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OPTIMUM DESIGN OF TRUSSES USING SUBOPTIMIZATION

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1. INTRODUCTION

In general, as the scale of construction of structures becomes larger, their configuration becomes more complicated and the number of analytical member elements becomes greater. In addition, the number of design variables and constraints increase, and numerical calculations for their optimum design become more difficult. Therefore, it would be better to carry out an optimum design using suboptimization, so that the number of design variables and constrains can be decreased.

A study on an optimum design using suboptimization was presented by S. Okubo[1),2)]. By using his method, variables and constraints can be decreased by one variable and one constraint for every member element. Other studies were presented by D. Kavlie and J. Moe[3)], and U. Kirsch, M. Reiss, U. Shamir[4)]. In their studies, end forces of each member are calculated by doing a structual analysis of the whole structure, and by using external loads corresponding to the end forces, an optimization problem of the substructures can be solved. Then, the design variables representing each submember are chosen, and optimization of the whole structure will be reached. Where the submember represents upper chord, lower chord, diagonal, etc., the present method of suboptimization is different from the above-mentioned methods, in that the design variables are divided into two groups, namely, the variables $\boldsymbol{x}$ of individual sections of elements and the variables $\boldsymbol{y}$ of sections common to the whole structure. The optimum values of $\boldsymbol{x}$ are obtained as functions of $\boldsymbol{y}$ by suboptimization, then the optimum values of the variables of the sections common to the whole structure are calculated by the SUMT (Sequence of Unconstrained Minimization Technique) method.

In the optimum design of the whole structure, the constraints contain only the upper and lower limits of values of the design variables, and the number of the design variables $\boldsymbol{y}$ can be greatly decreased. With this method, in the case of a small number of elements, computer time may not decrease so much.

The optimum values of the design variables are able to be used immediately in an actual bridge design. Since the optimum values of the variables are in general not integers, the integral value could be obtained by the branch and prune method. Although computer time is considerably increased, values of an objective function would not increase so much by rounding up to the next integer.

In a statically determinate structure, when not governed by deflections, the optimum values of the design variables are obtained when the working stresses reach the full allowable stresses. It would be enough if only the method of fully stressed design could be applied to the suboptimization. To illustrate this method, studies on the optimum design of I-section simple girders and continuous girders were published previously by one of the authors[5),6)].

The present paper describes an optimum design of trusses carried out by the SLP (Sequence of Linear Programming) method, applied to the suboptimization.

2. METHOD OF OPTIMUM DESIGN USING SUBOPTIMIZATION

A method of optimum design using suboptimization is given as follows: design variables are divided into two groups, the variables $\boldsymbol{x}$ and $\boldsymbol{y}$, as stated previously. The optimum values of $\boldsymbol{x}$ are obtained as a function of $\boldsymbol{y}$ by suboptimization using the SLP method, which is expressed by

$$\boldsymbol{x}=h(\boldsymbol{y}) \quad \cdots\cdots (1)$$

Constraints and an objective function common to the whole structure, are given as functions of $\boldsymbol{y}$ by substituting Eq. (1) into the constraints and

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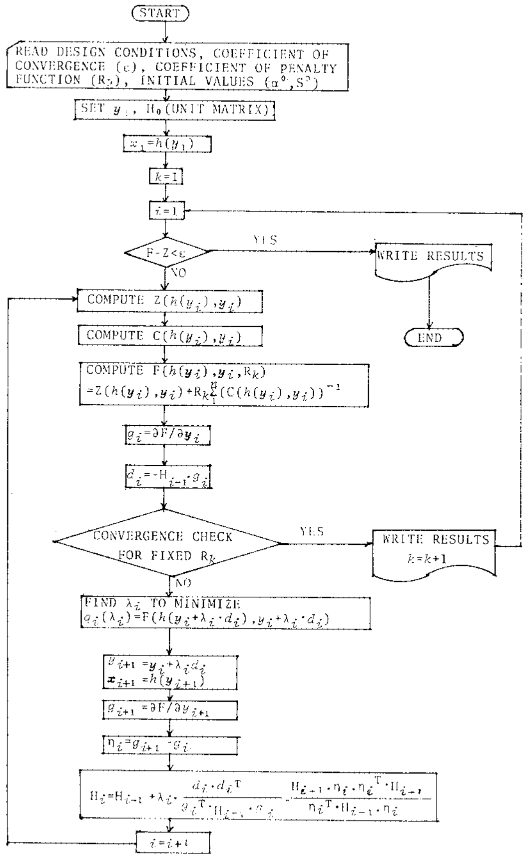


Fig. 1 Flow chart for optimum design of truss.

the objective function, and the optimum values of $\boldsymbol{y}$ are calculated by the SUMT method for the optimum design. Unconstrained minimization is made by a method suggested by Daividon-Fletcher-Powell[7),8)], which is shown by a flow chart in Fig. 1, in which the term of $g=\partial F/\partial \boldsymbol{y}$ is a gradient of the penalty function, F, with respect to $\boldsymbol{y}$.

Now, a structure with two analytical member elements, marked by I and II, is treated. The variables $\boldsymbol{x}$ belonging to I or II are called $\boldsymbol{x}_{\mathrm{I}}$ or $\boldsymbol{x}_{\mathrm{II}}$, respectively, and the constraints belonging to I or II are given as follows with $g_{i\mathrm{I}}$ or $g_{i\mathrm{II}}$:

$$g_{i\mathrm{I}}(\boldsymbol{x}_{\mathrm{I}}, \boldsymbol{x}_{\mathrm{II}}, \boldsymbol{y})=0 \quad (i_{\mathrm{I}}=1, \cdots, m_{\mathrm{I}}) \ , \quad \cdots\cdots(2\cdot\mathrm{a})$$

$$g_{i\mathrm{II}}(\boldsymbol{x}_{\mathrm{I}}, \boldsymbol{x}_{\mathrm{II}}, \boldsymbol{y})=0 \quad (i_{\mathrm{II}}=1, \cdots, m_{\mathrm{II}}) \ . \quad \cdots\cdots(2\cdot\mathrm{b})$$

