PROC. OF JSCE
No. 341, January 1984
[Technical Note]

VIBRATION OF AN INITIALLY DEFLECTED WEB PLATE UNDER PERIODIC BEAM BENDING

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1. INTRODUCTION

It is a well-known phenomenon that the initial deflection of plate girder webs induces additional plate deflections under the action of girder bending moments. If the bending moments are caused by the girder vibration, the action of the bending moments becomes periodic, resulting in the dynamic magnification. This vibration of the web plate will yield the possibility of the initiation of fatigue failure at the welded joint connecting the web and flange plates, and/or of the generation of undesired acoustic radiation. In this paper, the vibrational behavior of a plate with the initial imperfection subjected to sinusoidally varying inplane bending moment is examined from this point of view.

The influence of the initial imperfections has already been discussed in the post buckling problem[1] or the effective width problem[2], and has been studied recently in connection with the initiation of fatigue cracks by Maeda and Okura[3]. On the other hand, the out-of-plane vibration of plate under periodic edge thrusts has been attracting the interest of many researchers[4] and has been studied in relation to the parametric excitation.

Some other dynamic thrust problems are studied to check the impulsive edge thrust effect by Tassel[5] and Ueda and Yao examined the impulsive effect on the notch of steel plates using the Newmark's β method.

The quantitative evaluation of the amplitude of the parametric and/or forced vibrations has not been done so far because of the difficulty in defining the interaction effect of the nonlinearity between the deflection and thrust and the parametric and forced excitation. In order to get more practical value for the dynamic response of the plates, a numerical analysis must be carried out. In the analysis, the effect of the geometrical nonlinearity is taken into account, but the effect of plastification and structural damping is not included. It is true that the damping capacity has not been known so clearly, but these assumptions are made mainly to emphasize the effect of the geometrical nonlinearity and without these damping effects, the dynamic responses are expected to be more crucial. Of course, there will exist inherent errors due to the numerical integration in the calculated dynamic response and it is expected to have some kind of damping effects. But, since the main purpose of this analysis is to get practical values of the response approximately, the presentation of more accurate values is not intended in this paper.

2. NUMERICAL ANALYSIS METHOD

In the analysis, the geometrical nonlinearity is taken into account by the incremental formulation and is evaluated by the piecewise linear and iterative calculation procedure. The second order terms are included in the strain-displacement equations, and the linear relation is assumed for the stress-strain equations that are obtained by the Murray-Wilson's[7] method. The strain-displacement relation for thin plates is given by the theory based on the von Karman's and Kirchhoff's assumption. Then, the element equations for a triangular finite element are derived using the finite virtual displacement and the principle of stationary potential energy. The dynamic response is calculated by the Newmark's β method. The displacement functions used here are as follows:

$$u=\alpha_1+\alpha_2 x+\alpha_3 y$$
$$v=\alpha_4+\alpha_5 x+\alpha_6 y$$

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$$\left.\begin{aligned} w=&\beta_1\zeta_i+\beta_2\zeta_j+\beta_3\zeta_k+\beta_4\zeta_i\zeta_j+\beta_5\zeta_j\zeta_k+\beta_6\zeta_k\zeta_i \\ &+\beta_7(\zeta_i\zeta_j^2-\zeta_i\zeta_j^2)+\beta_8(\zeta_j\zeta_k^2-\zeta_k^2\zeta_j) \\ &+\beta_9(\zeta_k\zeta_i^2-\zeta_i\zeta_k^2) \end{aligned}\right\} \quad \cdots\cdots(1)$$

where x and y coordinates are fixed to an element; u and v are the displacements in the x and y direction; w is the displacement perpendicular to the x-y plane and is expressed by the area coordinate ζ_i, ζ_j, and ζ_k; α_i ($i=1\sim6$) and β_i ($i=1\sim9$) are the generalized coordinates.

3. MODEL AND PARAMETERS

Web plates are simulated by an initially deflected plate which is simply supported along the vertical sides and is constrained against rotation along the upper and lower sides to examine the clamping effect of the flanges. But the inplane displacements are not restrained on four sides. With the aid of the symmetry, the half panel of the plate is analysed and is divided into sixty four triangular elements as shown in **Fig. 1**.

