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PROPOSAL OF A TIME-DEPENDENT YIELD FUNCTION AND CHARACTERISTICS OF YIELDING OF SOFT ROCKS

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1. INTRODUCTION

The influence of time on yielding may be great significance for viscoelastic materials. Some criteria related to yielding of viscoelastic materials have been proposed so far. Reiner et al.[1] generalized von Mises' yield criterion of elastic materials to include viscoelastic materials, by assuming that yielding may occur when the distortional work which is conserved as elastic energy reaches a critical value. Furthermore, Reiner[2] discussed the above criterion to explain the phenomena such that the yield stress increases as the loading rate is increased and yielding occurs under constant stresses. Olszak[3] propounded a failure criterion that failure of some viscoelastic materials depends not only on the stored energy but also on the dissipative energy. By taking account of the difference between the influence of the stored energy and that of the dissipative one, Tateishi et al.[4] studied yield criteria of viscoelastic-plastic materials. As the above-mentioned yield criteria are related to viscoelastic materials whose yielding is independent of hydrostatic pressure, they should be mainly applied to materials such as metals.

On the other hand, Akai et al.[5] investigated the constitutive equation which can describe the time-dependent characteristics of soft rocks on the basis of the elastic-viscoplastic theory given by Perzyna.[6),7)] Hirai et al.[8] studied a yield function and a constitutive equation of soft rocks by use of elastic-plastic theory. In order to extend the yield function proposed by Hirai et al. to incorporate the effect of time, the investigation will have to be made about the yield criterion of viscoelastic-plastic materials. Further, it has been required that the yield criterion of materials whose plastic behavior is independent of hydrostatic pressure is generalized to that of materials such as soft rocks whose plastic behavior depends on hydrostatic pressure.

The objective in this paper is to deduce a suitable yield criterion of soft rocks to elucidate phenomena associated with yielding. For this purpose, a yield function which consists of the energy related to distortion and that to dilatation is proposed on the basis of energy criterion; and furthermore, each energy is divided into the stored and dissipative energies. The stress-strain relationship is derived from the proposed yield function by use of the special histories given by Gurtin et al.[9] The discussion is made concerning the relationship between the proposed yield function and the mechanical behavior for some histories. The availability of the proposed yield function is examined through experimental results of a soft rock.[5]

2. PROPOSAL OF A TIME-DEPENDENT YIELD FUNCTION OF SOFT ROCKS

Before the time-dependent yield function of soft rocks is investigated, let us summarize the stress-strain relationship of viscoelastic materials. With reference to rectangular Cartesian coordinates, the constitutive equation of viscoelastic materials is expressed in the form[10]

$$e_{ij}(t)=\int_0^t J_1(t-\tau)\frac{\partial s_{ij}(\tau)}{\partial \tau}\,d\tau \quad \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

$$\varepsilon_{kk}(t)=\int_0^t J_2(t-\tau)\frac{\partial \sigma_{kk}(\tau)}{\partial \tau}\,d\tau \quad \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

where $e_{ij}(t)$ are components of the deviatoric strain, $\varepsilon_{kk}(t)$ is the trace of the strain $\varepsilon_{ij}(t)$, $s_{ij}(t)$ are components of the deviatoric effective stress, $\sigma_{kk}(t)$ is the trace of the effective stress $\sigma_{ij}(t)$, $J_1(t)$ and $J_2(t)$ are creep functions related to distortion and dilatation respectively and t is time.

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Gurtin et al.[9] show that the stress-strain relationship and the work of viscoelastic materials are approximately elastic for very fast and very slow strain histories, which are called an accelerated and a retarded strain histor es respectively. Following the method given by Gurtin et al., let us consider the very fast and very slow stress histories, which are referred to as an accelerated and a retarded stress histories respectively. For a prescribed stress history $\sigma_{ij}(t)$, an accelerated and a retarded stress histories are defined respectively as follows.[9]

$$\sigma^{\lambda}_{ij}(t)=\sigma_{ij}(\lambda t)\,,\quad \lambda>1 \qquad (3)$$

$$\sigma^{\lambda}_{ij}(t)=\sigma_{ij}(\lambda t)\,,\quad \lambda<1 \qquad (4)$$

Eq. (3) means that the rate of the stress in the accelerated history is λ times as large as that in the prescribed history. The meaning similar to the content mentioned-above for eq. (3) is applied to eq. (4). Let $\varepsilon_{ij}(\lambda, t)$ be the strains at time t for both an accelerated and a retarded stress histories defined by eqs. (3) and (4). For an accelerated stress history, eqs. (1) and (2) take the relations

$$\lim_{\lambda\to\infty} e_{ij}(\lambda, t/\lambda)=J_1(0)s_{ij}(t) \qquad (5)$$

$$\lim_{\lambda\to\infty} \varepsilon_{kk}(\lambda, t/\lambda)=J_2(0)\sigma_{kk}(t) \qquad (6)$$

