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ULTIMATE STRENGTH DESIGN CRITERIA FOR TWO-HINGED STEEL ARCH STRUCTURES

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Design criteria of two-hinged, parabolic, steel arches based on the ultimate limit state design concept are presented. The design criteria on the planar ultimate strength are formulated for arch ribs. The ultimate strength design criteria are specified by critical cross sectional forces and critical stresses analyzed by the first order elastic analysis. Variable, cross sectional effect of arch ribs is also discussed from a viewpoint of the ultimate strength designing.

1. INTRODUCTION

Numerous research results on instability of arches have been reported so far[1]. The third edition of SSRC Guide[4] summarizes the then (1976) available experimental and analytical data and their relations to design applications. However, the design applications of arches are not based on geometrical and material nonlinear instability (i.e., ultimate strength or load carrying capacity) but elastic instability of them.

In the last decade, the design practice for various types of steel structures, which has based previously on allowable stress principles and linear theory, have been changed to limit states design rules to obtain more rational designs. As for steel arch structures, numerous researches on their planar ultimate stength have been performed in Refs. 2), 5), 6), 7), 8), 9), 10) and 11). Extensive findings concerning them have summarized in Ref. 12). Some design criteria based on the planar ultimate strength of parabolic steel arch ribs were presented. Ref. 6) presented practical formulas for the prediction of the ultimate strength expressed in terms of critical, bending moment and axial thrust, based on the ultimate strength of the two-hinged arch ribs. They have idealized uniform sandwich cross sections (composed of the two flanges separated by a web of negligible area). Practical formulas for the ultimate load intensity of the two-hinged and the fixed arch ribs were presented in Ref. 5). The formula is expressed in the function of the normal thrust.

In this paper, based on the ultimate strength results of two-hinged, parabolic, steel arches with thin-walled box cross sections analyzed by the authors so far[7),8),11)], the planar ultimate strength design criteria are presented for the arch ribs. The numerical approach used was based on the finite element technique and the modified Newton-Raphson procedure using the incremental load method and the tangent

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modulus method. It considered the influence of finite deformations, spread of yielding zones in cross sections and along the longitudinal axis, welding residual stresses and unloading due to strain reversal. The design criteria are specified by cross sectional forces and stresses at a quarter point of the rib calculated by the first order elastic analysis. The effect of a concentrated loading is examined from a viewpoint of the ultimate strength designing. The design criteria which take into account in a direct manner the effect of variable cross section of the arch ribs is also presented.

2. SCOPE OF APPLICATION

Generally, the major structural properties of an arch rib that affect the ultimate strength are its rise/span ratio, slenderness ratio, yield stress level, cross sectional properties, and distribution pattern and intensity of residual stresses. Among them, the cross sectional properties are standardized here because of the smaller influence on the ultimate strength as determined by the preceding investigations. The dimension of a thin-walled box cross section adopted as reference are shown in the inset of Fig.1, wherein H,B = depth and width of the reference cross section, t_w=thickness of a web plate, A_f, A_w=cross sectional area of a flange plate and a web plate, and r_x, k=radius of gyration and core radius, respectively. The distribution and intensity of residual stresses are also fixed as shown in Fig.1 based on the results of Ref. 8). For the range of the structural parameters adopted in the analysis, the proposed formulas are valid in the following range;

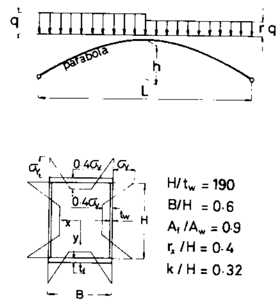

Fig. 1 Arch Geometry, Loading and Reference Cross Section.

λ=100~300 ; h/L=0.1~0.3 ; σ_Y=240~480 N/mm^2

E=210 000 N/mm^2

in which λ=slenderness ratio of an arch rib which is given by the ratio of the curvilinear length of arch axis L_s to the radius of gyration of the cross section r_x, σ_Y=yield stress level, and E=Young's modulus. The distribution pattern of load q is given by the loading parameter r-which is considered to vary from 0 to 0.99 herein as shown in Fig.1. The influence of a concentrated load is considered, corresponding to the line load given by Japanese Specification of Highway Bridges[3]. In this case the concentrated load Q_c is taken as $50(1-r)q/3$, in which q is Kg/m and the constant 50/30 has the dimension of meters.