The expressions of functions of only $\boldsymbol{y}$ are given by

$$g_i(\boldsymbol{y})=0 \quad (i=1, \cdots, m) \ , \quad \cdots\cdots(2\cdot\mathrm{c})$$

and the other constraint conditions are given by

$$\boldsymbol{x}\geqq 0 \ , \quad \cdots\cdots(2\cdot\mathrm{d})$$

$$\boldsymbol{y}\geqq 0 \ . \quad \cdots\cdots(2\cdot\mathrm{e})$$

In general, a problem of an optimum design is expressed by the following equation:

$$Z=f(\boldsymbol{x}_{\mathrm{I}}, \boldsymbol{x}_{\mathrm{II}}, \boldsymbol{y}) \quad \text{minimize,} \quad \cdots\cdots(3)$$

under the constraints given by Eqs. (2·a), (2·b), (2·c), (2·d) and (2·e). In Eqs. (2·a) and (2·b), as far as the objective function and constraints are concerned, the variation of sectional dimensions belonging to individual members has no effect on those belonging to the other elements. For the objective function and constraints, the groups of the variables $\boldsymbol{x}_{\mathrm{I}}$ and $\boldsymbol{x}_{\mathrm{II}}$ belonging to the element I and II, respectively, are independent of each other. In the case of a statically determinate structure, the change of its sectional dimensions does not have an effect on the distribution of the internal stresses. Then, for this case, too, as to the objective function and constraints, the groups of the variables $\boldsymbol{x}_{\mathrm{I}}$ and $\boldsymbol{x}_{\mathrm{II}}$ are independent of each other. That is, Eq. (2·a) is a function of only $\boldsymbol{x}_{\mathrm{I}}$ and $\boldsymbol{y}$. Similarly, Eq. (2·b) is of only $\boldsymbol{x}_{\mathrm{II}}$ and $\boldsymbol{y}$. Therefore, the constraints are given by:

$$g_{i\mathrm{I}}(\boldsymbol{x}_{\mathrm{I}}, \boldsymbol{y})\leqq 0 \quad (i=1, \cdots, m_{\mathrm{I}}) \ , \quad \cdots\cdots(4\cdot\mathrm{a})$$

$$g_{i\mathrm{II}}(\boldsymbol{x}_{\mathrm{II}}, \boldsymbol{y})\leqq 0 \quad (i=1, \cdots, m_{\mathrm{II}}) \ . \quad \cdots\cdots(4\cdot\mathrm{b})$$

And the objective function Z is expressed by

$$Z=Z_{\mathrm{I}}+Z_{\mathrm{II}}=f_{\mathrm{I}}(\boldsymbol{x}_{\mathrm{I}}, \boldsymbol{y})+f_{\mathrm{II}}(\boldsymbol{x}_{\mathrm{II}}, \boldsymbol{y}) \quad \text{minimize,} \quad \cdots\cdots(5)$$

where

$$\left.\begin{array}{l} Z_{\mathrm{I}}=f_{\mathrm{I}}(\boldsymbol{x}_{\mathrm{I}}, \boldsymbol{y}) \\ \text{and} \\ Z_{\mathrm{II}}=f_{\mathrm{II}}(\boldsymbol{x}_{\mathrm{II}}, \boldsymbol{y}) \end{array}\right\} \quad \cdots\cdots(5\cdot\mathrm{a})$$

show the objective functions of the element I, II, respectively.

In case of a statically indeterminate structure, the stress constraints are given by:

$$\sigma_J-\sigma_{aJ}=0 \quad (J=\mathrm{I}, \mathrm{II}) \ , \quad \cdots\cdots(6)$$

$$\sigma_J=S_J/A_J \ , \quad \cdots\cdots(7)$$

$$\boldsymbol{S}=[\boldsymbol{I}-\bar{\boldsymbol{S}}(\bar{\boldsymbol{S}}^T\boldsymbol{\rho}\bar{\boldsymbol{S}})^{-1}(\bar{\boldsymbol{S}}^T\boldsymbol{\rho})]\boldsymbol{S}_0 \ , \quad \cdots\cdots(8)$$

where

- σ_J: the normal component of working stress in the J-th member element,
- σ_{aJ}: the normal component of allowable stress in the J-th member element,
- A_J: the cross sectional area of the J-th member element,
- $\boldsymbol{S}$: the vector of normal force of the statically indeterminate structure with the element S_J due to any loads,
- $\boldsymbol{S}_0$: the vector of normal force of the statically determinate structure with the element S_{0J} due to any loads,

$\bar{S}$: the matrix of normal force of the statically determinate structure due to the statically indeterminate force $X_i=1$,

X: the vector of the statically indeterminate force with the element S_i,

$\boldsymbol{\rho}$: the matrix of flexibility with the element $\rho_J=l_J/EA_J$.

Since the value of A_J is obtained by a fully stressed design method, A_J may be regarded as a constant value. If a truss height is constant, the member length l_J and the stress of J-th member $\bar{S}_J$ also have a constant values. By Eq. (8), $A_{\rm I}$ and $A_{\rm II}$ are independent of each other for S. Therefore, it can be approximately regarded that the change of the sectional areas $A_{\rm I}$ and $A_{\rm II}$ does not have an effect on the distribution of internal stresses, and Eq. (6) is a function of $\boldsymbol{x}_{\rm I}$ and $\boldsymbol{y}$. Then, Eqs. (4·a) and (4·b) will be obtained. When the non-negative value ξ_i^2 is added to the left side of the constraint $g_i(\boldsymbol{x}, \boldsymbol{y})\leqq 0$, an equality is given as expressed by the following equation:

