

FATIGUE STRENGTH OF PLATE GIRDER IN BENDING CONSIDERING OUT-OF-PLANE DEFORMATION OF WEB

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When a thin-walled plate girder is subjected to repeated bending, it is possible that fatigue cracks may be initiated at the toe on the web side of fillet welds connecting the compression flange to the web. They are caused by the secondary bending stress produced by out-of-plane movement of the web under repeated in-plane bending, due to its unavoidable initial deflection. In this paper, first, the relation between in-plane bending stress and secondary bending stress is formulated. Secondly, substituting the fatigue strength of fillet welds subjected to secondary bending stress into the relation, the fatigue strength for the above fatigue cracks is expressed in in-plane bending stress. It is given as a function of the web slenderness ratio, and the influence of various factors on the fatigue strength is discussed.

1. INTRODUCTION

As pointed out in Refs. 1)~5), when a thin-walled plate girder is subjected to repeated loading, it is possible that fatigue cracks may be initiated at the toe on the web side of fillet welds around the web panel. They are caused by the secondary bending stress at the toe on the web side of fillet welds, which is produced under an in-plane load by the out-of-plane deformation of the web with initial deflections.

In AASHTO Specification⁶⁾, the limits of the web slenderness ratio of transversely stiffened plate girders are 163 in the allowable stress design and 192 in the load factor design for a steel with the yielding stress of 248.2 MPa. The use of considerably slender web plates is approved in the latter design. The limits of the web slenderness ratio in the load factor design are shown in a provision to prevent the initiation of the above-mentioned fatigue cracks. They were proposed by taking into consideration the results of fatigue tests of plate girders with transverse stiffeners⁷⁾. But, they are not directly based on the fatigue strength of fillet welds subjected to secondary bending stress. On the other hand, in the case of BS5400⁸⁾, fatigue is dealt with in its Part 10. Judging from the answer to the authors' paper⁹⁾ contributed to the conference held at University College, Cardiff in 1980, the fatigue cracks due to out-of-plane deformation of the web seems not to be considered there.

As for thin-walled plate girders in repeated bending, as pointed out in Ref.10), it is possible that the following three types of fatigue cracks may be initiated.

a) Type 1 fatigue cracks at the toe on the web side of fillet welds connecting the compression flange to the web, which are due to out-of-plane deformation of the web.

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b) Type 2 fatigue cracks at the toe on the web side of fillet welds connecting the vertical stiffener to the web.

c) Type 3 fatigue cracks at fillet welds connecting the tension flange to the web.

In the case of type 2 and 3 fatigue cracks, their fatigue strengths are given^{11,12}, and it is possible to estimate the initiation of them by comparing the flange stress calculated by the beam theory with the fatigue strength. On the other hand, in the case of type 1 fatigue cracks, the fatigue strength of fillet welds subjected to secondary bending stress is clarified¹³. But, it is impossible to estimate the initiation of them based on this fatigue strength, because the relation between in-plane bending stress and secondary bending stress is not formulated. The authors have already made it clear by the finite out-of-plane deformation analysis of rectangular plates in in-plane bending by the finite element method that not only the magnitudes of the initial deflections but also their shapes will influence greatly an increase in secondary bending stress, resulting in the initiation of the type 1 fatigue cracks^{10,14}.

In this paper, first, the relation between load and secondary bending stress of rectangular plates in in-plane bending is formulated. Secondly, substituting the fatigue strength of fillet welds subjected to secondary bending stress into the relation, the fatigue strength of type 1 fatigue cracks is expressed in in-plane bending stress. It is given as a function of the web slenderness ratio, and the influence of various factors on the fatigue strength is discussed.

2. FORMULATION OF FINITE OUT-OF-PLANE DEFORMATION OF RECTANGULAR PLATES IN IN-PLANE BENDING

(1) Relation between Load and Out-of-plane Deflection in the Case of Simple Supports at All Edges

As shown in Fig.1, a rectangular plate of $a \times b$ under in-plane bending is simply supported in the out-of-plane direction at all the edges. The initial deflection w_0 and the additional deflection w are respectively assumed as follows :

$$w_0 = \sin(\pi x/a) \{ e_{01} \sin(\pi y/b) + e_{02} \sin(2\pi y/b) \}, \dots\dots\dots (1)$$

$$w = \sin(\pi x/a) \{ e_1 \sin(\pi y/b) + e_2 \sin(2\pi y/b) \}, \dots\dots\dots (2)$$

where e_{01} and e_{02} =the mode components of the initial deflection, and e_1 and e_2 =the mode components of the additional deflection. Substituting Eqs.(1) and (2) into the Marguerre's plate differential equations¹⁵, and applying the Galerkin method to them, the relation between load and out-of-plane deflection can be obtained as follows¹⁶ :

$$\frac{\sigma_o}{\sigma_e} = \frac{9 \pi^2}{32} \theta_3 \frac{e_1}{e_{02} + e_2} + \frac{9 \pi^2}{32} (1 - \nu^2) \left(\frac{3}{4} \theta_5 \frac{q_1}{t_w^2} \frac{e_{01} + e_1}{e_{02} + e_2} + 3 \theta_7 \frac{q_2}{t_w^2} \frac{e_{01} + e_1}{e_{02} + e_2} + 3 \theta_8 \frac{q_3}{t_w^2} \right), \dots\dots\dots (3)$$

