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A PROBABILISTIC BASIS FOR FRACTILE-BASED STRUCTURAL DESIGN

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For rational reliability based structural design, it is important to accomplish the target probability of failure as accurate as possible. From this standpoint, this paper presents a probabilistic basis for the fractile-based structural design, and demonstrates the advantage of the method. Numerical examples indicate the superiority of the proposed fractile method to the second moment method. It is also noted that, by the independent determination of the design points from the linearization procedure, the proposed method has proved much simpler than the improved second moment method proposed by Rackwitz et. al., retaining its accuracy at a similar level.

1. INTRODUCTION

A reliability-based structural design is manipulated practically in a form that specific design values are determined rationally for their corresponding random design variables such as load effects and strengths of structures, reflecting their own probabilistic distributions. This idea has recently been incorporated for practical use in some of the existing design specifications[1)~4)], and their specification writing formats appear to be somewhat different from the conventional format of the allowable stress design.

Since the direct use of the probability of failure as an index to control the safety of structures involves the difficulty to manipulate the design format of practical purpose[5)], an alternative method called the second moment method proposed by Cornell[6)], Lind[7)], and others has played an important role as a basis for reliability-based design formats of practical use.

Some of the existing design specifications mentioned have directly adopted the concept of the safety index proposed in the second moment method, and others, most of which are implemented in a form of the partial safety factor format, seem to be influenced strongly by the second moment method for determining the partial safety factors.

According to the second moment method, the target probability of failure P_{fD} is given by the safety index β as

$$P_{fD}=\Phi(-\beta) \qquad (1)$$

in which $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}\,dt$ is the cumulative probability distribution function of standard normal

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distribution. Since the second moment method employs some ristrictions as assumptions in its own formulations, however, the correspondence of P_{fD} with the realized probability of failure P_{fR} may not always be good.

In the previous paper[8], two of the authors have pointed out the fallacy of the second moment method including poor correspondence with the realized probability of failure and have proposed an alternative method called the fractile method which utilized the concept of exceeding probabilities to determine the design values of random design variables. It is roughly said that the proposed fractile method happens to resemble the implicit concept of the conventional allowable stress design format. It has also been made clear that the exceeding probability e called the fractile of each variables used as the measure of safety controls the realized probability of failure much better than the safety index β and hence can be preferably replaced instead of β as the means of determining design values.

However, the previous paper lacks to present the relationship, which is corresponding to Eq.(1) for the second moment method, between the target probability of failure and fractiles. So, it has not been clear how the fractile e_i of each random variable X_i can be selected when the target probability of failure P_{fD} is given. This relation is definitely needed to apply the proposed faractile method for practical design purpose.

This study presents an approximate relation between the target probability of failure P_{fD} and the fractile e_i which is derived on a theoretical basis similar to the second moment method. Then, the correspondence between P_{fD} and P_{fR} for the proposed fractile method is examined, using a variety of probabilistic distributions for each variable with distribution parameters being changed.

An advantage to use e_i as a measure of the probability of failure as opposed to the safety index β is that e_i can naturally reflect the information of the tail part of distributions of each variable which seriously affects the the realized probability of failure P_{fR}. Fot this reason, assuming that sufficient data are available for random variables concerned and thus the accuracy of parameter estimation is possible, an extended analysis is also made incorporating higher order moments of distributions to reflect theoretically the effects of the tail-shapes of real distributions, and then some numerical examples are given.

Relating to the higher order moments of distributions, Rackwitz[9], and Paloheimo and Hannus[10], have improved the second moment method in its own framework in which the effects of the tail-shapes of distributions are successfully incorporated in order to overcome the drawbacks of the original second moment method mentioned before. The accuracy and practical simplicity of the proposed fractile method are compared with this Rackwitz's method.

2. APPROXIMATED RELATION BETWEEN PROBABILITY OF FAILURE AND FRACTILES

It is postulated that the limit state of structures such as static collapse and excessive vibration is expressed mathematically as

$$Z_j = Z_j(x_1, x_2, \cdots\cdots, x_n) = 0, \quad j = 1, 2, \cdots\cdots, m \qquad (2)$$

in which each Z_j is a limit state function defined such that $Z_j>0$, $Z_j<0$, and $Z_j=0$ represent states of safe, failure and its critical condition, respectively, $X=(X_1, \cdots\cdots X_n)$ are design variables relating to the j-th limit state and are regarded as random variables, and m is a number of limit states to be considered in designing structures. Since the intended discussions are common to each of Z_j, subscript j is omitted for the simplicity of writing.