$$g_i(\boldsymbol{x}, \boldsymbol{y})+\xi_i^2=0 \quad (i=1, \cdots, m_p) , \qquad \cdots\cdots(9)$$

where

$$m_p=m_{\rm I}+m_{\rm II}+m . \qquad \cdots\cdots(9\cdot a)$$

A vector with ξ_i^2 for the i-th element is represented by Ξ, and then the Eqs. (4·a), (4·b) and (4·c) are expressed, respectively, as follows:

$$g_{i_{\rm I}}(\boldsymbol{x}_{\rm I}, \boldsymbol{y})+\Xi_{i_{\rm I}}=0 \quad (i_{\rm I}=1, \cdots, m) , \qquad \cdots\cdots(10\cdot a)$$

$$g_{i_{\rm II}}(\boldsymbol{x}_{\rm II}, \boldsymbol{y})+\Xi_{i_{\rm II}}=0 \quad (i_{\rm II}=1, \cdots, m_{\rm II}) , \qquad \cdots\cdots(10\cdot b)$$

and

$$g_i(\boldsymbol{y})+\Xi_i=0 \quad (i=1, \cdots, m) . \qquad \cdots\cdots(10\cdot c)$$

The optimum values of $\boldsymbol{x}_{\rm I}$ and $\Xi_{\rm I}$ are obtained using the SLP method by minimizing the function $Z_{\rm I}=f_{\rm I}(\boldsymbol{x}_{\rm I}, \boldsymbol{y})$ under the constraints of Eq. (10·a) for any constant value of $\boldsymbol{y}$. The values of $m_{\rm I}$ variables within $\boldsymbol{x}_{\rm I}$ and $\Xi_{\rm I}$ take positive values, and $m_{\rm I}$ is equal to the number of Eq. (10·a). The values of the variables $\boldsymbol{x}$ and Ξ, with the same number as the number of the variables $\boldsymbol{x}_{\rm I}$ take zero. Here, $\boldsymbol{x}_{\rm I}^0$ and $\Xi_{\rm I}^0$ are defined as the optimum values of the variables $\boldsymbol{x}_{\rm I}$ and $\Xi_{\rm I}$. Similarly, the optimum values $\boldsymbol{x}_{\rm II}^0$ and $\Xi_{\rm II}^0$ of $\boldsymbol{x}_{\rm II}$ and $\Xi_{\rm II}$, respectively, are obtained by minimizing the function $Z_{\rm II}=f_{\rm II}(\boldsymbol{x}_{\rm II}, \boldsymbol{y})$ under the constraints of Eq. (10·b).

From Eqs. (10·a), (10·b), (10·c) and (5·a), the Lagrange's functions are expressed as follows:

$$\phi_{\rm I}(\boldsymbol{x}_{\rm I}, \boldsymbol{y}, \boldsymbol{\xi}_{\rm I}, \boldsymbol{\lambda}_{\rm I})=f_{\rm I}(\boldsymbol{x}_{\rm I}, \boldsymbol{y}) +\boldsymbol{\lambda}_{\rm I}^T(g_{\rm I}(\boldsymbol{x}_{\rm I}, \boldsymbol{y})+\Xi_{\rm I}) , \qquad \cdots\cdots(11\cdot a)$$

$$\phi_{\rm II}(\boldsymbol{x}_{\rm II}, \boldsymbol{y}, \boldsymbol{\xi}_{\rm II}, \boldsymbol{\lambda}_{\rm II})=f_{\rm II}(\boldsymbol{x}_{\rm II}, \boldsymbol{y}) +\boldsymbol{\lambda}_{\rm II}^T(g_{\rm II}(\boldsymbol{x}_{\rm II}, \boldsymbol{y})+\Xi_{\rm II}) . \qquad \cdots\cdots(11\cdot b)$$

When these functions are differentiated with respect to $\boldsymbol{x}$, $\boldsymbol{\xi}$ and $\boldsymbol{\lambda}$, and then, the optimum values $\boldsymbol{x}^0$, $\boldsymbol{\xi}^0$ and $\boldsymbol{\lambda}^0$ are substituted for the differentiated functions, the following equations are given for any values of $\boldsymbol{y}$,

$$\left.\begin{aligned} &\partial\phi_{\rm I}/\partial\boldsymbol{x}_{\rm I}=0, \quad \partial\phi_{\rm I}/\partial\boldsymbol{\xi}_{\rm I}=0, \quad \partial\phi_{\rm I}/\partial\boldsymbol{\lambda}_{\rm I}=0, \\ &\partial\phi_{\rm II}/\partial\boldsymbol{x}_{\rm II}=0, \quad \partial\phi_{\rm II}/\partial\boldsymbol{\xi}_{\rm II}=0, \quad \partial\phi_{\rm II}/\partial\boldsymbol{\lambda}_{\rm II}=0 . \end{aligned}\right\} \qquad \cdots\cdots(12)$$

Therefore, the following relations are obtained:

$$\boldsymbol{x}_{\rm I}=h_{\rm I}(\boldsymbol{y}), \quad \boldsymbol{\xi}_{\rm I}=\eta_{\rm I}(\boldsymbol{y}), \quad \boldsymbol{\lambda}_{\rm I}=\zeta_{\rm I}(\boldsymbol{y}), \qquad \cdots\cdots(1\cdot a)$$

$$\boldsymbol{x}_{\rm II}=h_{\rm II}(\boldsymbol{y}), \quad \boldsymbol{\xi}_{\rm II}=\eta_{\rm II}(\boldsymbol{y}), \quad \boldsymbol{\lambda}_{\rm II}=\zeta_{\rm II}(\boldsymbol{y}). \qquad \cdots(1\cdot b)$$

Then, by substituting Eqs. (1·a) and (1·b) into Eq. (3) the following expression is obtained:

$$Z=f_{\rm I}(h_{\rm I}(\boldsymbol{y}), \boldsymbol{y})+f_{\rm II}(h_{\rm II}(\boldsymbol{y}), \boldsymbol{y}) . \qquad \cdots\cdots(13)$$

The constraints become only Eqs. (2·c) and (2·e). It is then, necessary to obtain the optimum solutions of the variables $\boldsymbol{y}$ by minimizing Eq. (13) under the constraints of Eqs. (2·c) and (2·e). So, the optimum values $\boldsymbol{y}^0$ of the variables $\boldsymbol{y}$ are obtained by the SUMT method.

