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ULTIMATE STRENGTH FORMULA FOR CENTRAL-ARCH-GIRDER BRIDGES

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1. INTRODUCTION

A central-arch-girder bridge, a box girder bridge with an arch over the median strip of the bridge deck as shown in **Fig. 1**, is a new type of bridge having economic and aethetic merits. One of the important problems from the view point of its strength is how to determine the ultimate strength of the bridge controlled by lateral instability of the central arch. Particulary in the current design practice a simple but ultimate strength oriented procedure for determining effective buckling length of the arch is required. This paper shows that the ultimate strength of the central-arch-girder bridge can be predicted by column strength formulas with a modified effective length parameter.

2. PROPOSED ULTIMATE STRENGTH FORMULAS

The strength of braced or unbraced steel arches which fail by lateral instability under in-plane uniform loads can be predicted by column strength formulas such as the ones shown below which are taken from the Japanese Specifications for Highway Bridges[1),2)].

$$\left.\begin{array}{l}\sigma_{u0}/\sigma_y=1-0.136\lambda-0.3\lambda^2 \\ \qquad \text{for} \quad 0\leq\lambda\leq1.0 \\ \sigma_{u0}/\sigma_y=1.276-0.888\lambda+0.176\lambda^2 \\ \qquad \text{for} \quad 1.0\leq\lambda\leq2.52\end{array}\right\} \quad \cdots\cdots(1)$$

In this case the slenderness ratio term of the column is replaced by a slenderness parameter for the arch, λ, and the axial load term by the axial force of the arch at its springings, N_u. The slenderness parameter, λ, for the arch is defined as[1)]

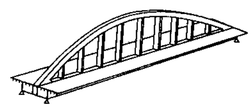

Fig. 1 General view of a central-arch-girder bridge.

$$\lambda=(K_e K_l K_\beta L/\pi r_y)\sqrt{\sigma_y/E} \quad \cdots\cdots\cdots\cdots\cdots\cdots(2)$$

where r_y is the radius of gyration of the arch rib section about centroidal principal axis, y, σ_y is the yield stress of the lowest strength steel used in the arch rib and E is the modulus of elasticity. The ultimate unit strength, σ_{u0}, for an arch is defined as the tangential thrust, N_u, divided by an area, A, which is the weighted average cross-sectional area of the arch rib over its entire curved length, L. The tangential thrust N_u is determined from a linear theory for the loaded arch.

The terms K_e, K_l and K_β are effective length factors. The coefficient K_e is 0.5 for the laterally clamped condition and 1.0 for the laterally hinged one. The coefficient K_l is 0.65 for the tilting hanger case and 1.0 for non-tilting hangers. The coefficient K_β is given by $K_\beta=1-\beta+(2r_y\beta/K_e a)$. The term β denotes a ratio of the length of braced portion to the total length of the arch rib and the term a denotes the distance between twin arch ribs. Since β equals zero for an arch without bracing system, K_β equals 1.0 for an isolated single arch.

When λ has been determined from Eq. (2) one proceeds next to Eq. (1) and determines σ_{u0}. Then, for a parabolic arch, for instance, the uniformly distributed load per unit length of the arch span at the ultimate state, p_u, is written in terms of σ_{u0} as follows:

$$p_u=2A\sigma_{u0}/l\sqrt{(1/16)(l/f)^2+1} \quad \cdots\cdots\cdots\cdots\cdots(3)$$

in which l and f denote the span and the rise of the arch respectively.

This paper proposes the use of Eq. (1) for the prediction for the ultimate strength of central-arch-girder bridges with λ modified so that it

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will apply to such a new type of bridge. Considering the effect of lateral flexural rigidity of the hangers, EI_H, on the increase in lateral flexural rigidity of the arch rib, EI_{yA}, an effective moment of inertia of the arch cross section is written as

$$\bar{I}_{yA}=I_{yA}(1+C\cdot i_H) \quad \cdots\cdots(4)$$

in which

$$i_H=(I_H/I_{yA})(L^3\cdot l/f^3\cdot l_p) \; . \quad \cdots\cdots(5)$$

In these equations, the term i_H is a non-dimensional parameter representing the flexural stiffness of the hangers and the term C is a constant controlling the contribution rate of i_H. The term i_H depends on the stiffness of the arch, I_{yA}/L^3. that of the hanger, I_H/f^3 and the number of hangers, l/l_p, where l_p denotes the spacing of the hangers.