$$\frac{\sigma_o}{\sigma_e} = \frac{9 \pi^2}{32} \theta_4 \frac{e_2}{e_{01} + e_1} + \frac{9 \pi^2}{32} (1 - \nu^2) \left(\frac{3}{4} \theta_6 \frac{q_2}{t_w^2} \frac{e_{02} + e_2}{e_{01} + e_1} + 3 \theta_7 \frac{q_1}{t_w^2} \frac{e_{02} + e_2}{e_{01} + e_1} + 3 \theta_8 \frac{q_3}{t_w^2} \right), \dots\dots\dots (4)$$

where σ_o =the in-plane bending stress, $\sigma_e = \pi^2 E / [12(1 - \nu^2)\beta^2]$, $\beta = b/t_w$, E =the Young's modulus, ν =the Poisson's ratio, t_w =the thickness of the web plate, $\theta_3 \sim \theta_8$ =the coefficients depending on the aspect ratio a/b , $q_1 = (e_{01} + e_1)^2 - e_{01}^2$, $q_2 = (e_{02} + e_2)^2 - e_{02}^2$, and $q_3 = (e_{01} + e_1)(e_{02} + e_2) - e_{01}e_{02}$.

(2) Relation between Load and Out-of-Plane Deflection Considering Rotational Restraint at Unloaded Edges

When a rectangular plate is subjected to in-plane bending, the out-of-plane deflection on the compressive side becomes larger than that on the tensile side. Accordingly, the relation between the load and the out-of-plane deflection at the location of $y = b/4$ from the compressive edge will be formulated. Here, letting e denote the additional out-of-plane deflection at this location, e is given by

$$e = e_1/\sqrt{2} + e_2. \dots\dots\dots (5)$$

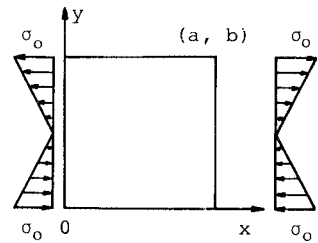


Fig.1 Rectangular plate in in-plane bending.

Investigating each term on the right side of Eqs.(3) and (4), it is revealed that when the additional deflection is small, the respective first terms are dominative, and that as it increases, they converge into a buckling coefficient, and on the contrary, an increase in the respective second terms becomes remarkable. Considering such behavior, Eqs.(3) and (4) can be approximated by the following equation, when either of e_{o1} or e_{o2} exists for the initial deflection :

$$\frac{\sigma_o}{\sigma_e} = k_{cr} \eta_o + \frac{3}{4}(1 - \nu^2)\theta\xi_o, \dots\dots\dots (6)$$

where

$$\eta_o = \frac{\sqrt{(e/t_w + e_o/t_w)e/t_w + r_1^2(e_o t_w)^2 - r_1 e_o t_w}}{e/t_w + e_o/t_w}, \quad \xi_o = \left(\frac{e}{t_w} + \frac{e_o}{t_w}\right)^2 \left[1 - \left(\frac{e_o}{r_2 e + e_o}\right)^2\right],$$

$e_o = e_{o1}/\sqrt{2}$ or e_{o2} , $k_{cr} = (9\pi^2/32)\sqrt{\theta_3 \theta_4}$ = the buckling coefficient, $r_1 = \sqrt{\theta_3/(2\theta_4)}$ for $e_o = e_{o1}/\sqrt{2}$ or $\sqrt{\theta_4/(8\theta_3)}$ for $e_o = e_{o2}$, and θ and r_2 = the coefficients.

η_o can be obtained by combining Eq.(5) and the respective first terms on the right side of Eqs.(3) and (4). On the other hand, ξ_o is assumed as above, referring to the form^{17,18)} of an equation of the relation between load and out-of-plane deflection of rectangular plates in compression. When $r_2=1$, ξ_o takes the same form as the one of rectangular plates in compression. Determining the values of θ and r_2 in the same manner as mentioned in the subsequent Section 2. (5) for the relation between load and out-of-plane deflection given by Eqs.(3) and (4), substituting these values into Eq.(6), and comparing Eq.(6) with Eqs.(3) and (4), Eq.(6) approximates Eqs.(3) and (4) very well.

If the values of k_{cr} , θ , r_1 and r_2 , which are determined by taking into account the rotational restraint at the unloaded edges, are used, Eq.(6) will give the relation between load and out-of-plane deflection considering the rotational restraint at the unloaded edges. A method to determine the values of these coefficients is stated in the subsequent sections.

(3) Determination of k_{cr}

The differential equation for the buckling analysis of plates in in-plane bending is as follows :

$$\frac{D_w}{t_w} \nabla^4 w + \sigma_o \left(1 - 2\frac{y}{b}\right) \frac{\partial^2 w}{\partial x^2} = 0, \dots\dots\dots (7)$$

where $D_w = E t_w^3 / [12(1 - \nu^2)]$ and $\nabla^4 = \partial^4 / \partial x^4 + 2 \partial^4 / \partial x^2 \partial y^2 + \partial^4 / \partial y^4$. The following equation is assumed for the additional deflection :

$$w = Y(y) \sin(\pi x/a), \dots\dots\dots (8)$$

where $Y(y)$ = the function of y . Substituting Eq.(8) into Eq.(7), and eliminating $\sin(\pi x/a)$, an ordinary differential equation about the function $Y(y)$ is obtained. Transforming this equation into finite-difference equations, and solving numerically the eigenvalue problem defined by the equations, k_{cr} is determined.

