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COMPUTER SIMULATION STUDIES ON THE FATIGUE LOAD AND FATIGUE DESIGN OF HIGHWAY BRIDGES

By Chitoshi MIKI, Yuji GOTO**, Hitoshi YOSHIDA*** and Takeshi MORI*****

The live loads for fatigue design and maintenance of highway bridges are studied based on computer simulations. Simulations of traffic live loads are carried out under various traffic volumes and constitutions of vehicle types. The type of vehicle, the vehicle weight and the array and headway of vehicles are cosidered as random variables. Bending moment responses of simply-supported one-lane girders of various lengths are calculated under usual traffic conditions. Moment ranges are analysed by using the rain flow method. For the evaluation of fatigue damage, the equivalent moment range and the reduced number of L-20 or T-20 live load are presented.

1. INTRODUCTION

Recently, fatigue cracks have been observed in steel highway bridges in Japan[1),2)]. These have been attributed to the increased use of welded details and lighter weight bridges and to the old age of the bridges. These fatigue damages may be accelerated under current conditions of increased vehicle number and vehicle weight.

The design live load in the Japanese Specifications for highway bridges consists of T-load, L-load and TT-43 load[3)]. These live loads except TT-43 load were set in 1957. Later, many studies were done to accurately determine the design live load on highway bridges focused on overweight trucks and the maximum load[4)-8)]. In order to evaluate the instability, the large permanent displacement or the collapse of steel superstructures, the maximum load which occurred only once in the life of the bridge is the control variable. Kunihiro et al.[6),7)] investigated the influence of overweight trucks on the ultimate condition of bridges by observing traffic flows. Fujino et al.[8)] proposed a new design load which was based on the statistical evaluation of the maximum traffic load using computer simulation.

The Japanese Specifications for railroad bridges[9)] include regulations governing fatigue design. The fatigue design load was used in the design of suspension bridges in Honshu-Shikoku Bridge project[10)]. However, there are no rules on the fatigue design of highway bridges in the Japanese Specifications[3)] except on orthotropic steel decks. To date, only a few studies[11),12)] have been done on the fatigue loadings of highway bridges in Japan. The evaluation of the fatigue strength of highway bridges based on the maximum

* Member of JSCE, Dr. Eng., Tokyo Institute of Technology (2-12-1, O-okayama, Meguro-ku, Tokyo, 152)
** Member of JSCE, Laboratory of Nihon Dohro Kodan (1-4-1, Tadao, Machida-shi, Tokyo, 194)
*** Tokyo Institute of Technology (present adress, Sato Kogyo Co. Ltd., Nihonbashi, Chuo-ku, Tokyo, 103)
**** Member of JSCE, M. Eng., Tokyo Institute of Technology

stress by the T or L-load and the number of vehicles in the bridge's life may provide very conservative results.

The fatigue damage of a steel bridge is caused by stress repetitions of long time. Many fatigue studies indicate that the stress range is the factor most likely to govern the fatigue strength of structural details. Therefore, in order to evaluate the fatigue life of steel bridges, the effect of repetitive loads under usual traffic conditions is essential rather than the unusual maximum load experienced in the bridge's lifetime. A stress range is calculated from the stress-time history of a bridge which includes the effects of vehicle weight, configuration, and the sequence and spacing between successive vehicles. The number and spacing of axles varies with vehicle type. The effect of a particular load will vary depending on the length of the bridge and the types of members in the bridge. On a bridge of short length, the passage of each axle may cause one stress cycle, then the number of stress cycles may be more than the number of vehicles. On a bridge of long length, the probability of having many closely spaced heavy vehicles simultaneously increases, and may result in a large stress range.

The aim of this paper is to present the concepts of fatigue load and fatigue design of highway bridges which can be utilized to provide appropriate safety levels for fatigue failure. From the viewpoint of fatigue, the primary differences between highway and railroad bridges are the types of vehicles and the extent of vehicle over-loading. In this study, traffic flow in one lane of the bridge is expressed as the traffic volume and the constitution of traffic. In the simulation of traffic loads, the sequence and weight of vehicles and the spacing of successive vehicles are the random variables. To evaluate fatigue damage, the concept of equivalent stress range and moment range and reduced number of design live load cycles are introduced.