The realized probability of failure P_{fR} can be obtained by integrating the joint probability density function $f_{X_1, \cdots\cdots, X_n}(x_1, \cdots\cdots, x_n)\, dx_1, \cdots\cdots dx_n$ of design variables $X_1, \cdots\cdots X_n$ in the failure region of $Z(X)<0$. In general, the boundary of $Z(X)=0$ is a nonlinear function with respect to $X_i\,(i=1, \cdots\cdots, n)$ and further X_i itself is not always independent each other and, even if independent, is subject to a wide variety of distribution shapes. However, an approximated probability of failure can be obtained by linearizing the limit state function at the point on the boundary and utilizing the well-known assumptions that all the X_i's are mutually independent and are normally distributed.[9]

Expanding the function Z into Taylor series at a point $\boldsymbol{X}^0$ which satisfies $Z(\boldsymbol{X}^0)=0$ and preserving the only linear terms, the linearized function Z^0 is given as

$$Z^0(\boldsymbol{X})=\sum_{i=1}^{n}\frac{\partial Z}{\partial X_i}\bigg|_{X^0}(X_i-X_i^0) \quad (3)$$

Utilizing the assumptions mentioned before, Z^0 is also normally distributed and its mean $\overline{Z}^0$ and variance $\sigma_{Z^0}^2$ are given respectively as

$$\overline{Z}^0=\sum_{i=1}^{n}\frac{\partial Z}{\partial X_i}\bigg|_{X^0}(\overline{X}_i-X_i^0) \quad (4)$$

$$\sigma_{Z^0}^2=\sum_{i=1}^{n}\left(\frac{\partial Z}{\partial X_i}\bigg|_{X^0}\sigma_{X_i}\right)^2 \quad (5)$$

in which $\overline{X}_i$, $\sigma_{X_i}^2$ are the mean and variance of each variable X_i respectively.

The probability that Z^0 is not positive is given as

$$P_r[Z^0\leqq 0]=\Phi\left(-\frac{\overline{Z}^0}{\sigma_{Z^0}}\right) \quad (6)$$

which can be regarded as an approximate probability of failure, and hence is called the target (prescribed), probability of failure, P_{fD}, that is

$$P_{fD}=\Phi\left(-\frac{\overline{Z}^0}{\sigma_{Z^0}}\right) \quad (7)$$

Meanwhile by selecting a point $\boldsymbol{X}^{0*}=(X_1^{0*},\ X_2^{0*},\ \cdots\cdots,\ X_n^{0*})$ which satisfies $Z^0(\boldsymbol{X}^{0*})=0$ and may not necessarily be equal to $\boldsymbol{X}^0$, the linearized limit state can be rewritten as

$$\overline{Z}^0(\boldsymbol{X})=\sum_{i=1}^{n}\frac{\partial Z}{\partial X_i}\bigg|_{X^0}(X_i-X_i^{0*}) \quad (8)$$

and $\overline{Z}^0$ as

$$\overline{Z}^0=\sum_{i=1}^{n}\frac{\partial Z}{\partial X_i}\bigg|_{X^0}(\overline{X}_i-X_i^{0*}) \quad (9)$$

The fractile e_i is defined for corresponding X_i^{0*} as

$$e_i=\begin{cases} P_r[X_i\leqq X_i^{0*}] & when\ \dfrac{\partial Z}{\partial X_i}\bigg|_{X^0}>0 \\ P_r[X_i\geqq X_i^{0*}] & when\ \dfrac{\partial Z}{\partial X_i}\bigg|_{X^0}\leqq 0 \end{cases} \quad (10)$$

Eq. (10) indicates that e_i is a lower fractile for variables such as strength of structures and an upper fractile for variables such as load effect. For this context, e_i is tacitly presumed very small. The assumption that each X_i is normally distributed leads to

$$\overline{X}_i-X_i^{0*}=\begin{cases} -\Phi^{-1}(e_i)\sigma_{X_i} & when\ \dfrac{\partial Z}{\partial X_i}\bigg|_{X^0}>0 \\ \Phi^{-1}(e_i)\sigma_{X_i} & when\ \dfrac{\partial Z}{\partial X_i}\bigg|_{X^0}\leqq 0 \end{cases} \quad (11)$$

in which $\Phi^{-1}(\bullet)$ is the inverse of $\Phi(\bullet)$ defined for Eq. (1).