Using Eqs. (2) and (4), the slenderness parameter, λ_L, for the central-arch-girder bridge is written as

$$\lambda_L=\lambda/\sqrt{1+Ci_H} \; . \quad \cdots\cdots(6)$$

At this stage the value C was determined empirically from the results of preliminary computer analyses. Using Eqs. (1) and (6) and two computed results of $(\lambda, i_H, \sigma_u/\sigma_y)=(2.0, 300, 0.298)$ and $(2.0, 700, 0.413)$, we obtain $C=0.001\,72$ and $C=0.001\,88$. Averaging these two values of C, we have $C=0.0018$. Then, Eq. (6) becomes

$$\lambda_L=\lambda/\sqrt{1+0.001\,8\,i_H} \; . \quad \cdots\cdots(7)$$

The term λ in Eq. (7) is determined from Eq. (2). Since the effective length factors for the central arch of the bridge become $K_e=0.5$, $K_t=0.65$ and $K_\beta=1.0$ from the definitions, Eq. (2) can be rewritten as

$$\lambda=(0.325L/\pi r_y)\sqrt{\sigma_y/E} \; . \quad \cdots\cdots(8)$$

When λ_L has been determined from Eqs. (7), (8) and (5), the ultimate unit strength for the central-arch-girder bridge, σ_u can be determind from Eq. (1) by replacing λ and σ_{u0} with λ_L and σ_u respectively. The uniformly distributed ultimate load, p_u, for the bridge is also calculated from Eq. (3) by replacing σ_{u0} with σ_u.

3. FORMULA PREDICTIONS AND COMPUTER SIMULATIONS

The applicability of the proposed formulas is examined by comparisons with computer analyses for various theoretical models of practical proportions. A finite element method developed for spatial elasto-plastic behavior of thin walled frames and arches[3] is used to generate the data for comparison.

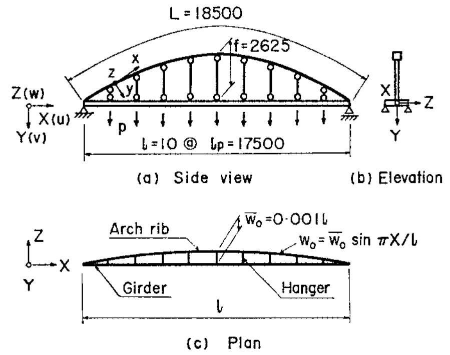

Fig. 2 Dimensions of the theoretical model (in cm).

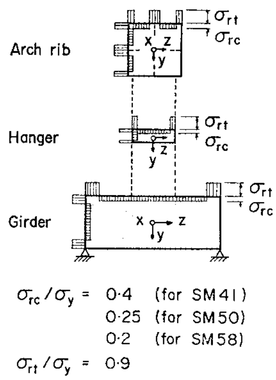

Fig. 3 Cross sections and residual stress distributions.

Figs. 2 and 3 show geometries and dimensions of the theoretical models used for comparison. In **Fig.** 2, small circles at both ends of the hangers show hinges rotatable with respect to Z-axis. The rotation with respect to X-axis is constrained at both ends of the hangers to resist against out-of-plane bending of the hangers caused by out-of-plane displacement of the arch. The stress-strain relation of the material used in the models is assumed to be an elastic-perfectly plastic type. The cross-sectional dimensions of each model are determined so that the parameters λ and i_H can cover the wide range of those values which may be encountered in actual bridges, while the other quantities are kept in approximately constant values corresponding to those of existing bridges. Ultimate capacities of the theoretical models are determined by the growth of unbounded horizontal and rotational displacements of the arch in the computer solution.

