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[Technical Note]

ON THE VISUALIZATION OF PLANAR DISPLACEMENT FIELDS OF A GRANULAR MEDIUM

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1. INTRODUCTION

Applications of photogrammetric principles and methods have not only been used for making topographic maps, but also utilized for non-topographic close-range photogrammetry, for instance to measure the sculptures in monuments, the box girder deformation, the limb remnant models, and so forth. However, the measurement using non-metric camera had formerly not been practical, because of the difficulty to adjust the systematic errors of that camera in photogrammetric analogue restituting plotters, and then only metric camera had been served for the purpose of photogrammetric measurement. The problem of adjusting those errors has recently been solved by the use of the comparator with an electronic computer or the analytical plotter. Thus non-metric cameras are now capable of being utilized in close-range photogrammetry, for example to measure the mine-roof instability, the aircraft movement, the sand displacement due to shear[1], the shield excavation model, and the like.

This report presents results of measurements obtained with the application of photogrammetric false parallax (i.e. so-called the Cameron effect) which has long been recognized as an accurate and expedient method of measuring particle movements in a plane parallel to the photographic image plane. Discussion will be made of the measured displacement fields of a granular medium in relation to earth pressure characteristics which become important in the practice of excavating tunnels at shallow depths, because soil deformations around the tunnel and the ground surface settlements caused by tunnel excavation have not fully been understood at the present stage.

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2. STEREO-PHOTOGRAMMETRIC METHOD FOR MEASURING DISPLACEMENT FIELDS

(1) Principle of False Parallax

The principle of false parallax measurement is described below, with reference to **Fig. 1**. Let an object A be displaced to position A^* in the horizontal direction (X). Then the corresponding images of the points A and A' can be seen as a and a' on the phtographs i and $i+1$, respectively. Therefore, the parallax at the moving point A, denoted by P_{XA}, is given by

$$P_{XA}=aa' \quad \cdots\cdots (1)$$

The parallax at an unmoved reference point, denoted by P_{XGA}, is represented by

$$P_{XGA}=O_iO_{i+1} \quad \cdots\cdots (2)$$

Hence, the displacement component (Δ_{XA}) of the point A is obtained as

$$\Delta_{XA}=K\cdot\delta_{XA}=K(P_{XGA}-P_{XA}) \quad \cdots\cdots (3)$$

where δ_{XA} is the corresponding displacement component on a 35 mm film, and K is its film scale-factor. If an object A moves in arbitrary direction on $X-Y$ plane, then the displacement component (Δ_{YA}) should also be measured. This

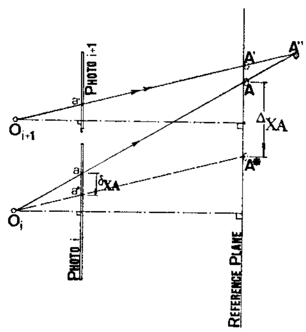

Fig. 1 The principle of false parallax.

displacement component is able to be expressed in the form:

$$\Delta_{YA}=K\cdot\delta_{YA}=K(P_{YGA}-P_{YA}) \quad\cdots\cdots\cdots\cdots\cdots(4)$$

where δ_{YA} is the displacement component in the vertical direction Y on the 35 mm film, P_{YA} is the Y-parallax at the point A, and P_{YGA} is the Y-parallax at a stationary reference point chosen near point A.

(2) Photography

For false-stereophotography, use was made of a non-metric camera (35 mm Contax RTS) with a format of 35 mm by 22 mm, together with a lens (Planar) having a forcal length of 59 mm. With this camera being fixed at a given station, photographs were successively taken of the vertical cross-section of a granular medium in the course of lowering the bottom plate (hereafter called the lowering panel) on which the granular medium was resting (for details of the test setup, reference should be made to section 3). Subsequently, the photography used in this study is explained in some detail.