2. CUMULATIVE FATIGUE DAMAGE

The number of cycles to failure (N) is related to a stress range (S_r) as follows.

$$(S_r)^m \cdot N = C \qquad (1)$$

The linear damage rule proposed by Miner[13] suggests that damage from variable loadings is given by

$$\sum \frac{n_i}{N_i} = 1 \qquad (2)$$

Eqs. (1) and (2) can be used to derive an equivalent stress range (S_{eq}) corresponding to the total variable stress cycles (N_t) as follows.

$$S_{eq} = (\sum (S_{ri})^m \cdot f_i)^{\frac{1}{m}} \qquad (3)$$

S_{ri} : stress range, n_i : frequency of occurrence of S_{ri}, $N_t = Z n_i$

N_i : number of cycles to failure corresponding to S_{ri}

f_i : relative frequency of occurrence of S_{ri} $(=n_i/N_t)$

The relationship expressed by Eqs. (1), (2) and (3) can be visualized graphically as shown in **Fig. 1**.

The relationship between load (L) and stress range (S_r) can be considered linear, then

$$S_r = \alpha\beta L \qquad (4)$$

The factor β, the elastic constant, relates load and stress to a particular location on the structure. The factor α is the ratio of the actual stress range to the calculated stress range under the same load and less than one. Conservative values of α of approximately 0.8 for transverse members and 0.7 for longitudinal members in multiple beams bridge were determined from field studies and used in the AASHTO Specifications[14]. However, many field measurements suggest that the value of α is less than 0.5[15].

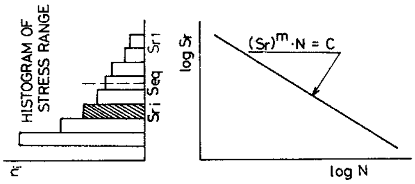


Fig. 1 Linear Damage Law.

Miner's rule yields the following relationship when expressed in terms of Eqs. (1), (2) and (4),

$$\frac{\alpha^m}{C}\cdot N_t\cdot\sum\frac{n_i}{N_t}\cdot(\beta L_i)^m=1 \quad\cdots\cdots (5)$$

The design stress cycles (N_d) corresponding to the design stress range (S_{rd}) provides the same fatigue damage condition as the variable stress (S_{ri}) cycles which occur in the life of bridge. Since

$$(S_{rd})^m\cdot N_d=C \quad\cdots\cdots (6)$$

is given by Eq. (1), and substitution into Eq. (5) yields

$$N_d=N_t\cdot\left(\alpha\frac{S_{eq}}{S_{rd}}\right)^m \quad\cdots\cdots (7)$$

The value N_v is the total number of vehicles which across in the life of bridge, then the fatigue damage caused by the passage of one vehicle is

$$N_r=\frac{N_t}{N_v}\left(\alpha\frac{S_{eq}}{S_{rd}}\right)^m \quad\cdots\cdots (8)$$

In this study, the value N_r is defined as the reduced number of stress cycles corresponding to one vehicle.

The relationship between stress range and moment range may be considered linear, so therefore, the equivalent moment range (M_{eq}) may be expressed

$$M_{eq}=\left(\sum(M_{ri})^m\cdot f_i\right)^{\frac{1}{m}} \quad\cdots\cdots (9)$$

and the reduced number of moment cycles may be expressed

$$N_r=\frac{N_t}{N_v}\left(\alpha\frac{M_{eq}}{M_{rd}}\right)^m \quad\cdots\cdots (10)$$

where, M_{ri} is the variable moment range and M_{rd} is the design moment range. Many fatigue test data suggest that the slope of the S_r-N curve of weldments close to $-1/3$. In this study, the value of $m=3$ is used. Consequently, the equivalent stress range and moment range are defined as the root-mean-cube of the variable stress range and varible moment range, respectively.