Fig. 4 shows the ultimate strength of the theoretical models with variation of i_H and λ values. The ultimate strengths of the models, σ_u, coincide with those of corresponding single

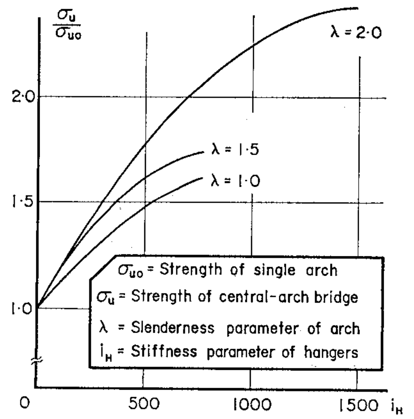

Fig. 4 Effects of hanger stiffness on the ultimate strength.

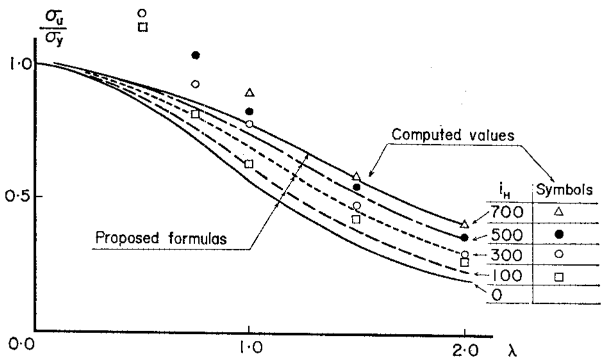

Fig. 5 Comparison of the formula predictions with computer analyses (SM41, $f/l=0.15$).

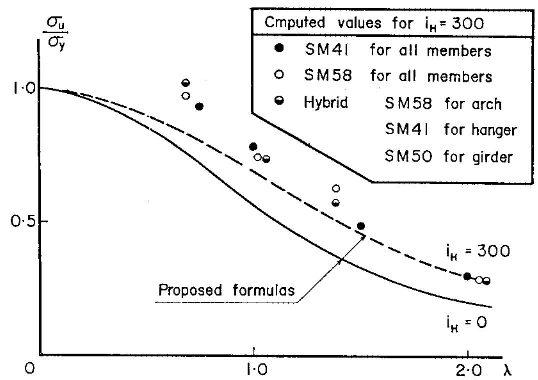

Fig. 6 Comparison of the formula predictions with computer analyses ($f/l=0.15$).

arches, σ_{u0}, when i_H equals zero, and remarkably increase with the increase in i_H values. It is also notable that the effect of i_H on the ultimate strength of the models is much greater for the models with the more slender arches.

In **Fig. 5**, the formula predictions are compared with the results of the computer analyses for the theoretical models with various λ and i_H values. **Fig. 6** also shows the comparison of the ultimate strength for the models made of high strength steel (SM58) and for the hybrid models made of SM41, SM50 and SM58 steels, of which yield stresses are $E/\sigma_y=875$, 656 and 456, respectively. It is seen from these figures that the suggested formulas can provide fairly good predictions for the ultimate strength of the theoretical models except in the range of λ less than 1.0. The stocky arches with such small values of λ attains fully plastic state prior to overall failure of the models and after that stage further loads are sustained mainly by the girder. Since the formulas are based on the strength of the arch, their predictions underestimate the strength of the total structure. Considering local buckling is likely to occur in such a fully plastic arch member, it is reasonable and conservative to accept the underestimation of the formulas in this range of λ.