3. DESIGN FORMULATIONS FOR ARCH RIBS

Useful findings that are brought out by the numerous numerical analyses performed so far[7),8),11)] with respect to general behavior of two-hinged steel arch structures with uniform cross section loaded to the ultimate state in their plane are as follows; 1) The load carrying capacity of the steel arch ribs depends chiefly on the slenderness ratio, the yield stress level of the material, the rise/span ratio, the unsymmetry of the distributed loads, and the concentrated load placed on a quarter point of the arch rib;[7),11)] 2) The effect of the variation of cross sectional proportions on the ultimate load intensity nondimensionalyzed by the full plastic load q_p is not significant, if the column slenderness ratio parameter $\bar{\lambda}$ for arch ribs is equal with each other (in which $\bar{\lambda}=\lambda\sqrt{\sigma_Y/E}/\pi$)[8),11)] The full plastic load q_p produces the squash axial force at the springing evaluated by the 1st order elastic analysis and given by

$$q_p=\frac{A_a\,\sigma_Y}{\sqrt{\left\{\dfrac{n-2}{2}\right\}^2+\left\{\sum_{i=1}^{n}\dfrac{5\,L\,l_1\,l_2}{8\,h}(l_1^2+3\,l_1\,l_2+l_2^2)\right\}^2}} \quad \cdots\cdots (1)$$

in which n=number of the nodes of an arch rib, $l_1=(i-l)/n$, $l_2=l-l_1$, i=order number of the nodes of an arch rib, A_a=cross sectional area of an arch rib. If the intensity of residual stresses is higher than a certain value, their effect on the load carrying capacity is not a function of their intensity but is nearly a

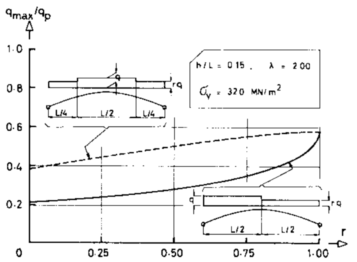

Fig. 2 Comparison of the Ultimate Strength under Symmetrical Loading with those under Unsymmetrical Loadings.

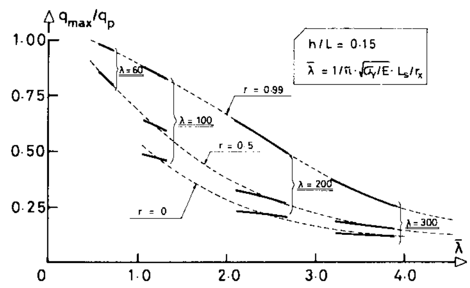

Fig. 3 Influence of Slenderness Ratio Parameter on the Ultimate Strength for Various Values of Slenderness Ratio and Yield Stress Level.

constant[8]; 3) The ultimate load intensity increases in proportion to the cross sectional area[11].

In addition to these findings, the following items are examined herein. Fig.2 shows the difference between the ultimate load for a given arch subjected to symmetrical and unsymmetrical distributed loadings as shown in the inset. As is obvious from the figure, the unsymmetrical distributed loading pattern gives a lower load carrying capacity for the arch than the symmetrical loading. That is, loadings with unsymmetrical distribution patterns are critical loading conditions for arches. Therefore, the unsymmetrical loadings are adopted hereafter in examining the ultimate strength design criteria. Typical results for the influence of the slenderness ratio parameter $\bar{\lambda}$ on the load carrying capacity are shown in Fig.3 for various values of the slenderness ratio $\lambda$and the yield stress level σ_Y. In this figure, the dotted line is shown for the influence of the slenderness ratio in which a certain value of the yield stress level (σ_Y=320 N/mm^2) is adopted and the solid line for the influence of the yield stress level in which a certain value of the slenderness ratio (λ=200) is adopted. It can be observed from Fig.3 that the difference between the two kinds of curves is practically small. Therefore, from a practical viewpoint it can be concluded that the influences of the slenderness ratio λ and the yield stress level σ_Y are collectively evaluated by the so-called column slenderness ratio parameter $\bar{\lambda}$.