(4) Determination of r_1

The first term on the right side of Eq.(6) can also be obtained by substituting Eqs.(1) and (2) into the following equation¹⁹⁾ and by applying the Galerkin method to it :

$$\frac{D_w}{t_w} \nabla^4 w = -\sigma_o \left(1 - 2\frac{y}{b}\right) \frac{\partial^2 (w_o + w)}{\partial x^2}. \dots\dots\dots (9)$$

Substituting Eqs.(1) and (8) into Eq.(9), and eliminating $\sin(\pi x/a)$, an ordinary differential equation about the function $Y(y)$ is obtained. Transforming this equation into finite-difference equations, and solving the simultaneous linear equations made by the equations, the relation between load and out-of-plane deflection is obtained.

On the other hand, solving $\sigma_o/\sigma_e = k_{cr} \eta_o$ about r_1 , the following equation is obtained :

$$r_1 = \frac{\frac{e}{e_o} - \left(\frac{1}{k_{cr}} \frac{\sigma_o}{\sigma_e}\right)^2 \left(\frac{e}{e_o} + 1\right)}{\frac{2}{k_{cr}} \frac{\sigma_o}{\sigma_e}}. \dots\dots\dots (10)$$

Substituting the above-mentioned relation between load and out-of-plane deflection into Eq.(10), τ_1 is almost constant, and the value at $e/e_0=1$ may be used for τ_1 .

(5) Determination of θ and τ_2

To determine the values of θ and τ_2 , the results of the finite element method analysis^(10,14) will be used. A mesh division of a rectangular plate is shown in Fig.2. Considering symmetry of the plate, its half is divided into 144 elements. The unloaded edges are free to move in the in-plane direction, and for boundary conditions in the out-of-plane direction at these edges, the following cases are treated :

- (I) both edges simply supported, and
- (II) both edges fixed.

The loaded edges are simply supported in the out-of-plane direction, and the in-plane bending is applied so that the membrane stress varies linearly at these edges. Substituting the values of σ_o/σ_e and e/t_w obtained by the finite element method analysis into the terms of $\sigma_o/\sigma_e - k_{cr} \eta_o$ and ξ_o , the relation between the two terms is obtained. It becomes almost linear at a certain value of τ_2 in ξ_o , regardless of the magnitudes of the initial deflections, and the values of θ are determined by applying the least squares method to it.

The values of k_{cr} , τ_1 , τ_2 and θ are shown in Table 1. The curves of load versus out-of-plane deflection given by Eq.(6) and the results of the finite element method analysis are shown in Fig.3. It is seen that Eq. (6) is very close to the values of the finite element method analysis.

(6) Relation between Secondary Bending Stress and Out-of-Plane Deflection

The relation between secondary bending stress σ_b/σ_e at the point $(a/2, 0)$ in Fig.1 and out-of-plane deflection is assumed as follows, and the acceptability of this equation will be explained :

$$\sigma_b/\sigma_e = S\eta_b + \sqrt{A^2 \xi_b^2 + B^2} - B, \dots\dots\dots (11)$$

where

$$\eta_b = \frac{(1+2\tau_1)\eta_o + \tau_2(1-\eta_o)}{\eta_o + 2\tau_1} \frac{e}{t_w}, \quad \xi_b = \left(\frac{e}{t_w} + \frac{e_o}{t_w} \right)^2 \left[1 - \left(\frac{e_o}{r_4 e + e_o} \right)^2 \right],$$

and S , τ_3 , τ_4 , A and B =the coefficients.

The first term on the right side of Eq.(11) is obtained as follows. The relation between secondary bending stress and out-of-plane deflection is given by

$$\sigma_b/\sigma_e = C_1 e_1/t_w + C_2 e_2/t_w \dots\dots\dots (12)$$

where C_1 and C_2 = the coefficients. Eliminating e_1 and e_2 in Eq.(12) by Eq.(5) and the respective first terms on the right side of Eqs.(3) and (4), the following equation is obtained when e_{o1} only exists :

$$\frac{\sigma_b}{\sigma_e} = \frac{\sqrt{2} C_1 \eta_o + 2 C_2 \tau_1}{\eta_o + 2 \tau_1} \frac{e}{t_w} \dots\dots\dots (13)$$

Here, let S denote the coefficient in front of e/t_w in Eq.(13) in the case of $e_{o1}/t_w=0$, and let $\tau_3=2 \tau_1 C_2/S$. Eliminating C_1 and C_2 in Eq.(13) with these coefficients, the first term on the right side of

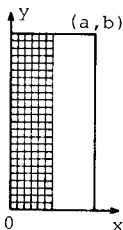


Fig.2 Mesh division.

Table 1 Values of coefficients.

a/b	(I)						(II)							
	1/3		0.5		1.0		1/3		0.5					
e_o	$e_{o1}/\sqrt{2}$	e_{o2}	$e_{o1}/\sqrt{2}$	e_{o2}	$e_{o1}/\sqrt{2}$	e_{o2}	$e_{o1}/\sqrt{2}$	e_{o2}	$e_{o1}/\sqrt{2}$	e_{o2}				
k_{cr}	33.78		25.63		27.13		43.72		39.46					
r_1	0.44	0.50	0.41	0.61	0.28	0.90	0.52	0.57	0.46	0.64				
r_2	0.35	0.60	0.65	1.0	1.0	0.35	0.35	0.50	0.65	0.64				
θ	5.07		5.25		6.52		6.51		8.01					
S							86.69		62.31					
r_3							1.03		1.28		1.08		1.36	
r_4							0.7		0.6		1.3		0.7	
A							11.70				6.14			
B							5.29				1.65			

- (I) Both edges simply supported
- (II) Both edges fixed

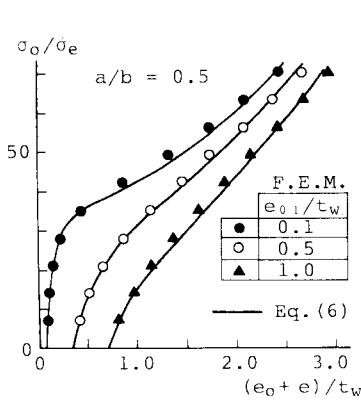


Fig. 3 σ_o/σ_e versus $(e_o + e)/t_w$ [Case (II)].