In the current design practice for central-arch-girder bridges, the effective length, l_e, of the central arch determined by the Specifications is usually modified by multiplying an empirical reduction factor to take the effect of hangers flexural stiffness into account. And then the validity of the empirical effective length is checked by comparing it with the effective length determined as $l_E=\sqrt{EI_{yA}/H_{cr}}$ by using the critical horizontal thrust of the arch, H_{cr}, from an eigenvalue analysis for the total structure. Thus determined effective lengths of the central arch of existing two bridges are summarized in **Table 1** together with those determined by the proposed formulas. The value 0.6 was used for the reduction factor in the actual design of these bridges. The factors of safety for the actual bridges, ν_D, are computed from the results of the ultimate strength analyses previously reported[4]. Namely, ν_D is determined as the ratio of the total ultimate loads of the analyses to the sum of design dead loads and live loads. The factors of safety, ν, for other cases are calculated from ν_D assuming that they are inversely proportional to the ultimate unit strengths, σ_u determined by Eq. (1) in correspondence to the respective effective lengths. From this table one can realize that the values from the Specifications are too conservative and the values determined by other three procedures are more reasonable. It is notable that the factors of safety determined by the suggested formulas show consistent values for two different bridges.

Table 1 Effective lengths l_e and factors of safety ν.

Procedures \ Items (Name of Bridges)	Izumi-Ohtsu Bridge $l=172.57$ m, $i_H=915$				Nanko-Suiro Bridge $l=169.3$ m, $i_H=170$			
	l_e (m)	λ_L	σ_u/σ_y	ν	l_e (m)	λ_L	σ_u/σ_y	ν
by Specifications	68.8	0.90	0.64	2.7	65.8	0.98	0.58	2.5
as Designed	41.3	0.54	0.84	2.0	39.5	0.59	0.82	1.8
as Eigenvalue problem	39.5	0.51	0.85	2.0	42.6	0.64	0.79	1.8
by Formulas	36.0	0.47	0.87	1.9	48.1	0.72	0.75	1.9

Another significant advantage of the proposed procedure is that these values required for stability design can be obtained without tedious eigenvalue analyses and a large scale digital computer.

4. APPLICABILITY AND LIMITATIONS OF THE FORMULAS

Generality of the theoretical models used for the computer simulations was studied and the results are summarized below without detailed numerical data.

(1) When the number of hangers changes between 5 to 19 or the rise-to-span ratio of the arch varies from 0.1 to 0.2, the parameter i_H in Eq. (7) is still effective to predict the ultimate strength of the models.

(2) Variations of the height to width ratio of the arch cross section from 1.0 to 2.0 and variations of the residual stress distribution patterns of the member do not appreciably affect the ultimate strength of the models.

(3) Changes in the in-plane flexural stiffness and the torsional stiffness of the girder do not show appreciable influence on the ultimate strength of the model. Though changes in the lateral flexural stiffness of the girder show slight influence on the ultimate strength of the model, the lateral flexural stiffness of the girder assumed for the theoretical models are conservative enough comparing with those of actual bridges.

(4) Though the rotational fixity of the arch at its ends depends on the flexural stiffness of the end portion of the girder with respect to the bridge axis, X, it is confirmed numerically that this imperfection of the rotational fixity is not a problem in actual bridges and the end of the arch can be considered to be clamped so far as lateral buckling of arch is concerned.

Though the theoretical models studied can be thought, from these results, to be representative of actual central-arch-girder bridges, the applicability of the formula is subjected to the limitations for which it was derived[1]. Particularly, local instability should be checked by proper methods.

5. CONCLUSIONS

Extending the slenderness parameter concept, it is shown that the ultimate strength of central-arch-girder bridges subjected to in-plane distributed loads can be predicted by column strength formulas. The proposed formulas are shown to be sufficiently accurate for preliminary design purposes. Since the proposed procedure is based on the analogy between a column and an arch, the column strength formula which is used for predictions of σ_u need not necessarily be Eq. (1). Curve 2 of the SSRC (Structural Stability Research Council) multiple column curves or curve c of the ECCS (European Convention for Construction Steelwork) multiple column curves or other similar curves can be used for the prediction of σ_u[1].

The numerical computations were carried out by the digital computer FACOM M-200 of the Computer Center, Kyushu University. Assistance of Mr. Hiroyuki Tanaka (Toda Construction Co. Ltd., former student of Kumamoto University) in numerical computations is much appreciated.

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