It can be proved as follows whether the values $\boldsymbol{x}_{\rm I}^0$, $\boldsymbol{x}_{\rm II}^0$ and $\boldsymbol{y}^0$ determined by the above-mentioned method give an optimum solution or not. In the same manner as Eq. (9), the expressions of (14) are given by Eqs. (2·a), (2·b) and (2·c):

$$\left.\begin{aligned} &g_{i_{\rm I}}(\boldsymbol{x}_{\rm I}, \boldsymbol{x}_{\rm II}, \boldsymbol{y})+\xi_{i_{\rm I}}^2=0 \quad (i=1, \cdots, m_{\rm I}) , \\ &g_{i_{\rm II}}(\boldsymbol{x}_{\rm I}, \boldsymbol{x}_{\rm II}, \boldsymbol{y})+\xi_{i_{\rm II}}^2=0 \quad (i=1, \cdots, m_{\rm II}) , \\ &g_i(\boldsymbol{y})+\xi_{iY}^2=0 \quad (i=1, \cdots, m) . \end{aligned}\right\} \qquad \cdots\cdots(14)$$

Then, the Lagrange's function is expressed by Eq. (14) as follows:

$$\phi(\boldsymbol{x}_{\rm I}, \boldsymbol{x}_{\rm II}, \boldsymbol{y}, \boldsymbol{\xi}_{\rm I}, \boldsymbol{\xi}_{\rm II}, \boldsymbol{\xi}_Y, \boldsymbol{\lambda}_{\rm I}, \boldsymbol{\lambda}_{\rm II}, \boldsymbol{\lambda}_Y) =f(\boldsymbol{x}_{\rm I}, \boldsymbol{x}_{\rm II}, \boldsymbol{y})+\boldsymbol{\lambda}^T g(\boldsymbol{x}_{\rm I}, \boldsymbol{x}_{\rm II}, \boldsymbol{y})+\Xi) , \qquad \cdots\cdots(15)$$

where

$$(\boldsymbol{\lambda}=(\lambda_{\rm I1}\cdots\lambda_{{\rm I}m_{\rm I}},\ \lambda_{\rm II1}\cdots\lambda_{{\rm II}m_{\rm II}},\ \lambda_{Y1}\cdots\lambda_{Ym})^T) .$$

Then, Eq. (15) is rewritten by Eqs. (4) and (5) as follows:

$$\begin{aligned} &\phi(\boldsymbol{x}_{\rm I}, \boldsymbol{x}_{\rm II}, \boldsymbol{y}, \boldsymbol{\xi}_{\rm I}, \boldsymbol{\xi}_{\rm II}, \boldsymbol{\xi}_Y, \boldsymbol{\lambda}_{\rm I}, \boldsymbol{\lambda}_{\rm II}, \boldsymbol{\lambda}_Y) \\ &\quad =f_{\rm I}(\boldsymbol{x}_{\rm I}, \boldsymbol{y})+f_{\rm II}(\boldsymbol{x}_{\rm II}, \boldsymbol{y}) \\ &\qquad +\boldsymbol{\lambda}_{\rm I}^T(g_{\rm I}(\boldsymbol{x}_{\rm I}, \boldsymbol{y})+\Xi_{\rm I}) \\ &\qquad +\boldsymbol{\lambda}_{\rm II}^T(g_{\rm II}(\boldsymbol{x}_{\rm II}, \boldsymbol{y})+\Xi_{\rm II})+\boldsymbol{\lambda}_Y^T(g(\boldsymbol{y})+\Xi_Y) . \end{aligned} \qquad \cdots\cdots(16)$$

Assuming that $\boldsymbol{y}$ is constant and differentiating Eq. (16) with respect to $\boldsymbol{x}_{\rm I}$, $\boldsymbol{x}_{\rm II}$, $\boldsymbol{\xi}_{\rm I}$, $\boldsymbol{\xi}_{\rm II}$, $\boldsymbol{\lambda}_{\rm I}$ and $\boldsymbol{\lambda}_{\rm II}$ the following expressions are obtained:

$$\left.\begin{aligned}\partial\phi/\partial\boldsymbol{x}_{\mathrm{I}}&=\partial f_{\mathrm{I}}(\boldsymbol{x}_{\mathrm{I}},\boldsymbol{y})/\partial\boldsymbol{x}_{\mathrm{I}}\\&\quad+\boldsymbol{\lambda}_{\mathrm{I}}^{T}(\partial g_{\mathrm{I}}(\boldsymbol{x}_{\mathrm{I}},\boldsymbol{y})/\partial x_{\mathrm{I}})\,,\\\partial\phi/\partial\boldsymbol{\lambda}_{\mathrm{I}}&=g_{\mathrm{I}}(\boldsymbol{x}_{\mathrm{I}},\boldsymbol{y})+\Xi_{\mathrm{I}}\,,\\\partial\phi/\partial\boldsymbol{\xi}_{\mathrm{I}}&=2\boldsymbol{\lambda}_{\mathrm{I}}\begin{bmatrix}\xi_{\mathrm{I}1}&&0\\&\ddots&\\0&&\xi_{\mathrm{I}m_{\mathrm{I}}}\end{bmatrix}\,,\end{aligned}\right\}\quad\cdots\cdots(17\cdot\mathrm{a})$$

