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BASIC COMPRESSIVE STRENGTH OF STEEL PLATES FROM TEST DATA

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This paper describes a data-base approach to the ultimate compressive strength of unstiffened plates loaded in uniaxial compression. A total 793 individual test results of plates is surveyed and stored into the Numerical Data-Base for the Ultimate Strength of Steel Structures (NDSS). Statistical assessments are made for the plate test results to compare with the available strength formulas. And the effects of initial imperfections such as residual stress and initial out-of-flatness are discussed. Then strength formulas are proposed for two groups, "with residual stress" and "without residual stress", to explain more accurately the test data.

1. INTRODUCTION

Buckling and post-buckling behavior of rectangular plates loaded in compression on two opposite edges have been investigated extensively[30),31)] since Bryan obtained the first buckling solution in 1891. In 1932, von Kármán derived an approximate formula for the effective width of simply supported edges. In 1945, Winter[32),34)] applied the effective width concept to light-gage steel design. Since then various formulas have been suggested for the ultimate strength and design of the cold-formed sheet and welded plate using the effective width concept.

Experimental investigations have been made of the local buckling of steel plate elements to obtain the strength reduction due to initial imperfections in medium thickness plate, and the strength increases in thin plates.

Extensive tests have been conducted in United Kingdom. They include model welded box columns of the Forth Bridge Towers[7)], and the studies of Dwight · Harrison[1)] on welded box columns, and of Dwight · Ractliffe[6),11)], Moxham[14),15)], Little[16)] on single plates and box columns. In Japan, tests have been made by Fujita[2)], Yoshiki[5)], Yokoo[8)], Uchiyama[9)], Tanaka[10)], Fukumoto[12)], Nishino[1),13)], Hasegawa[17)]. Little[39)], Massonnet[53)] and Usami · Fukumoto[23),25)] have investigated the local and overall interaction behavior of box columns.

Theoretical ultimate strength approaches to the elasto-plastic behavior of compressed plates with initial imperfections have been made in Japan by Okamura · Yoshida[48)], Watanabe[49)], Komatsu · Kitada · Miyazaki[50)], Ueda[55)] and Fujita[54)]. Komatsu and Kitada[51),52)] made ultimate strength analyses with initial imperfection of $\sigma_{rc}=0.4\,\sigma_y$ (σ_{rc} : compressive residual stress, σ_y : yield stress) and $w_0=b/150$ (w_0 : initial out-of-flatness, b : plate width), which are typical of the worst initial imperfections. They proposed a lower bound design curve and compared it with the available test data and the existing design formulas in the specifications.

Faulkner[46)] made an extensive review of effective plating and a listing of the important effective width formulas,

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and Hasegawa and Usami[56] introduced more recent effective width formulas.

A numerical data-base for the ultimate strength of steel structures has been established at Nagoya University[40)~42)].

This paper makes a survey of the ultimate compressive strength of plates which appeared in the available test reports. Comparisons are made statistically between the reviewed test data and the existing theoretical and empirical formulas for the ultimate strength of plates.

2. PLATE TEST DATA AND DATA-BASE

(1) Number of Test Data

The purpose of this paper is to review and store the data of uniformly compressed plates with clearly defined support conditions. The test data may be categorized by the cross-sectional shapes and the support conditions as (a) single plates, (b) square boxes and (c) cruciform-shapes. Test data for rectangular boxes are also stored for comparison with the square box test results. Tubular box sections are also surveyed and included in the data-base. The test data for cruciform and rectangular box are not analyzed herein, and the data for the compression flange of H and box beams are not included because of the small number of test data and the unidentified support conditions.

Table 1 shows a total of 793 individual test results which are categorized by cross-sectional shapes, fabrication process, method of testing and country of origin. In Europe, 476 tests are further classified into 323 of U. K., 148 of West Germany and 5 of Sweden. The data stored in the data-base are from Refs. 1) –29) except that test data in Ref. 3) have not been included in the analysis hereafter because of the unidentified edge condition.

Table 1 Number of Plate Data.

