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[Technical Note]

ON THE FRICTION FACTOR OF ALLUVIAL STREAMS

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1. GENERAL

A number of methods have already been proposed to determine the average velocity of an alluvial stream (see e.g. the reviews in 2), 4), 18) etc.) Yet none of them has been generally accepted, and research is still going on: the works 1), 5) to 8), 10) to 12) and 16) are but a few of recent contributions to the field. These works, however, diverge substantially in their approach to the problem, while their methods are more at the level of academics than of the practising engineers. The purpose of the present note is to suggest a practicable and yet reasonably reliable method for the prediction of the flow velocity corresponding to those early stages of sediment transport where the bed is covered by sand waves (ripples or dunes) while the suspended load is negligible. It will be assumed that the turbulent flow is in equilibrium and that it can be treated as two-dimensional.

Consider the Darcy-Weisbach friction factor

$$f=8\left(\frac{v_*}{v}\right)^2 \quad \cdots\cdots(1)$$

where v is the average velocity of the flow and v_* is the shear velocity. For the stages mentioned this friction factor can be treated, in accordance with convention, as a sum of the "pure friction component" f' and the "bedform component" f'':

$$f=f'+f'' \quad \cdots\cdots(2)$$

As is well known, f' can be evaluated as

$$f'=8\left[\frac{1}{\kappa}\ln\left(A\frac{h}{k_s}\right)\right]^{-2} \quad \cdots\cdots(3)$$

where κ is Von Karman constant, h flow depth, k_s height of the effective surface roughness and A is a known function of the Reynolds number v'_*k_s/ν which reduces into $\simeq 11.00=\text{const}$ when $v'_*k_s/\nu > \simeq 70$ (see e.g. 18)). The evaluation of f'', is less certain and it is still under investigation. If bedforms are represented by triangles having the height Δ and the base length Λ and if it is assumed that their downstream face is inclined by a constant angle (which in the case of a tranquil flow is equal to the angle of repose) then the geometry of bedforms can be specified by their "steepness" Δ/Λ and their "relative length" Λ/h. Accordingly, the value of f'' which is determined by the bedform geometry, must be given by

$$f''=\phi\left(\frac{\Delta}{\Lambda};\frac{\Lambda}{h}\right) \quad \cdots\cdots(4)$$

Since the bedforms (sand waves) emerge as a result of the progress of the two-phase motion the dimensionless bedform characteristics Δ/Λ and Λ/h which appear in (4) must be certain functions of the dimensionless variables X, Y and Z determining the two-phase motion en masse. (See the expressions of X, Y and Z in the List of Symbols.) Considering this, the bedform component f'' can be expressed also as

$$f''=\bar{\phi}(X, Y, Z) \quad \cdots\cdots(4)'$$

The variables X, Y and Z may not be used in the study of friction factor (or for that matter in any study related to the two-phase motion) explicity: sometimes they appear in the form of some combinations (such as e.g. $\Xi=X^2/Y$) or some functions (e.g. as Y_{cr} which can be interpreted as the modified Shields' function $\Phi(\Xi)$). The most important variable in (4)' is Y (for the steepness Δ/Λ of sand waves varies most intensively with Y/Y_{cr}[19]), and therefore f'' is often plotted by using Y as abscissa.

It follows that the study of f'' can be carried out either according to (4), (as has been done e.g. in Ref. 1), 5), 10) and 11) or according to (4) (as e.g. in Refs. 7), 8), 14) to 17)).

In present note f'' is treated as implied by the function (4) and the aim of the laboratory measurements reported herein is to contribute to the determination of this function for the range of parameters stated in the next section.