3. SIMULATION OF LOAD

Fig. 2 shows the flow of computer simulation. Traffic volume and constitution of traffic are input data in the simulation of the traffic flow in a one lane road. In this study, the passage of 20 000 vehicles is simulated in each model of traffic flow shown in section (2).

(1) Time headway of vehicles

First of all, the array of vehicles with random spacing in one lane is simulated. The distribution of the time headway (the time interval between the head of the first vehicle and the head of the second vehicle) is assumed as an Erlang distribution. The probability density function for Erlang distribution with freedom n is

$$f(t)=\lambda e^{-\lambda t}\cdot(\lambda t)^{n-1}/(n-1)! \quad\cdots\cdots (11)$$

mean : $E(t)=n/\lambda$, variance : $\mathrm{Var}(t)=n/\lambda^2$

Many previous studies suggest that the value of $n=3$ is adequate to estimate the distribution of time headway[16)-18)]. Mean value of time headway is given as

$$E(t)=3\,600/Q \quad\cdots\cdots (12)$$

$E(t)$: (sec.), Q : (vehicles/hour)

where, Q is the traffic volume in one hour. The value of Q is the representative parameter of the density of traffic (closeness of of vehicles). The traffic flow is simulated under Q is 500, 1 000, 1 500 and 2 000 vehicles/hour in one lane. The traffic volume of 2 000 approaches maximum capacity on one lane.

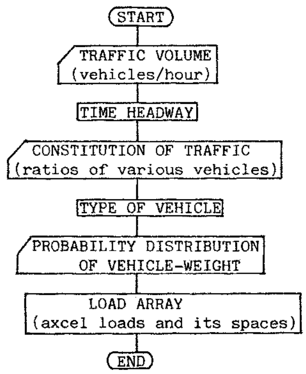


Fig. 2 Flow of Simulation.

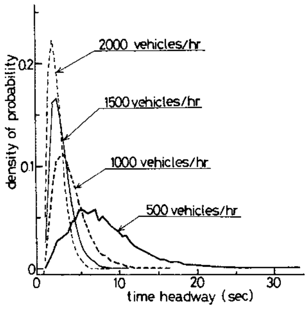


Fig. 3 Distributions of Time Headway.

Table 1 Constitution of Traffic for Simulation.

(%)

traffic model	C	ST	LT	LLT	TT
A	10	5	25	50	10
B	25	5	25	37	8
C	50	5	20	20	5
D	65	5	15	12	3
E	75	12	10	2	1

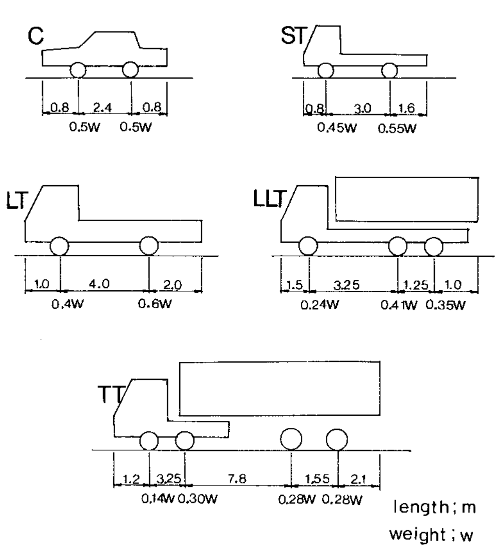


Fig. 4 Configurations and Axle Weight Distributions of Model Vehicles.

The speed of a vehicle varies with the type of vehicle and traffic volume. However, the speed of vehicle is fixed for each traffic volume because of the one lane road. The speed is 100 km/hour for $Q=500$ and 1 000 ; 70 km/hour for $Q=1\,500$; and 50 km/hour for $Q=2\,000$. Fig. 3 shows the distributions of time headways obtained by computer simulation.