As for a retarded stress history, eqs. (1) and (2) have the forms

$$\lim_{\lambda\to 0} e_{ij}(\lambda, t/\lambda)=J_1(\infty)s_{ij}(t) \qquad (7)$$

$$\lim_{\lambda\to 0} \varepsilon_{kk}(\lambda, t/\lambda)=J_2(\infty)\sigma_{kk}(t) \qquad (8)$$

Eqs. (5) to (8) imply that, for an accelerated and a retarded stress histories, the stress-strain relationship of viscoelastic materials approaches that of elastic ones. For the creep functions,[10] it is satisfied generally that

$$J_i(0)\leqq J_i(t)\leqq J_i(\infty)\,,\quad (i=1, 2) \qquad (9)$$

Using eq. (9), it is found from eqs. (1), (2) and (5) to (8) that

$$J_1(0)s_{ij}(t)\leqq e_{ij}(t)\leqq J_1(\infty)s_{ij}(t) \qquad (10)$$

$$J_2(0)\sigma_{kk}(t)\leqq \varepsilon_{kk}(t)\leqq J_2(\infty)\sigma_{kk}(t) \qquad (11)$$

where $e_{ij}(t)$ and $\varepsilon_{kk}(t)$ are given by eqs. (1) and (2) respectively. Eqs. (10) and (11) represent the upper and lower bound formulae of the strain of viscoelastic materials for a prescribed stress history. In what follows, an accelerated history means an accelerated stress or strain histories, according as the boundary condition is prescribed by stress or strain. The definition similar to an accelerated history above-mentioned is applied to a retarded history. From results given by Gurtin et al. and the above discussion, it is found that for both an accelerated and a retarded histories, the stress-strain relationship of viscoelastic materials approaches that of elastic ones.

According to Christensen,[10] if the motion starts at time $t=0$ and stress and strain are $\sigma_{ij}(t)=\varepsilon_{ij}(t)=0$ for $t<0$, the stored energy related to distortion, S_d and the rate of the dissipative energy to distortion, D_d are expressed as

$$S_d=s_{ij}(t)\int_0^t J_1(t-\tau)\frac{\partial s_{ij}(\tau)}{\partial\tau}\,d\tau -\frac{1}{2}\int_0^t\int_0^t J_1(t-\tau,\, t-\eta)\frac{\partial s_{ij}(\tau)}{\partial\tau}\frac{\partial s_{ij}(\eta)}{\partial\eta} \times d\tau\, d\eta \qquad (12)$$

$$D_d=\frac{1}{2}\int_0^t\int_0^t \frac{\partial}{\partial t}J_1(t-\tau,\, t-\eta)\frac{\partial s_{ij}(\tau)}{\partial\tau}\frac{\partial s_{ij}(\eta)}{\partial\eta} \times d\tau\, d\eta \qquad (13)$$

where the creep functions are assumed as

$$J_i(\tau, \eta)=J_i(\tau+\eta)\,,\quad (i=1, 2) \qquad (14)$$

If the difference between the influence of the stored energy and that of the dissipative one is taken into account in yielding, a yield function may be assumed from eqs. (12) and (13) in the form

$$f=R_d-R=0 \qquad (15)$$

where f is a yield function and R is a work-hardening parameter and

$$R_d=S_d+\phi\int_0^t D_d\,dt \qquad (16)$$

where ϕ is a material constant to be determined from experimental results. When $\phi=0$ and $\phi=1$ in eq. (15) with eq. (16), eq. (15) with eq. (16) is reduced to the yield function proposed by Reiner et al.[1),2)] and that by Olszak[3)] respectively. Although Tateishi et al.[4)] propounded a yield function similar to eq. (15) with eq. (16), the yield function given by Tateishi et al. is based on the sum of distortion and dilatation energies instead of the distortion energy in eq. (16). Therefore, since eq. (15) with eq. (16) is considered to be a generalization of yield functions proposed so far, the meaning of the constant ϕ will have to be investigated in the respect of the mechanical behavior of yielding. For this point, the investigation will be made later when the yield function of soft rocks is discussed in section 5.