2. EXPERIMENTAL MEASUREMENTS AND RESULTS

The present measurements were carried out in a variable slope glass-walled flume: length=

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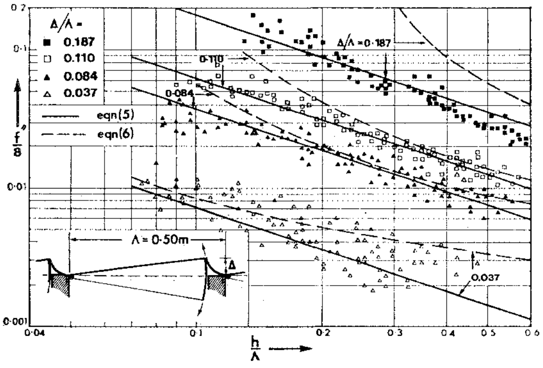

Fig. 1 Bedform Component of the Friction Factor as a Function of Sand Wave Steepness and Relative Sand Wave Length.

20 m, width = 0.76 m, height = 0.60 m (Graduate Hydraulic Lab., Queen's Univ.). The flume bed consisted of consecutive triangular bed forms simulating sand waves; $\Lambda = 0.50$ m, Δ adjustable (**Fig. 1**). The size D of the almost uniform sand glued on the surface of the bed forms, and thus the "skin roughness" k_s, was varied. The following values were used in combination with each other

D (mm): 3.12, 1.22, 0 (smooth bed forms)

Δ/Λ : 0.187, 0.110, 0.084, 0.037

The intervals of the slope S and the flow depth h were $0.002 \le S \le 0.005$ and $0.037\ \text{m} \le h \le 0.30$ m (smallest h when smallest Δ). For each configuration of the bed the largest h was determined so as to have two-dimensional conditions for at least 80% of the flume width b.

It follows that the present measurements were conducted for the following ranges of the dimensionless variables of the function (4)

$$0.037 \le \Delta/\Lambda \le 0.187$$
$$1.67 \le \Lambda/h \le 13.50$$

Since the steepest sand waves are ripples having $\Delta/\Lambda \simeq 0.2$ (see e.g. 18)) the range of Δ/Λ values above covers reasonably well the steepnesses of all realistic bedforms. As is well known, the relative size of dunes can be approximated by $\Lambda/h \simeq 2\pi \simeq 6.28$[18] and therefore the middle part of the present range of Λ/h can be assumed to represent the dunes; its lower and upper parts representing some ripples and sand bars respectively.

Thus v_* and v (needed for f) were computed as $\sqrt{gSh}$ and $Q/(bh)$. Since the flow past the bed form surfaces was always rough turbulent, f' was evaluated from (3) independently of $v'_* k_s/\nu$ by using $A = 11.00$. Following Ref. 9), $k_s = 2D$ was adopted. The total number of runs was 527. A detailed description of the experiments is given in Ref. 15).

The values of $f'' = f - f'$ corresponding to constant Δ/Λ are plotted versus Λ/h in **Fig. 1**. The solid lines on this graph represent the relation

$$f'' = 8\left[\frac{1}{2}\left(\frac{\Delta}{\Lambda}\right)^2\right]\cdot\frac{\Lambda}{h} = 4\frac{\Delta^2}{\Lambda h} \quad \cdots\cdots\cdots\cdots(5)$$

suggested in Refs. 7) and 17), the broken lines being the graphs of

$$f'' = \left[3.3\log\left(\frac{\Lambda h}{\Delta^2}\right) - 2.3\right]^{-2} \quad \cdots\cdots\cdots\cdots(6)$$

proposed in Ref. 14).

3. CONCLUSIONS

From the evaluation of present results, as well as from Refs. 7) and 14), it appears that the variables Δ/Λ and Λ/h determine f'' by combining themselves into the single dimensionless complex (variable)

$$\left(\frac{\Delta}{\Lambda}\right)^2\cdot\frac{\Lambda}{h} = \frac{\Delta^2}{\Lambda h} \quad \cdots\cdots\cdots\cdots(7)$$

and thus that (4) implies

$$f'' = \phi\left(\frac{\Delta^2}{\Lambda h}\right). \quad \cdots\cdots\cdots\cdots(8)$$

