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A MAN-MACHINE INTERACTIVE APPROACH TO COST ALLOCATION: A GAMING ANALYSIS

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1. INTRODUCTION

In the field of public works, there has been mounting concern about how to reconcile conflicting interests among the different parties involved. Among a variety of conflict problems is the well known problem: how to split the total costs of a joint project among different users. This problem, which is generally called "cost allocation" is the major concern of this paper.

The water resources field, for example, offers a set of typical cost allocation problems and has extensive literature on this theme. Many approaches have been proposed, tested and modified therein, and some of them appear to have gained extensive publicity and application in this field. The most conspicuous among them is *the Separable Cost Remaining Benefit* (*SCRB*) *Method*[1),2)]. This method, whose origin dates back to the early 1950's when a subcommittee of the Federal Interagency River Basin Committee recommended the *SCRB*, has been further developed in other countries to constitute the legal basis of present cost allocation procedures. In Japan it is prescribed by law that the allocation of costs should principally be performed by applying the *SCRB*-based procedure.

Though it is so widely applied, both theoretical and empirical studies have shown that *SCRB* has some crucial drawbacks and inconsistencies. Among them, the criticism that the conventional methods including *SCRB* fail to handle the bargaining feature of cost allocation has provoked the development of a new approach in the water resources field. This approach owes its theoretical basis to what has been developed as the theory of cooperative games. It was not until quite recently, however, that a systematic assessment was made of the implications and applicability of a cluster of game theoretic methods for cost allocation in water resources management. From the viewpoint of equity and fairness and common sense, Young, Okada, and Hashimoto[3)~6)] have identified a set of basic principles that ought to be embodied in cost allocation. They have then proceeded to a systematic check of both conventional and game theoretic methods against the basic principles. They have concluded that the conventional methods including *SCRB* and some game theoretic methods fail to satisfy some of the basic principles and only a couple of lesser known methods from game theory, i.e. *the Weak Nucleolus* (*WN*) and *the Proportional Nucleolus* (*PN*), proved to be more appropriate.

These points were illustrated by their application to a cost allocation problem among a group of Swedish municipalities developing a joint municipal water supply. The development of the above study has motivated another type of approach. Ståhl[7)] has implemented an empirical approach called "gaming" to the Swedish cost allocation problem. He claimed that any allocation method based on preselected norms may not be accepted by participants. His approach was characterized by his position that the participants ought to be given as much of a free hand as possible in their bargaining with the others to find a final compromise. "Invite players to the same table and let them play with the others", given a set of cost data on "going alone" and "going together". This was his idea.

The author[8)] has pointed out that the extent to which a cost allocation method has application may largely depend on the level or scope in which cost allocation is discussed and so there cannot be only one allocation method but rather many, and he claimed that if a cost allocation enters in the project implementation phase as is commonly the case, it becomes no more than a financial analysis and so demands a normative approach. Admittedly, there is another extreme situation in which empirical approach finds application. Suppose there has not yet been any established

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cost allocation procedure whatsoever and one desires to identify those rules or norms which pattern what may turn out to be a normative procedure in the future. Ståhl's approach[7] may be justified for this type of extreme situation. In practice, however, it appears more natural to assume that even when no procedure has yet been determined some minimum set of agreements or norms should be set a priori on which to base the negotiation game among them in order to find what may finally be developed into their cost allocation procedure. It is in this very sense that the author[8),11] has developed a "prescriptive-empirical approach" to cost allocation which is intended to go halfway between the normative end and the empirical end. There certainly are natural situations which demand this type of approach. The situations may include (i) when some or all of participants fail to understand the implications and validity of a normative method represented by *WN* and *PN*; (ii) when some or all are reluctant to accept the set of norms as they are, though they may allow some basic ones to be retained; and (iii) when with a set of norms proposed by the project manager or some of the participants, they desire to obtain a deeper understanding of what is implied by the application of these norms to cost allocation or they may even intend to add to the original set of norms. Very likely the situations may be compounded. The author[8),12] has noted that a prescriptive-empirical approach to cost allocation which deals with such a situation ought to serve both educational and problem-finding purposes. By education is meant an intention to get those who are ignorant to acknowledge some normative wisdom and principles. Problem-finding underlies a position which allows for latitude in individual experience, change, and trial and error.

This study extends the author's former studies[8),10),11] in the following points:

(i) The new approach is designed to incorporate a microcomputer in the procedure of cost allocation as an aid in supplying participants with information and explanations for the ongoing cost allocation gaming.

(ii) The information is all visualized and colored to appear on the screen of a color display unit linked with the microcomputer.

(iii) The rationales for employing a microcomputer, and not a large computer are economy, and ease and handiness with which to gain an access to, and to develop interactive dialogues with the computer. This is increasingly true as innovation in the microcomputer industry progresses year by year at a conspicuously high speed.

(iv) After conducting experiments a number of times with participants seated before the computer, we will closely analyze the results from both macroscopic and microscopic viewpoints. This will lead on to a systematic check of the potentials and limits of the proposed approach.