When soft rocks are subjected to high hydrostatic pressure, it is observed that appreciable permanent change occurs.[11)] This phenomenon may suggest that not only the distortion energy but also the dilatation energy may play an important role in yielding of soft rocks. Then, by analogy with the form of R_d given by eq. (16), it is set that

$$R_v=S_v+\phi\int_0^t D_v\,dt \qquad (17)$$

where the stored energy related to dilatation, S_v and the rate of the dissipative one to dilatation, D_v are expressed respectively in the form

$$S_v=\frac{1}{3}\sigma_{ii}(t)\int_0^t J_2(t-\tau)\frac{\partial\sigma_{jj}(\tau)}{\partial\tau}d\tau -\frac{1}{6}\int_0^t\int_0^t J_2(t-\tau,\ t-\eta)\frac{\partial\sigma_{ii}(\tau)}{\partial\tau}\frac{\partial\sigma_{jj}(\eta)}{\partial\eta} \times d\tau\, d\eta \quad\cdots\cdots(18)$$

$$D_v=\frac{1}{6}\int_0^t\int_0^t \frac{\partial}{\partial t}J_2(t-\tau,\ t-\eta)\frac{\partial\sigma_{ii}(\tau)}{\partial\tau}\frac{\partial\sigma_{jj}(\eta)}{\partial\eta} \times d\tau\, d\eta \quad\cdots\cdots(19)$$

Making a modification of the yield function given by Mróz et al.[12], Hirai et al.[8] proposed a yield function of soft rocks as an elastic-plastic material in the following form

$$f=I_2+\alpha I_2^{1/2}+\beta I_1^2+\gamma I_1-\kappa=0 \quad\cdots\cdots(20)$$

where I_2 is the second invariant of the deviatoric stress defined as $I_2=s_{ij}(t)s_{ij}(t)/2$, I_1 is the first invariant of the stress defined as $I_1=\sigma_{ii}(t)$, α, β and γ are material constants and κ is a work-hardening parameter and will be expressed in terms of the plastic work related to dilatation and that to distortion, as will be shown in eq. (44). Since the stress-strain relationship of viscoelastic materials is approximately elastic for a retarded history, it is considered that a yield function of viscoelastic-plastic materials for a retarded history approaches that of elastic-plastic ones. Then, the yield function given by eq. (20) may be assumed to be that of viscoelastic-plastic materials for a retarded history. The extension of the yield function eq. (20) proposed for elastic-plastic materials may be made for viscoelastic-plastic ones on the basis of energy criterion. The energy of viscoelastic materials for a retarded history approaches that of elastic ones and the invariants I_1^2 and I_1 in eq. (20) are related to dilatation and distortion elastic energies respectively. If eqs. (16) and (17) are incorporated into the yield function of viscoelastic-plastic materials, a time-dependent yield function of soft rocks may be assumed in comparison with eq. (20) in the form

$$f(t)=R_d+aR_d^{1/2}+bR_v-cR_v^{1/2}-R=0 \quad\cdots\cdots(21)$$

where a, b and c are material constants and R is a work-hardening parameter which does not depend on time.

For a prescribed stress history $\sigma_{ij}(t)$, let $f_\lambda(t)$ be yield functions at time t for an accelerated and a retarded stress histories given by eqs. (3) and (4) respectively. For an accelerated stress history, it follows from eq. (21) that

$$\lim_{\lambda\to\infty} f_\lambda(t/\lambda)=M_d+aM_d^{1/2}+bM_v-cM_v^{1/2}-R =0 \quad\cdots\cdots(22)$$

where

$$M_d=J_1(0)s_{ij}(t)s_{ij}(t)/2 \quad\cdots\cdots(23)$$

$$M_v=J_2(0)\sigma_{ii}(t)\sigma_{jj}(t)/6 \quad\cdots\cdots(24)$$

For a retarded stress history, it is seen from eq. (21) that

$$\lim_{\lambda\to 0} f_\lambda(t/\lambda)=N_d+aN_d^{1/2}+bN_v-cN_v^{1/2}-R =0 \quad\cdots\cdots(25)$$

where

$$N_d=J_1(\infty)s_{ij}(t)s_{ij}(t)/2 \quad\cdots\cdots(26)$$

$$N_v=J_2(\infty)\sigma_{ii}(t)\sigma_{jj}(t)/6 \quad\cdots\cdots(27)$$

Eqs. (22) and (25) mean that for an accelerated and a retarded stress histories, the yield function of viscoelastic-plastic materials eq. (21) approaches that of elastic-plastic ones similar to eq. (20).

3. INFLUENCE OF LOADING RATE ON YIELD STRESSES

In order to investigate the characteristics of yielding of soft rocks, it is instructive to adopt a series of experimental data of a soft sedimentary rock performed by Akai et al.[5),13)] The static yield stress shown in the second paper[5)] is defined for the case of the constant strain rate 10^{-8}/min which is considered to correspond to a retarded history. Then the elastic constants for the retarded history should be determined in application of the strain rate 10^{-8}/min. However, since the elastic constants obtained in the first paper[13)] by Akai et al. are those for the case of the strain rate 3.5×10^{-6}/min, the elastic constants for the strain rate 3.5×10^{-6}/min are assumed to be those for a retarded history in the present paper for convenience.