(2) Constitution of traffic

The type of vehicles in the array is determined by Monte Carlo simulation, which is based on the constitution of traffic (ratio of various vehicles). For simplicity, the vehicles were lumped into five categories as follows :

(C) : motor car

(ST) : small truck

(LT) : large truck with two axles, (bus)

(LLT) : large truck with three axles

(TT) : trailer truck, (semi-trailer truck)

Fig. 4 shows the configurations and the axle weight distributions of the model vehicles. In the Japanese Specifications for highway bridges[3], the configurations of LT, LLT and TT is T-load, 25 tonf truck with three axles and TT-load, respectively. However, the axle weight distributions of LT and LLT were based on the results of measurements on the Tomei Expressway[19].

Table 1 shows the constitution of traffic according to the model used in this study. The ratios of TT and LLT trucks decrease in the order of A, B, C, D and E. Type-A is modeled the traffic flows on Tomei Expressway after midnight, the major part of this traffic model is constituted by heavy trucks (LT, LLT and TT). Type-C is the model of the usual traffic flows on the Tomei Expressway. Type-E is the model of the traffic flows on the urban expressways such as the Tokyo Metropolitan Expressway and the Hanshin Expressway, the major part of this traffic flow is constituted by motor cars.

(3) Vehicle weight data

Field measured data taken on March 10 and 11, 1983 at Nihon-Daira on the Tomei Expressway are available[19] to decide the distributions of the weights of LT, LLT and TT trucks. All truck weight measurements have been done at the same location since 1972, review of these data shows no significant changes[19].

Considering the scatter of measured data, normal distributions are applied for the weight of LT and

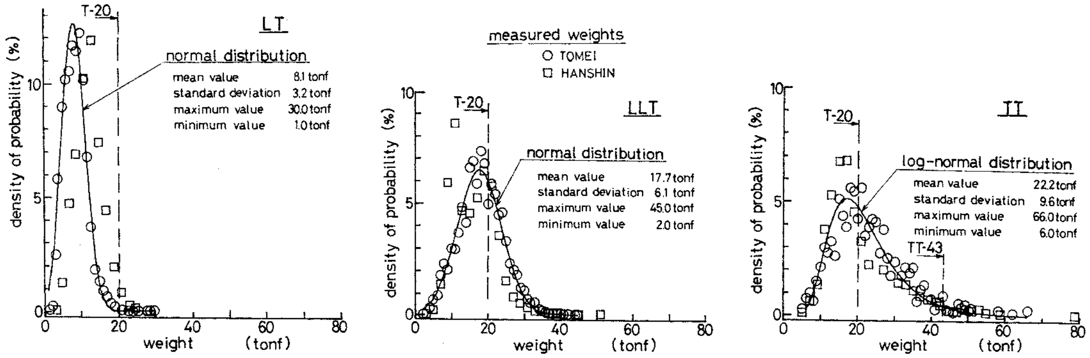


Fig. 5 Distributions of Vehicle Weight.

LLT trucks and log-normal distribution is applied for that of TT trailer trucks. The probability density function of each weight is illustrated in Fig. 5 with the measured weights on the Tomei Expressway. The maximum and minimum values of each distributions adopt measured data. Measured weights on the Hanshin Expressway[20] are also plotted in Fig. 5. There is no significant weight difference between trucks on the Tomei Expressway and Hanshin Expressway. These measured data indicate that the weight of many 3-axles trucks, almost one-half of them, are over T-20 load of the Specifications[3] and the weights of some trailers are over even TT-43 load of the Specifications[3]

The distributions of weights of motor car (C) and small truck (ST) are the same normal distributions as those in Fujino et al.'s study[8], and those are

C : normal distribution, mean 1.2 tonf, variance 0.6 tonf^2, maximum 4.0 tonf, minimum 0.5 tonf

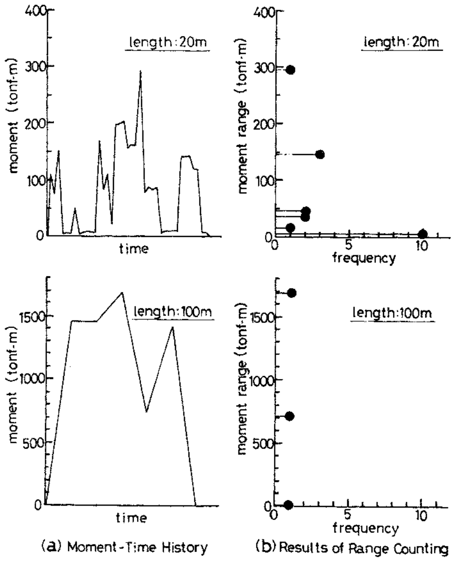
ST : normal distribution, mean 3.1 tonf, variance 3.2 tonf^2, maximum 8.0 tonf, minimum 0.8 tonf

