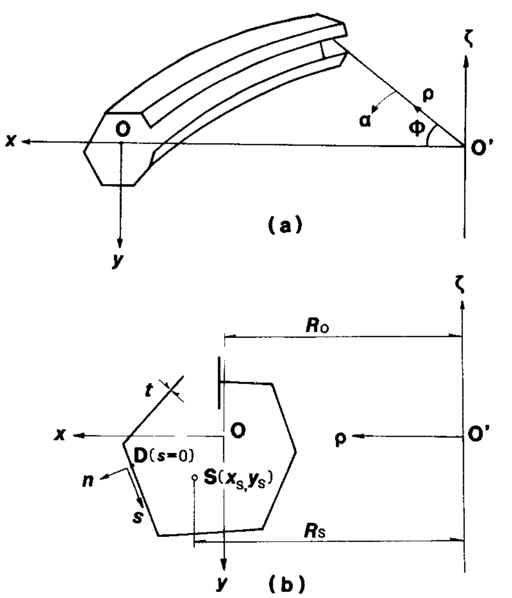

Fig. 1 (a) General view and coordinate system.
(b) Geometry of thin-walled section.

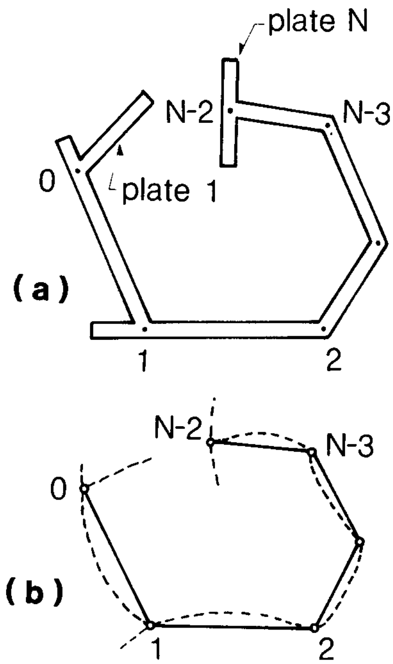

Fig. 2 Degrees of freedom for distortion.

$\cdots$, θ_{N-3} are defined by the relative rotation or the change of the angle between adjacent plate elements. Fig. 3 shows the definition of the relative rotation θ_i, which is assumed positive when the plate $i+1$ rotates counterclockwise from the plate $i+2$.

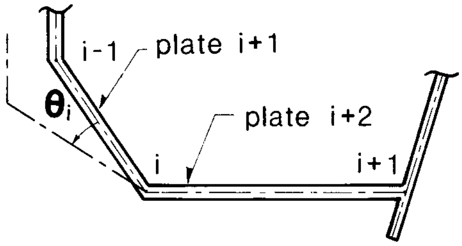

Fig. 3 Definition of cross sectional distortion.

4. STRAIN FIELD OF CROSS SECTIONAL DISTORTION

(1) Initial Displacements and Stresses

If any cross section of a curved beam remains undistorted during deformations produced by various loads, the displacement of any point in the section is completely specified by $u_S^{(0)}$, $v_S^{(0)}$, $w_S^{(0)}$ and $\varphi^{(0)}$, in which the former three components denote the displacements of the shear center $S(x_s, y_s)$ in the x, y and α directions, respectively, and the last, $\varphi^{(0)}$, denotes the rotation of the section about the shear center. These four components are termed the initial displacements of the distortion problem in the present theory.

In the conventional theory for the thin-walled beam of open cross section, the displacements produce the axial strain $\varepsilon_\alpha^{(0)}$ and the shear strain $\gamma_{s\alpha}^{(0)}$ due to St. Venant torsion, and all other strains are neglected. Corresponding to these strains, the initial axial stress $\sigma_\alpha^{(0)}$ and shear stress $\tau_{s\alpha}^{(0)}$ are given by the relations

$$\sigma_\alpha^{(0)}=E\varepsilon_\alpha^{(0)}, \quad \tau_{s\alpha}^{(0)}=G\gamma_{s\alpha}^{(0)} \quad \cdots\cdots (2)$$

in which E is Young's modulus and G is shear modulus.

(2) Additional Displacements and Strains due to Distortion

When the cross section distorts the total displacement of the beam will be described by the modified rigid-body movements $u_S^{(0)}+u_s$, $v_S^{(0)}+v_s$, $w_S^{(0)}+w_s$ and $\varphi^{(0)}+\varphi$ plus the displacements due to relative rotations of the plate elements, $\theta_i(i=1, 2, \cdots, N-3)$. The unknowns u_s, v_s, w_s and φ are termed the additional displacements of the distortion problem.