Type of Profile	Europe	North America	Japan	Total
Single Plate	362		55	417
Welded Square Box	74	8	93	175
Square Tube			49	49
Welded Rectangular Box	20		6	26
Rectangular Tube			22	22
Cruciform	20	12	72	104
Total	476	20	297	793

(2) Data Items

The data-base of plates consists of a) σ_{cr} : maximum average compressive stress, b) σ_y : yield stress, c) width-thickness ratio with the following additional items :

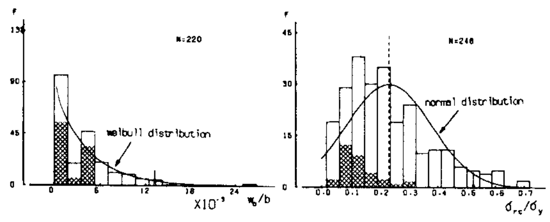
1) Cross-sectional shape···single plate, square box column, rectangular box section and cruciform section.
2) Dimensions of plate···thickness, width, length, aspect ratio and cross-sectional area.
3) Fabrication method···welded built up, cold-formed, mechanical cutting, gas cutting. After treatment···as-delivered, annealed and gas heated.
4) Yield stress···nominal yield stress, measured yield stress, method of test (compression or tension).
5) Support condition···unloaded and loaded edges···simple support, clamped, free and others.
6) Initial imperfections···measured residual stress and measured initial out-of-flatness.
7) Other measured items···load-average strain curve and average strain at the maximum load.
8) References···reference number, reference and the country where the test was conducted.
9) Identification···name of specimens and sequential number.

Measured values of Young's modulus E and Poisson's ratio ν are not usually given in the references, and so the values adopted for these material properties are $E=206$ kN/mm^2 (2. 1×10^6 kgf/cm^2) and $\nu=0.3$, respectively.

(3) Material Properties of Test Specimens and Initial Imperfections

Material Properties···Of the $N=789$ test specimens, 13% are of high tensile steels ($\sigma_y>430$ N/mm^2) and the rest are of ordinary steels. Mean value of the ratio of the measured to nominal yield stress, $\sigma_y)_a/\sigma_y)_n$, is 1. 13. The coupon test data include values for cold-formed square tubes which have high yield stress due to cold-forming. When these values ($N=71$) are excluded in the data, the yield stress ratio becomes 1. 09. The thicknesses of tensile coupons are from 2. 1 mm to 22. 3 mm, with a mean value of 7. 1 mm. The tests have been made therefore for comparatively thin plates.

Initial Out-of-Flatness··· **Fig. 1** (a) shows a histogram of the (w_0/b) values for the $N=220$ cases where



(a) Initial Out-of-Flatness (b) Residual Stresses

Fig. 1 Histogram of Initial Imperfections.

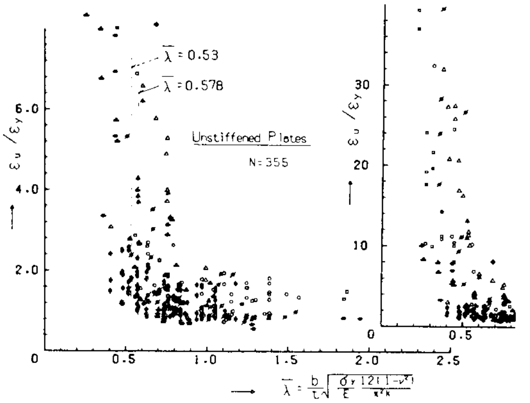


Fig. 2 Edge Strains at the Ultimate Strength.

measurements are available, in which w_0 is the maximum out-of-flatness or the out-of-flatness of the plate center and b is the width of the plate.

The shaded areas are for the initial out-of-flatness of plates bent by cold pressing. A fitted Weibull distribution is shown by a solid curve. For 91 out of the 220 cases, the w_0 value is larger than $b/250$, the most common value prescribed by codes as a fabrication tolerance.

Residual Stress…Fig. 1 (b) shows a histogram of the ratio of the measured residual compressive stress σ_{rc} (which is uniformly distributed over much of the plate) to the measured yield stress σ_y for the $N = 248$ cases where measurements are available. The mean value and standard deviation of σ_{rc}/σ_y are $M = 0.23$' and $S = 0.145$, respectively. The shaded areas are for $N = 32$ high strength steels ($\sigma_y > 430\ N/\mathrm{mm}^2$). The magnitude of the residual compressive stress may not be influenced by the yield stress of base metal and thus the σ_{rc}/σ_y values for high strength steels become small as shown in Fig. 1 (b). The mean and standard deviation of the σ_{rc}/σ_y values of the histogram excluding the shaded areas are $M = 0.25$ and $S = 0.15$, respectively.

(4) Average Compressive Strain at the Maximum Load

There are significant differences in the load-shortening curves for different width-thickness ratios of plates, steel grades and initial imperfections.

The average compressive stress σ—strain ε relationships of compressed plates can be classified schematically into the following three patterns: a) for high b/t ratios, a linear elastic $\sigma-\varepsilon$ relationship may be held to the peak load and then followed by abrupt descending curve, b) for low b/t ratios, a linear $\sigma-\varepsilon$ relationship is followed by gradual change of slope to a yield plateau and then strength can be increased in the strain hardening range, and c) for medium b/t ratios, the $\sigma-\varepsilon$ relationship is in between a) and b), i. e., the strength gradually decrease beyond the peak plateau.