In order to obtain a definite photo-scale, a level-babble with a curvature of 1.5 m was used to level the camera, and the view-finder was utilized to maintain the camera axis perpendicular to the vertical cross-sectional plane of a granular medium. The 35 mm film-scale was thereafter evaluated to be 1 : 25, based on the result that the reference grid intervals of 60 mm on the test setup were measured to be 2.4 mm on the 35 mm film. The principal distance of the camera used was evaluated to be 60 mm based on the fact that the object distance for the photography was adopted to be $D=1\,500$ mm as shown in **Fig. 2**.

A false parallax (i.e. the Cameron effect) is observed at a moving object on a pair of aerial photographs having an initial and a terminal position. Similarly, a false parallax is given by a stereopair of photographs taken of "before" and "after" movement of a granular substance

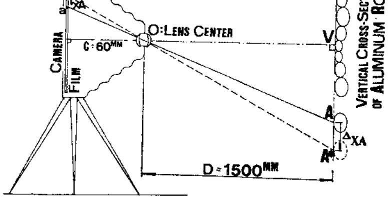

Fig. 2 Schematic drawing of photography and vertical cross-section of aluminum rods used.

on a plane. The enlarged RC paper prints at the scale of 1 : 4 obtained from the 35 mm original successive films showing each step of settling phenomenon associated with tunnel excavation were stereo-scopically measured with a parallax-bar and a mirror stereoscope.

(3) Assessment of Accuracies

It can be assumed that particle displacement magnitudes obtained from the false parallax method have the following errors:

- σ_1: Image displacement error in 35 mm film due to the tilting of the optical axis of camera;
- σ_2: Image displacement error in the enlarged RC paper print due the tilting of the projectional easel plane (i.e. similar to error to σ_1);
- σ_3: Lens distortions in the 35 mm camera and in the projector for enlarging photographs;
- σ_4: Non-planarity in 35 mm film;
- σ_5: Shrinkage and expansion errors in 35 mm film and in enlarged RC paper print;
- σ_6: Non-parallelismic error due to the measuring parallax-bar direction (i.e. the axis of connecting line between two mess-marks) dissonant with the grid axis of 60 mm intervals on the vertical plane of the granular medium; and
- σ_7: Personal observational error in the false stereo-measurement.

If the angle of maximum camera tilt (δ_θ) is equal to 1°, σ_1 is assessed as follows.

$$\sigma_1=\max(\Delta x'-\Delta x)=\Delta x'\left\{1-\frac{\cos\theta}{\cos(\theta-\delta_\theta)}\right\}$$
$$=\pm 0.03\,\text{mm} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(5)$$

where $\Delta x'$ (≤ 10 mm) is the maximum particle displacement along vertical grid axis, Δx is the displacement along camera axis tilted, and θ is the angle spanning the maximum distance R (≤ 250 mm), as indicated in **Fig. 3**. If the distortions in photographing camera lens and in enlarging projector lens are the same characteristics, measuring a bundle of rays from the projector or measuring the projected image is corrected by the reciprocity law. Even if there are still some distortion differences between the camera and the projector, measured parallax (i.e. particle displacement in a direction) is accurately provided more than enough, as the expected accuracy in this test is ± 0.5 mm of a tolerance error on the cross-sectional plane of aluminum rods. Hence, the errors of σ_3 and σ_4 do not exert such a decisive influence on the issue of this test. The error σ_5 is judged here to be negligibly small. In view of **Fig. 4** the error

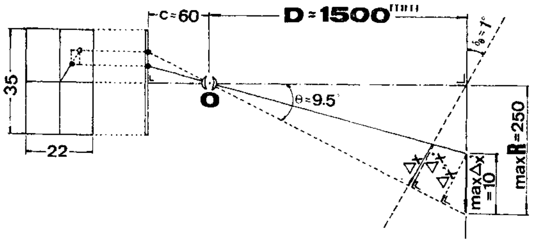

Fig. 3 Image displacement error due to the tilting of the optical axis of camera.