2. DESIGN OF GAMING

(1) Problem identified

Let us assume that three cities now contemplate undertaking a joint water supply project. Their primary concern is with how to allocate the total costs. So we have three players and not more. We will limit the number of players to three, because (i) three players are the minimum condition for a coalition to be formed; (ii) the displaying of information in more than three dimensions entails technical difficulties; and (iii) a three-player game is considered the prototype of a coalition game. Each player may be allowed to go alone which would cost him what is termed an individual cost of attaining the goal. Or he may contemplate going together with one of the rest, in order to form a coalition against the last one who is forced to go alone. The cost of so doing is called a coalition cost or a joint cost. The data on all of these costs to be estimated in advance are given in **Table 1**.

Table 1 Input cost data.

INDIVIDUAL	COALITION	GRAND COALITION
C (A)=6.5 C (B)=4.2 C (C)=1.5	C (AB)=10.3 C (AC)= 8.0 C (BC)= 5.3	C (ABC)=10.6

(UNIT: 10^8 Yen)

(2) Microcomputer system implemented

With costs and functions taken into account, a choice has been made to utilize the Sharp MZ-80 K2 microcomputer system, which consists of a green computer (main module), a dual floppy disk, a dot printer, a color monitor and interface units to link them together (see **Fig. 1**).

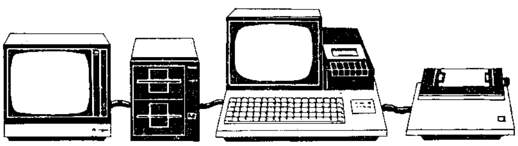

Fig. 1 Microcomputer system.

(3) Minimum norms built in

Depending on whether players go alone or together, basic norms which have been reduced to minimum requirement are to be built in our approach. One is the self-evident balance condition that a total of costs assigned to each be equal to the entire costs of the grand coalition project to be participated in by all three cities. If no coalition is formed, the remaining condition is the principle of individual rationality which dictates that none of the participants be worse off by participating in the grand project. Extension of this principle is made to the case in which a coalition is contemplated by two of the three; that is, the principle of group rationality; which prescribes that a group which could contemplate forming a coalition be not worse off by participating in the grand coalition.

To formulate the above conditions in mathematical terms.

Self-evident condition:

$$X_A+X_B+X_C=C(ABC) \qquad (1)$$

Individual rationality:

$$X_A \leqq C(A);\ X_B \leqq C(B);\ X_C \leqq C(C). \qquad (2)$$

Group rationality:

$$\left.\begin{array}{ll} X_A+X_B \leqq C(AB) & \text{if cities } A \text{ and } B \text{ go together;} \\ X_B+X_C \leqq C(BC) & \text{if cities } B \text{ and } C \text{ go together;} \\ X_A+X_C \leqq C(AC) & \text{if cities } A \text{ and } C \text{ go together.} \end{array}\right\} \qquad (3)$$

In the above X_i denotes the cost to be allocated to city i (i being A, B, or C) and $C(i)$ or $C(S)$ represents the costs of the participant(s) as specified by the symbol parenthesized (S being AB, BC or AC). We assume that these characteristic functions (cost parameters) have somehow been determined.

It is noted that the collection of the above three conditions gives the concept of core, a well known concept from cooperative game theory as the basis of fairness and equity in bargaining and negotiation. Since it is assumed that "going alone" and "going together" are mutually exclusive in our cost allocation gaming and so only one of the two conditions, individual or group rationality is set to hold, there is no guarantee for a compromise solution to satisfy core always.

(4) Pre-gaming guidance

By inviting three players to the game as the representatives of the three cities, we begin by supplying players with some guiding information which includes (i) cost data and possible coalition patterns; (ii) description of some minimum norms to base the game, i.e. self-evident condition, individual rationality and group rationality with the concept of core also illustrated for reference (see **Photo 1**); and (iii) basic rules and procedures for the gaming.

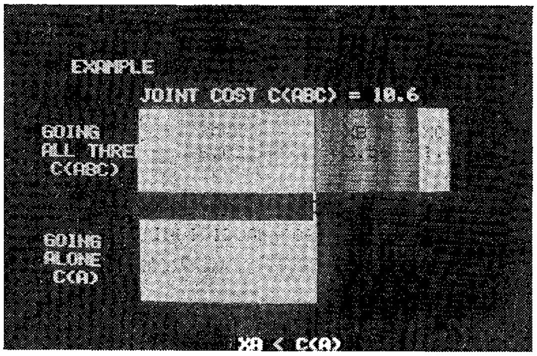

Photo 1 Pre-gaming guidance information.