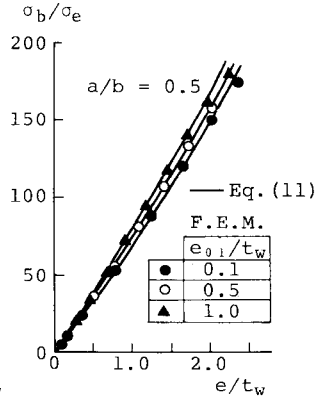


Fig. 4 σ_b/σ_e versus e/t_w [Case(II)].

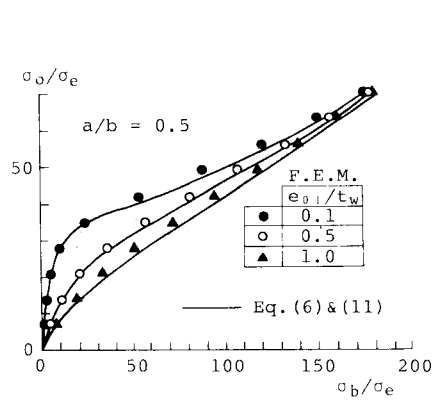


Fig. 5 σ_o/σ_e versus σ_b/σ_e [Case (II)].

Eq.(11) is obtained. On the other hand, when e_{o2} only exists, such a term is obtained in the same manner. The coefficient S is, from the definition, an increase in secondary bending stress per a unit out-of-plane deflection at $y = b/4$ of the buckled form, and its values are determined by the buckling analysis in Section 2. (3).

The coefficient r_3 is determined by solving $\sigma_b/\sigma_e = S\eta_b$ about r_3 as follows :

$$r_3 = \frac{(1/S)(\sigma_b/\sigma_e) \eta_o + 2 r_1}{(e/e_o)(e_o/t_w) 1 - \eta_o} - (1 + 2 r_1) \frac{\eta_o}{1 - \eta_o} \dots \dots \dots (14)$$

Substituting the relation between secondary bending stress and out-of-plane deflection obtained in Section 2. (4) into Eq.(14), r_3 is almost constant, and the value at $e/e_o = 1$ may be used for r_3 .

The coefficients r_4 , A and B are determined as follows. Substituting the values of σ_b/σ_e and e/t_w obtained by the finite element method analysis into the terms of $\sigma_b/\sigma_e - S\eta_b$ and ξ_b , the relation between the two terms is obtained. It can be expressed in a curved line at a certain value of r_4 in ξ_b , regardless of the magnitudes of the initial deflections, and the values of A and B are determined by applying the least squares method to it.

The values of the coefficients S , r_3 , r_4 , A and B are also shown in Table 1. The comparison of σ_b/σ_e versus e/t_w curves given by Eq.(11) with the results of the finite element method analysis is shown in Fig.4. The relation between σ_o/σ_e and σ_b/σ_e is obtained by Eqs.(6) and (11). Its comparison with the results of the finite element method analysis is shown in Fig.5.

3. RELATION BETWEEN LOAD AND SECONDARY BENDING STRESS CONSIDERING BOTH INITIAL DEFLECTION AND RESIDUAL STRESS

(1) Relation between Load and Out-of-Plane Deflection Considering Both Initial Deflection and Residual Stress

The fundamental plate differential equations for the finite out-of-plane deformation of thin plates which have the initial deflection and the residual stress together, are given as follows²⁰⁾ :

$$\frac{1}{E} \nabla^4 F = \left[\frac{\partial^2 (w_o + w)}{\partial x \partial y} \right]^2 - \frac{\partial^2 (w_o + w)}{\partial x^2} \frac{\partial^2 (w_o + w)}{\partial y^2} - \left\{ \left(\frac{\partial^2 w_o}{\partial x \partial y} \right)^2 - \frac{\partial^2 w_o}{\partial x^2} \frac{\partial^2 w_o}{\partial y^2} \right\}, \dots \dots \dots (15)$$

$$\frac{D_w}{t_w} \nabla^4 w = \frac{\partial^2 (F_o + F)}{\partial y^2} \frac{\partial^2 (w_o + w)}{\partial x^2} - 2 \frac{\partial^2 (F_o + F)}{\partial x \partial y} \frac{\partial^2 (w_o + w)}{\partial x \partial y} + \frac{\partial^2 (F_o + F)}{\partial x^2} \frac{\partial^2 (w_o + w)}{\partial y^2}, \dots \dots \dots (16)$$

where F = the stress function and F_o = the stress function for residual stress. Eqs.(15) and (16) show that the initial deflection and residual stress interact each other. Generally, the initial deflections specified in a specification for bridges or measured in fatigue tests are considered to be the ones at the time when the

equilibrium condition is reached after the initial deflection and residual stress have interacted each other. The fundamental plate differential equations defined by such initial deflections for the finite out-of-plane deformation of thin plates will be derived in the next place.