(1) Cross Sectional Force Expression

a) For the Reference Cross Sectional Rib

From the viewpoint of design practice, it is desirable that the design formulas of arch ribs are expressed by the critical cross sectional forces or stresses calculated by the 1st order elastic analysis, even if mathematical expressions of formulas become slightly complicated. Accordingly the calculated results for the ultimate strength of the arches are reviewed again. Figs.4,5, and 6 show typical examples of the interaction curves between the nondimensional maximum axial force N^{1st}_{max}/N_Y and bending moment

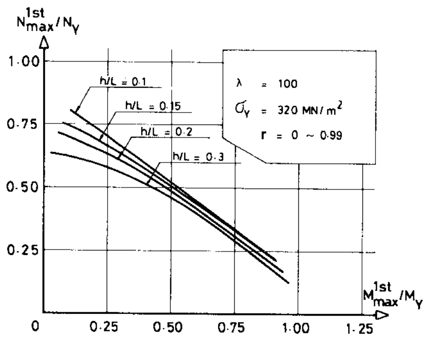

Fig. 4 Relationship Between the Maximum Force and Bending Moment (for λ= 100).

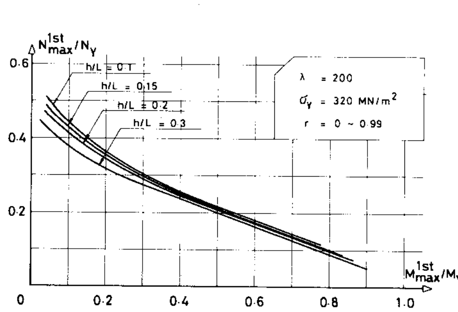

Fig. 5 Relationship Between the Maximum Force and Bending Moment (for λ=200).

M^{1st}_{max}/M_Y at a quarter point of arch ribs. Here, these maximum forces are calculated by the 1st order elastic analysis for the arch under the ultimate load, and N_Y, M_Y are the squash axial force and yield bending moment respectively. From these figures, it can be observed that the interaction curves may describe a quadratic line when the maximum axial force is greater than a certain critical value, while the interaction curves show linear functional relationship when the maximum axial force is less than the critical value. This critical value is defined as n_{cr} in this paper. By considering these results, the following design formulas are proposed herein :

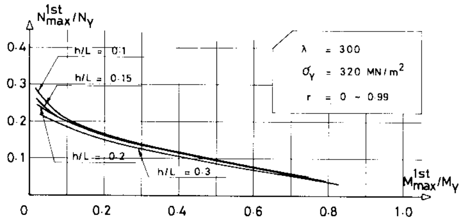

Fig. 6 Relationship Between the Maximum Force and Bending Moment (for λ=300).

$$f=1 \quad \cdots\cdots (2\cdot a)$$

$$a\left[\frac{M^{1st}_{max}}{M_Y}\right]^2+b\left[\frac{M^{1st}_{max}}{M_Y}\right]+c\left[\frac{N^{1st}_{max}}{N_Y}\right]=f \quad \text{for} \quad \frac{N^{1st}_{max}}{N_Y}\geqq n_{cr} \quad \cdots\cdots (2\cdot b)$$

$$\alpha\left[\frac{M^{1st}_{max}}{M_Y}\right]+\beta\left[\frac{N^{1st}_{max}}{N_Y}\right]=f \quad \text{for} \quad \frac{N^{1st}_{max}}{N_Y}<n_{cr} \quad \cdots\cdots (2\cdot c)$$

where a,b,c, α, and β are coefficients depending on $\bar{\lambda}$ and h/L. These coefficients and n_{cr} are calculated by substituting the maximum axial force N^{1st}_{max} and bending moment M^{1st}_{max} which are analyzed for various cases of the slenderness ratio, the yield stress level, the rise/span ratio, the unsymmetry of distributed load and the concentrated load placed on a quarter point of the arch ribs into Eqs.2. Furthermore by applying the regression analysis on statistics for the calculated values on n_{cr}, α, and β the relationship between these coefficients and $\bar{\lambda}$, h/L can be expressed by functional formulas as follows :