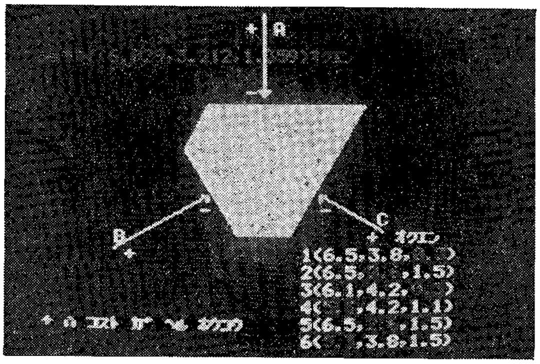

Photo 2 Gaming provisional results on display (1).

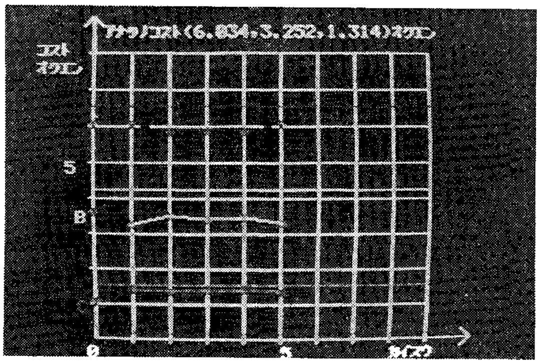

Photo 3 Gaming provisional results on display (2).

(5) Gaming

Gaming starts by asking each of the players to choose between "going alone" and "going together". Suppose all choose to go alone. Then players are asked to specify their "satisfactory level" for their share of the costs. Since it is designed to keep them from knowing what the others aspire to as their satisfactory levels, they are asked to report to the gaming operator by ballot.

With the satisfactory levels thus fed in, the computer immediately tells them about what the automatically reconciled solution is. If all find it acceptable, which is rather unlikely in a very early stage of the gaming, we terminate the gaming and this solution is taken as their final compromise solution. Otherwise we go on with the same procedure until all agree to finalize the gaming.

(6) Theoretical basis and its formulation

Once satisfactory levels are specified by either individual players or a group of players forming a coalition, the problem of finding a (provisional) compromise solution may easily be formulated as a multi-objective programming problem.

If no coalition is formed, the problem is written as:

$$\text{Minimize } X_A, \quad \cdots\cdots (4)$$

$$\text{Minimize } X_B, \quad \cdots\cdots (5)$$

$$\text{Minimize } X_C, \quad \cdots\cdots (6)$$

subject to

$$X_A \leqq C(A);\ X_B \leqq C(B);\ X_C \leqq C(C), \quad \cdots\cdots (7)$$

$$X_A + X_B + X_C = C(ABC), \quad \cdots\cdots (8)$$

where inequality constraints come from individual rationality with X_A, X_B and X_C and $C(A)$, $C(B)$, $C(C)$ and $C(ABC)$ as defined before.

If a coalition is formed, there are two levels for the players in the group to go through in reaching a (provisional) compromise. With a coalition formed by, say, A and B, the level-one problem is formulated as:

$$\text{Minimize } X_A + X_B, \quad \cdots\cdots (9)$$

$$\text{Minimize } X_C, \quad \cdots\cdots (10)$$

subject to

$$X_A + X_B \leqq C(AB) \quad \cdots\cdots (11)$$

$$X_C \leqq C(C) \quad \cdots\cdots (12)$$

$$X_A + X_B + X_C = C(ABC). \quad \cdots\cdots (13)$$

The first inequality condition of Equation (11) dictates that group rationality should hold for a coalition (AB), whereas individual rationality needs to hold for an individual player C as expressed by the second inequality condition of Equation (12).

On finding a provisional compromise solution for a coalition (AB) and an individual player C, as explained later, the level-two problem is to determine how to split between them the costs X_{AB} as assigned collectively to players A and B at level one. This lower level problem is played by the two, A and B who formed a coalition at the upper one. This is expressed as:

$$\text{Minimize } X_A, \quad \cdots\cdots (14)$$

$$\text{Minimize } X_B, \quad \cdots\cdots (15)$$

subject to

$$X_A \leqq C(A);\ X_B \leqq C(B), \quad \cdots\cdots (16)$$

$$X_A + X_B = X_{AB}. \quad \cdots\cdots (17)$$

Again, the inequality conditions of Equation (16) are the expression of individual rationality to hold for A and B.

(7) Compromise finding algorithm

As is clear from the formulations above, the problem has been converted into a multi-objective programming problem, to which a variety of techniques have been so far developed to locate a most acceptable solution (or "satisfying" solution), not an optimal solution from a single objective viewpoint. Among many candidates a technique of goal programming with an explicit assumption of L-shaped utility function[12] has been singled out. This may easily be advocated by all the players who may be more likely to compromise when they see everyone's goal better balanced than otherwise.

By "balanced" we mean that the extent to which the achievement of one's objective is remote from his satisfactory level needs to be as close as possible to the extent to which the achievement of the other's objective is remote from his (the other's) satisfactory level. It is noted that another level called a permissible level is defined as the level the corresponding goal ought to reach at least. We take either individual or group rationality to stand for the permissible level for each player or a group, respectively.