Let s_i be the counterclockwise curvilinear coordinate of the point $i(x_i, y_i)$ located at the junction of the plate $i+1$ and $i+2$, as shown in Fig. 4. Further let $P(x, y)$ denote a point located on the center line of an arbitrary wall, and let u and v denote the additional displacements of P in the x and y directions. Then

$$\left.\begin{aligned} u&=u_s-(y-y_s)\varphi-\sum_{i=1}^{N-3}(y-y_i)\theta_i\mu_i \\ v&=v_s+(x-x_s)\varphi+\sum_{i=1}^{N-3}(x-x_i)\theta_i\mu_i \end{aligned}\right\} \quad \cdots\cdots (3)$$

where $\mu_i=\begin{cases} 1 & \text{if} \quad s\leq s_i \\ 0 & \text{if} \quad s>s_i \end{cases}$

For further development we define the direction cosines l and m as follows:

$$\left.\begin{aligned} l&=\cos(x, n)=\frac{\partial x}{\partial n}=\frac{\partial y}{\partial s} \\ m&=\cos(y, n)=\frac{\partial y}{\partial n}=-\frac{\partial x}{\partial s} \end{aligned}\right\} \quad \cdots\cdots (4)$$

Let ξ and η denote the displacements of P in n and s directions, respectively. Then, from simple geometry we find

$$\left.\begin{aligned} \xi&=u_sl+v_sm-h_s\varphi-\sum_{i=1}^{N-3}r_{si}\mu_i\theta_i \\ \eta&=-u_sm+v_sl+h_n\varphi+\sum_{i=1}^{N-3}r_{ni}\mu_i\theta_i \end{aligned}\right\} \quad \cdots\cdots (5)$$

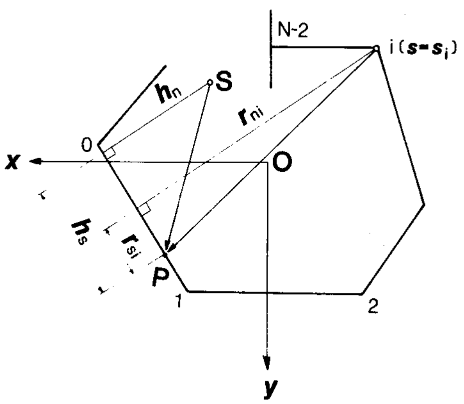

Fig. 4 Geometry of cross section.

where

$$h_s = -(x - x_s)m + (y - y_s)l, \qquad h_n = (x - x_s)l + (y - y_s)m \quad (6)$$

$$r_{si} = -(x - x_i)m + (y - y_i)l, \qquad r_{ni} = (x - x_i)l + (y - y_i)m \quad (7)$$

Referring to Fig. 4, we find that h_s and h_n are, respectively, s and n components of the vector drawn from S to P, and that r_{si} and r_{ni} are those of the vector drawn from i to P.

We shall assume that the effect of the shear strain $\gamma^*_{s\alpha}$ along the center line of the wall on deformations is extremely small and can be neglected. That is

$$\gamma^*_{s\alpha} = \frac{1}{\rho}\frac{\partial \eta}{\partial \alpha} + \rho\frac{\partial}{\partial s}\left(\frac{w}{\rho}\right) \quad (8)$$

where w is the additional axial displacement of P. Substituting η of Eq. (5) into Eq. (8) and integrating with respect to s, we obtain

$$w = w_s - (x - x_s)\left(\frac{u_s'}{R_s} - \frac{w_s}{R_s}\right) - (y - y_s)\frac{v_s'}{R_s} + \omega\left(\frac{\varphi'}{R_s} - \frac{v_s'}{R_s^2}\right) + \sum_{i=1}^{N-3}\frac{\rho}{R_s}(\psi_i - \psi_{is})\frac{\theta_i'}{R_s} \quad (9)$$

where

$$\omega = R_s\rho\int_S^{s_S}\frac{1}{\rho^2}h_n ds, \qquad \psi_i = R_s^2\int_S^{s_i}\frac{1}{\rho^2}r_{ni}\mu_i ds \quad (10)$$

and ψ_{is} is the value of ψ_i at the shear center $S(s = s_s)$. In Eq. (9) and throughout the present paper, the prime denotes differentiation with respect to α, that is $(\ )' = d(\ )/d\alpha$.