The patterns of experimental points in **Fig. 1** suggest that the (simpler) linear function (5) is a more realistic representation of (8) than the logarithmic function (6). With regard to the physics of the phenomenon, this means that the energy loss due to sand waves is mainly because of the sudden enlargement and the separation of flow at their abrupt downstream surfaces (the hypothesis leading to (5)), rather than because of the manifestation of sand waves as a "generalized roughness" (motivation leading to the logarithmic form (6)). Indeed, the logarithmic law is valid only if the size of the elements forming the boundary roughness is "small" in comparison to the external dimensions of flow (see e.g. Ref. 13)). But this is not necessarily so in the case of the actual dunes (whose height Δ may, in some instances, reach e.g. one third of the flow depth h). Nor was the height Δ of the bed forms used in the present experiments always "small". Observe from **Fig. 1** that the divergence between the lines representing the logarithmic form (6) and the data systematically increases when Δ/h increases (i.e. when Δ/Λ increases and h/Λ decreases).

Adopting $\kappa = 0.4$, $A = 11$, $k_s = 2D$ and using the form (5) one arrives at the following expression for v

$$v = \left\{\left[2.50\ln\left(5.5\frac{h}{D}\right)\right]^{-2} + \frac{1}{2}\frac{\Delta^2}{\Lambda h}\right\}^{-1/2}\cdot\sqrt{gSh}\ . \quad \cdots\cdots\cdots\cdots(9)$$

Here the multiplier of $v_* = \sqrt{gSh}$ is the dimensionless Chézy coefficient which reflects the influence of both skin friction (first term) and sand waves (second term). Although this relation has been actually tested for the laboratory data alone, the exploratory calculations suggest that it is capable of giving realistic results for large scale alluvial streams as well.

4. NUMERICAL EXAMPLE

Figure 2 shows (with a 10-times exaggerated depth-to-width ratio) a typical cross-section of the Beaver river in Alberta, Canada (**Fig. 2.1** in Ref. 3)); slope $S=0.000\,25$, free surface bankfull width $B=180$ ft $\simeq 55$ m, average bankfull flow depth $h=9.4$ ft$\simeq 2.87$ m, grain size of the bed sand $D=D_{50}=0.5$ mm. (The width-to-depth ratio $B/h\simeq 19$ is sufficiently large and the flow can be treated as two dimensional).

Since the sand waves have not been measured, first Λ and Δ must be estimated. This can be done e.g. with the aid of the experimental curves in 18), 19) and 20). Using the given values above (and also $\rho=1\,000$ kg/m³, $\nu=10^{-6}$ m²/s and $\gamma_s/\gamma=1.65$) we compute

$$X=41.95\ ;\quad Y=0.87\ ;\quad Z=5\,740$$

and determine (from the Shields' diagram)

$$Y_{cr}=0.031\ .$$

The present value of X is larger than $\simeq 30$ and therefore the sand waves are dunes (**Fig. 3**), their average length being (according to Ref. 18))

$$\Lambda \simeq 2\pi h \simeq 18 \text{ m}\ .$$

The value of Δ can be predicted with the aid of the graph in **Fig. 4** (reproduced from 19)). Since $Z \gg 100$ we use the curve C_4 and for $\eta = Y/Y_{cr}=28$ we determine $\delta=\Delta/\Lambda\simeq 0.044$. Hence

$$\Delta \simeq 0.044 \cdot \Lambda \simeq 0.8 \text{ m}\ .$$

Substituting the values of S, h, D, Λ and Δ in (9) one computes

$$v=0.96 \text{ m/s}$$

which gives for the bankfull flow rate

$$Q=v\cdot Bh\simeq 151 \text{ m}^3\text{/s}\ .$$

(The actually measured bankfull flow rate is 5 000 ft³/s$\simeq$142 m³/s (p. 19 in Ref. 3)).