For a retarded history, the shear and bulk moduli μ and K of the soft rock tested by Akai et al. can be assumed to take approximately

$$\mu=4.0\times10^3\ \mathrm{kgf/cm^2}\ (3.92\times10^2\ \mathrm{MPa}) \quad\cdots(28)$$

$$K=7.0\times10^3\ \mathrm{kgf/cm^2}\ (6.86\times10^2\ \mathrm{MPa}) \quad\cdots(29)$$

By use of eqs. (7), (8), (28) and (29), it follows that

$$J_1(\infty)=1/(2\mu)=1.25\times10^{-4}\ \mathrm{cm^2/kgf}\ (1.28\times10^{-3}\ \mathrm{MPa^{-1}}) \quad\cdots\cdots(30)$$

$$J_2(\infty)=1/(3K)=4.76\times10^{-5}\ \mathrm{cm^2/kgf}\ (4.86\times10^{-4}\ \mathrm{MPa^{-1}}) \quad\cdots\cdots(31)$$

Although it is considered that it is difficult to decide strictly the elastic constants for an accelerated history, in view of the experimental data of the soft rock, the instantaneous elastic shear modulus μ_i may be assumed approximately to be

$$\mu_i = 8.0\times10^3\ \text{kgf/cm}^2\ (7.84\times10^2\ \text{MPa}) \cdots(32)$$

Then it leads from eqs. (5) and (32) that

$$J_1(0) = 1/(2\mu_i) = 6.25\times10^{-5}\ \text{cm}^2/\text{kgf}\ (6.38\times10^{-4}\ \text{MPa}^{-1}) \cdots\cdots(33)$$

Assuming that the form of displacement is separated into time and space variables, Christensen[10] showed that

$$J_2(t)/J_1(t) = \omega \cdots\cdots(34)$$

where ω is real constant. Using eqs. (30), (31), (33) and (34) leads to

$$J_2(0) = 2.38\times10^{-5}\ \text{cm}^2/\text{kgf}\ (2.42\times10^{-4}\ \text{MPa}^{-1}) \cdots\cdots(35)$$

Let us consider the sample of a soft rock which is subjected to a compressive stress σ_3' in the axial direction under confining pressures $\sigma_1' = \sigma_2'$ in a triaxial compression test. The compressive stresses are taken to be positive. In triaxial tests, eqs. (22) and (25) can be written as

$$\lim_{\lambda\to\infty} f_\lambda(t/\lambda) = J_1(0)q^2/3 + a\{J_1(0)/3\}^{1/2}q + 3bJ_2(0)/2p^2 - c\{3J_2(0)/2\}^{1/2}p - R = 0 \cdots\cdots(36)$$

$$\lim_{\lambda\to0} f_\lambda(t/\lambda) = J_1(\infty)q^2/3 + a\{J_1(\infty)/3\}^{1/2}q + 3bJ_2(\infty)/2p^2 - c\{3J_2(\infty)/2\}^{1/2}p - R = 0 \cdots\cdots(37)$$

where the mean principal stress and the deviatoric stress are defined as $p = (2\sigma_1' + \sigma_3')/3$ and $q = \sigma_3' - \sigma_1'$ respectively. In triaxial tests, the yield function eq. (20) is expressed in the form

$$f = q^2/3 + \alpha/3^{1/2}q + 9\beta p^2 - 3\gamma p - \kappa = 0 \cdots\cdots(38)$$

To investigate the influence of time on yielding, it may be assumed that the yield function eq. (38) is obtained for a retarded history, as mentioned in section 2. Then, it is found from eqs. (37) and (38) that

$$\left.\begin{aligned} a &= \alpha\{J_1(\infty)\}^{1/2} \\ b &= 6\beta J_1(\infty)/J_2(\infty) \\ c &= \gamma J_1(\infty)\{6/J_2(\infty)\}^{1/2} \\ R &= \kappa J_1(\infty) \end{aligned}\right\} \cdots\cdots(39)$$

For the yield function eq. (38), the material constants α, β and γ and the work-hardening parameter at the initial yielding κ_0 was reported as[8]

$$\left.\begin{aligned} \alpha &= 54.6\ \text{kgf/cm}^2\ (5.35\ \text{MPa}) \\ \beta &= 0.149 \\ \gamma &= 42.9\ \text{kgf/cm}^2\ (4.20\ \text{MPa}) \\ \kappa_0 &= 0 \end{aligned}\right\} \cdots\cdots(40)$$