The underlying idea as graphed in **Fig. 2** is that the well-balanced solution should lie, if possible, precisely on the line connecting between the two points, those representing each one's satisfactory level and permissible level, respectively; otherwise it ought to be as close as possible to the line. This line is termed a "goal vector." If one cannot know which situation is to occur, linear programming needs to be made use of to calculate what is regarded as a well-balanced solution. Since one may easily prove that only the former situation takes place for our three dimensional problem as is clear from **Fig. 3**, the solution is analytically identified with the point which is the intersection of the goal vector and the plane for the self-evident

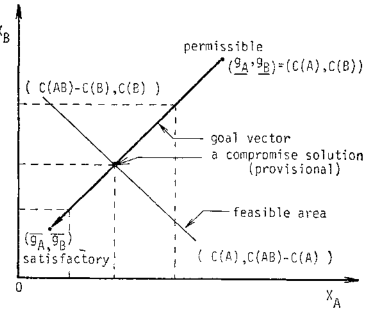

Fig. 2 Well-balanced solution on two-goal space.

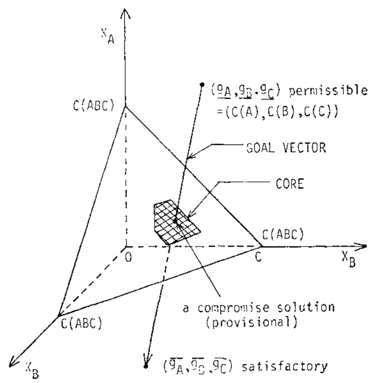

Fig. 3 Compromise solution located on three-goal space.

total-cost balance condition.

In consequence, a well-balanced solution which we conceive as a provisional compromise solution is given by the following formuli:

If no coalition is formed,

$$\hat{X}_i=\bar{g}_i+(\lambda_i/\Sigma\lambda_i)\cdot(C(ABC)-\Sigma\bar{g}_i)$$
$$\text{for } i=A,\ B \text{ and } C, \quad \cdots\cdots(18)$$

where $\lambda_i=C(i)-\bar{g}_i$ with $C(i)$ which is player i's individual cost taken as his permissible level and $\bar{g}_i$ represents his satisfactory level.

If a coalition is formed by A and B, by way of explanation, the level-one allocation is given as:

$$\hat{X}_{AB}=\bar{g}_{AB}+(\lambda_{AB}/(\lambda_{AB}+\lambda_C))\cdot(C(ABC)-(\bar{g}_{AB}+\bar{g}_C)) \quad \cdots\cdots(19)$$

for a coalition (AB); and

$$\hat{X}_C=\bar{g}_C+(\lambda_C/(\lambda_{AB}+\lambda_C))(C(ABC)-(\bar{g}_{AB}+\bar{g}_C)), \quad \cdots\cdots(20)$$

for the remaining individual C.

With X_{AB} given as such, the level-two allocation reads as:

$$\hat{X}_i=\bar{g}_i+(\lambda_i/\Sigma\lambda_i)(\hat{X}_{AB}-\Sigma\bar{g}_i), \quad \cdots\cdots(21)$$

for $i=A$ and B for the example of A and B forming a coalition.

A question arises in applying the above allocation formulae: how to specify one's satisfactory level which seems to be largely subjective. This is precisely what motivated the man-machine interactive approach to be proposed in this paper. That is, to get around this difficulty, participants are asked to take part in a series of bargaining games and by so doing they are expected to learn gradually what seems to be a reasonable claim. From this viewpoint the following visual information system has been designed.

(8) Man-machine interactive dialogues

By programming these formuli into the microcomputer and on feeding it with the data on the satisfactory levels specified by the players, the solution is instantaneously calculated to appear on the screen. The players can also have access to some other visual information as illustrated by **Photos** 2 and 3. The former photo shows the players where the solution is located either in or out of core. If players want to keep track of the series of their past provisional compromise solutions which they have so far not accepted in hope of finding a better one, the diagram as shown in the second photo serves their purpose. The idea is to get the players seated before the computer and let them play with the others by developing interactive dialogues with the rest of the participants with the machine as the media of communication.

3. RESULTS OF EXPERIMENTS

(1) Design of experiments

By thus developing a microcomputer based information system for the cost allocation gaming, a total of 60 students were invited to the forum, with the results of 20 cases. The students, undergraduate and graduate, were from the Department of Civil engineering, Tottori University. Each time three students were asked to be seated before the computer with the author as the operator and referee for the gaming. They were allowed to play not more than eight rounds. The limit to eight rounds is grounded on the assumption that if three of all are allowed to try two courses of action, namely, going alone or going together with someone, the number of possible outcomes is $2^3=8$.