The locations of both the shear center S and the origin D of the coordinate s are chosen to satisfy the following properties:

$$\int_A\frac{R_s}{\rho}\omega dA = 0, \qquad \int_A\frac{R_s}{\rho}\omega x dA = 0, \qquad \int_A\frac{R_s}{\rho}\omega y dA = 0 \quad (11)$$

The additional axial strain ε_α of any fiber due to distortion is

$$\varepsilon_\alpha = \frac{1}{\rho}\left(\frac{\partial w}{\partial \alpha} + u\right) = \bar{\varepsilon}_\alpha + \sum_{i=1}^{N-3}\left\{(\psi_i - \psi_{is})\frac{\theta_i''}{R_s^2} - \frac{R_s}{\rho}(y - y_i)\mu_i\frac{\theta_i}{R_s}\right\} \quad (12)$$

where $\bar{\varepsilon}_\alpha$ represents the rigid body strain component included in the additional strain ε_α and is given by

$$\bar{\varepsilon}_\alpha = \frac{R_s}{\rho}\{\varepsilon - (x - x_s)\kappa_y - (y - y_s)\kappa_x + \omega\Theta\}$$

$$\varepsilon = \frac{1}{R_s}(w_s' + u_s), \qquad \kappa_x = \frac{1}{R_s^2}(v_s'' + R_s\varphi), \qquad \kappa_y = \frac{1}{R_s^2}(u_s'' - w_s'), \qquad \Theta = \frac{1}{R_s^2}\left(\varphi'' - \frac{v_s''}{R_s}\right)$$

Since we have neglected the shear strain $\gamma^*_{s\alpha}$ in Eq. (8), the only shear strain considered is $\bar{\gamma}_{s\alpha}$ due to St. Venant torsion:

$$\gamma_{s\alpha} = \bar{\gamma}_{s\alpha} = 2\frac{R_s^2}{\rho}\left(\frac{\varphi'}{R_s} - \frac{v_s'}{R_s^2}\right)n \quad (13)$$

5. ADDITIONAL STRESS DUE TO DISTORTION

(1) Additional Axial Stress

The additional axial stress σ_α due to distortion is, from Hooke's Law,

$$\sigma_\alpha = E\varepsilon_\alpha \quad (14)$$

The additional stress system must be self-equilibrating since the initial stresses, $\sigma_\alpha^{(0)}$ and $\gamma_{s\alpha}^{(0)}$ in Eq. (2), are already in equilibrium with the external loads. Therefore, at any section the additional stress resultants are zero:

$$\int_A\sigma_\alpha dA = 0, \qquad \int_A\sigma_\alpha y dA = 0, \qquad \int_A\sigma_\alpha x dA = 0, \qquad \int_A\sigma_\alpha\omega dA = 0 \quad (15)$$

Since it is well known that in a thin-walled open section beam St. Venant torque is neglected as opposed to warping torque[13], the fourth condition in Eq. (15) indicates that the additional stress system is also independent of twisting moment.

Substituting Eq. (14) into Eqs. (15) and performing the indicated integrations, we get the quantities ε, χ_x, χ_y and Θ in terms of θ_i to eliminate $\bar{\varepsilon}_\alpha$ from Eq. (12). Finally, substituting ε_α thus obtained into Eq. (14), we get

$$\sigma_\alpha = E\frac{R_s}{\rho}\sum_{i=1}^{N-3}\left(\Psi_i\frac{\theta_i''}{R_s^2} - \Phi_i\frac{\theta_i}{R_s}\right) \qquad (16)$$

where

$$\left.\begin{aligned}\Psi_i &= \frac{\rho}{R_s}(\phi_i - \phi_{is}) - C_{i0} - C_{iy}x - C_{ix}y - C_{i\omega}\omega \\ \Phi_i &= (y - y_i)\mu_i - D_{i0} - D_{iy}x - D_{ix}y - D_{i\omega}\omega\end{aligned}\right\} \qquad (17)$$

The eight coefficients C_{i0}, C_{iy}, $\cdots$, $D_{i\omega}$ are known constants, which have been determined so that Eq. (16) satisfies Eq. (15).