Thus, substitution of eqs. (30), (31) and (40) into eq. (39) leads to

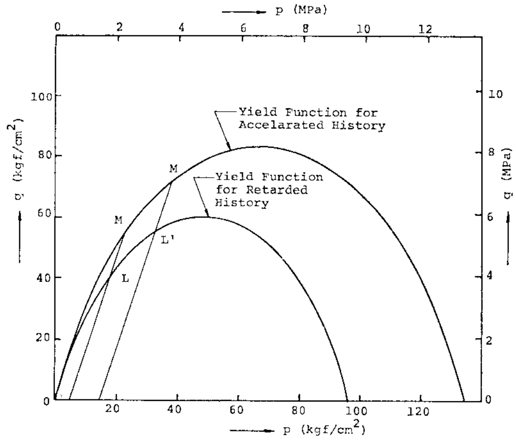

Fig. 1 Yield Functions for Accelerated and Retarded Histories (from 5)).

$$\left.\begin{aligned} a &= 0.61\ (\text{kgf/cm}^2)^{1/2}\ (0.191\ \text{MPa}^{1/2}) \\ b &= 2.35 \\ c &= 1.90\ (\text{kgf/cm}^2)^{1/2}\ (0.597\ \text{MPa}^{1/2}) \\ R_0 &= 0 \end{aligned}\right\} \cdots(41)$$

where R_0 denotes the work-hardening parameter at the initial yielding. Substitution of eqs. (33), (35) and (41) into eq. (36) leads to the yield function at the initial yielding for an accelerated history, as shown in **Fig. 1**. The yield function eq. (37) for a retarded history, as shown in **Fig. 1**, is assumed to be identical with the yield function of eq. (38). As the leading rate changes from a retarded history to an accelerated one, that is, the loading rate is increased, the yield stress increases; e.g., $L\to M$ and $L'\to M'$ in **Fig. 1**. As compared with the data reported by Akai et al.[5], it is found that the results shown in **Fig. 1** explain properly the relationship between the yield stress and the loading rate.

4. INFLUENCE OF LOADING RATE ON STRESS-STRAIN RELATIONSHIPS

For viscoelastic-plastic materials, Naghdi et al.[14] showed that the normality of the plastic strain rate vector for the yield surface does not necessarily hold. However, since it is proved that the property of viscoelastic materials approaches that of elastic ones for an accelerated and a retarded histories, the normality rule relevant to elastic-plastic materials can be applied to viscoelastic-plastic ones for these special histories.

According to Prager,[15] the plastic strain rate for elastic-plastic materials can be written in the form

$$\dot{\varepsilon}_{ij}^{(p)}=\frac{1}{A}\frac{\partial f}{\partial \sigma_{ij}}\frac{\partial f}{\partial \sigma_{kl}}\dot{\sigma}_{kl} \quad \cdots\cdots(42)$$

where

$$A=-\left(\frac{\partial f}{\partial \varepsilon_{mn}^{(p)}}+\frac{\partial f}{\partial \kappa}\frac{\partial \kappa}{\partial \varepsilon_{mn}^{(p)}}\right)\frac{\partial f}{\partial \sigma_{mn}} \quad \cdots\cdots(43)$$

and $\dot{\varepsilon}_{ij}^{(p)}$ are components of the plastic strain rate and κ is a work-hardening parameter. For soft rocks as elastic-plastic materials, Hirai et al.[8] proposed a form for the rate of the work-hardening parameter κ in the form

$$\dot{\kappa}=\phi_1\sigma_{ii}\dot{\varepsilon}_{jj}^{(p)}/3+\phi_2 s_{ij}\dot{\varepsilon}_{ij}^{(p)} \quad \cdots\cdots(44)$$

where ϕ_1 and ϕ_2 are material constants, which for the soft rock tested by Akai et al.[5] was obtained as

$$\left.\begin{aligned}\phi_1&=-1.12\times10^4\,\text{kgf/cm}^2\ (-1.10\times10^3\,\text{MPa})\\ \phi_2&=\ \ 3.73\times10^3\,\text{kgf/cm}^2\ (\ 3.66\times10^2\,\text{MPa})\end{aligned}\right\} \quad \cdots\cdots(45)$$

As the yield function for a retarded history given by eq. (37) is assumed to be identical to eq. (38), the stress-strain relationship for the retarded history can be seen in the paper written by Hirai et al.[8]

The yield function for an accelerated history is represented by eq. (36), which can be altered as

$$F=q^2/3+\alpha'/3^{1/2}q+9\beta'p^2-3\gamma'p-\kappa'=0 \cdots(46)$$

where

$$\left.\begin{aligned}\alpha'&=a/\{J_1(0)\}^{1/2}\\ \beta'&=bJ_2(0)/\{6J_1(0)\}\\ \gamma'&=c\{J_2(0)/6\}^{1/2}/J_1(0)\\ \kappa'&=R/J_1(0)=\kappa J_1(\infty)/J(0)\end{aligned}\right\} \quad \cdots\cdots(47)$$