Substituting Eq. (11) into Eq. (9) leads to

$$\overline{Z}^0=-\sum_{i=1}^{n}\left|\frac{\partial Z}{\partial X_i}\bigg|_{X^0}\right|\Phi^{-1}(e_i)\,\sigma_{X_i} \quad (12)$$

Then, the target probability of failure of Eq. (7) is expressed using Eqs. (5) and (12) as

$$P_{fD}=\Phi\left[\frac{\sum_{i=1}^{n}\left|\frac{\partial Z}{\partial X_i}\right|_{X^0}\Phi^{-1}(e_i)\,\sigma_{X_i}}{\left\{\sum_{i=1}^{n}\left(\frac{\partial Z}{\partial X_i}\Big|_{X^0}\sigma_{X_i}\right)^2\right\}^{1/2}}\right] \quad\cdots\cdots (13)$$

which finally relates P_{fD} with fractiles e_i in the same level of approximation as the second moment method.

Incidentally, the relation given as

$$\mu^{(k)}=\frac{k!}{2^{k/2}\left(\frac{k}{2}\right)!}\sigma^k \qquad k=2,4,\cdots\cdots \quad\cdots\cdots (14)$$

holds between the even-order central moment $\mu^{(k)}(k=2,4,\cdots\cdots)$ and the standard deviation σ for normally distributed variables.[11] Thus, Eq. (13) can alternatively be rewritten

$$P_{fD}=\Phi\left[\frac{\sum_{i=1}^{n}\left|\frac{\partial Z}{\partial X_i}\right|_{X^0}\Phi^{-1}(e_i)\sqrt[k]{\frac{2^{k/2}\left(\frac{k}{2}\right)!}{k!}\mu_i^{(k)}}}{\left\{\sum_{i=1}^{n}\left(\frac{\partial Z}{\partial X_i}\Big|_{X^0}\sqrt[k]{\frac{2^{k/2}\left(\frac{k}{2}\right)!}{k!}\mu_i^{(k)}}\right)^2\right\}^{1/2}}\right] \quad\cdots\cdots (15)$$

3. DETERMINATION OF DESIGN VALUES

The condition for a set of fractiles $e=(e_1,\cdots\cdots,e_n)$ to satisfy the given target probability of failure P_{fD} can be obtained by Eq. (13) when linearizing point is designated for a nonlinear limit state function. An appropriate selection*) for respective e_i's from acceptable e which satisfies Eq. (13) makes it possible to determine each design values X_i^* reflecting the actual distributions of variables, as

$$X_i^*=F_{X_i}^{-1}(P_i) \quad\cdots\cdots (16)$$

in which

$$P_i=\begin{cases} e_i & when \ \frac{\partial Z}{\partial X_i}\Big|_{X^0}>0 \\ 1-e_i & when \ \frac{\partial Z}{\partial X_i}\Big|_{X^0}\leq 0 \end{cases} \quad\cdots\cdots (17)$$

and $F_{X_i}(\bullet)$ is an actual cumulative distribution function of X_i and $F_{X_i}^{-1}(\bullet)$ is its inverse.

Discussed hitherto is a rather general method to determine the design values using fractiles for arbitrary nonlinear limit state functions. For practical application, however, it is common that design variables can be separated explicitly into 2 groups, that is, strength $(\partial Z/\partial X_i>0)$ and load effect $(\partial Z/\partial X_i<0)$ represented as R and S respectively, and then it is regarded as safe, failure and its critical condition when $R>S$, $R<S$ and $R=S$, respectively. Although studies[12,13] have been made so far for the treatment of the nonlinear function with respect to R and S such as $Z=\ln R-\ln S$, it is rather essential for most of the actual designs that the boundary between safe region and failure region on the $R-S$ plane is linear, in other words, $Z=0$ is the linear relation between R and S. Partly for this reason, a linear limit state function with two design variables $Z=R-S$ is used for a numerical exmaple to demonstrate the effectiveness of the proposed fractile method.