The weight of vehicle is distributed to each axle in proportion to the ratio shown in Fig. 4. Consequently, the array of loads are obtained.

4. CYCLIC MOMENT AND COUNTING METHOD

The influence line is used to obtain the response of bending moment by the passage of an array of loads at the center of a simply supported one lane girder. The stress response of any bridge component is composed of two contributions ; a static and dynamic response. A vehicle crossing a bridge having a perfectly smooth, straight, horizontal surface will produce a nearly static response[21].

The calculation was done for girders of 10, 20, 50 and 100 m long. Fig. 6(a) illustrates the moment-time history in the girders of 20 and 100 m long by the passage of the same array of 23 vehicles. A completely different response occurs in accordance with the length of girder. Because the stress range, not the maximum stress, is the primary factor for the fatigue strength of bridge members, moment range has to be analysed from moment-time history. The rain flow counting method is recognized as the most appropriate



(a) Moment-Time History (b) Results of Range Counting

Fig. 6 Examples of Moment-Time History and Range Counting.

method for the evaluation of fatigue damage[22]. Fig. 6 (b) illustrates the results of counting to the moment-time history in Fig. 6 (a) by using the rain flow counting method.

5. THE MAXIMUM MOMENT AND EQUIVALENT MOMENT RANGE

In Fig. 7, examples of histograms for the moment ranges of a 20 m long girder obtained by computer simulations are presented. The superscript A, C and E of moment (M) denotes the model of the constitution of traffic, and the subscript (max) and (eq) denotes the maximum moment range and the equivalent moment range, respectively. The maximum moment range obtained by rain-flow method is equal to the maximum moment occurred by the passage of this array of vehicles in this study (see Fig. 6). The maximum moment and equivalent moment range vary with the traffic volume and the constitution of traffic.

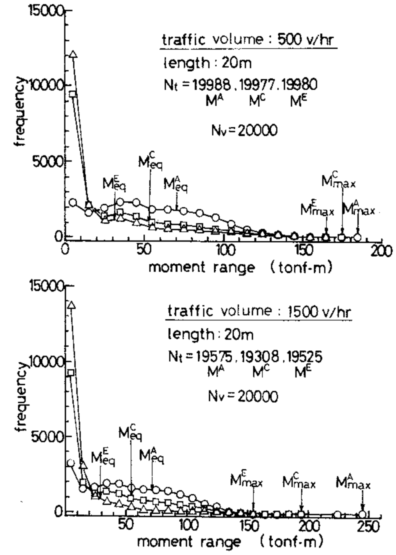


Fig. 7 Examples of Histograms for Moment Range.

Fig. 8 shows the maximum moment and the equivalent moment range of each girder. The values of M_T and M_L in Fig. 8 are the maximum moments caused by T-20 and L-20 design live loads. In the calculation of M_L, the lane width is assumed to be 3 m. All of the maximum moments except some of the 100 m girders are higher than the moment by L-20 load (M_L) regardless of the traffic volume and the constitution of traffic. Overweight trucks are responsible for these results. The probability of having many heavy trucks simultaneously on a bridge of short length is very low, so therefore, the maximum moment is determined by the maximum load in the array of loads. The value of the maximum moment increases as the ratio of the heavy trucks (LT, LLT and TT) in traffic flows increases and the traffic volume increases.