$a=2.509-1.689\,\bar{\lambda}$;
$b=-1.213+1.605\,\bar{\lambda}-0.135\,\bar{\lambda}^2$;
$c=(1.824-0.914\,\bar{\lambda}+0.376\,\bar{\lambda}^2)(0.82+1.2\,h/L)$;
$\alpha=1/m_p$; $\beta=(m_p-m_{cr})/(m_p\,n_{cr})$;
$m_p=1.172-0.0469\,\bar{\lambda}$;
$n_{cr}=(1-b m_{cr}-a m_{cr}^2)/c$;
$m_{cr}=m_p$ for $(a m_p^2+b m_m-1)/a<0$ or
$m_{cr}=m_p-\sqrt{(a m_p^2+b m_p-1)/a}$ for $(a m_p^2+b m_p-1)/a\geqq 0$

For demonstration of the accuracy of the ultimate strength design formula - Eqs.(2) - proposed herein, comparison with the exact values calculated by the ultimate strength analysis are made for various cases of the vertical loading condition ($r=0\sim0.99$), rise/span ratio ($h/L=0.1\sim0.3$), slenderness ratio ($\lambda=100\sim350$), and yield stress level ($\sigma_Y=240\sim480$ N/mm²). Some investigated results are shown in Figs.7, 8, and 9. The solid lines in these figures illustrate the interaction curves between N^{1st}_{max} and M^{1st}_{max} given by the proposed design formula and ⊗ and ⊙ marks show the analyzed results for $h/L=0.1$ and 0.3, respectively. It will be seen from these figures that the results predicted by the proposed design formula and the analyzed ones agree fairly well. In addition to them, the effect of the concentrated load Q_c applied on a quarter point of the arch ribs, in which the concentrated load corresponds to the line load introduced by the Japanese Specification of Highway Bridge[3] is examined. The influence on the ultimate strength is shown in Fig.10 and the accuracy of the design formula is tabulated for typical example in Table 1. Here, q_{max} is the ultimate load intensity of an arch subjected to distributed load, q^*_{max} is the distributed load intensity in the ultimate state of the arch under the distributed load and the concentrated load Q_c, f(DESIGN) is Eq. 2a, and f(ANALYSIS) is the f-value calculated by substituting N^{1st}_{max} and M^{1st}_{max} values for the arch under q^*_{max} and Q_c into Eqs.2 b and 2 c. Therefore, by comparing f(ANALYSIS) and f(DESIGN) the applicability of the design formula for the arch under not only the distributed load but also the concentrated load can be checked. From the results investigated herein, it can be seen that, with the investigated range, the proposed

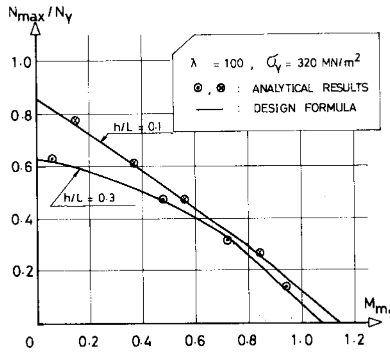

Fig. 7 Comparison of Results Predicted by the Design Formula with those Calculated by the Ultimate Strength Analysis (λ=100).

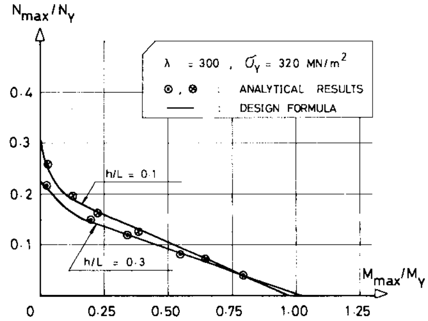

Fig. 9 Comparison of Results Predicted by the Design Formula with those Calculated by the Ultimate Strength Analysis (λ=300).

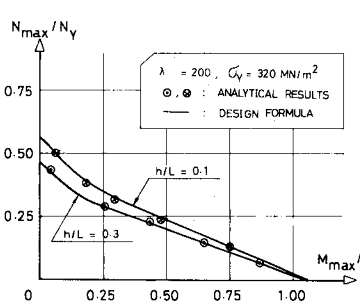

Fig. 8 Comparison of Results Predicted by the Design Formula with those Calculated by the Ultimate Strength Analysis (λ=200).