If players found still hard to compromise within an allowed number of eight rounds, they were asked to rate each of their former provisional compromise solutions. This rating by each player is reported only to the operator, who

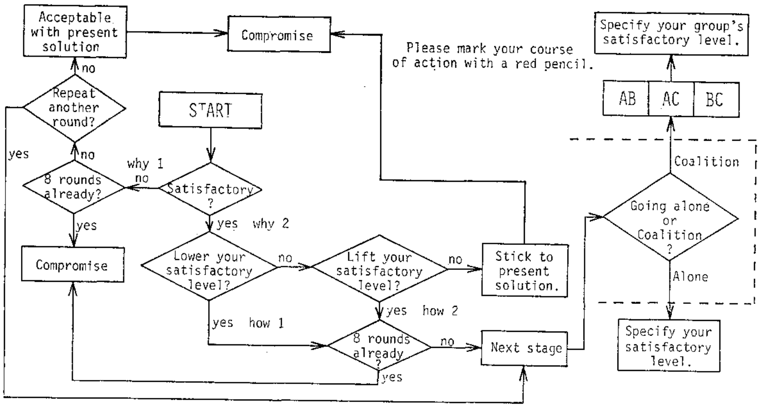

Fig. 4 Questionnaire.

singles out one of the solutions that is rated "averagedly highest" by the participants. By "averagedly highest" is meant the solution for which the rating is averaged over the three players to rank the highest.

In each round of the game, the players were asked to fill out a questionnaire on the following items: (i) the reason for either accepting or not accepting the current provisional compromise solution, (ii) selection of courses of action, i.e. gong alone or going together; and the latter being the case, with whom?, and (iii) specification of one's satisfactory level (changeable in each round) (see **Fig. 4**).

When the game was over, they were questioned about (iv) the degree to which their understanding had improved of the key built-in allocation normalities, and (v) the rating of their preference among a given set of reference methods of cost allocation, that is, *SCRB*, *Shapley Value*, *Nucleolus*, *Weak Nucleolus* (*WN*) and *Proportional Nucleolus* (*PN*).

Table 2 Empirical results.

Case	A	B	C
1	5.817	3.553	1.230
2	5.745	3.687	1.168
3	6.065	3.362	1.173
4	5.900	3.500	1.200
5	5.723	3.756	1.121
6	5.970	4.316	1.313
7	5.937	3.675	0.988
8	5.620	3.747	1.233
9	5.900	3.494	1.206
10	5.802	3.659	1.139
11	5.649	3.689	1.262
12	6.084	3.327	1.188
13	5.934	3.577	1.089
14	5.658	3.767	1.175
15	5.927	3.688	0.988
16	5.709	3.637	1.254
17	5.851	3.508	1.241
18	5.381	3.919	1.299
19	5.911	3.685	1.003
20	5.548	3.791	1.261

(UNIT: 10^8 Yen)

(2) Macroscopic Analysis of Results

Table 2 lists the twenty cases of empirical results. For analytical convenience, by dividing the range of values into three the final compromise values were classed into three categories, "high" denoted by H (unfavorable), "medium" denoted by M, and "low" denoted by L (favorable.) **Table 3** and **Fig. 3** show the frequency distributions of the final compromise values for three players. Study of this table immediately indicates that:

(i) Players A and C outrank B in the number of those who finally accepted a relatively high cost or relatively unfavorable allocation.

(ii) Comparatively, player B outranks players A and C in the number of those who fall into the middle category.

(iii) As for players A and C the number of those who enjoy a relatively low cost is the smallest, whereas it is relatively large for player B.

(iv) Collectively players A and C share a pattern of right-downward distribution, as against B which is patterned by its cone-shaped distribution.

(v) To reinterpret, a macroscopic feature of the results is characterized by seemingly favorable values for player B against A and C. This may be justified by the fact that player B is the strongest position to claim a relatively lower cost for him on the ground that the contribution of A and C to their joint venture and the grand project in increasing economical efficiency is minor compared to that of B, as is clearly structured in the cost input data in **Table 1**. This means that the

Table 3 Compromise values categorized.

CASE	A	B	C
1	M	L	M
2	M	M	L
3	H	L	M
4	M	L	M
5	L	H	L
6	H	L	H
7	H	M	L
8	L	H	H
9	M	L	M
10	M	M	L
11	L	H	H
12	H	L	M
13	H	M	L
14	L	H	M
15	H	H	L
16	L	M	H
17	M	L	H
18	L	H	H
19	H	M	L
20	L	H	H

L: low value, M: medium, H: high

Table 4 Player's characters categorized.

CASE	A	B	C
1	W	M′	S
2	M′	M′	W
3	W	M′	W
4	W	W	W
5	W	W	S
6	S	S	S
7	W	S	S
8	M′	W	W
9	W	S	W
10	W	W	M′
11	S	W	M′
12	W	S	S
13	W	M′	S
14	S	S	S
15	W	M′	S
16	S	M′	W
17	W	S	W
18	S	W	W
19	W	W	M′
20	S	W	M′

S: strong bargainer, M′: medium, W: weak

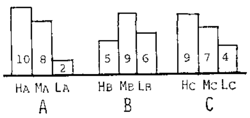

Fig. 5 Histograms of the distribution of compromise solutions.

outlined distribution pattern of compromise values is determined largely by the structure of the cost input data.