Let w_r and F_r respectively denote the out-of-plane deflection and stress function at the time when the equilibrium condition is reached after the initial deflection has been varied by the residual stress. Furthermore, let w_a and F_a respectively denote the additional out-of-plane deflection and stress function for a stress produced by loading. Using these parameters, w and F are expressed as follows :

$$w = w_a + w_r - w_o, \dots\dots\dots (17)$$

$$F = F_a + F_r - F_o \dots\dots\dots (18)$$

Substituting Eqs.(17) and (18) into Eqs.(15) and (16), and eliminating w_o and F_o by using the relation that Eqs.(15) and (16) also hold on the unloaded conditions of $w_a=0$ and $F_a=0$, Eqs.(15) and (16) are respectively transformed as follows :

$$\frac{1}{E} \nabla^4 F_a = \left\{ \frac{\partial^2 (w_r + w_a)}{\partial x \partial y} \right\}^2 - \frac{\partial^2 (w_r + w_a)}{\partial x^2} \frac{\partial^2 (w_r + w_a)}{\partial y^2} - \left\{ \left(\frac{\partial^2 w_r}{\partial x \partial y} \right)^2 - \frac{\partial^2 w_r}{\partial x^2} \frac{\partial^2 w_r}{\partial y^2} \right\}, \dots\dots\dots (19)$$

$$\begin{aligned} \frac{D_w}{t_w} \nabla^4 w_a - \left(\frac{\partial^2 F_r}{\partial y^2} \frac{\partial^2 w_a}{\partial x^2} - 2 \frac{\partial^2 F_r}{\partial x \partial y} \frac{\partial^2 w_a}{\partial x \partial y} + \frac{\partial^2 F_r}{\partial x^2} \frac{\partial^2 w_a}{\partial y^2} \right) \\ \underbrace{\dots\dots\dots} \\ = \frac{\partial^2 F_a}{\partial y^2} \frac{\partial^2 (w_r + w_a)}{\partial x^2} - 2 \frac{\partial^2 F_a}{\partial x \partial y} \frac{\partial^2 (w_r + w_a)}{\partial x \partial y} + \frac{\partial^2 F_a}{\partial x^2} \frac{\partial^2 (w_r + w_a)}{\partial y^2} \dots\dots\dots (20) \end{aligned}$$

Seemingly, Eqs.(19) and (20) show that the terms on the residual stress marked with a wave line are added to the Marguerre's fundamental plate differential equations defined by the initial deflection w_r and the additional out-of-plane deflection w_a . If the initial deflection of Eq.(1), the additional deflection of Eq. (2) and the residual stress of rectangular distribution such as shown in Fig.6 are assumed for w_r , w_a and F_r , respectively, θ_3 in Eq. (3) and θ_4 in Eq. (4) have only to be replaced by $\theta_3 - |\sin(2 \pi c / b) / (2 \pi c / b)| \sigma_{rc} / \sigma_e$ and $\theta_4 - |\sin(4 \pi c / b) / (4 \pi c / b)| \sigma_{rc} / \sigma_e$, respectively. These coefficients are related only to k_{cr} and τ_1 in Eq.(6).

When F_r is generally given by a function of only y , k_{cr} and τ_1 are respectively determined as follows. k_{cr} is determined by the following equation instead of Eq.(7) :

$$\frac{D_w}{t_w} \nabla^4 w_a - \left\{ \frac{\partial^2 F_r}{\partial y^2} - \sigma_o \left(1 - 2 \frac{y}{b} \right) \right\} \frac{\partial^2 w_a}{\partial x^2} = 0. \dots\dots\dots (21)$$

τ_1 is determined by substituting the relation between load and out-of-plane deflection obtained by the following equation instead of Eq.(9) into Eq.(10) :

$$\frac{D_w}{t_w} \nabla^4 w_a - \frac{\partial^2 F_r}{\partial y^2} \frac{\partial^2 w_a}{\partial x^2} = - \sigma_o \left(1 - 2 \frac{y}{b} \right) \frac{\partial^2 (w_r + w_a)}{\partial x^2}. \dots\dots\dots (22)$$

The values of k_{cr} and τ_1 obtained by Eqs. (21) and (22) for the residual stress shown in Fig.6 are listed in Table 2. It is seen that as the residual stress increases, k_{cr} decreases, and that τ_1 varies depending on the magnitude of the residual stress.

(2) Relation between Secondary Bending Stress and Out-of-Plane Deflection Considering Both Initial Deflection and Residual Stress

The values of S considering the residual stress are determined by Eq.(21), and the values of τ_3 are determined by substituting the relation between secondary bending stress and out-of-plane deflection obtained by Eq.(22) into Eq.(14). The values of S and τ_3 for the residual stress shown in Fig.6 are listed in Table 2. It is seen that as the residual stress increases, S decreases, and that τ_3 varies depending on the magnitude of the residual stress.

The influence of the residual stress on the coefficients A , B and τ_4 will be examined by the results of the finite element method analysis. In the finite element method analysis program¹⁷⁾ considering the residual stress, after the equilibrium condition has been calculated for a given initial deflection and a given residual

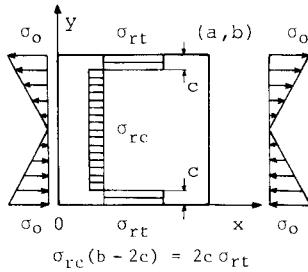


Fig.6 Rectangular plate in in-plane bending with residual stress.

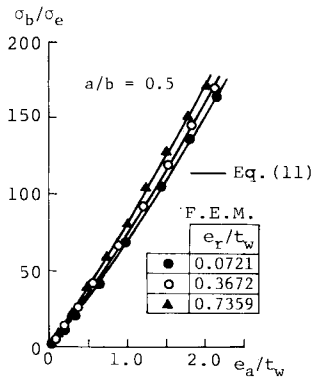


Fig.7 σ_b/σ_e versus e_a/t_w [Case(II)].