$$\left.\begin{aligned}\partial\phi/\partial\boldsymbol{x}_{\mathrm{II}}&=\partial f_{\mathrm{II}}(\boldsymbol{x}_{\mathrm{II}},\boldsymbol{y})/\partial\boldsymbol{x}_{\mathrm{II}}\\&\quad+\boldsymbol{\lambda}_{\mathrm{II}}^{T}(\partial g_{\mathrm{II}}(\boldsymbol{x}_{\mathrm{II}},\boldsymbol{y})/\partial\boldsymbol{x}_{\mathrm{II}})\,,\\\partial\phi/\partial\boldsymbol{\lambda}_{\mathrm{II}}&=g_{\mathrm{II}}(\boldsymbol{x}_{\mathrm{II}},\boldsymbol{y})+\Xi_{\mathrm{II}}\,,\\\partial\phi/\partial\boldsymbol{\xi}_{\mathrm{II}}&=2\boldsymbol{\lambda}_{\mathrm{II}}\begin{bmatrix}\xi_{\mathrm{II}1}&&0\\&\ddots&\\0&&\xi_{\mathrm{II}m_{\mathrm{II}}}\end{bmatrix}\,.\end{aligned}\right\}\quad\cdots(17\cdot\mathrm{b})$$

Then, the following expressions are obtained by substituting the optimum values $\boldsymbol{x}_{\mathrm{I}}^{0}$, $\boldsymbol{x}_{\mathrm{II}}^{0}$, $\boldsymbol{\xi}_{\mathrm{I}}^{0}$, $\boldsymbol{\xi}_{\mathrm{II}}^{0}$, $\boldsymbol{\lambda}_{\mathrm{I}}^{0}$ and $\boldsymbol{\lambda}_{\mathrm{II}}^{0}$ under $\boldsymbol{y}=$const.:

$$\left.\begin{aligned}&\partial\phi/\partial\boldsymbol{x}_{\mathrm{I}}=0,\quad\partial\phi/\partial\boldsymbol{\lambda}_{\mathrm{I}}=0,\quad\partial\phi/\partial\boldsymbol{\xi}_{\mathrm{I}}=0,\\&\partial\phi/\partial\boldsymbol{x}_{\mathrm{II}}=0,\quad\partial\phi/\partial\boldsymbol{\lambda}_{\mathrm{II}}=0,\quad\partial\phi/\partial\boldsymbol{\xi}_{\mathrm{II}}=0.\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots(18)$$

Then, the following expression is obtained by substituting Eqs. (1·a), (1·b) and (18) into Eq. (16):

$$\begin{aligned}&\phi(\boldsymbol{x}_{\mathrm{I}},\boldsymbol{x}_{\mathrm{II}},\boldsymbol{y},\boldsymbol{\xi}_{\mathrm{I}},\boldsymbol{\xi}_{\mathrm{II}},\boldsymbol{\xi}_{Y},\boldsymbol{\lambda}_{\mathrm{I}},\boldsymbol{\lambda}_{\mathrm{II}},\boldsymbol{\lambda}_{Y})\\&\quad=f_{\mathrm{I}}(h_{\mathrm{I}}(\boldsymbol{y}),\boldsymbol{y})+f_{\mathrm{II}}(h_{\mathrm{II}}(\boldsymbol{y}),\boldsymbol{y})\\&\qquad+\boldsymbol{\lambda}_{\mathrm{I}}^{0T}(g_{\mathrm{I}}(\boldsymbol{x}_{\mathrm{I}}^{0},\boldsymbol{y})+(\xi_{\mathrm{I}1}^{02},\cdots,\xi_{\mathrm{I}m_{\mathrm{I}}}^{02})^{T})\\&\qquad+\boldsymbol{\lambda}_{\mathrm{II}}^{0T}(g_{\mathrm{II}}(\boldsymbol{x}_{\mathrm{II}}^{0},\boldsymbol{y})+(\xi_{\mathrm{II}1}^{02},\cdots,\xi_{\mathrm{II}m_{\mathrm{II}}}^{02})^{T})\\&\qquad+\boldsymbol{\lambda}_{Y}^{T}(g(\boldsymbol{y})+\Xi_{Y})\\&\quad=f_{\mathrm{I}}(h_{\mathrm{I}}(\boldsymbol{y}),\boldsymbol{y})+f_{\mathrm{II}}(h_{\mathrm{II}}(\boldsymbol{y}),\boldsymbol{y})\\&\qquad+\boldsymbol{\lambda}_{Y}^{T}(g(\boldsymbol{y})+\Xi_{Y})\,.\quad\cdots\cdots\cdots\cdots(19)\end{aligned}$$

Since $\boldsymbol{y}^{0}$ is the optimum values of $\boldsymbol{y}$ and satisfies Eq. (5) under the constraints of Eqs. (2·c), (2·d) and (2·e), the following relations are obtained:

$$\left.\begin{aligned}&\partial\phi/\partial\boldsymbol{y}=\partial f(\boldsymbol{y}^{0})/\partial y+\boldsymbol{\lambda}_{Y}^{0T}(\partial g(\boldsymbol{y}^{0})/\partial\boldsymbol{y})=0\,,\\&\partial\phi/\partial\boldsymbol{\lambda}_{Y}=g(\boldsymbol{y}^{0})+\Xi_{Y}^{0}=0\,,\\&\partial\phi/\partial\boldsymbol{\xi}_{Y}=2\boldsymbol{\lambda}_{Y}^{0T}\begin{bmatrix}\xi_{Y1}^{0}&&0\\&\ddots&\\0&&\xi_{Ym}^{0}\end{bmatrix}=0\,.\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots(20)$$

Then, $\boldsymbol{x}_{\mathrm{I}}^{0}$, $\boldsymbol{x}_{\mathrm{II}}^{0}$ and $\boldsymbol{y}^{0}$ can be optimum values.