Since the material constants α', β', and γ' in eq. (47) can be expressed by eqs. (33), (35) and (41), it follows that

$$\left.\begin{aligned}\alpha'&=77.2\,\text{kgf/cm}^2\ (7.57\,\text{MPa})\\ \beta'&=0.149\\ \gamma'&=60.5\,\text{kgf/cm}^2\ (5.93\,\text{MPa})\end{aligned}\right\} \quad \cdots\cdots(48)$$

The work-hardening parameter κ' in eq. (47) can be written in terms of the work-hardening parameter κ in eq. (44). Substituting eqs. (43) to (48) into eq. (42) leads to the relationship between the deviatoric plastic strain and stress for an accelerated history as follows:

$$\begin{aligned}\dot{d}^{(p)}/\dot{q}=&(2/3q+\alpha'/3^{1/2})\{(2/3+2\beta')q\\ &+\alpha'/3^{1/2}+6\beta'\sigma_1'-\gamma'\}\{J_1(0)/J_1(\infty)\}/\\ &[(2\beta'\phi_1+2/3\phi_2)q^2+\{3\phi_1(4\beta'\phi_1-\gamma'/3)\\ &+\alpha'\phi_2/3^{1/2}\}q+3\phi_1\sigma_1'(6\beta'\sigma_1'-\gamma')]\end{aligned} \quad \cdots\cdots(49)$$

where $\dot{d}^{(p)}$ is the deviatoric plastic strain rate

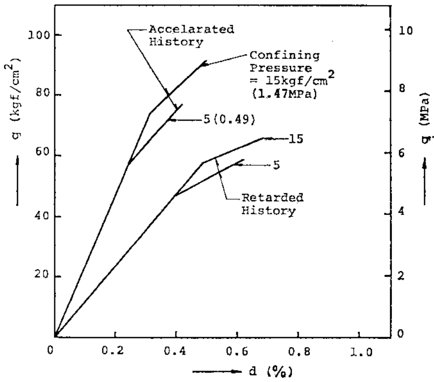

Fig. 2 Relationships between Stress and Strain for Accelerated and Retarded Histories (from 5)).

defined as $\dot{d}^{(p)}=(2/3)|\dot{\varepsilon}_{33}^{(p)}-\dot{\varepsilon}_{11}^{(p)}|$. The stress-strain relationship in viscoelastic range for both an accelerated and a retarded histories are expressed by eqs. (5) to (8). Combining the viscoelastic and plastic strains for an accelerated and a retarded histories leads to the total stress-strain relationships, as shown in **Fig. 2**. It is found from **Fig. 2** that (1) the yield point for an accelerated history is higher than that for a retarded history. (2) The gradient of curves for an accelerated history is steeper than that of curves for a retarded history. The results shown in **Fig. 2** have the tendency similar to those reported by Akai et al.[5] Kobayashi[16),17)] showed the stress-strain curves of sandstone for various loading rates. It is noticed that there are some relationships mentioned in **Fig. 2** in the data given by Kobayashi.

The stress-strain-time relationship for a prescribed stress history is illustrated schematically by the curve OLM in (d, q, t) space in **Fig. 3**.

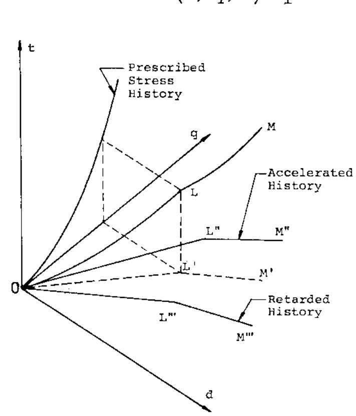

Fig. 3 Stress-strain-time Relationships for Prescribed, Accelerated and Retarded Histories.

The point L corresponds to the initial yielding. The curve OL′M′ is the projection of the curve OLM on (d, q) plane. For both an accelerated and a retarded histories which correspond to the prescribed stress history, the stress-strain relationships are indicated by the curves OL″M‴ and OL‴M‴ respectively. It is shown that the curve OL′M′ exists generally between the curves OL″M″ and OL‴M‴.