Linearization is not needed for $Z=R-S$, and Eq. (13) leads to

$$P_{fD}=\Phi\left[\frac{\Phi^{-1}(e_R)\sigma_R+\Phi^{-1}(e_S)\sigma_S}{\sqrt{\sigma_R^2+\sigma_S^2}}\right] \quad\cdots\cdots (18)$$

A set of e_R and e_S which satisfies Eq. (18) is selected appropriately and then design values R* and S*

* Since there is only one equation (13) for n-variables of e to satisfy, selection for the respective value of e_i is rather arbitrary. Although the unique selection for e_i is made possible from the optimality consideration reflecting the difference of the number of data available, it may be preferably left to individual judgment for practical implementation.

defined as

$$P_r[R \leqq R^*] = e_R$$
$$P_r[S \geqq S^*] = e_S \quad \cdots\cdots (19\cdot\text{a, b})$$

are determined using the actual distributions as given by Eq.(16). Although actual design is said satisfactory when the following inequality condition as

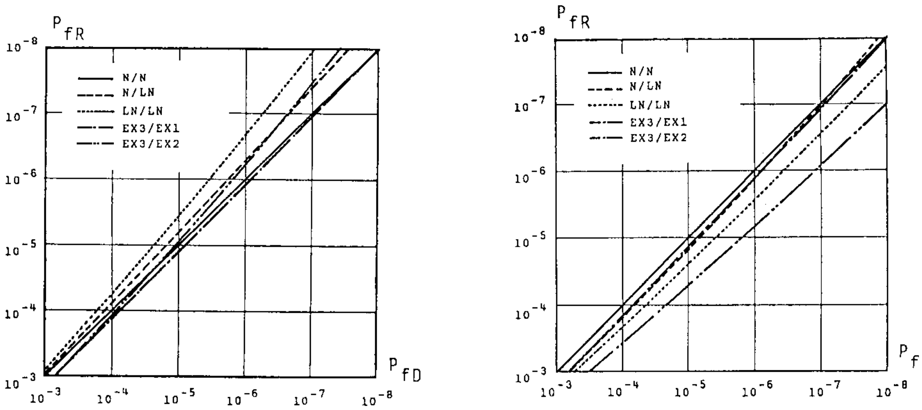
$$S^* \leqq R^* \quad \cdots\cdots (20)$$

is satisfied, the realized probability of failure P_{fR} is evaluated, assuming that $S^* = R^*$ is accomplished in design, by

$$P_{fR} = \int_{-\infty}^{\infty} f_S(t) F_R(t)\, dt \quad \cdots\cdots (21)$$

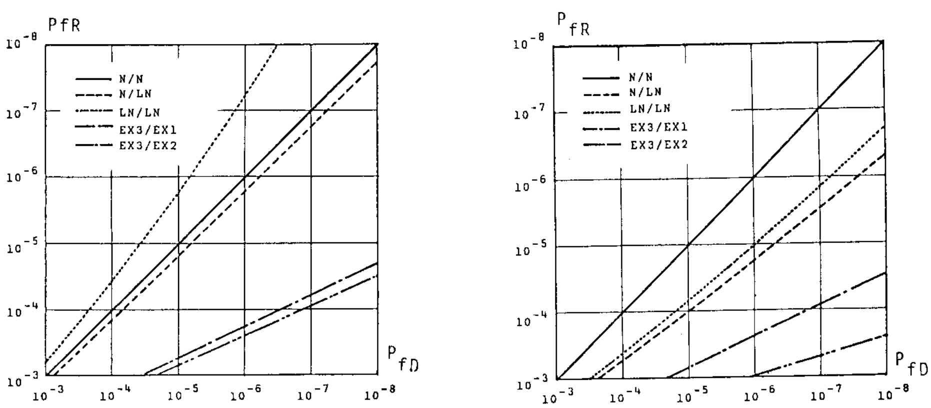
When the difference between P_{fR} evaluated by Eq. (21) and the prescribed P_{fD} in Eq. (18) is found to be smaller, the proposed fractile method is recognized as better method to control the probability of failure.

The realized probability of failure of Eq.(21) naturally differs depending on a selection of (e_R, e_S). For numerical demonstration in this study $e_R = e_S = e$ is assumed for a variety of mathematical probabilistic distributions on R and S. The results are shown in **Fig.1** in which N/N et.al. represent the combinations of distributions of R/S and among them N, LN, EX1, EX2 and EX3 indicate normal,



(a) V_R=0.10, V_S=0.10 (b) V_R=0.10, V_S=0.20

Fig. 1 Correspondence between Target and Realized Probabilities of Failure: Fractile Method.