In general, the Lagrange's function is expressed as follows:

$$\phi(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda})=f(\boldsymbol{x},\boldsymbol{y})+\boldsymbol{\lambda}^{T}\cdot g(\boldsymbol{x},\boldsymbol{y})\,.\quad\cdots\cdots\cdots(21)$$

According to Kuhn-Tucker's theorem, a necessary and sufficient condition for minimizing the objective function Eq. (5) under the constraints of Eqs. (2·a), (2·b), (2·c), (2·d) and (2·e) at the point $\boldsymbol{x}^{0}$ and $\boldsymbol{y}^{0}$, is that the vector $\boldsymbol{\lambda}^{0}$ can be found, which satisfies the following unequality:

$$\left.\begin{aligned}&\boldsymbol{x}^{0}\geqq0\,,\\&\boldsymbol{y}^{0}\geqq0\,,\\&\boldsymbol{\lambda}^{0}\geqq0\,,\\&\phi(\boldsymbol{x}^{0},\boldsymbol{y}^{0},\boldsymbol{\lambda})\leqq\phi(\boldsymbol{x}^{0},\boldsymbol{y}^{0},\boldsymbol{\lambda}^{0})\leqq\phi(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda}^{0})\,.\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots(22)$$

3. EXAMPLES OF OPTIMUM DESIGN OF TRUSSES USING SUBOPTIMIZATION

(1) Example I: Two-panel truss

Example I is of an optimum design using a method proposed in the present study, the optimum design of a two-panel truss, as shown in **Fig. 2**, is carried out. In this figure, ①, ②, ③ and ④ show the number of members, and (1), (2) and (3) in **Fig. 2** show the classification of member sections. In this case a load of $P=$ 2 000 tons is applied to the panel point of the center span.

a) Design variables

Among design variables, the grade of material S, the upper flange plate and web plate thickness T_u, T_w, respectively, of the classification of the member section (1), the web plate thickness T_w of the classification (2) and the flange and web plate thickness T_u, T_w, respectively, and the flange width B_u of the classification (3), are related to $\boldsymbol{x}$, and the truss height H and the truss chord width B are related to $\boldsymbol{y}$. Concerning S, the steel of 41 kg/mm² in tensile strength is expressed by 4, the steel of 50 kg/mm² is expressed by 5 and 58 kg/mm² by 6. The following equation can be used to calculate T of the classification of the member section (1):

$$T_l=T_u\times(B+8)/B\,,\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(23)$$

and to calculate T_u and T_l of the classification of the member section (2)

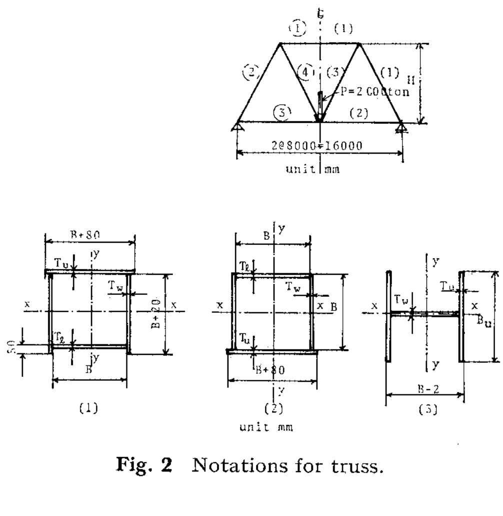


Fig. 2 Notations for truss.

$$T_u = T_w \times (B+4)/(B+8)\ , \quad \cdots\cdots(24\cdot a)$$

and

$$T_l = T_w \times (B+4)/B\ . \quad \cdots\cdots(24\cdot b)$$

b) Constraints

In the case of suboptimization, the constraints contain the limit of stress, the limit of deflection, the upper and lower limits of values of the design variables at the classifications of the member section, the limit of ratio of plate width to thickness for prevention from local buckling, the limit of slenderness ratio of $l/r_y \leqq l/r_x$ where r_x and r_y are the radius of gyration about the $\boldsymbol{x}$ and $\boldsymbol{y}$ axis, respectively, and $l/r \leqq 120$ at the section (1); the limit of ratio of plate width to thickness below 80, the limit of slenderness ratio of $l/r \leqq 200$ at the section (2); the limit of ratio of plate width to thickness of $B_u/T_u \leqq 32$ and $B_w/T_w \leqq 80$, the limit of slenderness ratio of $l/r \leqq 200$ at the section (3). In the case of overall optimization, the constraints are only the upper and lower limits of the variables H and B.

The allowable stresses are a function of the discrete variable S, but it is considered to be a continous function as shown in the following equations:

$$\left.\begin{aligned} &\mathrm{FG1} = 2S^2 - 23S + 80\ , \\ &\mathrm{FG2} = -13S + 145\ , \\ &\sigma_{ua2} = 1.7S^2 - 10.7S + 24\ , \\ &\sigma_{ua3} = 150S^2 - 3\,050S + 16\,500\ , \end{aligned}\right\} \quad \cdots\cdots(25)$$

and

$$\sigma_{ta} = 100S^2 - 400S + 1\,400\ . \quad \cdots\cdots(26)$$

The compressive allowable stress without considering local buckling, σ_{cag}, is obtained from the following equations:

$$\left.\begin{aligned} &l/r \leqq \mathrm{FG1}: \quad \sigma_{cag} = \sigma_{ta}\ , \\ &\mathrm{FG1} < l/r \leqq \mathrm{FG2}: \quad \sigma_{cag} = \sigma_{ta} - \sigma_{ua2}(l/r - \mathrm{FG1})\ , \\ &\mathrm{FG2} < l/r: \quad \sigma_{cag} = 12\,000\,000/\{\sigma_{ua3} + (l/r)^2\}\ . \end{aligned}\right\} \quad \cdots\cdots(27)$$

The compressive allowable stress against local buckling σ_{cal} is obtained from the following equations:

$$\gamma_d = 0.35S^2 - 8.75S + 69.0\ , \quad \cdots\cdots(25\cdot a)$$

$$\left.\begin{aligned} &c_d \leqq \gamma_d: \quad \sigma_{cal} = \sigma_{ta}\ , \\ &c_d > \gamma_d: \quad \sigma_{cal} = 2\,200\,000(1/c_d)^2\ , \end{aligned}\right\} \quad \cdots\cdots(28)$$

where

$c_d = B/T$

B: the width of a plate,

T: the thickness of a plate.