(2) Transverse Bending Moment

Let a segment be cut out from a thin-walled curved beam by two adjacent sections perpendicular to its axis. We treat it as a polygonal frame consisting of N members of flexural rigidity EI_s as shown in Fig. 5 (a), and examine its bending moments M_s which produces transverse stress σ_s in each plate element. The beam may be unstiffened or stiffened by equally spaced transverse frames as shown in Fig. 5 (b). If the spacing angle α_c of the stiffening frames is small, their effect could be accounted for by increased plate rigidity. Where the stiffening frames are spaced widely apart, however, transverse bending moment produced by distortion may be almost carried by each frame whose flexural rigidity is calculated for the section including effective width of plate elements. The bending moment M_s is related to the rotations $\theta_i (i=1, 2, \cdots, N-3)$ by the expression

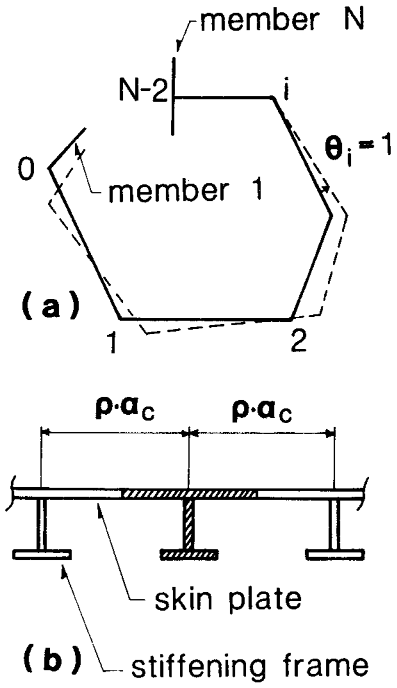

Fig. 5 Bending of transverse frame.

$$M_s = \sum_{i=1}^{N-3} M_i\theta_i \qquad (18)$$

where M_i is the bending moment of the frame associated with a unit value of the rotation $\theta_i = 1$.

6. EQUILIBRIUM EQUATIONS AND BOUNDARY CONDITIONS

(1) Virtual Work Equation

We denote the n, s, α components of external forces per unit surface area on a thin-walled curved beam by p_n, p_s, p_α, respectively, and the n, s, α components of external forces per unit area on the boundary cross section at $\alpha=0$ and $\alpha=\phi$ by $\bar{p}_n$, $\bar{p}_s$, $\bar{p}_\alpha$, respectively. The internal forces developed in the distorted beam are the sum of the known initial stress and the unknown additional stress system, except that the effect of the additional shearing stress on the distortion is extremely small and can be neglected. Therefore, the equilibrium condition for the distorted beam is expressed by the principle of virtual work as follows :

$$\int_0^\phi\int_s\int_n\{(\sigma_\alpha^{(0)} + \sigma_\alpha)\delta\varepsilon_\alpha + \sigma_s\delta\varepsilon_s + \tau_{s\alpha}^{(0)}\delta\gamma_{s\alpha}\}\rho\,dn\,ds\,d\alpha - \int_0^\phi\int_s(p_n\delta\xi + p_s\delta\eta + p_\alpha\delta w)\rho\,ds\,d\alpha$$
$$-\left[n_\alpha\int_A(\bar{p}_n\delta\xi + \bar{p}_s\delta\eta + \bar{p}_\alpha\delta w)dA\right]_0^\phi = 0 \qquad (19)$$

where $\int_s(\ )ds$ indicates the integration taken over the entire length of the wall, $\int_n(\ )dn$ over the thickness t of the wall, $n_\alpha=-1$ at $\alpha=0$ and $n_\alpha=1$ at $\alpha=\phi$.

If $\delta\bar{\xi}$, $\delta\bar{\eta}$, $\delta\bar{w}$, $\delta\bar{\varepsilon}_\alpha$ and $\delta\bar{\gamma}_{s\alpha}$ denote the variations of the rigid body displacements and corresponding strains included in those of the additional displacements and strains due to distortion, the initial stresses and external forces must satisfy the virtual work equation given by

$$\int_0^{\Phi}\int_s\int_n(\sigma_\alpha^{(0)}\delta\bar{\varepsilon}_\alpha+\tau_{s\alpha}^{(0)}\delta\bar{\gamma}_{s\alpha})\rho dndsd\alpha-\int_0^{\Phi}\int_s(p_n\delta\bar{\xi}+p_s\delta\bar{\eta}+p_\alpha\delta\bar{w})\rho dsd\alpha$$
$$-\left[n_\alpha\int_A(\bar{p}_n\delta\bar{\xi}+\bar{p}_s\delta\bar{\eta}+\bar{p}_\alpha\delta\bar{w})dA\right]_0^{\Phi}=0 \quad (20)$$