(a) V_R=0.10, V_S=0.10 (b) V_R=0.10, V_S=0.20

Fig. 2 Correspondence betwen Target and Realized Probabilities of Failure: Second Moment Method.

logarithmic normal, extreme-I type (Gumbel), extreme-II type (Frechet) and extreme-III type (Weibul) distributions, respectively. Symbols V_R and V_s are the coefficients of variation of R and S respectively. In a similar way, computations are also made in comparison for the second moment method and the results are shown in Fig. 2 plotting P_{fD} of Eq.(1) as abscissa. Comparison of the results indicates that the superiority of the proposed fractile method is obvious from the view of the correspondence between P_{fD} and P_{fR}. As has been pointed out in the previous paper, the difference between P_{fD} and P_{fR} becomes greater when the actual distributions of each variables deviate more from the normal distribution, particularly influenced by the tail-shapes of distributions. The reason why the accuracy of the proposed fractile method is superior to the second moment method results from the fact that the very tail-shapes of actual distributions are effectively incorporated in the determinations of design values, as evident from Eq. (16). Incidentally, the second moment method can be understood to determine the design values for a particular combination of e_R and e_s of Eq. (18) by

$$\begin{aligned} R^* &= \overline{R} + \Phi^{-1}(e_R)\sigma_R \\ S^* &= \overline{S} - \Phi^{-1}(e_S)\sigma_S \end{aligned} \qquad (22\cdot\text{a,b})$$

instead of Eq.(19) combined with Eq.(16) in the proposed fractile method.

4. INCLUSION OF HIGHER ORDER MOMENTS OF DISTRIBUTIONS

Eq.(18) itself is nothing but a different expression of the second moment method and thus exact only for the case that all the variables are normally distributed. As mentioned before, the accuracy of the target probability of failure P_{fD} is expected to be improved when the information of the tail-shapes of distributions can be further incorporated into the approximate equation (18). Higher order central moments of each variable can successfully be used for this purpose, reflecting the shapes of distributions far from the center of the distributions more effectively than the second moment. For this reason, Eq.(14) is substituted into the standard deviations σ_R and σ_S in Eq.(18), and thus P_{fD} is expressed as

$$P_{fD} = \Phi\left[\frac{\Phi^{-1}(e_R)\sqrt[k]{\dfrac{2^{k/2}\left(\frac{k}{2}\right)!}{k!}\mu_R^{(k)}} + \Phi^{-1}(e_S)\sqrt[k']{\dfrac{2^{k'/2}\left(\frac{k'}{2}\right)!}{k'!}\mu_S^{(k')}}}{\left\{\left(\sqrt[k]{\dfrac{2^{k/2}\left(\frac{k}{2}\right)!}{k!}\mu_R^{(k)}}\right)^2 + \left(\sqrt[k']{\dfrac{2^{k'/2}\left(\frac{k'}{2}\right)!}{k'!}\mu_S^{(k')}}\right)^2\right\}^{1/2}}\right]$$

$$k,\ k'=2,4,6,\cdots \qquad (23)$$

incorporating the higher order central moments $\mu_R^{(k)}$ and $\mu_S^{(k')}$. It should be noted that $\mu^{(k)}$ does not coincide with $\dfrac{k!}{2^{k/2}\left(\frac{k}{2}\right)!}\sigma^k$ for arbitrary distributions, in general.

Remembering that only one side of the tails of distribution which is lower side for strength and upper side for load effect substantially influences the probability of failure, information of the other side should be excluded for the evaluation of $\mu^{(k)}$. Thus, here in this study, a convenient method is used to evaluate higher order moments, in which only the substantial half side from the median of the distribution is considered as expressed by

$$\mu^{(k)} = \begin{cases} 2\int_{X^M}^{\infty}(x-X^M)^k f_x(x)dx & when\ X^* > X^M \\ 2\int_{-\infty}^{X^M}(x-X^M)^k f_x(x)dx & when\ X^* \leq X^M \end{cases} \qquad (24)$$

in which X^M is the median of variable X. In other words, Eq.(24) is understood such that $\mu^{(k)}$ is evaluated for the fictitious symmetric distribution composed of the substantial half side of the actual distribution. It is for this reason that k and k' in Eq.(23) (also Eq.(15)) are counted only by even numbers.