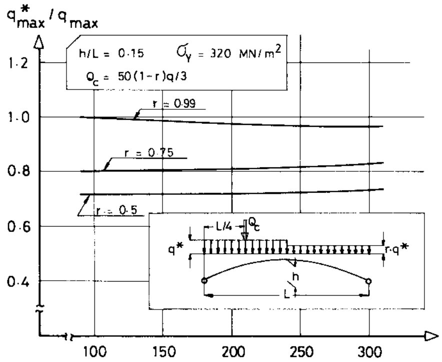

Fig. 10 Influence of the Concentrated Load Q_c on the Ultimate Strength.

design formula always provides slightly conservative evaluation -f(ANALYSIS)≥1- for the ultimate strength of the arch ribs under the concentrated load. The applicability of the proposed design formula is checked by varying the yield stress level also. The typical results are listed in **Table 2** where the notation used are similar to those in **Table 1**. From **Table 2**, fairly good agreement may be found between the f-values given by the proposed design formula and by the ultimate strength analysis.

Table 1 Comparison of f-Results by the Design Formula with those by the Ultimate Strength Analysis (for Application of concentrated load).

r	$\bar{\lambda}$	q^*_{max}/q_{max}	M^{1st}_{max}/M_y	N^{1st}_{max}/N_y	f(ANALYSIS)/f(DESIGN)
0.5	1.24	0.719	0.831	0.333	1.098
	2.48	0.715	0.754	0.158	1.115
	3.73	0.736	0.640	0.090	1.056
0.76	1.24	0.801	0.586	0.479	1.086
	2.48	0.814	0.532	0.237	1.147
	3.73	0.829	0.610	0.130	1.247
0.99	1.24	1.000	0.130	0.746	1.124
	2.48	0.978	0.077	0.471	1.135
	3.73	0.968	0.047	0.243	1.139

h/L=0.15, σ_y=320 N/mm², Q_c=50(1−r)q/3.

The maximum difference between the results given by the proposed design formula and by the ultimate strength analysis is 7.8% on conservative side and 4.2% on risky side within the practical range-r=0~0.9, σ_Y=240~480N/mm², λ=100~300. Moreover, the design formula yield conservative estimation for the cases of 127/140=90% in all the results discussed herein. Therefore, it may be concluded that the design formulas proposed by Eq.2 evaluate the ultimate strength of two-hinged steel arch ribs accurately enough for practical purpose.

b) For Arbitrary Cross Sectional Ribs

By the 1st order elastic analysis, the bending moment of arches is in proportion to the load intensity and

Table 2 Comparison of f-Results by the Design Formula with those by the Ultimate Strength Analysis (for various values of slenderness ratio and yield stress level).

λ	σ_y (N/mm²)	$\bar{\lambda}$	M^{1st}_{max}/M_y	N^{1st}_{max}/N_y	f(ANALYSIS)/f(DESIGN)
100	240	1.076	0.934	0.217	1.009
			0.659	0.427	1.003
			0.106	0.751	1.185
	280	1.162	0.932	0.214	1.023
			0.645	0.418	1.011
			0.105	0.744	1.172
	320	1.242	0.914	0.212	1.023
			0.628	0.407	1.013
			0.105	0.744	1.174
	360	1.318	0.898	0.206	1.021
			0.613	0.397	1.016
			0.105	0.744	1.180
200	240	2.152	0.865	0.102	0.998
			0.624	0.216	1.051
			0.058	0.573	1.125
	280	2.324	0.839	0.099	0.995
			0.589	0.204	1.049
			0.054	0.529	1.125
	320	2.484	0.811	0.095	0.986
			0.558	0.193	1.044
			0.049	0.479	1.104
	360	2.636	0.787	0.092	0.981
			0.529	0.183	1.034
			0.046	0.453	1.130
300	240	3.228	0.776	0.061	0.940
			0.533	0.124	0.993
			0.034	0.328	1.129
	280	3.486	0.741	0.058	0.932
			0.493	0.117	0.999
			0.029	0.307	1.211
	320	3.726	0.709	0.056	0.930
			0.458	0.107	0.989
			0.026	0.249	1.119
	360	3.954	0.681	0.054	0.930
			0.428	0.100	0.994
			0.023	0.241	1.211

$h/L=0.15$, $\bar{\lambda}=(\lambda/\pi)\sqrt{\sigma_y/E}$

span length, and the axial force is in proportion to the load intensity, approximately. The yield moment is in proportion to the section modulus. Therefore, the design formulas for arch ribs with arbitrary cross sections should be considered for the above-mentioned factors.