The width-thickness ratio b/t is expressed as a modified plate slenderness

$$\bar{\lambda}=\sqrt{\sigma_y/\sigma_e}=b/t\sqrt{(\sigma_y/E)\cdot 12(1-\nu^2)/\pi^2 k} \quad \cdots\cdots (1)$$

in which σ_e is the critical stress given by the linear buckling theory and k is a buckling coefficient which varies with the aspect ratio of the plate (a/b) and the support conditions along the edges ($k=4$ for the flanges of square box sections with $a/b>1$ and $k=0.425$ for the flange outstands of H sections).

The commonly specified values of the width-thickness limits which will ensure adequate plastic hinge rotations in mild steel box sections ($b/t=30$) and the flanges of H sections ($b/t=10$) correspond to $\bar{\lambda}=0.578$[44] (see Fig. 2).

There are 355 cases of load-shortening curves available in the surveyed references. Figure 2 shows the nondimensional edge strain $\varepsilon_u/\varepsilon_y$ at the ultimate plate strength plotted against the plate slenderness $\bar{\lambda}$. The symbols used for the test points, which are summarized in Tables 2 and 3, allow the different groups to be identified.

The effective-width concept, which was proposed originally by von Kármán, predicts failure when the plate edge stress reaches the yield stress when no residual stress is present (i. e. $\varepsilon_u/\varepsilon_y=1$). However, if the tensile residual stress in welded built-up sections is equal to the yield stress, then the nondimensional edge strain $\varepsilon_u/\varepsilon_y$ at failure should be equal to 2.

From Fig. 2, the lower bound of the $\varepsilon_u/\varepsilon_y$ values is in the range of 0.8–2.0 for $\bar{\lambda}>0.7$. For $\bar{\lambda}<0.7$, the values of $\varepsilon_u/\varepsilon_y$ increase with a large scatter, and the edge strain may reach the initial strain hardening strain at $\varepsilon_{st}/\varepsilon_y=9-17$.

3. STRENGTH CHARACTERISTICS AND TESTING PARAMETERS

In order to compare the plate strengths of different cross-sectional shapes, with different fabrication processes and supported conditions, the ratios of the test points to those of a basic plate strength curve are compared[11),12)]. The measured yield stress and cross-sectional dimensions are used to nondimensionalize the plate strengths and plate slenderness $\bar{\lambda}$, and the basic strength curve is obtained from

$$\sigma_{cr}/\sigma_y=0.984/\bar{\lambda}-0.292/\bar{\lambda}^2+0.0334/\bar{\lambda}^3 \cdots\cdots (2)$$

This equation represents the mean strength curve obtained through a regression analysis of the test points ($N=146$) for the welded square box sections alone.

Fig. 3 demonstrates the scatter of the test points from Eq. (2), especially in the range of $\sigma_{cr}/\sigma_y>1(\bar{\lambda}\leqq\bar{\lambda}_0=0.579)$ where the average compressive stress σ_{cr} exceeds the yield stress.

(1) Single Plates

A test set-up to satisfy simply supported conditions along the unloaded edges is quite troublesome and various efforts have been made[15)].

Fig. 3 shows the test results of 413 of the single plate tests plotted in the σ_{cr}/σ_y (maximum average stress/actual yield stress) and $\bar{\lambda}$ coordinates. Symbols of the test points are summarized in Table 2 showing different fabrication conditions, yield stress and aspect ratio α. In Fig. 3, Eq. (2), von Kármán's formula and Euler's $1/\bar{\lambda}^2$ are given for reference. Among the 413 test data, 313 were simply supported along the unloaded edges and 101 were clamped. 72 points were for $\alpha=a/b<1$. No test data were available for the clamped edges with $\alpha<1$. The test values of α ranged from 0.167 to 6.0.

Coefficients k in Eq. (1) for plates subjected to pure compression are

$k=4.0$ for unloaded edges simply supported and $\alpha\geqq1$

$=\alpha^2+1/\alpha^2+2$ for $\alpha<1$

$k=6.97$ for unloaded edges clamped and $\alpha\geqq1$

The crosses + and arrows ↓ indicate the mean M and mean minus two standard deviations (M–2S) of all the single plate test points which fall into each 0.1 interval of $\bar{\lambda}$, where M=mean value and S=standard deviation. The

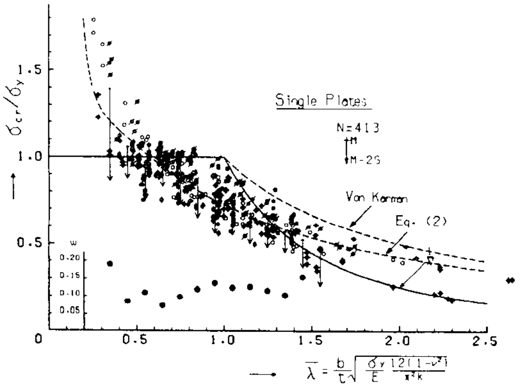


Fig. 3 Test Results of Single Plates.