Subtracting Eq. (20) from Eq. (19) gives the required equation governing the distortion problem of the thin-walled curved beam as follows :

$$\int_0^{\Phi}\int_s\int_n\left[\sigma_\alpha\delta\varepsilon_\alpha+\sigma_s\delta\varepsilon_s+\sum_{i=1}^{N-3}\sigma_\alpha^{(0)}\left\{(\psi_i-\psi_{is})\frac{\delta\theta_i''}{R_s^2}-\frac{R_s}{\rho}(y-y_i)\mu_i\frac{\delta\theta_i}{R_s}\right\}\right]\rho dndsd\alpha$$
$$-R_s\sum_{i=1}^{N-3}\int_0^{\Phi}\left(m_{\alpha i}\delta\theta_i+m_{\psi i}\frac{\delta\theta_i'}{R_s}\right)d\alpha-\sum_{i=1}^{N-3}\left[n_\alpha\left(\bar{T}_i\delta\theta_i+\bar{M}_{\psi i}\frac{\delta\theta_i'}{R_s}\right)\right]_0^{\Phi}=0 \quad (21)$$

For simplicity, we introduce the quantities with respect to the external forces defined by

$$m_{\alpha i}=\int_s\frac{\rho}{R_s}(p_sr_{ni}-p_nr_{si})\mu_ids,\qquad m_{\psi i}=\int_s\frac{\rho^2}{R_s^2}p_\alpha(\psi_i-\psi_{is})ds \quad (22)$$

$$\bar{T}_i=\int_A(\bar{p}_sr_{ni}-\bar{p}_nr_{si})\mu_idA,\qquad \bar{M}_{\psi i}=\int_A\frac{\rho}{R_s}\bar{p}_\alpha(\psi_i-\psi_{is})dA \quad (23)$$

Furthermore, with respect to the initial stress $\sigma_\alpha^{(0)}$ and the initial shear flow $q^{(0)}$ produced by both bending and warping torque of the beam, new quantities are defined by

$$M_{\psi i}^{(0)}=\int_A\frac{\rho}{R_s}\sigma_\alpha^{(0)}(\psi_i-\psi_{is})dA,\qquad M_{xi}^{(0)}=\int_A\sigma_\alpha^{(0)}(y-y_i)\mu_idA \quad (24)$$

$$T_i^{(0)}=\int_sq^{(0)}r_{ni}\mu_ids \quad (25)$$

The above quantities $m_{\psi i}$, $M_{\psi i}^{(0)}$ and $T_i^{(0)}$ are related as follows :

$$T_i^{(0)}=-\frac{1}{R_s}M_{\psi i}^{(0)\prime}-m_{\psi i} \quad (26)$$

Now performing the integration of the first and second terms in Eq. (21), we get

$$\left.\begin{aligned}&\int_0^{\Phi}\int_s\int_n\sigma_\alpha\delta\varepsilon_\alpha\rho dndsd\alpha=R_s\sum_{i=1}^{N-3}\int_0^{\Phi}\left(M_{\psi i}\frac{\delta\theta_i''}{R_s^2}-M_{\Phi i}\frac{\delta\theta_i}{R_s}\right)d\alpha\\&\int_0^{\Phi}\int_s\int_n\sigma_s\delta\varepsilon_s\rho dndsd\alpha=R_s\sum_{i=1}^{N-3}\int_0^{\Phi}F_{si}\delta\theta_id\alpha\end{aligned}\right\} \quad (27)$$

where

$$M_{\psi i}=\sum_{j=1}^{N-3}\left(EI_{ij}\frac{\theta_j''}{R_s^2}-EJ_{ij}\frac{\theta_j}{R_s}\right),\qquad M_{\Phi i}=\sum_{j=1}^{N-3}\left(EJ_{ji}\frac{\theta_j''}{R_s^2}-EK_{ij}\frac{\theta_j}{R_s}\right),\qquad F_{si}=\sum_{j=1}^{N-3}f_{ij}\theta_j \quad (28)$$

The quantities I_{ij}, J_{ij}, K_{ij} and f_{ij} are geometrical properties of the cross section defined as follows :

$$I_{ij}=\int_A\frac{R_s}{\rho}\Psi_i\Psi_jdA,\qquad J_{ij}=\int_A\frac{R_s}{\rho}\Psi_i\Phi_jdA,\qquad K_{ij}=\int_A\frac{R_s}{\rho}\Phi_i\Phi_jdA,\qquad f_{ij}=\int_s\frac{M_iM_j}{EI_s}ds \quad (29)$$

Finally, integrating Eq. (21) by parts, we arrive at the following virtual work equation :

$$R_s\sum_{i=1}^{N-3}\int_0^{\Phi}\left(\frac{M_{\psi i}''}{R_s^2}-\frac{M_{\Phi i}}{R_s}+F_{si}-\frac{T_i^{(0)\prime}}{R_s}-\frac{M_{xi}^{(0)}}{R_s}-m_{\alpha i}\right)\delta\theta_id\alpha+\sum_{i=1}^{N-3}\left[(M_{\psi i}+M_{\psi i}^{(0)}-n_\alpha\bar{M}_{\psi i})\frac{\delta\theta_i'}{R_s}\right.$$
$$\left.+\left(-\frac{M_{\psi i}'}{R_s}+T_i^{(0)}-n_\alpha\bar{T}_i\right)\delta\theta_i\right]_0^{\Phi}=0 \quad (30)$$