Then, the compressive allowable stress σ_{ca} is obtained from the following equation:

$$\sigma_{ca} = \sigma_{cag} \times \sigma_{cal}/\sigma_{ta}\ . \quad \cdots\cdots(29)$$

The tensile allowable stress σ_{ta} is obtained from Eq. (26). The ratio, γ, of plate width to thickness for prevention from buckling of compression members is expressed as a function of S as follows:

$$\gamma = -8S + 88.0\ . \quad \cdots\cdots(30)$$

A limit of the ratio of plate width to thickness is set up for tension members as 80, and the ratio of width to thickness of a free outstanding leg is limited below 16.

c) Objective function

The objective function Z consists of material and fabrication costs, and is expressed by

$$\begin{aligned} Z &= \sum_k \sum_l \tilde{H}_{kl} \cdot (\mathrm{SMH}) + \sum_i \sum_j H_{ij} \cdot (\mathrm{SMH}) \\ &\quad + \sum_j \rho V_j \cdot C \cdot (\mathrm{CM}) \\ &= Z_1(\mathrm{SMH}) + Z_2(\mathrm{SMH}) + Z_3(\mathrm{CM}) \\ &= (\mathrm{CM}) \times (Z_1\mu + Z_2\mu + Z_3)\ , \end{aligned} \quad \cdots\cdots(31)$$

where

ρ: the unit weight of steel material,

C: the coefficient for unit cost of the steel material,

(CM): the unit cost of the steel material,

(SMH): the unit cost for one man hour,

H_{ij}: the man hour of the i-th manufacturing operation of the j-th element which is a function of the design variables,

$\tilde{H}_{kl}$: the man hour of the k-th manufacturing operation of the l-th element which is a fixed value,

$\mu = (\mathrm{SMH})/(\mathrm{CM})$.

C is considered a function of T (thickness of plate) and S, and is expressed in the form of an equation with C_1 and C_2. C_1 in C which is a function of S, according to the "Prime Costs of Steel Highway Bridges in Japan in 1979",[9] is expressed as follows:

$$C_1 = 0.125S^2 - 0.955S + 2.820\ , \quad \cdots\cdots(32)$$

and C_2 which is a function of T is expressed as follows:

$$C_2 = 0.034\,8T^2 - 0.084\,5T + 1.209\,1\ . \quad \cdots\cdots(33)$$

Then, the following expression is obtained:

$$C = C_1 \times C_2\ . \quad \cdots\cdots(34)$$

Z_1, Z_2 and Z_3 are divided by CM to be dimensionless.

Then, it can be considered that only μ is related to costs. H_{ij} is obtained from the "Prime Costs of Steel Highway Bridges in Japan in 1972"[10] which were determined by the method of least squares from the latest actual examples at bridge fabricating shops in Japan, and can be expressed as follows:

$$\left.\begin{aligned} &H_{1j}=3W_t\cdot HA(S)/T\ ,\\ &H_{2j}=0.055N_B\cdot HA(S)\\ &H_{3j}=3.5\times\{1.0+(0.01B-0.5)\}\times HA(S)\\ &\quad\left(\begin{array}{l}\text{if } B \text{ is below 50 cm,}\\ B \text{ is equal to 50 cm}\end{array}\right),\\ &H_{4j}=0.58L_1\cdot HA(S)\ ,\\ &H_{5j}=0.58L_2\cdot HA(S)\ ,\\ &H_{6j}=0.28A_r\cdot HA(S)\ ,\end{aligned}\right\} \quad\cdots\cdots(35)$$

where the suffix i of H_{ij} shows marking-off, hole boring, cutting of member edges, shop butt welding, shop fillet welding and shop painting, and W_t=the weight of members, N_B=the number of holes, B=the plate width, L_1=the total welded length of butt welds equivalent to 6 mm fillet, L_2=the same length as L_1 of fillet welds, $\bar{L}_1$=the total actual welded length of butt welds, $\bar{L}_2$=the total actual welded length of fillet welds, A_r=the surface area, $HA(S)$=the coefficient of man hours depending on S, and can be expressed by

$$HA(S)=0.085S^2-0.725S+2.54\ . \quad\cdots\cdots(36)$$

L_1 and L_2, assumed as a function of T, are calculated by the following equations:

$$\left.\begin{aligned} &L_1=\bar{L}_1\times\eta_1(T)\ ,\\ &L_2=\bar{L}_2\times\eta_2(T)\ ,\end{aligned}\right\} \quad\cdots\cdots(37)$$

$$\left.\begin{aligned} &\eta_1(T)=1.2T^2+3.8T+1.3\ ,\\ &\eta_2(T)=0.0476T^2+0.1952T+0.7572\ .\end{aligned}\right\} \quad\cdots\cdots(38)$$

N_B is calculated by the following equation:

$$N_B=A_g\times\sigma_a/\rho_s\ , \quad\cdots\cdots(39)$$

where

A_g: the sectional area of a member,
σ_a: the allowable stress,
ρ_s: the yield strength of a high tensile bolt depending on S, which can be calculated by the following equation:

$$\rho_s=-335S^2+3955S-6580\ . \quad\cdots\cdots(40)$$

$\tilde{H}_{kl}$ is calculated by the following expression:

$$\tilde{H}_{kl}=7.0A_{l2}+56+35/NP\ , \quad\cdots\cdots(41)$$

where

NP: the number of panels.