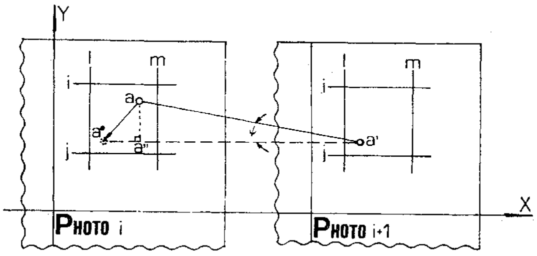

Fig. 4 Non-parallelismic error.

σ_6 is evaluated as follows:

$$\sigma_6 = \ddot{K}(P_{XA} - P_{XA'}) = \ddot{K}(1-\cos\psi)aa'$$
$$= \pm 0.05\,\text{mm} \quad \cdots\cdots (6)$$

where $\ddot{K}=4$ is the scale-factor of the enlarged paper prints, the distance aa' ($\approx a'a'' \leq 250$ mm) is the distance between the messmarks (i.e. the measuring marks of parallax-bar) on the glass plates, and the angle ψ is calculated from $\psi=$ arctan $(aa''/a'a'')$ = arctan (2.5 mm/250 mm) $\approx$ 0.6°. The error σ_7 (i.e. $\sigma_{\Delta X}$ or $\sigma_{\Delta Y}$) is determined as follows.

$$\sigma_7 = \sigma_{\Delta X} = \sqrt{\sigma_G^2 + \sigma_X^2} = 4\sqrt{0.02^2 + 0.03^2}$$
$$= \pm 0.15\,\text{mm} \quad \cdots\cdots (7)$$

where $\sigma_{\Delta X}$ (or $\sigma_{\Delta Y}$) is determined by applying the law of error propagation in Eq. (3) (or (Eq. (4)), and σ_G and σ_X are the standard deviations of 10 parallax measurements at a grid intersection and a particle of aluminum rod, respectively.

Consequently, the maximum total error σ_{TX} (or σ_{TY}) in Δ_X (or Δ_Y) is assessed from

$$\sigma_{TX} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \cdots + \sigma_7^2}$$
$$= \pm 0.2\,\text{mm} \ (\approx \sigma_{TY}) \quad \cdots\cdots (8)$$

3. EXPERIMENTATION

This section outlines the material, test apparatus and test procedure used in the present study.

(1) The Granular Material Used

The side (front) wall of the container was taken away during the experiment, in order to eliminate the friction between the wall of the container and the material contained in it. This was realized by adopting aluminum rods whose diameters were 3, 5, and 9 mm, with a length of 50 mm. The model ground was formed by mixing these three kinds of aluminum rod at a rate of 8 : 7 : 3 in weight.

(2) The Test Apparatus and Test Procedure

The test apparatus has a lowering panel which is situated at the center of the bottom plate of the container and is able to be moved up and down. The width of lowering panel (B) was 120 mm. The model ground was formed by accumulating the aluminum rods until the ground height (H) was equal to 2.5 B. The front surface of the ground was carefully adjusted to be precisely vertical by utilizing a level-babble and a plumb-bob.

Exposure interval for photography was selected to be every 1 mm settlement which the settlement of the lowering panel ranges from 0 to 30 mm, and thereafter the exposure interval was chosen to be every 2 mm-settlement when the settlement of the lowering panel exceeds 30 mm. The settling rate of the lowering panel was chosen to be 1 mm/min. Settlements of the lowering panel and loads acting on it (earth pressure times loaded area) were measured with a 0.01 mm graduation dial gage and a load cell, respectively, together with an automatic recording unit.

4. RESULTS AND DISCUSSION

(1) Relation between Earth Pressure and Panel Settlement

Fig. 5 shows that earth pressure (P) rapidly decreases with increasing settlement of the panel ($\Delta = \Delta_Y$) and takes a minimum value at Δ from 2.0 to 2.5 mm, and thereafter P gradually increases to an approximately constant value as Δ increases. It is also seen from **Fig. 5** that, for the conditions of $H/B \geq 1.0$, the height of the

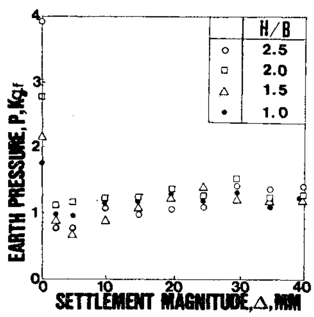

Fig. 5 Plot of earth pressure vs. settlement of panel.