(2) Governing Differential Equations and Boundary Conditions

Since Eq. (30) must hold for any choice of virtual displacement $\delta\theta_i$, we obtain the governing differential equations and the boundary conditions for the distortion problblm as follows :

Equilibrium Equation :

$$\sum_{j=1}^{N-3}\left\{\frac{1}{R_s^4}EI_{ij}\theta_j''''-\frac{1}{R_s^3}E(J_{ij}+J_{ji})\theta_j''+\left(f_{ij}+\frac{1}{R_s^2}EK_{ij}\right)\theta_j\right\}-\frac{1}{R_s}T_i^{(0)\prime}-\frac{1}{R_s}M_{xi}^{(0)}-m_{\alpha i}=0$$

$(i=1,\ 2,\ \cdots,\ N-3)$ ·· (31)

<u>Boundary Conditions</u> (at $\alpha=0$ and at $\alpha=\phi$) :

$$\left.\begin{array}{lll}\theta_i=\overline{C}_i & \text{or} & \frac{1}{R_s}M'_{\varphi i}=T_i^{(0)}-n_\alpha\overline{T}_i \\ \theta'_i=\overline{D}_i & \text{or} & M_{\varphi i}=n_\alpha\overline{M}_{\varphi i}-M_{\varphi i}^{(0)}\end{array}\right\}\quad (i=1,\ 2,\ \cdots,\ N-3) \qquad (32)$$

where $\overline{C}_i$ and $\overline{D}_i$ are prescribed values at the boundaries.

(3) Equilibrium Equation for Curved Beam with Discrete Stiffening Frames

When the beam is stiffened transversely by discrete frames positioned at $\alpha=\alpha_k(k=1,\ 2,\ \cdots,\ K)$, we must use the following virtual work equation in place of Eq. (31) :

$$\sum_{j=1}^{N-3}\left[\int_0^\phi\left\{\frac{1}{R_s^4}EI_{ij}\theta_j'''' - \frac{1}{R_s^3}E(J_{ij}+J_{ji})\theta_j''+\frac{1}{R_s^2}EK_{ij}\theta_j\right\}\delta\theta_i d\alpha+\sum_{k=1}^{K}\frac{1}{R_s}f_{ij}\theta_j(\alpha_k)\delta\theta_i(\alpha_k)\right]$$

$$-\int_0^\phi\left(\frac{1}{R_s}T_i^{(0)\prime}+\frac{1}{R_s}M_{xi}^{(0)}+m_{\alpha i}\right)\delta\theta_i d\alpha=0 \qquad (i=1,\ 2,\ \cdots,\ N-3) \qquad (33)$$

7. NUMERICAL CALCULATIONS AND EXPERIMENTAL RESULTS

(1) Dimensions and Loadings for the Models

Curved and straight beam models, summarized in Table 1, were fabricated from 2.3 mm-thick steel plates, and theoretical and experimental results were compared. The models were chosen to represent the proportions of guideway structures for the suspended monorail system[5]. Dimensions of the curved model A are shown in Fig. 6 ; two longitudinal stringers of inverted T-section were welded to the lower flanges of the beam, and equidistant transverse frames of T-section were also positioned by welding along the span. Since the cross section is consisted of nine wall elements including the bottom stringers, the degree of freedom for the cross sectional distortion is 6.

The ends of each model were held within rigid restraining frames, which could be adjusted by means of steel rollers to provide simple support condition. Each model was acted upon by jacking loads consisting of equal vertical point loads above the parallel stringers for two loading conditions, i. e., a vertical jack load of $P=2@150$ kgf (2.94 kN) applied at midspan (Case 1) and the same load applied at the framed section close to midspan (Case 2).

(2) Numerical Calculation Method

For curved beams all the geometrical properties of the cross section are different from those for straight beams, owing to their initial curvature. However, when we analyze the curved structures with minimum radius of curvature limited, such as horizontally curved bridges and guideways for traffic use, it is well known that the effects of curvature on the section properties are sufficiently small and practically negligible. Neglecting the effects of curvature, the cross section shown in Fig. 6 have a vertical axis of symmetry and some geometrical properties which are odd functions of y in Eq. (29) become zero. In this case

Table 1 Summary of experimental models.