The cross sectional forces acting on an arch rib with arbitrary cross section are given by :

$$\frac{A^{1st}_{max}}{A_Y}=\frac{N^{1st}_{max}}{N_Y}\cdot\frac{A^{1st}_{max}}{N^{1st}_{max}}\cdot\frac{N_Y}{A_Y} \quad (3\cdot a)$$

$$\frac{B^{1st}_{max}}{B_Y}=\frac{M^{1st}_{max}}{M_Y}\cdot\frac{B^{1st}_{max}}{M^{1st}_{max}}\cdot\frac{M_Y}{B_Y} \quad (3\cdot b)$$

in which A^{1st}_{max}, B^{1st}_{max}=axial force and bending moment at a quarter point of an arch rib calculated by 1st order elastic analysis for the rib with an arbitrary cross section under the ultimate load, N^{1st}_{max}, M^{1st}_{max}=those for the rib with the reference cross section under the ultimate load, A_Y, B_Y= squash axial force and yield bending moment of an arbitrary cross section, and N_Y, M_Y=those of the reference cross section. N^{1st}_{max} and M^{1st}_{max}/L are practicaly in proportion to the ultimate load intensity for the arch rib with the reference cross section. From the item 3) in the previous paragraph (1) of this chapter 3, the ultimate load intensity increases in proportion to the cross sectional area. Namely, N^{1st}_{max} and M^{1st}_{max}/L are also in proportion to the cross sectional area of the reference cross section. Similary, A^{1st}_{max} and B^{1st}_{max}/L^* are in proportion to a cross sectional area of an arbitrary cross section. Therefore, $A^{1st}_{max}/N^{1st}_{max}$ and $(LB^{1st}_{max})/(L^*M^{1st}_{max})$ are approximately equal to a ratio of a cross sectional area of an arbitrary cross section to that of the reference cross section. On the contrary, N_Y/A_Y and M_Y/B_Y are equal to the ratio of the reference cross sectional area to an arbitrary cross sectional area and to the ratio of the section modulus of the reference cross section to that of an arbitrary cross section. Therefore, considering above-mentioned relation, it becomes possible to define the criteria in the form of the critical stresses as follows :

$$\frac{N^{1st}_{max}}{N_Y}=\frac{A^{1st}_{max}}{A_Y} \quad (4\cdot a)$$

$$\frac{M^{1st}_{max}}{M_Y}=\frac{B^{1st}_{max}}{B_Y}\cdot\frac{k^*}{L^*}\cdot\frac{L}{k} \quad (4\cdot b)$$

in which k^*, k are the core radius of an arbitrary and the reference cross section, respectively. L^* is a span length of an arbitrary cross sectional arch. For an arbitrary cross sectional arch, Eqs.(4) should be substituted into Eqs.(2). Detail examinations of the ultimate strength design formula for arbitrary cross sectional arches proposed herein are performed in the paragraph on "Critical Stress Expressions" later on.

(2) Critical Stress Expressions

For structural design it is more conventional to express the design criteria by the critical stresses (or allowable stresses into safety factor). The critical stresses σ_{aa} due to maximum axial force and σ_{ba} due to maximum bending moment acting on an arch rib with arbitrary cross section are given by :

$$\frac{\sigma_{aa}}{\sigma_Y}=\frac{A^{1st}_{max}}{A_Y}\ ;\ \frac{\sigma_{ba}}{\sigma_Y}=\frac{B^{1st}_{max}}{B_Y} \qquad (5)$$

Substituting Eq.(5) into Eq.(4), it becomes possible to define the criteria in the form of the critical stresses as follows :

$$\frac{\sigma_{aa}}{\sigma_Y}=\frac{N^{1st}_{max}}{N_Y}\ ;\ \frac{\sigma_{ba}}{\sigma_Y}=\frac{M^{1st}_{max}}{M_Y}\cdot\frac{k}{L}\cdot\frac{L^*}{k^*} \qquad (6)$$