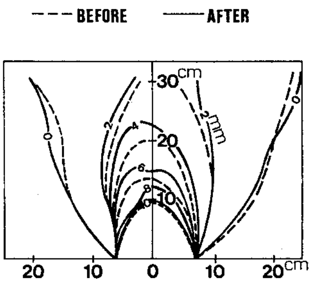

Fig. 6 Contours of equal settlement of granular medium.

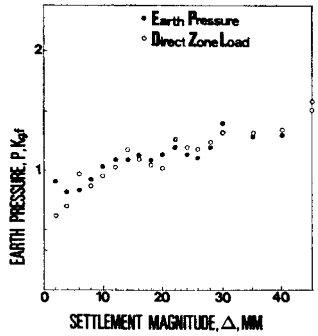

Fig. 7 Comparison between earth pressure and weight of direct zone at various settlements.

ground (H) does not exsert significance influences on the magnitude of minimum earth pressure (P_{min}) and on the magnitude of panel settlement Δ when $P=P_{min}$. Hence, if an overburden height $H_i/B \geq 1.0$, the overburden height does not offer influence on P. It may be appreciate here to mention that these findings obtained from the false parallax method are considered to be similar to the earth pressure characteristics suggested by Murayama and Matsuoka.

(2) Relation between Earth Pressure and Direct Zone

According to we apply in the theory of arch in terms of arch action, there is a close relationship between panel load called the weight of "direct zone" and panel load measured. The direct zone is defined here as the zone which is enclosed by the arch-shaped innermost curve above the lowering panel, as shown in **Fig. 6**. In this figure the solid curves represent contours of equal settlement at the 8 mm-settlement of the panel and the broken curves represent contours of equal settlement at the 10 mm-settlement of the lowering panel. As mentioned above, the earth pressure (P) is approximately constant with increasing settlement of the panel when Δ is more than 2.0 to 2.5 mm. Hence the weight of direct zone is equivalent to earth pressure. The settlement magnitudes are illustrated by the vertical components of aluminum rods settled. Direct zone is able to be identified from measured contours of equal settlements which are shown in **Fig. 6**. In this figure it can be said that the areas of two direct zones of 8 mm- and 10 mm-settlement contours are coincident with each other. **Fig. 7** compares the measured earth pressure (P) with the calculated weight of the direct zone, at a given settlement of the lowering panel. Here the weight of the direct zone was calculated by multiplying the measured area of it by the unit weight of granular medium of 2.26 gf/cm^3. It is seen from **Fig. 7** that, for $\Delta > 2$ mm, the creation of direct zone has already been completed, and P becomes equal to the weight of the direct zone. For $\Delta > 2$ mm, it is suggested that the direct zone has not fully been developed, in accordince with the dilatancy phenomenon and changing the angles of internal friction in the granular medium.

5. CONCLUDING REMARKS

The principal conclusions drawn from the present study are summarized as follows:

- The measuring reliability regarding displacement is estimated as the standard deviation of ± 0.2 mm;
- The method of false-stereophotogrammetry can accurately be measured in the whole displacement field of a granular medium;
- Earth pressure P is equivalent to the weight of the direct zone, that is,

$$P \approx \gamma \cdot A \cdot l \qquad (9)$$

where γ is the unit weight of ground material, A is the calculated area of the direct zone, and l is the length of aluminum rods used.

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REFERENCE

1) Murayama, S. and O. Inoue: Characteristics of Pasticle Displacements of Sand Subjected to Shear (in Japanese), Proceedings of JSCE, No. 309, pp. 79∼89, 1981.

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