The maximum initial deflection w_0 is chosen here to be two-hundred-fiftieth of the width being refered to the Specification of Japanese Highway Bridges. The influence of this value and applied periodic bending moment on the dynamic response is checked preliminarily by changing those values in a certain case. The results show that the dynamic response is approximately proportional to the initial deflection and applied periodic bending moment. The influence of the static bending moment is also checked in the range from zero to $0.3\,M_{cr}$, where M_{cr} is the critical bending moment for elastic buckling. This influence is not significant within that calculated range and possibly is absorbed in that of the initial deflection. Therefore, the static bending moment is not considered here. Finally, the major structural parameters adopted here are as follows:

width thickness ratio β : 250

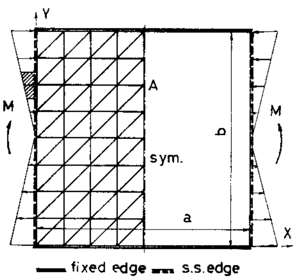

Fig. 1 Analysed model.

Table 1 Natural frequency of the plate.

Vibrational mode (m, n)	Natural circular frequency $\omega_{m,n}$ (rad/s)	Natural period T_{mn} (s)
(1, 1)	88.30	0.071 16
(1, 2)	211.10	0.029 76
(3, 1)	310.53	0.020 23
(1, 3)	394.24	0.019 54

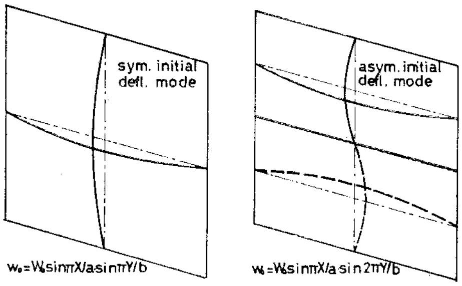

Fig. 2 Assumed initial deflection modes.

aspect ratio α : 1.0
periodic bending moment M_0 : $0.3\,M_{cr}$
damping factor δ : 0.0

Two modes of initial deflection which are symmetrical and unsymmetrical modes are considered here as shown in **Fig. 2**. The circular frequencies of the periodically varying bending moment load are $\omega_{1,1}/4$, $\omega_{1,1}/2$, $\omega_{1,1}$, $2\omega_{1,1}$, $\omega_{1,2}$, $\omega_{3,1}$, and $2\omega_{1,2}$, where $\omega_{n,m}$ is the natural circular frequency of the plate, and n and m indicate the numbers of the half waves along the X and Y axes of the vibration mode respectively, and are obtained as shown in **Table 1**. $\omega_{1,1}/4$ may roughly correspond to the natural circular frequency of the ordinary plate girders. Eight cycles of the periodically varying bending moment are applied, which turn out to be approximately enough for the vibration to reach the steady state. The thickness-width ratio β is fixed to be 250 here. In the case of smaller values for it, the increased stiffness of the plate will result in the decrease of the dynamic response and the higher speed vibration. The decreased dynamic response will reduce the damping effect because of the nonlinearity, and vice versa. But, it will be expected that an approximate values can be obtained by the calculated results and that the substantial vibrational behavior will not change. The time step used in the numerical integration is taken as one-sixteenth of the period of the applied bending moment.

4. RESULTS OF NUMERICAL INTEGRATION

(1) In the Case of Symmetrical Initial Deflection Mode

When the frequency of the periodic bending moment coincides with that of the asymmetrical mode $\omega_{1,2}$, the magnitude of vibration becomes significantly large, and thus the vibration is in resonance as shown by the history curve of **Fig. 3** at the point A shown in **Fig. 1**. Consequently, if the frequency of the bending moment is twice as much as $\omega_{1,2}$, the vibration is expected to grow violently by the parametric excitation. Contrary to expectation, however, the growth of the parametric resonance vibration is not obtained. The vibration has a tendency to change to the forced one as shown in **Fig. 4**. For the frequency of a quarter of the natural frequency, $\omega_{1,1}$ the vibration is the forced one as shown in **Fig. 5** and the dynamic magnification factor becomes only about one-tenth of that of the resonance vibration. **Fig. 6** shows the relationship between the dynamic magnification factor and the frequency of applied bending moment in 8-cycle applications at the point A by empty circles.

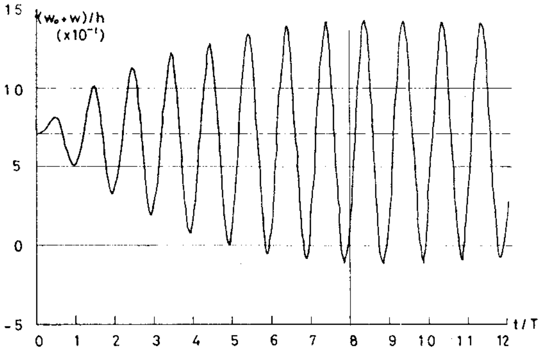

Fig. 3 Calculated history curve for the periodic bending $\omega_{1,2}$ in the case of the symmetrical initial deflection mode (in Resonance).