5. YIELDING CAUSED BY CREEP

In the triaxial compression test of a soft rock, consider the possibility that yielding may occur under constant stresses. The boundary condition of stress is specified as

$$\left.\begin{aligned}\sigma_1'(t)&=\sigma_2'(t)=\sigma_1'h(t)\\ \sigma_3'(t)&=\sigma_3'h(t)\end{aligned}\right\}\quad\cdots\cdots(50)$$

where σ_1' and σ_3' are positive constants and $h(t)$ is the Heaviside unit step function defined by

$$\left.\begin{aligned}h(t)&=0,\quad t<0\\ &\;\;1,\quad t>0\end{aligned}\right\}\quad\cdots\cdots(51)$$

The boundary condition given by eq. (50) belongs to an accelerated stress history. It is to be noticed that[10]

$$\left.\begin{aligned}&\frac{dh(t)}{dt}=\delta(t)\\ &\int_0^t g(t-\tau)\delta(\tau)d\tau=g(t)\end{aligned}\right\}\quad\cdots\cdots(52)$$

where $\delta(t)$ is the delta function and $g(t)$ is a continuous function of t.

By use of eqs. (14) and (52), the yield function given by eq. (21) for a triaxial compression test of eq. (50) can be written as

$$f(t)=U_d+aU_d^{1/2}+bU_v-cU_v^{1/2}-R=0\cdots\cdots(53)$$

where

$$\left.\begin{aligned}U_d&=q^2/3[2J_1(t)-J_1(2t)+\phi\{J_1(2t)-J_1(0)\}]\\ U_v&=3p^2/2[2J_2(t)-J_2(2t)+\phi\{J_2(2t)-J_2(0)\}]\end{aligned}\right\}$$
$$\cdots\cdots(54)$$

The yield function given by eq. (53) is represented in **Fig. 4**. As time t increases, the yield surface contracts from the yield function of an accelerated history toward that of a retarded one. Therefore, if the rock is subjected to the constant stresses below the yield stresses for an accelerated history, yielding will take place at the moment when eq. (53) is satisfied.

If it is assumed that the yield stresses p and q which satisfy eq. (53) in the limit $t\to\infty$ are equivalent to those for a retarded history, p and q in eq. (37), it follows that

$$\phi=0\quad\cdots\cdots(55)$$

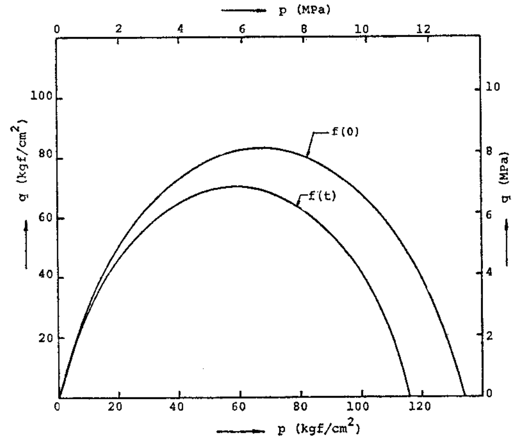

Fig. 4 Yield Function Dependent on Time (from 5)).

This implies that the stored energy only is associated with yielding of soft rocks. If, however, the above assumption is not pertinent to soft rocks, it is concluded that

$$\phi\neq 0\quad\cdots\cdots(56)$$

This means that the dissipative energy as well as the stored energy have to be taken into account for the criterion of yielding. In view of the fact that substitution of $a=b=c=0$ into eq. (21) leads to eq. (15), eq. (21) with $\phi=0$ and eq. (21) with $\phi=1$ are related to the yield criterion preposed by Reiner et al.[1),2)] and that by Olszak[3)] respectively. It is found that the meaning of the material constant ϕ in the proposed yield function can be made clear through the mechanical behavior in a retarded history and that in creep. For the soft rock experimented by Akai et al.[5)], it seems difficult to decide presently what value the material constant ϕ takes for lack of the experimental data to serve the present purpose. The further detailed investigation will be required for the determination of the material constant ϕ in the proposed yield function eq. (21) through many experimental results.

Before closing this section, it may be of interest to make a comparison with the work of Akai et al.[5)] The constitutive model established by Akai et al. is based on the theory proposed by Perzyna.[6),7)] Perzyna develops a constitutive equation for elastic-viscoplastic materials by postulating the existence of static and dynamic yield functions as well as the assumption of normality. The constitutive equation for visco-elastic-plastic (rather than elastic-viscoplastic) materials has been discussed in the present paper. The yield function of viscoeleastic-plastic materials reduces to that of elastic-plastic ones for two limiting cases, i.e., very fast and very slow loading conditions. The normality for the yield function of viscoelastic-plastic materials is hold for only two limiting cases.

The theory of viscoelastic-plastic materials discussed here is reduced to that of the classical linear viscoelasticity in one case and the invicid theory of elastic-plastic solids in another. However, Perzyna's constitutive equation may not reduce to that in the classical viscoelastic theory. Furthermore, the perfect plastic idealization is assumed in the viscoplastic constitutive equation given by Akai et al. In the present paper, the above assumption is not employed and the stress-strain relationship at the viscoelastic-plastic state is derived from the yield function based on the energy related to viscoelastic materials.