Model	Span Length	Radius of curvature	Interval of stiffening frames
A	200 cm	400 cm	182 mm
B	200	400	95
C	200	∞ (straight)	182

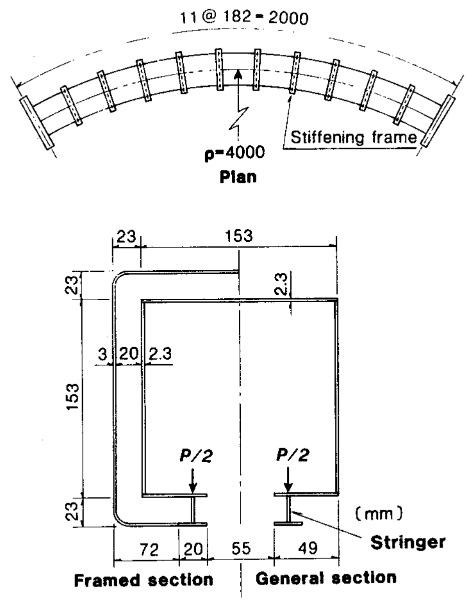

Fig. 6 Dimensions of Experimental model.

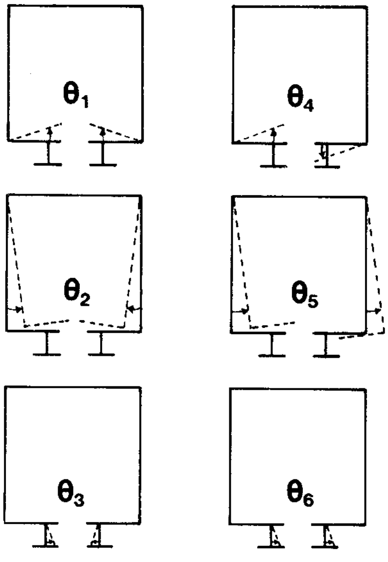

Fig. 7 Symmetrical and antisymmetrical distortion.

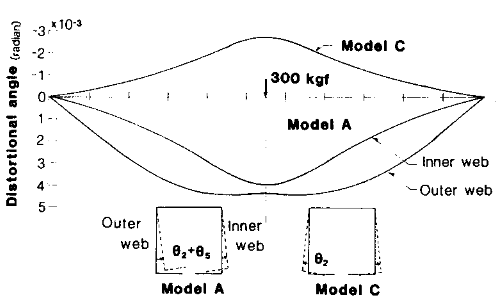

Fig. 8 Variation along the span of distortional angle.

we can separate the six degrees of distortional freedom into three symmetrical distortions ($\theta_1 \sim \theta_3$) and three antisymmetrical ones ($\theta_4 \sim \theta_6$) as illustrated in Fig. 7.

By using Eq. (34) and applying the Rayleigh-Ritz Method we can obtain the unknown displacements θ_i, and then we introduce the results into Eqs. (16) and (18) to find the longitudinal and transverse stresses.

(3) Comparison of Results and Some Considerations

Fig. 8 gives the calculated variation along the span of the distortional angle $\theta_2 + \theta_5$ between the upper flange and the inner or outer web for the loading case 1. In the straight model C the antisymmetrical component θ_5 is zero and the split between two lower flanges is observed to be spreading, whereas it is closing in the curved model since the magnitude of relative rotation of the outer web is larger than that of the inner one.

In Fig. 9 and Fig. 10, the variations along the span of longitudinal stress in the bottom flanges of stringers with inverted T-section, corresponding to the loading cases 1 and 2, are plotted for the curved models A and B, respectively. The dotted lines in these figures indicate the initial stress calculated by assuming that the original shape of every cross secton is unaltered during deformation. The magnitude of the stress in the outer stringer is much larger than that in the inner one, owing to normal stress produced by the warping torsion. Even when the load is applied at the framed

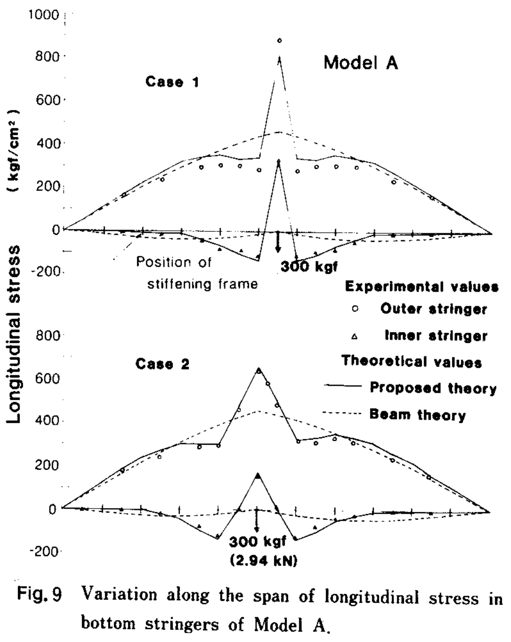

Fig. 9 Variation along the span of longitudinal stress in bottom stringers of Model A.