Table 2 Comparison of Test Results for Single Plates.

Unloading Edges	Fabrication Conditions		Exp)a/ Eq. (2) N	M	S	ω
Simply	W M (α≥1)	○	106	0.997	0.097	0.097
			124	0.999	0.113	0.113
	W M (α<1)	ø	4	0.904	0.080	0.088
			6	0.899	0.063	0.070
	W H	⊖	6	0.982	0.075	0.077
			6	0.982	0.075	0.077
	W* M	◆	8	1.102	0.093	0.084
			8	1.102	0.093	0.084
	C M (α≥1)	ø	52	1.128	0.090	0.080
			66	1.123	0.104	0.093
	C M (α<1)	⌖	57	0.946	0.172	0.182
			67	0.938	0.162	0.172
	A M	●	23	1.095	0.118	0.108
			29	1.041	0.152	0.146
	A H	●	6	1.098	0.071	0.065
			6	1.098	0.071	0.065
Clamped	W M	ϕ	46	0.925	0.084	0.090
			69	0.935	0.073	0.079
	C M	♦	20	1.058	0.095	0.090
			32	1.021	0.094	0.092
Total			328	1.012	0.132	0.130
			413	1.004	0.133	0.132

W = weld with both side, W* = weld with center,
C = as-cutting, A = annealed,
M = mild steel ($\sigma_y<430$ N/mm²),
H = high strength steel ($\sigma_y\geqq430$ N/mm²)

lower of the figure indicates the coefficient of variation ω of the test points over each $\bar{\lambda}=0.1$ interval.

Table 2 summarizes the statistical results, that is, M, S and ω of the ratio of the test results to Eq. (2) for different groups of unloaded edge conditions, welding and annealing conditions, aspect ratios $\alpha \gtreqless 1$ and steel grades $\sigma_y \gtreqless 430\ N/\mathrm{mm}^2$. W represents the presence of the residual stress caused by weld beads along the unloading edges. In **Table** 2, the upper line indicates the results obtained only for the test data which fall into the range of $\bar{\lambda} \geqq \bar{\lambda}_0 = 0.579$ for which Eq. (2) yields $\sigma_{cr}/\sigma_y \leqq 1$, and the lower line indicates the ratio for all the test data including those in the strain hardening range.

Single plate strengths are summarized as follows from **Fig.** 3 and **Table** 2 :

(a) There is a significant loss in plate strength due to the presence of the residual stresses (W). Plates with residual stress (W) possess strengths between 5% and 15% less than those without residual stresses (C : as-cut and A : annealed). The test points for the plates are always below von Kármán's Eq. (3) for $\bar{\lambda} \geqq 1.0$.

(b) Plates with clamped unloaded edges possess strengths between 5% and 15% higher than those with simply supported edges as shown in Refs. 6), 15) and 22). However, the elastic critical stress σ_e according to the linear buckling theory is 75% higher for clamped edges than for simply supported edges. Therefore, the clamped plate test data plots a little lower than the simply supported plate data when $k=6.97$ is used for the clamped plates.

(c) Test data for $\alpha<1$ is scattered in the vicinity of Eq. (2) curve for $\bar{\lambda}=0.5$ to 1.3. For $\bar{\lambda}>1.3$, however, the test data become close to the Euler curve, showing column-like behavior, and the plate strengths in the range are quite different from those of plates with $\alpha \geqq 1$.

(d) There is little difference in strength due to the steel grades in the $\sigma_{cr}/\sigma_y-\bar{\lambda}$ coordinates. This may be partly due to the limited number of test data for high strength steels in the present survey.

(2) Square Box Columns

Fig. 4 shows 224 test results for square boxes with uniform thickness. The value of k in Eq. (1) is 4.0 and b is the flange width between the centers of webs for both welded and tubular box sections. The aspect ratios of test plates range from $\alpha=3$ to 8.7.

Table 3 summarizes the statistical results as in **Table** 2 for welded and tubular square box sections. The plates are categorized into sub-groups with respect to fabrication conditions.

Plate strengths are summarized as follows from **Fig.** 4 and **Table** 3.

(a) There is a significant difference in plate strength due to the presence of the residual stresses (D). Plate with residual stress (D) possess strengths between 5% — 15% less than those of annealed plates (A), which is similar to the conclusion reached for single plates.