Table 2 Influence of residual stress on k_{cr} , S , τ_1 and τ_3 [Case(II)].

For $a/b = 1/3$

σ_{rc}/σ_e ($c/b=1/12$)	k_{cr}	S	e_0			
			$e_{01}/\sqrt{2}$		e_{02}	
			r_1	r_3	r_1	r_3
0	43.72	86.69	0.52	1.03	0.57	1.28
2	39.06	81.02	0.51	1.05	0.58	1.30
4	34.11	75.36	0.49	1.06	0.60	1.33
6	28.76	69.59	0.46	1.05	0.63	1.36

For $a/b = 0.5$

σ_{rc}/σ_e ($c/b=1/12$)	k_{cr}	S	e_0			
			$e_{01}/\sqrt{2}$		e_{02}	
			r_1	r_3	r_1	r_3
0	39.46	62.31	0.46	1.08	0.64	1.36
2	33.17	58.70	0.42	1.02	0.68	1.40
4	26.07	54.66	0.36	0.92	0.78	1.53
6	17.26	49.60	0.26	0.72	1.04	1.94

stress, the in-plane bending is applied. In the case of $c/b=1/12$ and $\sigma_{rc}/\sigma_e=4$ for the residual stress of rectangular distribution shown in Fig.6, calculations are carried out for the initial deflections of $e_{01}/t_w=0.1, 0.5$ and 1.0 . The aspect ratio a/b is 0.5 in each case. Letting e_r denote the out-of-plane deflection at $y=b/4$ at the time when the equilibrium condition is calculated for these residual stress and initial deflections, e_r/t_w becomes $0.0721, 0.3672$ and 0.7359 for $e_{01}/t_w=0.1, 0.5$ and 1.0 , respectively. Letting e_a denote the additional out-of-plane deflection at $y=b/4$

produced by loading of in-plane bending, the relation between σ_b/σ_e and e_a/t_w is obtained as shown in Fig.7. Replacing e_o and e in Eq.(11) with e_r and e_a , respectively, the relation this equation gives is also shown in the figure. The values on the column of $a/b=0.5$ and $e_o=e_{01}/\sqrt{2}$ in Case (II) in Table 1 are used for the coefficients S, A, B, r_1, r_3 and r_4 . It is seen from Fig.7 that Eq.(11) is very close to the values of the finite element method analysis. Accordingly, it is concluded that the residual stress does not influence the coefficients A, B and r_4 . Also, the term of $S\eta_b$ in Eq.(11) seems not to be much influenced by the residual stress, though the coefficients S and r_3 are influenced by the residual stress.

From the above discussion, to apply Eqs.(6) and (11) to the case the residual stress also exists, the out-of-plane deflection at the time when the equilibrium condition is reached after the initial deflection has been varied by the residual stress has only to be used for e_o , and the coefficients k_{cr} and r_1 considering the residual stress have only to be used.

4. FATIGUE STRENGTH CONSIDERING OUT-OF-PLANE DEFORMATION OF WEB

(1) Influence of Initial Deflection Shape

The authors revealed in Refs.10), 14) that not only the magnitudes of the initial deflections but also their shapes would influence the initiation of type 1 fatigue cracks. In the case of the web panels of aspect ratio $a/b=1$, the initial deflection of mode 1 in the x -direction does not much increase the secondary bending stress, but the ones of mode 2 and 3 do it greatly. Here, the mode 1,2 and 3 mean the shapes of one, two and three half-sine-waves, respectively. On the other hand, as for the initial deflection shapes in the y -direction, the mode 2 makes the largest secondary bending stress. For web panels which have the initial deflection of mode 2 or 3 in the x -direction, the relation between in-plane bending stress σ_o and secondary bending stress σ_b can be calculated by Eqs.(6) and (11). The values on columns of $a/b=0.5$ and $1/3$ in Case (II) in Table 1 are used in these equations corresponding to the initial deflections of mode 2 and 3,

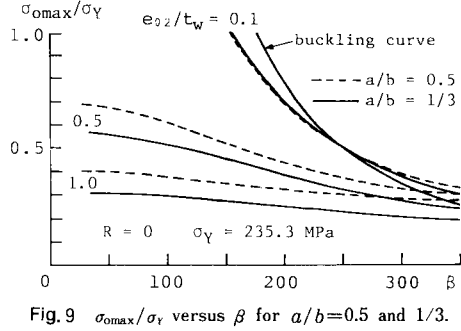
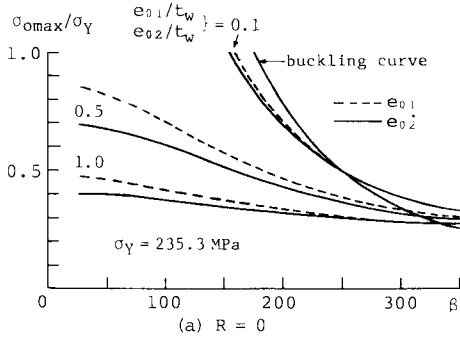


Fig.9 σ_{omax}/σ_Y versus β for $a/b=0.5$ and $1/3$.