(vi) As far as a set of compromise values for players A, B and C are concerned, the pattern which occurs most frequently has proved to be (H_A, L_B, M_C), indicating that A, B and C belong to category H, L and M, respectively. Again, this agrees with the general trend of favorable results for player B.

(vii) As a vehicle of comparison it might be of analytical interest to apply the above symbolic system to a set of the other cost allocation techniques. Then we get *Nucleolus* $=(H_A, L_B, L_C)$, $WN=(H_A, L_B, M_C)$, $PN=(H_A, L_B, H_C)$, *Shapley Value* $=(H_A M_B, L_C)$ and $SCRB=(H_A, M_B, L_C)$. It follows from this that all but *SCRB* and *Shapley Value* share the general trend of favorable results for B as seen from the above gaming experiments. The pattern which is closest to our experimental results has proved to be that of *WN*.

(viii) Another analytical interest is to check whether initial solutions affect what they have finally agreed on. This underlies our suspicion that much of the game might be determined by just a single "pushing" player who can pre-empt the others by claiming an exorbitantly favorable value for himself at the outset of the gaming. A statistical test has been done to examine the significance of the differences between initial and final compromise solutions for each player. It has been shown that no statistical significance is gauged in the manner the former values deviate from the latter. So we may conclude that repetitive rounds of gaming help players learn how they should act or react by forming a coalition where necessary, thus eventually converging onto a range of reasonable values, notwithstanding some minor exceptions of extreme values.

(3) Microscopic analysis

So far a macroscopic analysis of the results has been made. We now turn to more microscopic features of the results. In other words we intend to take a close-up of the above discussed question: how much does player's bargainability count in

A

Bargaining Character	Compromise value L	M	H	
S	5	0	1	6
M′	1	1	0	2
W	1	5	6	12
	7	6	7	

B

Bargaining Character	Compromise value L	M	H	
S	4	1	1	6
M′	2	3	1	6
W	1	2	5	8
	7	6	7	

C

Bargaining Character	Compromise value L	M	H	
S	4	3	1	8
M′	2	0	2	4
W	1	3	4	8
	7	6	7	

Fig. 6 Compromize values vs. player's characteristics.

gaming? We base our analysis again on the questionnaire results.

We start with the definition of player's characteristics of "strong", "medium" and "weak". A "strong" bargainer, S is defined as a person *who wants to have another round, hoping to gain more even if he feels that the current solution is rather satisfactory for him;* or *who always finds any solution unacceptable as a final compromise*; or *who immediately starts feeling unsatisfactory with the current solution, if it turns out to be less favorable for him than the preceding one.*

Likewise by definition, a "weak" bargainer, W is a person *who is ready to find some solution acceptable in a relatively early stage and rushes to compromise by giving up some of his present claim, even if he finds it not necessarily so satisfactory.* A "medium" bargainer, M is defined to be any of the rest, neither weak or strong.

In the above are underlined these paragraphs which we can identify from the questionnaire. By applying the above definitions to our players, each player in a gaming has been marked with "S", "M" or "W", as shown in **Table 4**. Comparison of this table with **Table 3** which lists the compromise values categorized as "H", "M" and "L", leads to **Fig. 6**. This figure shows the number of those players with a bargaining character categorized as "S", "M" or "W" against compromise values ranked as "H", "M" or "L". A mere glance at this table shows that be it player A, B or C, it is highly likely that those "strong" bargainers tend to enjoy relatively "low" costs (favorable values), whereas those "weak" ones end up with relatively "high" costs (unfavorable values). This tendency may not be, however, so clear for those who come under the category of "medium". A statistical test has been done to examine this hypothesis: a player's bargaining character affects what he will finally gain.

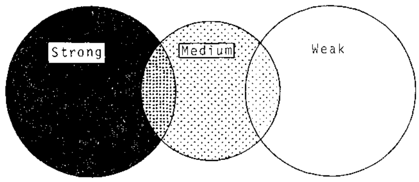

Fig. 7 Bargainer's characteristics discriminated.

The result partially supports this hypothesis with a significance level of five percent; that is, significance has been gauged between any two cases, one which was played by a "weak" A (B or C) and another played by a "strong" A (B or C, respectively), though no significance has been determined between any two cases where one of the two players was found to be "medium". This fact is illustrated in **Fig. 7**.

To conclude we may fairly say that microscopically a cost allocation gaming so defined is subject to the player's bargainability to a limited extent, if we compare a particular outcome with another.

(4) Complementary analysis

By reference to the results of the post-gaming questionnaires, some complementary analysis has been conducted with the following findings:

(i) 85 percent of those who had failed to understand the concept of core before they involved in the gaming, admitted that the implication of the concept became much clear to them.

(ii) 80 percent of those who finally gained an understanding of core found it a reasonable condition for cost allocation.