(b) Tubular sections may possess relatively high strength compared with welded sections.

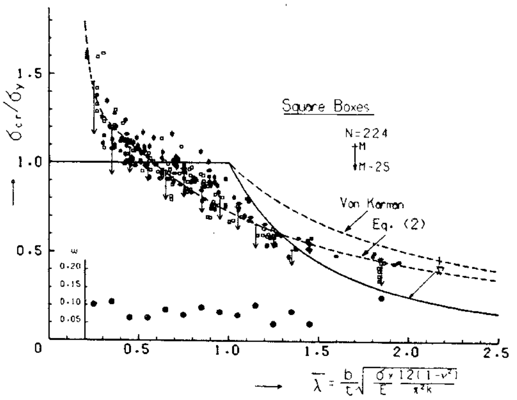


Fig. 4 Test Results of Square Box Columns.

Table 3 Comparison of Test Results for Square Box Columns.

Fabrication Conditions			Exp)$_a$/ Eq. (2) N	M	S	ω
Welded Boxes	D M	□	79 107	0.990 0.990	0.095 0.093	0.096 0.094
	D H	⊖	31 39	1.025 1.005	0.058 0.068	0.057 0.068
	B M	✚	1 1	0.996 0.996	-- --	-- --
	A M	■	14 26	1.128 1.057	0.079 0.103	0.070 0.097
	A H	●	1 2	1.225 1.128	-- 0.138	-- 0.123
Tubes	D M	⌽	19 38	1.093 1.042	0.066 0.083	0.060 0.080
	A M	⧫	4 11	1.247 1.082	0.078 0.147	0.063 0.136
Total			149 224	1.032 1.015	0.102 0.097	0.099 0.095

D = as-delivered. A = annealed.
B = gas-flame was run down the center line of four sides.
M = mild steel ($\sigma_y<430\ N/mm^2$).
H = high strength steel ($\sigma_y>430\ N/mm^2$)

(c) There is little difference in strength due to the steel grades in the $\sigma_{cr}/\sigma_y-\bar{\lambda}$ coordinates, which is similar to the conclusion reached for single plates.

(d) There is little difference in strength between single plate and square box tests for the same fabrication conditions.

4. COMPARISON WITH ULTIMATE PLATE STRENGTH FORMULAS

Various effective width formulas for plates have been proposed by many researchers which are based on test results and post-buckling strength analyses. Faulkner[46] made an extensive review of effective plating in the plate analysis of stiffened plates. Important effective width formulas were listed and some reviews were made of the statistical characteristics of the test data.

The effective width formulas shown here are for comparison with the test data. The formulas are expressed using the slenderness parameter$\bar{\lambda}$.

Von Kármán orginally suggested that Eq. (3) gives the maximum average compressive stress σ_{cr}.

$$\sigma_{cr}/\sigma_y = b_e/b = \sqrt{\sigma_e/\sigma_y} = 1/\bar{\lambda} \cdots\cdots (3)$$

Winter[33,34] suggested that Eq. (3) should be modified for light-gage steel design to

$$\sigma_{cr}/\sigma_y = b_e/b = 1/\bar{\lambda}\cdot(1-0.25/\bar{\lambda}) \cdots\cdots (4)$$

Lind[35] proposed Eq. (5) as a simple compromised based on experimental data for light gage steel plates ($N=336$).

$$\sigma_{cr}/\sigma_y = b_e/b = 0.86/\bar{\lambda} \cdots\cdots (5)$$

Möller[45] and Faulkner[46], respectively, proposed Eqs. (6) and (7), respectively

$$\sigma_{cr}/\sigma_y = 1/\bar{\lambda}\cdot(1-0.148/\bar{\lambda}^2) \cdots\cdots (6)$$

$$\sigma_{cr}/\sigma_y = 1/\bar{\lambda}\cdot(1.05-0.277/\bar{\lambda}) \cdots\cdots (7)$$

Dwight proposed Eq. (8) in which the average compressive residual stress σ_{rc} is accounted for

$$\sigma_{cr}/\sigma_y = 1/\bar{\lambda}\cdot(0.85-\sigma_{rc}/\sigma_y) \cdots\cdots (8)$$

Usami and Fukumoto[23] proposed that the ultimate plate strength of high strength steel based on the box column test results could be obtained from

$$\sigma_{cr}/\sigma_y = 0.75/\bar{\lambda} \cdots\cdots (9)$$

Table 4 summarizes the statistical results for M and S of the ratios of the test results for single plates and square boxes to Eqs. (1)–(6) and (9), respectively. In **Table 4**, the upper line indicates the results obtained when only the test data (N is given in the table) which fall into the range of $\bar{\lambda} \geqq \bar{\lambda}_0$ are used where $\bar{\lambda}_0$ is the limiting slenderness parameter for which $\sigma_{cr}=\sigma_y$ according to each formula. The lower line indicates the results obtained using all the test data ($N=340$ for plates and 224 for box sections).