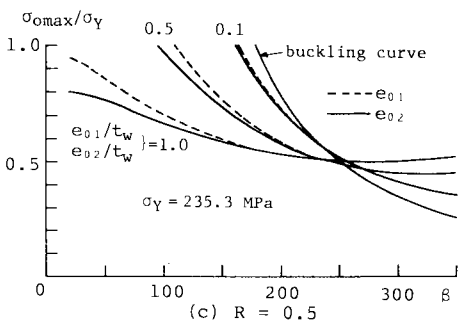
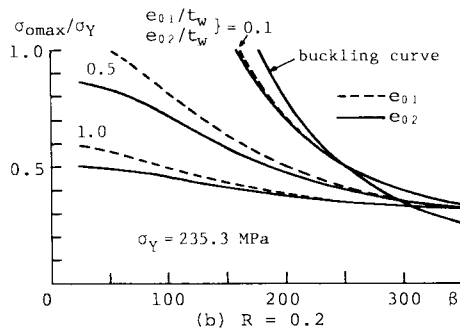


Fig.8 σ_{omax}/σ_Y versus β for $a/b=0.5$.

respectively.

The fatigue strength¹³⁾ for the secondary bending stress range at 2×10^6 cycles is about 166.7 MPa for steels of JIS-SS41, which have the minimum specified tensile strength of 402 MPa. Letting σ_{bmin} and σ_{bmax} denote the secondary bending stresses corresponding to the minimum and maximum in-plane bending stresses σ_{omin} and σ_{omax} , respectively, and solving Eqs.(6) and (11) under the condition of

$$\sigma_{bmax} - \sigma_{bmin} = 166.7, \dots\dots\dots (23)$$

the relation between the maximum in-plane bending stress σ_{omax} for the 2×10^6 cycles fatigue strength and the web slenderness ratio β can be obtained.

The relation between σ_{omax}/σ_Y and β in the case of $a/b=0.5$ is shown in Fig.8. Here, σ_Y is the specified yielding stress, which is 235.3 MPa for SS41 steels. R is the in-plane bending stress ratio defined by the following equation :

$$R = \sigma_{omin}/\sigma_{omax} \dots\dots\dots (24)$$

From the figure, the followings are pointed out :

- 1) The difference of σ_{omax}/σ_Y due to the components e_{01} and e_{02} of the initial deflection mode in the

- t -direction becomes smaller with an increase of the web slenderness ratio β .
- 2) σ_{omax}/σ_Y becomes larger with an increase of R . In the case of $R=0.5$, after β has exceeded by about 250, σ_{omax}/σ_Y for a large initial deflection becomes larger than that for a small initial deflection. Furthermore, σ_{omax}/σ_Y for $e_{01}/t_w=1.0$ in the case of $R=0.5$ starts to rise after β has exceeded by about 300. As β increases, the web becomes more slender and the out-of-plane deflection increases. With an increase in the out-of-plane deflection, the relation between σ_o and σ_b obtained by Eqs.(6) and (11) converges to the one given by

$$\sigma_o = (3/4)(1 - \nu^2)(\theta/A)\sigma_b, \dots\dots\dots (25)$$

This equation gives $\sigma_{omax}/\sigma_Y = 0.63, 0.78$ and 1.26 for $R=0, 0.2$ and 0.5 under the condition of Eq.(23), respectively. Accordingly, after σ_{omax}/σ_Y has decreased with an increase of β and has reached a minimum value, it increases and converges to a limiting value given by Eq.(25).

- 3) The buckling curve given by the following equation is also shown in Fig.8 :

$$\sigma_{omax} = 39.46 \sigma_e, \dots\dots\dots (26)$$

There exists the region of β where σ_{omax}/σ_Y for the 2×10^6 cycles fatigue strength is much smaller than the

buckling strength, depending on the magnitude of the initial deflection and stress ratio. For example, for $\beta = 200$, the buckling strength is $\sigma_{\text{omax}}/\sigma_Y = 0.78$, but the 2×10^6 cycles fatigue strength at $R=0$ is $\sigma_{\text{omax}}/\sigma_Y = 0.69, 0.43$ and 0.32 for $e_{0z}/t_w = 0.1, 0.5$ and 1.0 , respectively.

The relation between $\sigma_{\text{omax}}/\sigma_Y$ and β in the case of $a/b=1/3$ shows the same tendency as that in the case of $a/b=0.5$. However, as shown in Fig.9, $\sigma_{\text{omax}}/\sigma_Y$ for $a/b=1/3$ is always smaller than that for $a/b=0.5$ except for the case of $e_{0z}/t_w=0.1$. In the case of $e_{0z}/t_w=0.1$, $\sigma_{\text{omax}}/\sigma_Y$ for $a/b=1/3$ is slightly larger than that for $a/b=0.5$ in the region of β less than about 250.

(2) Influence of Residual Stress

As the web becomes thinner, the rotational restraint of the flanges to the web becomes larger, and the web can be regarded as fix-supported by the flanges¹⁰⁾. On the other hand, in actual plate girders, there exists the residual stress in their webs due to welding. As mentioned in Chapter 3, the residual stress makes the buckling coefficient k_{cr} in Eq.(6) decrease. Accordingly, the influence of the residual stress on the fatigue strength will be examined by decreasing the value of k_{cr} from the value for the fixed support at the unloaded edges. For the coefficients in Eqs.(6) and (11) except k_{cr} , the values of Case (II) in Table 1 are used.