The typical values of k/L adopted herein as the reference cross section are listed in Table 3. In order to inspect the accuracy of the ultimate strength design formulas expressed in the form of critical stresses for arbitrary cross sectional arch ribs - Eq.(6) -, four kinds of various proportioning of box cross sections are taken up and checked. The results are listed in Table 4. Columns 2, 5, 8 and 11 in Table 4 show $(B^{1st}_{max}/B_Y)/(M^{1st}_{max}/M_Y)$ calculated using the results of the ultimate strength analysis, directly. Those values are approximately equal to the ratio of the core radius of the reference cross section nondimensionalized by the span length L to that of an arbitrary cross section nondimensionalized by a span length L^* listed in Columns 3, 6, 9, and 12, respectively. Judging from those comparisons and referring Eq.4b, it can be said that the expressions for critical stresses proposed herein can evaluate the ultimate strength of arch ribs with sufficient accuracy.

Tadle 3 Core Radius of Reference Cross Section Nondimensionalyzed by the Span Length.

rise/span ratio	Slenderness Ratio			
	100	200	250	300
0.1	0.823×10^{-2}	0.412×10^{-2}	0.329×10^{-2}	0.274×10^{-2}
0.15	0.848	0.424	0.339	0.383
0.2	0.881	0.440	0.352	0.294
0.3	0.966	0.483	0.386	0.322

(3) Variable Cross Sectional Effects

Generally speaking, in proportioning steel plate girders, the size of the flange plates can be conveniently reduced in regions where the bending moments have decreased below the maximum values. In designing based on 1st analysis it may be advantageous in some cases of steel arch bridge structures under higher bending moments to use flange plate with reduced thicknesses where the bending moments have dropped appreciably, to make up the arch ribs. From a viewpoint of the ultimate strength designing the effect of the above-mentioned variable cross section of the ribs is examined and a design proposal for the effect is presented herein.

By examining stress conditions at the ultimate limit states for several types of variable cross sections, thickness change location of the flange plates in the arch rib without causing stress concentrations is determined herein and w_1 is a certain value of 0.8, as shown in Fig.11. Fig.12 and 13 show typical examples

Table 4 Examination of Accuracy of the Design Formula Expressed by the Form of the Critical Stresses (for various proportioning of box cross section).

λ	Type-A			Type-B			Type-C			Type-D		
(1)	$\frac{\bar{B}^{1st}_{max}}{\bar{M}^{1st}_{max}}$ (2)	$\frac{\bar{k}}{\bar{k}^*}$ (3)	$\frac{(2)-(3)}{(2)}$ (4)	$\frac{\bar{B}^{1st}_{max}}{\bar{M}^{1st}_{max}}$ (5)	$\frac{\bar{k}}{\bar{k}^*}$ (6)	$\frac{(5)-(6)}{(5)}$ (7)	$\frac{\bar{B}^{1st}_{max}}{\bar{M}^{1st}_{max}}$ (8)	$\frac{\bar{k}}{\bar{k}^*}$ (9)	$\frac{(8)-(9)}{(8)}$ (10)	$\frac{\bar{B}^{1st}_{max}}{\bar{M}^{1st}_{max}}$ (11)	$\frac{\bar{k}}{\bar{k}^*}$ (12)	$\frac{(11)-(12)}{(11)}$ (13)
100	0.960	0.966	0.6 %	1.004	1.000	0.4 %	1.075	1.054	2.0 %	1.228	1.157	5.8 %
150	0.966	0.966	0.0	1.010	1.000	1.0	1.070	1.052	1.7	1.201	1.155	3.8
200	0.971	0.965	0.6	1.005	1.000	0.5	1.059	1.052	0.7	1.170	1.155	1.3
250	0.966	0.965	0.0	0.993	1.000	0.7	1.040	1.052	1.1	1.139	1.155	1.4

$\bar{k}=k/L$, $\bar{k}^*=k^*/L^*$, $\bar{M}^{1st}_{max}=M^{1st}_{max}/M_y$, $\bar{B}^{1st}_{max}=B^{1st}_{max}/B_y$, $L=100$ m, $L^*=300$ m, $\sigma_y=360$ N/mm², $r=0.5$

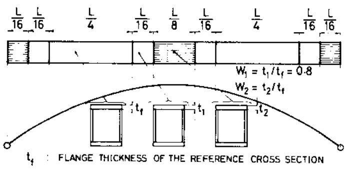

Fig. 11 Reference Thickness Change Location of Flange Plate.