In all cases, the equivalent moment range is well below the L-20 load moment (M_L), and these do not

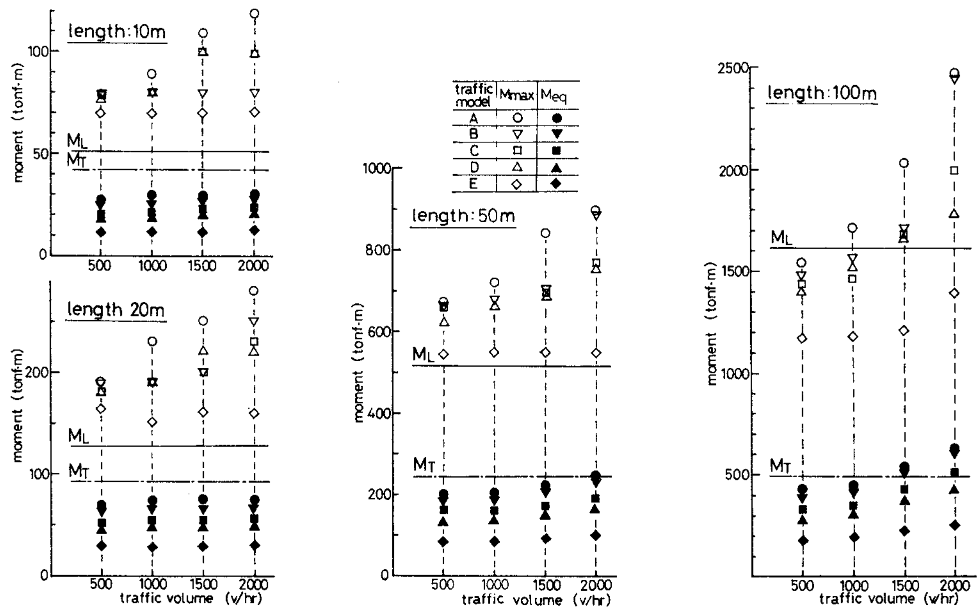


Fig. 8 Maximum Moment and Equivalent Moment Range.

exceed even the moment by one T-20 load (M_T) except some of the 100 m long girder. The value of equivalent moment range (M_{eq}) varies not so much with traffic volumes in each traffic constitution model, regardless of the increase of the maximum moment (M_{max}). These results suggest that the constitution of traffic is the primary variable determining the equivalent moment range.

The fatigue life of the bridge member is evaluated by using the equivalent stress range which is calculated from the equivalent moment range, the modulus of section and the factor α, and the fatigue life (S_r-N) curve corresponding to the joint.

6. REDUCED CYCLES

The number of cycles of design loads as L-20 or T-20 load which cause same fatigue damage as that by the passage of one vehicle in various vehicles was obtained by using Eq. (10). This number of cycles of L-20 or T-20 load is defined as the reduced cycles of L-20 load or T-20 load. Usually, the L-20 or T-20 load stress is known, so therefore, the evaluation of the fatigue damage can be done simply by using the reduced cycles of L-20 or T-20 load and traffic data on the bridge.

A value of α equal to 1.0 is used in this calculation for simplicity. The value of reduced cycles is proportional to the α of the power of m ($m=3$ in this study).

(1) Reduced cycles of L-20 load

Fig. 9 shows the reduced cycles of L-20 load (N_{rL}). The values of N_{rL} are extremely low and vary with the girder length and the constitution of traffic. For each girder, the ratio of the N_{rL} value of traffic model A to that of model E is approximately 10. The value of N_{rL} of a 100 m long girder is about 0.1 times as much as that of a 10 m long girder.

The number of cycles of L-20 load which corresponds to all loading cycles in the design life can be obtained by using N_{rL} values and the traffic data ; the constitution of traffic and traffic volumes on the bridge. Table 2 shows the calculation of the number of cycles of L-20 load which corresponds to the daily traffic flows in the first lane of the Tomei Expressway, east bound, at Yamato[19]. For convenience, one day was divided into eight time ranges of three hours (column ①). Column ② indicates the model of the constitution of traffic selected from Table 1 according to the traffic data. Column ③ shows the measured traffic volume during the time range of three hours. The values of N_{rL} in column ④ are derived from Fig. 9 and traffic data of columns ② and ③. In column ⑤, the values of ③ x ④ give the number of cycles of L-20 load to the traffic in each time range. The total value of column ⑤ (the value of 1114.9 for the length of 20 m and 125.1 for the length of 100 m) is the number of cycles of L-20 load to the one day traffic of 14 267 vehicles. The total number of cycles in the design life can be obtained by multiplying these values by number of days in the design life. For example, if the design life is 50 years, the total number of cycles of L-20 load would be