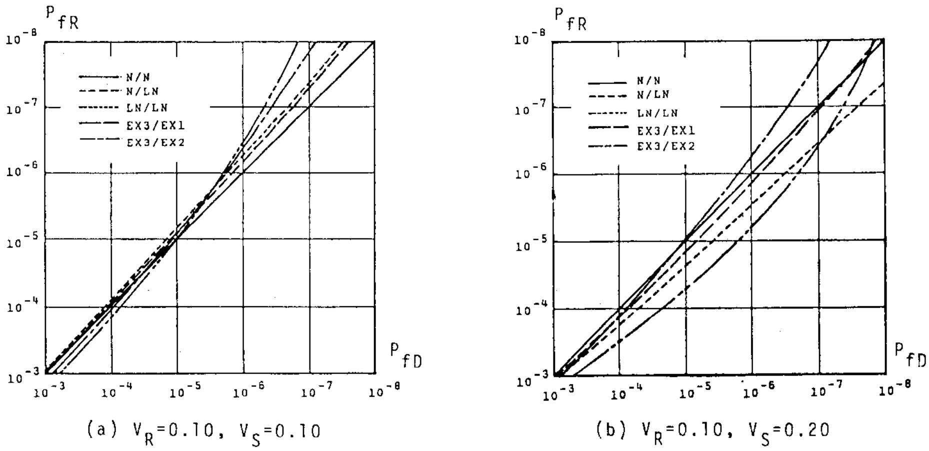


Fig. 3 Correspondence between Target and Realized Probabilities of Failure: Fractile Method with Higher Order Moment.

Numerical computation is made and the results with $k=k'=6$ are shown in Fig.3 for the same combinations of distributions and parameters as the previous examples. An improvement of the accuracy can be observed in comparison with Fig.1.

5. COMPARISON WITH RACKWITZ'S METHOD

A comparison between the second moment method and the proposed fractile method has been made and then an improvement of the accuracy for the fractile method has been tried, taking more precise information of tail-shapes of distributions into account. Since Rackwitz[9] and Paloheimo, et.al.[10] also have reflected the tail-shape sensitivity of distributions within the framework of the second moment method, as mentioned in Introduction, a further investigation is needed to make clear of the effectiveness and simplicity of the proposed method.

As Rackwitz has pointed out,[9] Paloheimo's method is interpreted such that the actual distribution of each variable is replaced to the normal distribution which has the same mean of variable and the same fractile integrated probability at a specified design value as those for the actual distribution, not to the normal distribution of the same mean and variance as used for the original second moment method. Denoting the mean and the variance of the replaced normal distribution as $\overline{X}_i^N$, σ_i^{N2} respectively, the statement above is expressed mathematically as

$$\overline{X}_i^N=\overline{X}_i$$
$$F_{X_i}(X_i^*)=P_i=\Phi\left(\frac{X_i^*-\overline{X}_i^N}{\sigma_i^N}\right) \qquad (25\cdot a,b)$$

in which P_i is defined by Eq.(17) and thus σ_i^N is given by

$$\sigma_i^N=\frac{X_i^*-\overline{X}_i}{\Phi^{-1}(P_i)}\,(\neq\sigma_i) \qquad (26)$$

Therefore, it is understood that Paloheimo's method adopts σ_i^N of Eq.(26) for the evaluation of the target probability of failure of Eq.(13).

Rackwitz has further improved the inclusion of the influences of the tail-shapes in a similar idea as Paloheimo, but with the difference that the replaced normal distribution has the same values of the fractile integrated probability and the probability density function at a specified design value as the actual distribution. This is expressed in place of Eq. (25) as

$$F_{X_i}(X_i^*)=P_i=\Phi\left(\frac{X_i^*-\overline{X}_i^N}{\sigma_i^N}\right)$$

$$f_{X_i}(X_i^*)=\varphi\left(\frac{X_i^*-\overline{X}_i^N}{\sigma_i^N}\right)\cdot\frac{1}{\sigma_i^N} \qquad \text{(27·a,b)}$$

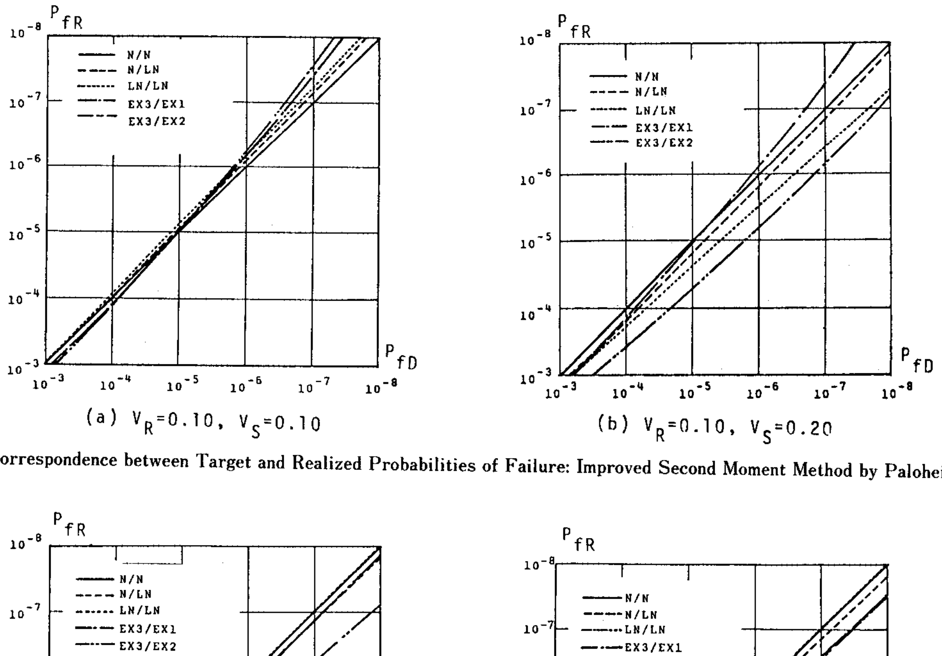
in which $\varphi(\bullet)$ denotes a standard normal probability density function. Thus, σ_i^N of Rackwitz's method is given by