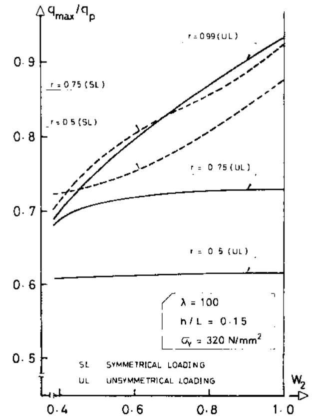

Fig. 12 Interaction Curves Between the Non-Dimensional Ultimate Load Intensity and Change of Flange Plate Thickness (for $\lambda=100$).

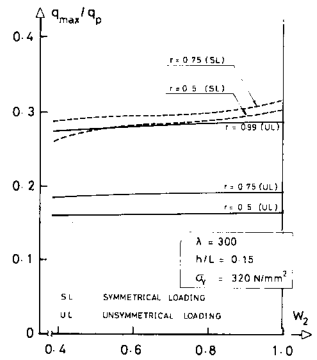

Fig. 13 Interaction Curves Between the Nondimensional Ultimate Load Intensity and Change of Flange Plate Thickness (for $\lambda=300$).

of the interaction curves between the nondimensional ultimate load intensity and the change of the flange plate thickness. From these figures it can be observed that the ultimate strength of the arch ribs with the variable cross sections under unsymmetrical loading is more critical than the strength under symmetrical loading. This phenomenon is similar to the case of the uniform cross sctional arch ribs as shown in Fig.2. The symmetrical loading patterns in Fig.12 and 13 are same as the patterns as shown in the inset of Fig.2. From Fig.12 and 13 it can be also observed that the application of the flange plates with reduced thickness is advantageous to the ultimate strength designing of the arches under the higher bending moments (i.e., loading case with lower value of r - the range of r in actually well-designed steel arch bridge structures may be from 0. 65 to 0. 85). By the ultimate strength analysis the load carrying capacities for $w_2=1$, shown in Figs.12 and 13, agree with those for the reference uniform cross section as shown in Fig.1. Considering these results the following formula can be presented as an ultimate strength design recommendation for two-hinged steel arch ribs with reduced flange thicknesses :

$$w_1=0.8\,;\; w_2\geq 0.8 \qquad (7)$$

where the reference thickness transition points are located as shown in Fig.11.

4. CONCLUSION

Practical design formulas based on the ultimate strength were presented for two-hinged steel arch structures. These design formulas were derived within the range of conventional structural dimensions of steel arch bridges. The major findings can be summarized as follows:

(1) The design citeria for the ultimate strength of the steel arch ribs can be expressed by the cross sectional forces - axial force and bending moment - or stresses at the section of a quarter point of the arch span. Consequently, the criteria are described independent of the distribution pattern of the distributed load and the intensity of a concentrated load. The design criteria can be formulated by the characteristic parameters of arch structures such as the slenderness ratio parameter and the rise/span ratio as shown in Eq.(2).

(2) If designers desire to examine the ultimate strength by using so-called allowable stresses, they must consider the core radius of cross sections in addition to the above-mentioned characteristic parameters as shown in Eq.(6).

(3) The ultimate strength design recommendation for the arch ribs with reduced flange thicknesses can be presented by Eq.(7), in direct manner, provided that the reference thickness transition points should be located as shown in **Fig.11**. After applying the design recommendation as shown in Eq.(7) to the ultimate strength designing, designers do not require to pay attention to the effect of the reduced flange thickness on the load carrying capacity.

In this paper, the safety factor or load factor is not refered to in proposing the design formulas. However, noting that the proposed criteria are established on the values calculated by the 1st order elastic analysis, it could be permissible to determine the allowable sectional forces and/or stresses by dividing those critical resultants specified in Eqs.(4) and/or (6) by the conventional safety factor or load factor.

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