The mean ratios compared with the von Kármán formula are low showing that this equation overestimates the test data. The Usami and Fukumoto formula is clearly very close to the mean value of the test points for $\bar{\lambda} > \bar{\lambda}_0$.

Faulkner approximated the coefficient of variation ω for his reviewed test data on simply supported plates from his mean curve (Eq. (7)) as

$$\omega = 0.05 \quad \text{for} \quad 0.26 \leqq \bar{\lambda} \leqq 1.32$$

$$\omega = 0.038\ \bar{\lambda} \quad \text{for} \quad \bar{\lambda} > 1.32$$

The lower part in **Fig. 4** indicates ω-values in the range of 0.049 to 0.101 for $\bar{\lambda}$ from 0.4 to 1.5, so that Faulkner's lowest ω-value is close to the lower bound.

Table 4 Comparison between Test Results and Various Design Curves.

	Single Plate (α≥1) N	M	S	Square Box N	M	S
Von Karman	122	0.783	0.093	52	0.758	0.069
	340	0.882	0.175	224	0.950	0.189
Lind	166	0.895	0.114	83	0.882	0.084
	340	0.938	0.156	224	0.986	0.161
Winter	246	0.958	0.110	130	0.962	0.096
	340	0.982	0.147	224	1.023	0.147
Möller	243	0.888	0.105	125	0.896	0.096
	340	0.933	0.156	224	0.985	0.165
Usami	213	1.006	0.130	110	1.000	0.093
	340	1.010	0.154	224	1.042	0.138

5. ULTIMATE PLATE STRENGTH FROM TEST DATA

Since there were significant losses and variations in strength due to the presence of the residual stresses, the plate strengths are categorized into two groups depending on the presence of the

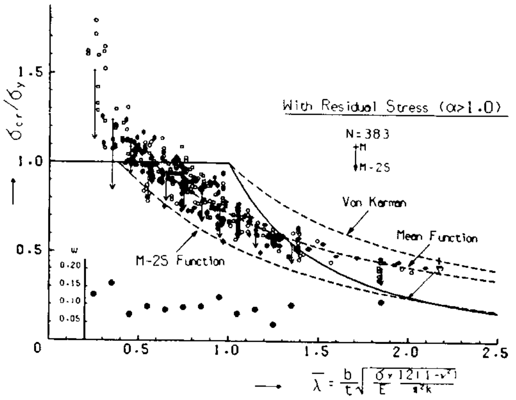


Fig. 5 Test Results and Proposed Design Curves "with residual stress" (W).

Table 5 Comparison between Test Results and Proposed Curve "with residual stress" (W).

				Exp)$_a$/ Eq. (10)			
				N	M	S	ω
Single Plates	Simply	W M	○	106	1.009	0.097	0.096
		W H	⊖	6	0.996	0.077	0.077
	Clamped	W M	ɸ	48	0.935	0.083	0.088
Welded Boxes		D M	□	79	1.002	0.096	0.095
		D H	⊟	31	1.038	0.059	0.056
Tubes		D M	ɸ	19	1.105	0.067	0.060
Total				289	1.004	0.097	0.096

residual stresses.

Ultimate plate strength for $\alpha<1$ are not analyzed hereafter because of insufficient test data.

(1) Plates with Residual Stress

Fig. 5 shows the results of 383 plate with residual welding stresses. The symbols of the test points are summarized in Table 5. Test data in this category are for single plates with weld beads along the unloaded edges, and for as-welded box columns.

In Fig. 5, the nearly equal length of the 2 S arrows in the range of $\bar{\lambda}>0.5$ may assume uniform variance of the test data through this range. A nonlinear regression analysis with an assumed uniform variance was therefore performed to obtained a mean curve of the test points. This mean curve is

$$\begin{aligned} &\sigma_{cr}/\sigma_y=0.968/\bar{\lambda}-0.286/\bar{\lambda}^2+0.0338/\bar{\lambda}^3 && \text{for } \bar{\lambda}\geqq 0.571 \\ &\sigma_{cr}/\sigma_y=1.0 && \text{for } \bar{\lambda}<0.571 \end{aligned} \quad \cdots\cdots (10)$$

and the standard deviation which is a root of conditional variance is $S=0.0871$.