$$\sigma_i^N=\frac{\varphi\{\Phi^{-1}(P_i)\}}{f_{X_i}(X_i^*)} \qquad (28)$$

and the target probability of failure of Eq. (13) is evaluated using Eq. (28)

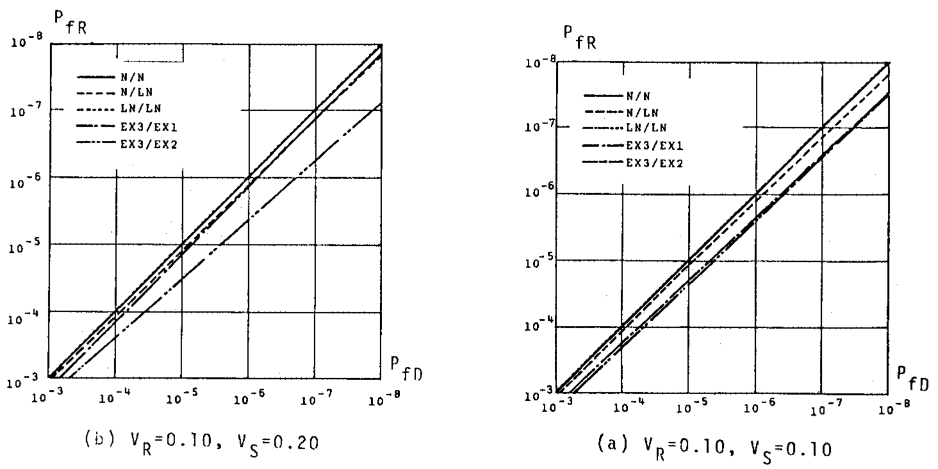
Both of the improved methods cited above have incorporated the fractile information of the actual distributions at the design values and therefore are similar to the proposed fractile method in the sense that the actual fractile probability is considered for the evaluation of Eq. (13).

For comparison with the proposed fractile method of Fig.3 which reflects higher order moments of distributions, numerical examples are shown for Paloheimo's method of Eq. (26) and Rackwitz's method of Eq. (28) in Fig.4 and Fig.5 respectively. Higher accuracy of the target probability of failure with the realized value is observed, to a certain extent, in Rackwitz's method in this limited computation. However, since Paloheimo and Rackwitz's methods take the design value at the same point as the linearization point of Eq.(3), simply according to the original second moment method, which gives the shortest distance between the origin and the failure domain in the normalized space[14], an iterative pro-



(a) V_R=0.10, V_S=0.10 (b) V_R=0.10, V_S=0.20

Fig. 4 Correspondence between Target and Realized Probabilities of Failure: Improved Second Moment Method by Paloheimo et.al.



(b) V_R=0.10, V_S=0.20 (a) V_R=0.10, V_S=0.10

Fig. 5 Correspondence between Target and Realized Probabilities of Failure: Improved Second Moment Method by Rackwitz.

cedure is necessary to evaluate the design values X_i^* as clear by the interaction between Eq. (13) and Eq. (26) or (28). On the other hand, the proposed fractile method has separated the evaluation of fractile e_i including the linearization with the determination of the design values X_i^*, as evident from Eq. (16) which is free from Eq. (13). Then, iteration is not required, and calculation turns to be much simpler.