(iii) According to the rating of five other alternative cost allocation methods, *SCRB* has been found to rank top, followed by *Shapley Value*, then by *PN*, then by *WN* and finally by *Nucleolus*.

(iv) The most popular criterion employed by participants for so rating was the ease of use and understanding and the minimum mathematical complexities which are entailed in a given method.

It should be underlined that this ease of use and understanding attracts practitioners and laymen. For this reason *SCRB* and *Shapley Value* tend to be rated above the rest. This may well explain why *SCRB* still gains so much popularity, despite of flaws it involves.

It should be noted that all participants have agreed that this type of experimental technique could serve the purpose of familiarizing people with the problem of cost allocation, leading them through the essential question of "what is fairness and equity" up to the increased understanding and support of some more prescriptive methods.

(5) SCRB as part of gaming

Last but not least, it also has been found that our prescriptive-empirical gaming approach offers a reinterpretation of *SCRB*. That is, it may easily be demonstrated that the *SCRB* based solution is no more than a special solution among a set of possible compromise solutions for our gaming. The point to be made is the assumption that each player who chose to go alone has agreed to take his own "separable cost" as a satisfactory level.

This may be interpreted as each of the participants claiming that he would be most satisfactory if his cost burdens are no greater than the incremental costs of his involvement in the project as the last participant. This claim has been known as marginality principle[3),4),5),6)] and has been exercised as a minimum obligation for participants in the practice of cost allocation. If this condition is lift, the last participant would be asked to leave the grand project, which in turn would leave him with no choice but to go alone, costing him more than his separable cost in common situations. On substituting one's separable cost, $SC(i)$ into his satisfactory level, $\bar{g}_i$ in Equation (18), we get for player A

$$\begin{aligned}\hat{X}_A&=\bar{g}_A+\{\lambda_A/(\lambda_A+\lambda_B+\lambda_C)\}(C(ABC)\\&\quad-(\bar{g}_A+\bar{g}_B+\bar{g}_C)),\\&=\bar{g}_A+\{(C(A)-\bar{g}_A)/(C(A)-\bar{g}_A+C(B)\\&\quad-\bar{g}_B+C(C)-\bar{g}_C)\}(C(ABC)-(\bar{g}_A+\bar{g}_B+\bar{g}_C)),\\&=SC(A)+(RB(A)/(RB(A)+RB(B)\\&\quad+RB(C))(C(ABC)-(SC(A)\\&\quad+SC(B)+SC(C))), \quad\cdots\cdots(22)\end{aligned}$$

which is precisely the allocation formula of *SCRB* with each one's individual costs being lesser than his benefits. Clearly, the same formula applies to players B and C, with A being replaced with B (or C) and both B and C replaced with C (or A) and A (or B), respectively. In the above SC stands for one's separable cost and RB denotes ones remaining benefit which is defined to be the difference between one's individual cost and separable cost. Therefore it holds for $i=A$, B and C that:

$$RB(i)=C(i)-SC(i)=C(i)-\bar{g}_i. \quad\cdots\cdots(23)$$

The above fact indicates that our new approach works very well for the purpose of gaining further insight into SCRB and offers a game-theoretic reinterpretation of what is implied by the method. If participants so desire, they can take the *SCRB* based solution for the gaming. It should also be noted that the above conclusion has been derived from the implicit assumption that the benefit of the project to each participant exceeds his individual costs. This assumption, however, is not vital but may rather be dispensed with to accomodate a more general setting for *SCRB*. That is, if the minimum of i's individual cost, $C(i)$ or i's benefit for the grand project, $B(i)$ is considered to represent the value of the grand project for i more adequately than i's individual cost $C(i)$ does, as is claimed by *SCRB*, we may fairly replace $C(i)$ with Min $(C(i), B(i))$ in Equation (2) to produce a generalized version of individual rationality. This is identical to saying that $C(i)$ is replaced with this minimum term in Equations (18), (22) and (23), assuming that no coalition is formed. From this we may as well conclude that with some slight modification this gaming analysis helps to develop a game theoretic interpretation of *SCRB* and that *SCRB* seems to offer some practical implications for this type of gaming.

4. CONCLUSION

In this paper a prescriptive-empirical approach has been presented to cost allocation with an aid of a microcomputer-based information system and it has been demonstrated how well our approach serves our purpose.

Some major findings may be summarized as:

(i) Despite minor differences among various cases, the outlined distribution pattern of compromise values is determined largely by the structure of the input cost data, and not so much by the bargainability of players. This is very much owing to those basic norms incorporated in the gaming which guide much of the direction of the gaming. This explains why the presented approach is called a "prescriptive-empirical" approach, not simply an empirical approach or a gaming.

(ii) From a microscopic point of view, however, this is not necessarily the same. One's bargaining power makes some difference. Therefore we may say that players are given a limited free hand as long as they stay within the predetermined conditions incorporated in the gaming procedure.

(iii) The microcomputer-aided approach has proved to be very effective and helpful in educating people who are not familiar with cost allocation.