The mean minus two standard deviation curve is expressed as,

$$\begin{aligned} &\sigma_{cr}/\sigma_y=-0.174+0.968/\bar{\lambda}-0.286/\bar{\lambda}^2+0.0338/\bar{\lambda}^3 && \text{for } \bar{\lambda}\geqq 0.389 \\ &\sigma_{cr}/\sigma_y=1.0 && \text{for } \bar{\lambda}<0.389 \end{aligned} \quad \cdots\cdots (11)$$

The dotted curves in Fig. 5 represent the mean curve (Eq. (10)) and the mean minus two standard deviations curve, respectively. Table 5 summarizes the mean function (Eq. (10)) for differently categorized sub-groups. The results in Table 2 are for $N=289$, the number of the $N=383$ test points which fall in the range $\bar{\lambda}\geqq\bar{\lambda}_0=0.571$. For $\bar{\lambda}<\bar{\lambda}_0$, $\sigma_{cr}/\sigma_y=1$ is assumed which ignores the strain hardening effect.

From Fig. 5 and Table 5, the proposed formula (Eq. (10)) agrees quite well with the mean value of the test points for $\bar{\lambda}>\bar{\lambda}_0$ for all the sub-groups except for the single plates with the unloaded clamped edges. The M–2S Function is close to the lower bound of the test data and even close to the Euler curve $\sigma_{cr}/\sigma_y=1/\bar{\lambda}^2$ when $\bar{\lambda}$ becomes large. The limiting slenderness parameter $\bar{\lambda}_0=0.389$ for the M–2S Function correspond closely to the lower bound of the scattered test points in the vicinity of $\sigma_{cr}/\sigma_y=1$. The plate strength curve proposed by Komatsu and Kitada[51] falls in between M and M–2S curves.

(3) Plates without Residual Welding Stress

Fig. 6 shows the results of 172 plate tests without residual stress. As-cut and annealed single plates and annealed square box sections are included in this category.

In the same way as for the plates with residual stresses, a nonlinear regression analysis with a uniform variance was used to develop the following mean function:

$$\begin{aligned} &\sigma_{cr}/\sigma_y=1.133/\bar{\lambda}-0.384/\bar{\lambda}^2+0.0468/\bar{\lambda}^3 && \text{for } \bar{\lambda}\geqq 0.658 \\ &\sigma_{cr}/\sigma_y=1.0 && \text{for } \bar{\lambda}<0.658 \end{aligned} \quad \cdots\cdots (12)$$

with the standard deviation $S=0.104$. The M–2S curve is

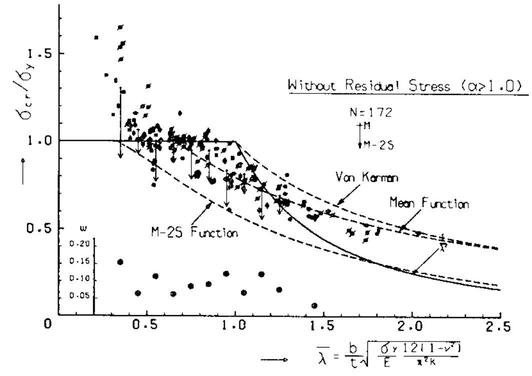


Fig. 6 Test Results and Proposed Design Curves "without residusl stress" (W/O).

Table 6 Comparison between Test Results and Proposed Curve "without residual stress" (W/O).

				Exp)$_a$/ Eq. (11) N	M	S	ω
Single Plates	Simply	A M	●	23	0.997	0.107	0.107
		A H	-●-	6	0.989	0.053	0.053
		C M	⌀	46	1.036	0.087	0.084
	Clamped	C M	◆	17	0.975	0.093	0.095
Welded Boxes		A M	■	13	1.036	0.075	0.073
		A H	-■-	1	1.123	--	--
Tubes		A M	◆	4	1.148	0.079	0.069
Total				110	1.021	0.094	0.093

$$\sigma_{cr}/\sigma_y=-0.208+1.133/\bar{\lambda}-0.384/\bar{\lambda}^2+0.0468/\bar{\lambda}^3 \quad \text{for } \bar{\lambda}\geqq 0.337$$
$$\sigma_{cr}/\sigma_y=1.0 \quad \text{for } \bar{\lambda}<0.337 \quad \cdots\cdots(13)$$

Table 6 summarizes the statistical results for the ratio of the test points to the mean curve (Eq. (12)) for differently categorized sub-groups. From Fig. 6, the M Function is close to the von Kármán's curve, when $\bar{\lambda}$ becomes large. The proposed formula (Eq. (12) agrees quite well with the value of the test points for $\bar{\lambda}>\bar{\lambda}_0=0.658$. The M–2S Function underestimates the test points for large $\bar{\lambda}$, since the uniform variance was assumed for much of the range.