6. DISCUSSION

All the results presented in this paper have been given in the form of the relations between the target and the realized probabilities of failure as shown in Fig.1～5. A reliability-based structural design may effectively be attained by better agreement between them, using a rather simpler method for the determination of design values for practical implementation. For this context, the proposed fractile method is superior in terms of accuracy to the original second moment method which only reflects the mean and variance of distributions, and also is superior to Paloheimo's and Rackwitz's methods in terms of simplicity.

In the proposed method, the linearization of the limit state function which indicates the boundary between the safe and failure domains is separated from the selection of the design point. The main interest of this study is given to the latter subject which is related to the mathematical representation after Eq. (8), characterized by the non-interactive treatment between Eqs. (13) and (16).

Eq. (13) itself is the result of linearization for the limit state function, and is essentially within the framework of the second moment method. However, the point X_i^0 of linearization is arbitrary in the proposed method, and secondly an intended design point X_i^{0*} can be assigned only with the consideration of the target probability of failure on the linearized surface, irrespective of X_i^0 and finally the design point X_i^* is determined directly from Eq. (16), which is naturally different both from X_i^0 and X_i^{0*}. On the other hand, the present idea of the second moment method is that the linearization point is located at the shortest distance between the failure boundary and the origin in the normalized space. As Cho has pointed out[15], this premise tends to overestimate the probability of failure, resulting in the oversafe design of structures, under certain conditions such as that the true boundary is highly nonlinear and so on.

The second moment method adopts this particular point of linearization also for the design points which are identical to the design values specified in design codes. This has been explained by the reason that failure is most likely to occur at this point. However, once the boundary is linearized, the design point need not be located at a point where failure is most likely to occur. For example, in the safeguard against the earthquake in Japanese specifications for steel highway bridges[16] or steel railway bridges[17], the lower bound values of static ultimate strength are adopted as resistance and the static earthquake intensity as low as 0.2g is taken for load effect with the factor of safety of only 1.0. It is not plausible to consider that this combination of values is most likely at the instant of failure, but both of the design variables seem to take larger values than specified ones in most cases of failure. Nevertheless, the proponents of the second moment method do not seem to argue that the currently specified design values should be increased by the same scale both for the resistance and the load.

Although the proposed fractile method places its emphasis on Eq. (16) combined with Eq. (13) which only reflects the mean and variance of the normal distribution for the evaluation of the intended fractile e_i, its own improved method is also presented by Eq. (23) and (24). For practical circumstances, however, it is not likely to find sufficient data enough to estimate the higher order moments accurately for all the design variables. The accuracy of estimating higher order moments with limited number of data is of great concern to be investigated as well as the influence of the excessive large (or small) values of data due to errors on higher order moments.

The both methods of Paloheimo and Rackwitz have been found similar to the proposed fractile method in the sense of utilization of the fractile probabilities at the very design values. While the present study has recommended the separate use of Eq. (13) for the evaluation of the intended fractile e_i and Eq. (16) for the determination of the design values X_i^*, Paloheimo and Rackwitz have related substantially Eq. (13) with Eq. (16), possibly contributing the higher accuracy of the realized probability of failure P_{fR} However, the idea itself remains to be within the framework of the second moment method, and the computations involve much complicated procedures. As seen by the difference between Fig.1 and Fig.5 compared with Fig.2, the proposed fractile method does not lose the accuracy and significantly relieve the computations, and moreover the method can directly be applied for the practical determinations of the design values, just similar to the way used for the conventional specification writing.

7. CONCLUDING REMARKS

For rational reliability based structural design, it is most important to preserve the consistent safety against failure. This can be attained firstly by establishing appropriately the target probability of failure for a design incident of concern, and next by evaluating properly the corresponding design values so as to minimize the error between the realized and the target probabilities of failure.

For this purpose, this paper has presented the fractile method which incorporates the tail-shapes of distributions of design variables into the evaluation of design values in a rational and simple fashion. The idea of the method is rather similar to the implicit concept of the conventional allowable stress design,[8], but contradict the prevailing notion of the original second moment method which utilizes essentially the information on the mean and the variance of distributions of design variables.

Although the second moment method has been improved by Paloheimo and Rackwitz, also reflecting the tail-shape information, the proposed method has been proved much simpler in the evaluation of design values, and has shown practically the same accuracy as the rather complicated improved second moment method. It should also be noted for the proposed fractile method that the design point (or the design values) can be selected, with some degrees of freedom, mainly from the consideration of the characteristics of each random design variables, free both from the linearization point and the most probable point of failure. It is further noted that higher order moments can be incorporated into the determination of design values, if desired, for further improvement of accuracy.

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