Note: The uncertainties arise when X is in the interval

$$\simeq 5.50 < X < \simeq 30;$$

i.e. when both ripples and dunes are present (**Fig. 3**). In this case we have two sand wave lengths and heights (Λ_r, Λ_d and Δ_r, Δ_d). How do these four sand wave dimensions affect (simultaneously) the value of the friction factor f? The answer to this question is not yet known, and the measurements currently carried out at Queen's University are aimed at finding it (**Fig. 5**).

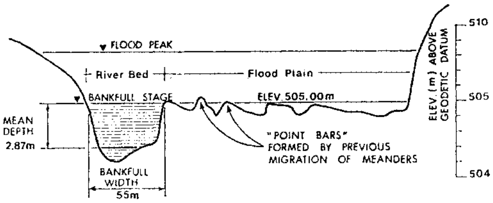

Fig. 2 Typical Cross Section of the Beaver River in Alberta, Canada.

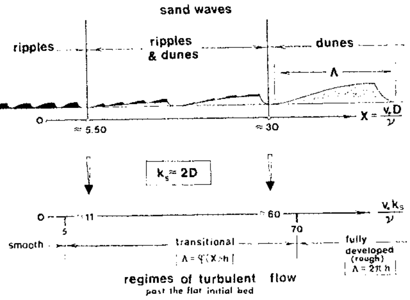

Fig. 3 Relation between Types of Sand Waves and the Regimes of Turbulent Flow Generating them.

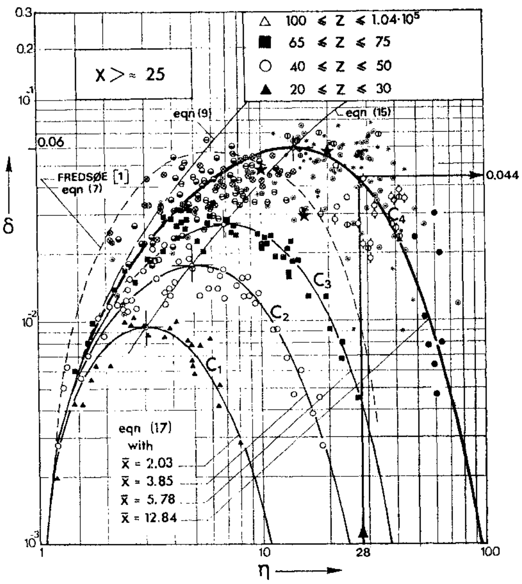

Fig. 4 Variation of Sand Wave Steepness with Relative Flow Intensity and the Dimensionless Flow Depth.

5. APPENDIX—LIST OF SYMBOLS

b: flume width
B: width of the free surface of a river
D: typical grain size of sand forming the bed surface roughness (usually D_{50})

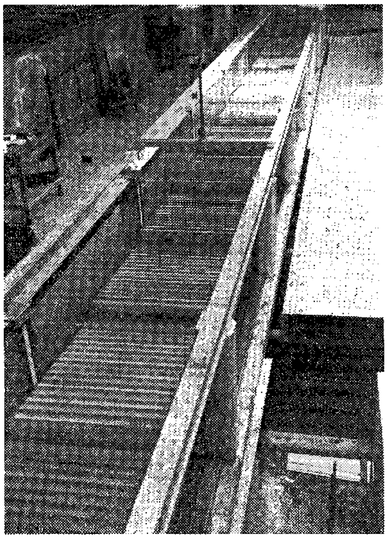

Fig. 5 Laboratory Flume with Dunes and Ripples Superimposed on them (Queen's University, Kingston, Canada).

f: Darcy-Weisbach friction factor
g: acceleration due to gravity
h: flow depth
k_s: height of the effective bed surface roughness ($\simeq 2D$)
Q: flow rate
S: slope of the uniform equilibrium flow
v: average velocity of flow
v_*: shear velocity ($=\sqrt{gSh}$)
x: flow direction
γ: specific weight of fluid
γ_s: specific weight of grains in fluid
ν: kinematic viscosity
ρ: fluid density
Δ, Λ: height and length of sand waves
ϕ: function of