(iv) By incorporating what has been agreed on through experimentation into the gaming procedure, we may expect to add to the normalities for guiding the game, thus eventually leading closer to a more normative type of methodology.

In spite of the benefits of our approach, there seems to be much room for extension and improvement. A list of technical difficulties to overcome includes: (i) how to visualize more than three dimensional information on the screen of a color monitor linked with a microcomputer if more than three players are involved in the cost allocation; and (ii) how to speed up the processing and display of information, and how to overlay one image on another in order to make the presentation of information more attractive and effective.

Another concern of ours is to invite practitioners and managers experienced in the business of cost allocation to play the game by themselves. By accommodating their advice and criticism we may develop a more applicable approach in line with the approach suggested here.

A step forward has already been taken with some encouraging results which will be presented in the author's next paper.

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費用割り振りのための人間・機械系対話型アプローチ：ゲーミング分析による一考察

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昭和59年3月

公共事業，中でも水資源開発の分野では，そこに参加する各主体にとっての大きな関心事の一つとして当該共同事業費全体のうちのどれくらいが各参加主体の負担すべき額になるのかという問題がある．この種の共同事業費の割り振り問題はつまるところ，(i) 各参加主体にとって当該プロジェクトがどれくらいの価値（便益）をもたらすのか，(ii) 各参加主体の参加が当該共同事業費にどの程度の負担増を強いているのか（換言すれば当該主体の参加が経済効率性にどの程度の寄与をしていると考えられるか）を明らかにすることであり，そのうえで費用割り振りにおける「公正さ」の基準を決めてやることである．

費用割り振りのための手法（費用割り振り法）としてよく知られ，現在わが国をはじめ世界各国で採用されている分離費用身替り妥当支出法（SCRB）は，主として多目的ダムの費用割り振り法として開発・考案されたものである．この手法は「身替り費用」や「分離費用」などの機会費用や当該事業の「便益」に準拠して導出した「公正さ」の規範により配分する考え方をとっている．これはその簡便さのゆえに広く使われるところとなっているが，その理論的意味づけが必ずしも明確でないこと，ならびに各主体の参加の動機づけと交渉力が「公正さ」の規範として明示的に組み込まれていないことがその欠点として指摘されてきた．

著者らはこのような欠点を補うための新しいアプローチとしてゲーム理論を用いた各種手法（シャプレイ値・仁など）をさらに拡張・改善し，弱仁（WN）や比例仁（PN）などの手法を開発しその有効性を明らかにしてきた．しかしSCRBを含めて上記のいずれの手法にも共通していえる短所は「公正さ」の基準があらかじめすべて規範的に決められている点である．

本研究ではこの点に着目し，従来の規範的なアプローチを補完するものとして，実験シミュレーション的なアプローチであるゲーミング手法の長所を組み入れた「半規範的・半実験的 (prescriptive-empirical)」な方法論を提示することを目的とした．すなわち本研究では費用割り振り問題を各参加者による交渉ゲームにみたてて実際にゲームを展開してもらうことにより最終的な合意に達する過程をシミュレートする方式（これを「人間・機械系対話型アプローチ」と称する．）の導入を試みた．ただし本ゲーミングにおいて最低限必要と考えられるルールを「公正さの最低限の規範」としてあらかじめ定めておく方式をとった．

またゲームを進めていくうえでの有効な補助手段としてマイクロコンピューターとカラーディスプレイ装置を用いた対話型の情報提示システムを導入した．さらにゲーミングにあたって進行の途中や終了後に各種項目についてアンケート調査を行うことにより参加者の判断，反応の根拠，ゲーム対戦相手の反応に対する思わく，交渉形態などの各種行動特性やパターンに関する情報を収集し，後でこれらのパターンとゲーミングの結果との関連について解析を行った．なおゲーミングでは3つの都市を参加主体とする広域水道事業を想定し，その共同事業の費用割り振り問題を取り上げた．その際，これら3市を代表する参加者（土木工学専攻の大学4年生と修士課程1·2年生）によるゲーミングを計20試合（3×20=60人）行った．

これらのゲーミングの結果を多角的に分析することにより，本研究で提示したアプローチは実際の割り振り法を開発・改良していくうえで有効なアプローチであることが示された．特に参加者が従来の規範的手法の意味づけや内容が理解しにくかったり，それに異議を唱えるとき，ならびに参加者の中で最低限の公正基準について合意が成立したものの唯一の解に絞り切れないでいる場合，などにこのゲーミング分析はきわめて適切な支援システムとなり得ることが明らかにされた．

さらに本研究では分離費用身替り妥当支出法による解が本ゲーミング手法（またはその拡張法）の特殊解と一致し得ることをも理論的に立証した．これにより分離費用身替り妥当支出法に対する一つの新しい解釈（ゲーム論的解釈）を導出することができ，この意味でも本研究のアプローチは今後の費用割り振り法を改善拡張していくうえで有用と考えられる．