$$\begin{aligned} &1\,114.9\times365\times50=2.0\times10^7 \text{ cycles : length of 20 m} \\ &125.1\times365\times50=2.3\times10^6 \text{ cycles : length of 100 m} \end{aligned} \qquad (13)$$

Since the traffic volume and constitution of traffic will change in the design life, these changes can be accounted for by altering the corresponding value of N_{rL}.

When the value of α of 0.5 is assumed, the total number of cycles of L-20 load is reduced as follows.

$$\begin{aligned} &2.0\times10^7\times(0.5)^3=2.5\times10^6 \text{ cycles : length of 20 m} \\ &2.3\times10^6\times(0.5)^3=2.9\times10^5 \text{ cycles : length of 100 m} \end{aligned} \qquad (14)$$

The influence of α is remarkable.

The fatigue design of bridge members can be done by using L-20 load, the total number of cycles in design life shown here and design fatigue life curve for each details.

(2) Reduced cycles of T-20 load

Fig. 10 shows the reduced cycles of T-20 load (N_{rT}). The constitution of traffic is the primary variable responsible for the value of N_{rT}. The influence of traffic volume is insignificant. If a traffic volume of

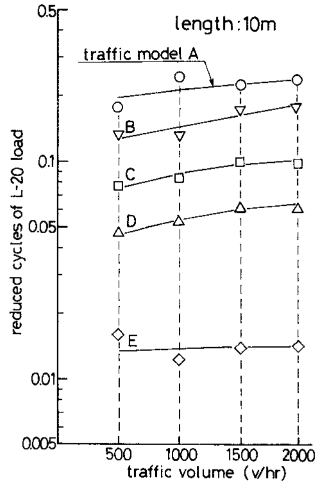


Table 2 Calculation of the Number of Cycles of L-20 Load.

① time	② traffic model	③ traffic volume (3hrs)	length:20m ④ N_{rL}	length:20m ⑤ ③x④	length:100m ④ N_{rL}	length:100m ⑤ ③x④
1- 4	A	681	0.160	108.8	0.018	12.3
4- 7	B	1520	0.120	182.4	0.014	21.3
7-10	(C+D)/2	2303	0.060	138.2	0.0065	15.0
10-13	C	1521	0.070	106.5	0.0080	12.2
13-16	C	2631	0.075	197.3	0.0082	21.6
16-19	D	3070	0.050	153.5	0.0053	16.3
19-22	C	1534	0.070	107.4	0.0080	12.3
22- 1	B	1007	0.120	120.8	0.014	14.1
total		14267		1114.9		125.1

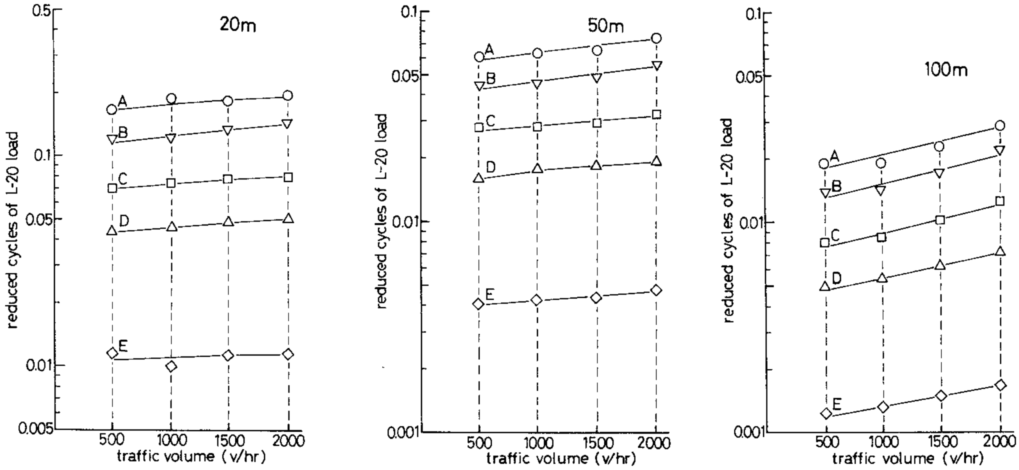


Fig. 9 Reduced Cycles of L-20 Load.