(4) Effect of Initial Imperfections

As shown in Figs. 1, $N=248$ of measured residual stresses have been reported in the surveyed references, and histograms showing variations of the measured data. The effect of the initial imperfections upon the ultimate plate strength has been analyzed in different investigators using the measured magnitude of the imperfections. Detailed statistical correlation of the initial imperfections with the ultimate plate strengths are not given in this paper because of the shortcomings of the enough data available. However, the ultimate strength data of plates which have large magnitude of the initial imperfections are shown in the following figures.

Initial Out-of-Flatness···From Fig. 5, the test points which have the initial measured maximum out-of-flatness $w_0>0.004\ b$ were chosen to investigate the reduction of plate strength due to the magnitude of the initial out-of-flatness. Fig. 7 shows the test results of 52 plates with $w_0>0.004\ b$ and with residual stress. The mean curve of Eq. (10) is also drawn for comparison. Values of $M=0.964$ and $S=0.0634$ were obtained by statistical manipulation of the ratio of the chosen test points to the mean curve (Eq. (10)). The test points are plotted considerably below the mean curve for $\bar{\lambda}<1.3$.

Similar results can be seen for the case without residual stress. $N=39$ with $w_0>0.004\ b$ were chosen from the $N=172$ tests shown in Fig. 6, and they were compared with the mean curve of Eq. (12). Values of $M=0.959$ and $S=0.0755$ were obtained from the ratio of the chosen test data to the mean curve of Eq. (12).

Residual Stresses···Plates with large magnitude of residual welding stress ($\sigma_{rc}\geqq 0.4\ \sigma_y$) which were chosen from the $N=383$ test results. There were $N=27$ of those mainly scattered near the limiting value $\bar{\lambda}_0=0.571$. The ratios of the chosen test points to the mean curve Eq. (10) are $M=0.994$ and $S=0.0944$, and no significant effect of the residual stress can be detected from these

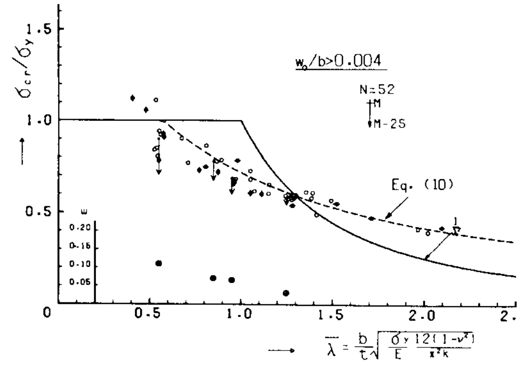


Fig. 7 Effect of Initial Out-of-Flatness "with residual stress" (W).

values. There are no test data available for plate with large initial imperfections $\sigma_{rc}/\sigma_y>0.4$ together with $w_0/b>0.004$.

Fig. 7 shows that the ultimate plate strength which is significantly influenced by the magnitude of the initial out-of-flatness in the medium range of $\bar{\lambda}$. Even when the $N=52$ test data for $w_0>0.004\,b$ are included, the scattered test points are closely bounded by the M−2 S curve (Eq. (11)).

When the test data with large initial imperfections are excluded, a regression analysis using the $N=306$ test data of Fig. 5 leads to the mean curve approximately by

$$\sigma_{cr}/\sigma_y=1.023/\bar{\lambda}-0.339/\bar{\lambda}^2+0.0458/\bar{\lambda}^3 \quad \cdots\cdots (14)$$

and with a standard deviation $S=0.0864$.

Eq. (14) is only slightly higher than Eq. (10) and there is no significant difference between them.

6. CONCLUSIONS

The following main conclusions have been drawn from the results of the experimental data-base approach.

(1) A numerical data-base system has been developed for experiments on steel plates. Details of cross-sectional shape, fabrication processes, material properties, initial imperfections, and others have been stored in the data-base, from a total of 793 plate tests conducted in Europe, North America and Japan. Various computational and statistical manipulation programs are available to evaluate the test data.

(2) No clear difference between plate strength by single plate test method and square box test method may be found from the test data.

(3) The plate strengths are categorized into two groups depending on the presence of the residual welding stresses. Two plate strength formulas which explain more accurately the test data are proposed from the statistical results based on the experimental data-base.

(4) Annealed plates show greater variations in strength than as-welded plates.

(5) Reduction of plate strength due to the magnitude of the initial out-of-flatness is predicted in the medium range of plate slenderness from the results of the experimental data-base approach.

(6) Further experimental investigations are needed for high strength steel plates with plate slenderness parameters near the limiting value $\bar{\lambda}_0$.

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