6. CONCLUDING REMARKS

On the basis of energy criterion, a time-dependent yield function represented by the stored energy and the dissipative one was proposed for soft rocks. The above yield function is able to describe properly the time-dependent behavior such that as the loading rate is increased, the yield stress increases and yielding occurs under constant stresses. For two limiting cases that the loading conditions are very fast and very slow, the stress-strain relationships derived from the proposed yield function explain adequately the experimental data containing the effect of loading rate.

By making certain assumptions between the mechanical behavior in a very slow loading and that in creep, it was shown that the yield function proposed here is reduced to those given by Reiner et al. and Olszak. Therefore, it was suggested that the yield function propounded here is a generalization of those proposed so far.

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和文要旨

軟岩の時間依存性の降伏関数の提案と降伏の特性

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静水圧が塑性挙動に影響を与えない粘弾・塑性体の降伏規準に関しては，いくつかの規準式が提案されている．しかしながら軟岩などのように降伏特性が静水圧に依存し得る粘弾・塑性体に対して，降伏規準をどのようにして決定すべきかという問題に関する研究は数少ないのが現状と思われる．本論文においてはエネルギー規準の立場から考察を行い，粘弾性体のエネルギーを体積変形とせん断変形に関する部分とに分解し，さらにそのおのおのを保存エネルギーと散逸エネルギーに分けて，それらを引数とする粘弾・塑性体の降伏関数を提案する．この提案された時間依存性の降伏関数は，Reiner, Olszak, 立石らによって従来提案されているエネルギー規準の考え方を軟岩に拡張したものであり，力学挙動との対応関係を明確にするために一つの材料定数を取り入れ，その力学的意味を検討している．

まず載荷速度の降伏応力に及ぼす影響を考察するために，ある与えられた応力履歴に対して Gurtin らによって示された加速履歴と遅延履歴という2つの極限の履歴を考え，それらについて提案された粘弾・塑性体の降伏関数がどのような形に変換されるかを調べている．遅延履歴の場合，すなわち非常にゆっくりした載荷条件のもとでは粘弾・塑性体の降伏関数は弾塑性体のそれに近づくことが示される．この場合，粘弾・塑性体の降伏関数は著者らが先に提案した弾塑性体のそれと同一であると仮定すると，粘弾・塑性体の降伏関数に含まれる材料定数を決定することができる．ここでは軟岩の実験結果を用いて材料定数を求めている．一方，加速履歴の場合すなわち非常に速い載荷条件のもとでは粘弾・塑性体の降伏関数は弾塑性体のそれに近づくことが示される．これらの結果をもとに加速履歴における降伏応力は遅延履歴のそれより大きくなることが示され，降伏応力に及ぼす載荷速度の影響を明らかにしている．また上記の結果は軟岩の実験事実を適切に説明していると考えられる．

次に応力-ひずみ関係に及ぼす載荷速度の影響を調べるために Gurtin らの加速・遅延履歴を用いる．これらの履歴において粘弾・塑性体の降伏関数は弾塑性体のそれに帰着し得ることから，Prager によって与えられた構成式を用いることが可能となる．粘弾・塑性体の遅延履歴における応力-ひずみ関係は通常得られる弾塑性体のそれと等しいと仮定され得るならば，遅延履歴に対して著者らが先に示した弾塑性体の構成式を用いることができる．また加速履歴における構成式については，遅延履歴に対して得られた結果をもとにして，Prager の構成式を用いて決定することができる．遅延履歴と加速履歴の場合を比べると，応力-ひずみ曲線の勾配は加速履歴の方が大きくなり，この傾向は軟岩の実験事実をよく表現していると思われる．

次に一定応力を受ける場合の粘弾性体に生じ得る降伏すなわちクリープ降伏について考察している．粘弾性体が一定応力を受ける場合の荷重条件は加速履歴に属するものと考えられる．この加速履歴に対する降伏応力以下の一定応力を載荷した場合，ある時間の経過後降伏が起き得ることが提案された降伏関数を用いて説明される．

提案された降伏関数に含まれる材料定数について，それは保存エネルギーと散逸エネルギーの降伏関数への関与する割合を表現していると考えられる．一方降伏に関する力学挙動について，遅延履歴に対する降伏応力とクリープ降伏応力との関係からその材料定数の力学的意味が明らかにされている．また本論文において提案された降伏規準と従来提案されている降伏規準との相互関係についても考察されている．結論として，エネルギー規準に基づいた時間依存性の降伏関数は，軟岩の時間効果を含む実験事実を適切に説明し得ることが示されている．