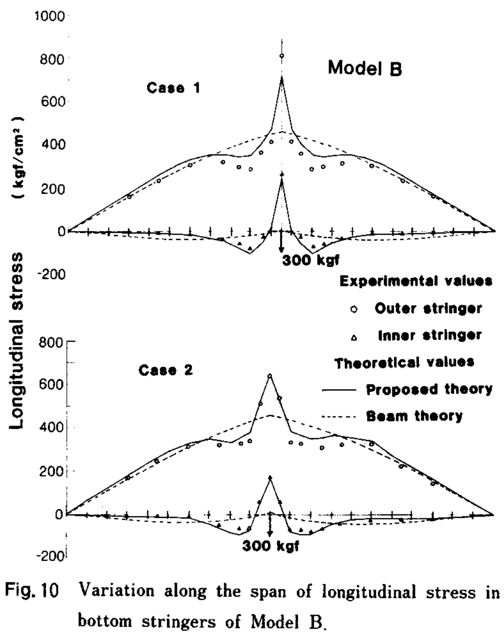

Fig. 10 Variation along the span of longitudinal stress in bottom stringers of Model B.

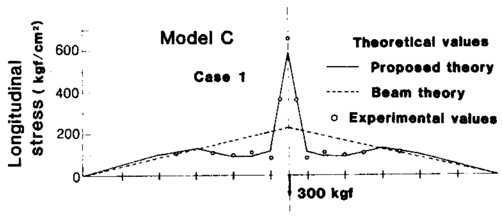

Fig. 11 Variation along the span of longitudinal stress in bottom stringers of Model C.

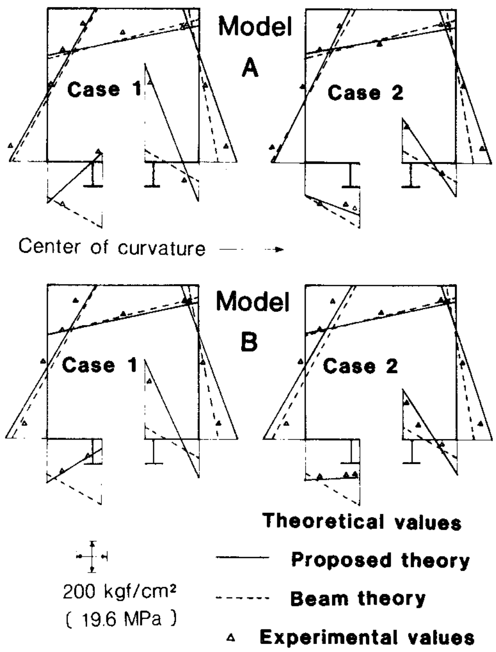

Fig. 12 Distribution of longitudinal stress at midspan cross section.

section (Case 2), significant additional stress develops due to both the cross sectional distortion and the local effect of concentrated forces. Fig. 11 gives the same stress variation for the loading case 1 in the model C.

Because of the absence of a transverse stiffening frame at midspan in each model, particular attention is concentrated on the longtudinal stress of midspan cross section, the results of which are shown in Fig. 12. When the contribution of the additional stress is taken into account, the stress on the inner bottom flange is not small, as we might have suspected had the distortion of the cross section been not considered.

8. CONCLUDING REMARKS

Formulation for the distortional behavior of thin-walled curved beams with open cross section has been presented. The conventional curved beam theory based on the assumption of rigid cross section has been modified, and the governing equation for the distortion problem has been derived by virtue of the principle of virtual work.

The definition for the distortion of cross section is that the deflection of each plate element in its own plane accompanied by its transverse bending, and open section curved beams with any degree of freedom for the distortion in accordance with the above definition can be treated. The equations and formula derived herein are sufficiently general to be applicable to beams with large curvature, including, of course, the straight beam as a special case. The beam is assumed to have constant cross section, but the effect of transverse stiffening frames is taken into consideration.

The theoretical and experimental studies have revealed that a considerable increase in the deflections and stresses may arise from the distortion in both curved and straight beams, and that the complicated distortional behavior of thin-walled curved beam can be predicted with reasonable accuracy by the presented analysis.

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