Dimensionless Combinations

$X=\frac{v_* D}{\nu}$: grain size Reynolds number
$Y=\frac{\rho v_*^2}{\gamma_s D}$: mobility number
$Z=h/D$: dimensionless flow depth
$\eta=Y/Y_{cr}$: relative intensity of sediment transporting flow
$\delta=\Delta/\Lambda$: sand wave steepness

Note: h, S, v and v_* are the space average values. Subscript cr signifies the "critical stage" (the initiation of sediment transport)

REFERENCES

1) Alam, M. Z. and J. F. Kennedy: Friction factors for flow in sand bed channels, Journal of the Hydraulics Division, ASCE, Vol. 95, HY 6, Nov. 1969.
2) Bogardi, J.: Sediment Transport in Alluvial Streams, Akademiai Kiado, Budapest, 1974.
3) Blench, T.: Mobile Bed Fluviology, T. Blench & Associates Ltd, Edmonton, Alta, 1966.
4) The Bed Configuration and Roughness of Alluvial Streams, Task Committee Rep., J.S.C.E., Tokyo 160, Nov. 1974.
5) Chiemeka, G. Ilo: Resistance to flow in Alluvial channels, Journal of the Hydraulics Division, ASCE, Vol. 101, HY 6, June 1975.
6) Davies, T. R. H.: Bedform Spacing and Flow Resistance, Journal of the Hydraulic Division, ASCE, Vol. 106, HY 3, March 1980.
7) Engelund, F.: Hydraulic Resistance of Alluvial Streams, Journal of the Hydraulics Division, ASCE, Vol. 92, HY 2, March 1966.
8) Fredsoe, J.: The Friction Factor and Height Length Relations in Flow over a Dune-covered Bed, Institute of Hydrodynamic Tech. Univ. of Denmark., Prog. Rep. 37, Dec. 1975.
9) Kamphuis, J. W.: Determination of Sand Roughness for Fixed Beds, Journal of Hydrauiic Research, IAHR, Vol. 12, No. 2, 1974.
10) Kishi, T.: Bed Forms and Hydraulic Relations for Alluvial Streams, Application of Stochastic Processes in Sediment Transport, Edited by H. W. Shen and H. Kikkawa, Water Res. Publications, Littleton, Colorado, U.S.A. 80161, 1980.
11) Klaasen, G. J.: Sediment Transport and Hydraulic Roughness in Relation to Bed Forms, Publication No. 213, Delft Hydraulic Laboratory, May 1979.
12) Pillai, C. R. S.: Effective Depth in Channels Having Bed Undulations, Journal of the Hydraulic Division, ASCE, Vol. 105, HY1, January 1980.
13) Schlichting, H.: Boundary Layer Theory, 6th Edition, McGraw-Hill, 1968.
14) Vanoni, V. A. and Li-San Hwang: Relation Between Bed Forms and Friction in Streams, Journal of Hydraulics Division, ASCE, Vol. 93, HY 3, May 1967.
15) Vatagodakumbura, S.: Forces Acting on the Bed Features of an Open Channel Flow, M.Sc. thesis, Queen's Univ., Kingston, Canada, April 1975.
16) Vittal, N., Ranga Raju, K. G. and Garde, R. J.: Resistance of Two Dimensional Triangular Roughness, Journal of Hydraulic Research, Vol. 15, No 1, 1977.
17) Yalin, M. S.: On the Average Velocity of Flow Over a Mobile Bed, La Houille Blanche, No. 1, Jan. 1964.
18) Yalin, M. S.: Mechanics of Sediment Transport, 2nd Edition, Pergamon Press, Oxford, 1977.
19) Yalin, M. S. and E. Karahan: Steepness of Sedimentary Dunes, Journal of the Hydraulics Division, ASCE, Vol. 105, HY 4, April 1979.
20) Yalin, M. S.: On the Determination of Ripple Length, Journal of the Hydraulics Division, ASCE, Vol. 103, HY, April 1977.

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