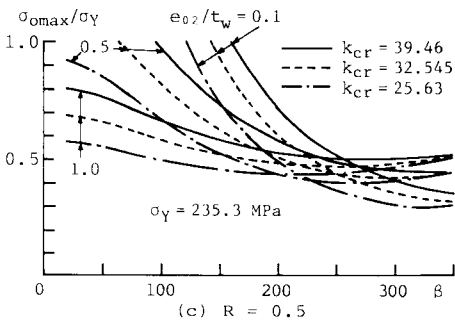
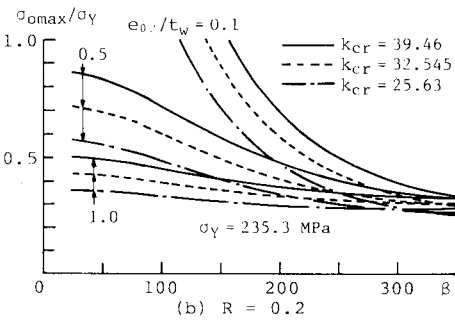
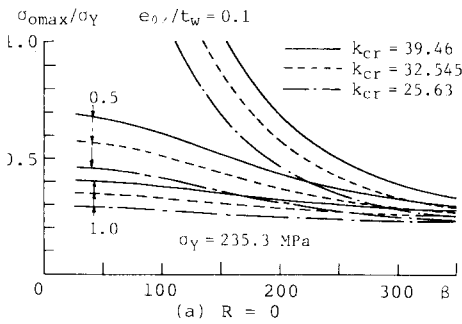


Fig.10 Influence of residual stress [$a/b=0.5$].

The relation between $\sigma_{\text{omax}}/\sigma_Y$ and β in the case of $a/b=0.5$ is shown in Fig.10. $k_{cr}=39.46$ and 25.63 in the figure correspond to the values of the buckling coefficient for the fixed and simple supports at the unloaded edges, respectively, and $k_{cr}=32.545$ is the mean value of the two. It is seen that as k_{cr} decreases, $\sigma_{\text{omax}}/\sigma_Y$ also does. The degree of the decrease of $\sigma_{\text{omax}}/\sigma_Y$ due to the residual stress becomes smaller with the increase of R and β . Especially, the curves for $e_{0z}/t_w=0.5$ and 1.0 at $R=0.5$ approach each other and rise with the increase of β . This is because they fast converge to the values given by Eq.(25) which is not related to k_{cr} .

(3) Influence of Yielding Stress

It is revealed by the results of fatigue tests of T-type fillet welds subjected to repeated out-of-plane bending that the 2×10^6 cycles fatigue strength of SM58 steels, which have the minimum specified tensile strength of 569 MPa, is almost the same as the one of SS41 steels¹³⁾. Accordingly, for steels of the yielding stress not exceeding 451.1 MPa which is the

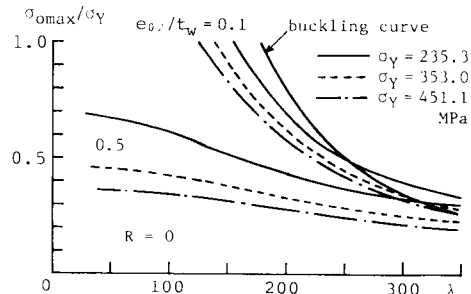


Fig.11 Influence of yielding stress [$a/b=0.5$].

specified yielding stress of SM58, the condition of Eq.(23) will be used.

The relation between $\sigma_{\text{omax}}/\sigma_Y$ and λ in the case of $a/b=0.5$ and $R=0$ is shown in Fig.11. Here, λ is defined as follows :

$$\lambda = \beta \sqrt{\sigma_Y} / 235.3 \dots \dots \dots (27)$$

It is seen from the figure that $\sigma_{\text{omax}}/\sigma_Y$ decreases with the increase of the yielding stress. This can be explained as follows.

Using Eq.(27), the terms σ_o/σ_e and σ_b/σ_e on the left side of Eqs.(6) and (11) are transformed into $1.264 \times 10^{-3} \lambda^2 (\sigma_o/\sigma_Y)$ and $1.264 \times 10^{-3} \lambda^2 (\sigma_b/\sigma_Y)$, respectively. The range of σ_b/σ_Y for the 2×10^6 cycles fatigue strength is $166.7/\sigma_Y$ from Eq.(23). Since $166.7/\sigma_Y$ decreases with the increase of σ_Y , $\sigma_{\text{omax}}/\sigma_Y$ also decreases.

Generally, using the web slenderness ratio multiplied by the square root of the yielding stress such as Eq.(27), the buckling strength can be expressed independently of the yielding stress. Even if such a parameter is used, however, the relation between fatigue strength and web slenderness ratio cannot be expressed independently of the yielding stress.

5. CONCLUSIONS

The relation between load and secondary bending stress of rectangular plates in in-plane bending was given by Eqs.(6) and (11). Using these equations, the fatigue strength of type 1 fatigue cracks due to out-of-plane deformation of the webs of thin-walled plate girders subjected to repeated bending was expressed in in-plane bending stress. This fatigue strength was given as a function of the web slenderness ratio, and the following features were pointed out.

(1) The influence of the initial deflection mode in the y -direction on the fatigue strength becomes smaller with the increase of the web slenderness ratio.

(2) As the stress ratio increases, the fatigue strength also increases. When the stress ratio is small, the fatigue strength for a large initial deflection is smaller than that for a small initial deflection. When the stress ratio is large, however, this is reversed in the region where the web slenderness ratio is large.

(3) When the web panel of aspect ratio 1 has the initial deflection of mode 2 or 3 in the x -direction, there exists the region where the fatigue strength becomes considerably smaller than the buckling strength depending on the magnitudes of the initial deflection and stress ratio.

(4) The residual stress reduces the fatigue strength. The degree of the reduction becomes smaller with the increases of the stress ratio and web slenderness ratio.

(5) Using the web slenderness ratio multiplied by the square root of the yielding stress, the buckling strength can be expressed independently of the yielding stress. But, even if such a parameter is used, the relation between fatigue strength and web slenderness ratio cannot be expressed independently of the yielding stress, because the fatigue strength of fillet welds subjected to secondary bending stress is almost constant regardless of an increase of the yielding stress.

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