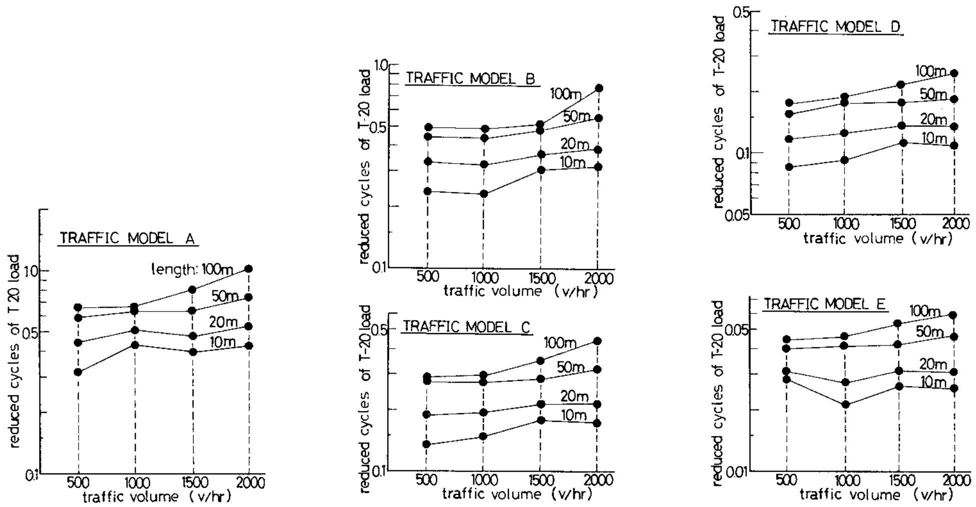


Fig. 10 Reduced Cycles of T-20 Load.

2 000 vehicles per hour is excepted due to rare traffic conditions, the conservative value of N_{rT} for each model of the constitution of traffic can roughly be estimated independent of girder length. These values are 0.7, 0.5, 0.3, 0.2 and 0.05 for model A, B, C, D and E, respectively. The ratio of heavy trucks (LT, LLT and TT) are 0.85, 0.7, 0.45, 0.3 and 0.13 for model A, B, C, D and E, respectively. The values of N_{rT} are substantially lower than the ratio of heavy trucks regardless of many illegal overweight trucks.

7. CONCLUDING REMARKS

The load for the fatigue design of highway bridge is studied based on computer simulations. The traffic flow is represented by the traffic volume and the constitution of traffic, and the type of vehicle, the weight of vehicle and the sequence and headway of vehicles are considered as random variables.

(1) Equivalent moment range which is a characteristic value for fatigue damage is defined as the root mean cube of the variable moment range. The equivalent moment ranges are significantly lower than the moment range by L-20 load and do not exceed even the moment range by single T-20 load in nearly all traffic conditions on girders.

(2) Reduced cycles of L-20 and T-20 loads which cause the fatigue damage equal to that caused by the passage of one vehicle in the traffic flow, are presented for the fatigue design. The values of the reduced cycles of L-20 load are extremely low and vary with the girder length and the constitution of traffic. The values of the reduced cycles of T-20 load are substantially lower than the ratio of the heavy truck number to the total vehicle number, regardless of some existence of illegal overweight trucks.

In order to provide more reliable results which can be used in the fatigue design of bridge members, additional studies are required. Further studies should investigate various dynamic effects and the effects of multiple vehicles on the bridge with multiple lanes. Also, more field measurements of stresses have to be performed to evaluate the value of α.

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