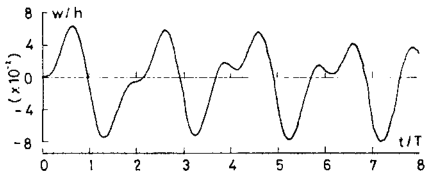

Fig. 4 Calculated history curve for the periodic bending $2\omega_{1,2}$ (parametric excitation frequency) in the case of the symmetrical initial deflection.

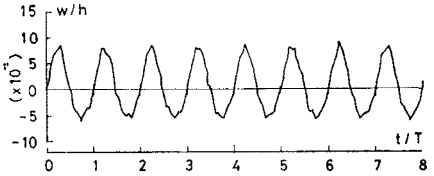

Fig. 5 Calculated history curve for the periodic bending $\omega_{1,2}/4$ in the case of the symmetrical initial deflection (forced vibration).

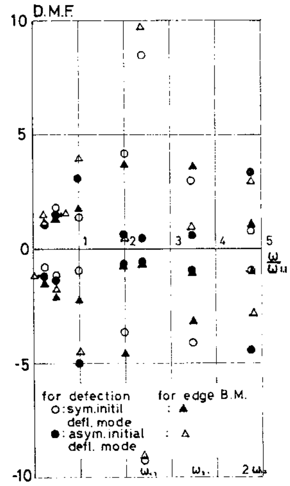

Fig. 6 Calculated dynamic magnification factor for the deflection and the edge bending moments (out-of-plane).

(2) In the Case of Asymmetrical Initial Deflection Mode

For the case where the initial deflection has the asymmetrical mode, the resonance is excited by the natural frequency of the symmetrical modes. The fundamental behaviors are quite similar to those for the symmetrical initial deflection mode, except the dynamic magnification factor and the frequency-resonance relationship. The dynamic magnification factor at the point A is illustrated by full circles in **Fig. 6** also.

In this case, the growth of the parametric resonance vibration is not again observed. This tendency is same as the case where the initial deflection has the buckling mode for the beam bending. This may be explained by the fact that the amplitude excited by the parametric bending moment does not develop beyond the initial deflection and that it does not become in resonance because of the damping effect of the nonlinearity and excitation factor. As seen in **Fig. 6**, the dynamic magnification factor is about 10 in the maximum value. It means that the damping ratio corresponds to 0.05 approximately in this analysis. Of course, this value stems from the integration error, but it must be noted that the actual structures have inherently many other damping capacities. For this problem, further analyses may be required in detail.

(3) Out-of-plane Bending Response at the Flange Connection

The constraining out-of-plane bending stresses are produced at the welded flange joints under the static and periodic beam bending moments. By the periodic bending moment, these stresses are applied repeatedly and are expected to cause the fatigue failure at the welded connection. In **Fig. 6**, the dynamic magnification factors of the stresses for the symmetrical and asymmetrical initial deflection mode shown by full and empty triangles respectively. These values are almost same as those of the deflection. In this analyzed case, the maximum bending stress reaches to about 72 MN/m^2.

5. CONCLUSIONS

In this paper, the dynamic behavior of a web plate, which has the initial deflections and which is applied by the periodical beam bending moments, is studied numerically by the dynamic elastic finite deformation analysis. From the results, the following conclusions may be drawn:

(1) For the plate with the symmetrical initial deflection mode, the plate vibration shows resonance when the frequency of the applied periodic beam bending moment coincides with the natural frequency $\omega_{1,2}$ of the symmetrical plate vibration mode. The dynamic magnification factor grows over 10.

(2) For the plate with the asymmetrical initial deflection mode, the plate vibration shows resonance when the frequency of the applied periodic beam bending moment coincides with the natural frequencies $\omega_{1,1}$ and $\omega_{1,3}$ of the asymmetrical plate vibration mode. In this case, the dynamic magnification factor becomes above 5.

(3) In the both cases of the symmetrical and asymmetrical initial deflection mode, the parametric resonance vibration is not obtained in this numerical analysis, even in the main resonance region of the applied periodic beam bending moment, that is, when the frequency of the applied bending moment is equal to a half of the natural frequencies of the plate.

(4) The dynamic magnification factor of the out-of-plane bending stress at the flange connection takes almost same value as that of the deflection.

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(*Received May 30, 1983*)