A_{l2} is calculated by the following expression:

$$A_{l2}=2.5+0.25A_{l1}\ . \quad\cdots\cdots(41\cdot a)$$

and

$$A_{l1}=0.5l-15.0\ (\mathrm{m})\ , \quad\cdots\cdots(41\cdot b)$$

$$A_{l1}\geqq 0\ , \quad\cdots\cdots(41\cdot c)$$

where

l: the length of a member (m).

Then, the optimum values of the variables $\boldsymbol{x}$ are obtained with a fixed value of $\boldsymbol{y}$ for each element, by solving the objective function under the above-mentioned constraints using the SLP method.

d) Overall optimization

Among the design variables, H and B are related to $\boldsymbol{y}$. Since Eq. (1) which is obtained by suboptimization, satisfies all of the constraints except Eq. (2·c), it would be enough if only the constraints could be satisfied Eq. (2·c) in overall optimization. The above-mentioned SUMT approach will be applied to the method of overall optimization.

As S is a discrete variable, the integral value is obtained by the branch and prune method.

e) Results of calculation and discussions

The optimum values of the variables, penalty function and objective function are given as shown in **Table 1** from the results of the optimum design by the present method. The penalty function, F, is shown by the following equation:

$$F(\boldsymbol{y},R_k)=Z(\boldsymbol{y})+R_k\sum_i\{g_i(Y)^{-1}\}$$
$$(i=1,\cdots,m_p) \quad\cdots\cdots(42)$$

where

$F(\boldsymbol{y},R_k)$: the penalty function,

Table 1 State of convergence for $L=16$ m.

K	IS	H (cm)	B (cm)	S_1	S_2	S_3	S_4	T_{u1} (cm)	T_{u2} (cm)	T_{u4} (cm)	T_{w1} (cm)	T_{w2} (cm)	T_{w3} (cm)	T_{w4} (cm)	B_u (cm)	F (1 000 yen)	Z (1 000 yen)	R_k
1	0	727.3	60.0	5.0	6.0	5.0	5.0	2.40	1.93	2.74	2.64	1.95	1.17	2.74	87.6	3 892.2	3 892.1	1.00
1	1	725.0	58.4	6.0	6.0	5.0	5.0	1.89	1.89	2.75	1.91	2.10	1.20	2.75	88.0	3 866.7	3 866.6	1.00
1	2	702.7	58.2	6.0	6.0	5.0	5.0	1.87	1.89	2.76	2.08	2.12	1.24	2.76	88.4	3 863.3	3 863.2	1.00
2	0	702.7	58.2	6.0	6.0	5.0	5.0	1.87	1.89	2.76	2.08	2.12	1.24	2.76	88.4	3 863.3	3 863.2	0.02
2	1	658.8	73.4	5.0	5.0	5.0	5.0	2.07	2.08	2.68	2.36	2.14	1.06	2.68	85.8	3 835.8	3 835.8	0.02
2	2	661.7	73.5	5.0	5.0	5.0	5.0	2.08	2.09	2.68	2.32	2.12	1.05	2.68	85.8	3 834.9	3 834.9	0.02

Table 2 Comparison of optimum values for span length of 16 m.

Case	H^0 (cm)	H (cm)	B (cm)	S_1	S_2	S_3	S_4	T_{u1} (cm)	T_{u2} (cm)	T_{u4} (cm)	T_{w1} (cm)	T_{w2} (cm)	T_{w3} (cm)	T_{w4} (cm)	B_u (cm)	F (1000yen)	Z (1000yen)
1	727.3	661.7	73.5	5.0	5.0	5.0	5.0	2.08	2.09	2.68	2.32	2.12	1.05	2.68	85.8	3 834.9	3 834.9
2	640.0	672.0	73.6	5.0	5.0	5.0	5.0	2.08	2.09	2.67	2.24	2.11	1.04	2.67	85.5	3 833.8	3 833.8
3	888.9	688.4	73.6	5.0	5.0	5.0	5.0	2.08	2.08	2.66	2.13	2.10	1.01	2.66	85.2	3 833.4	3 833.4

$Z(\boldsymbol{y})$: the objective function,
R_k: the coefficient of a penalty term,
$\boldsymbol{y}$: the variables of a section common to the whole structure.

In **Table 1**,
K: the number of repeat for a variable R_k,
IS: the number of repeat for a fixed R_k.

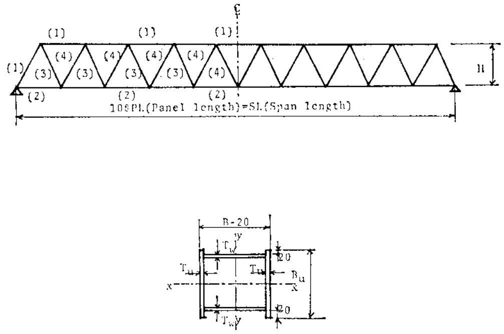
The suffix of the variables shows the number of the members. It can be seen that converging to the optimum values is very good and the number of repeats are few. In these results, the values converge at each value of 1.0 and 0.02 of R_k with two times of repeat. The results of a two-panel truss calculated by this method are summarized in **Table 2**. In this table, H^0 shows an initial value of H, and for the cases of the variables of H^0 the values of the design variables, the penalty and the objective functions are shown. The different optimum values for the different initial values are then shown in this table. α shows the initial value of a span length divided by a truss height. Case 1, Case 2 and Case 3 show the initial value $\alpha^0=2.2$, $\alpha^0=2.5$ and $\alpha^0=1.8$, respectively. Therefore, for Case 1, Case 2 and Case 3, H^0 is equal to 727.3(cm), 640(cm) and 888.9(cm), respectively. According to the results of the optimum design, the values of Case 2 are almost equal to the values of Case 3, so the optimum solutions converge to global minimum values, but in Case 1 the value of the objective function is a little larger than the values in the other cases. Thus, the penalty function, F, does not show a perfect convex function and it seems that its value has converged to a local minimum value. In this optimum design, SMH$=4\,000$(yen/hour), CM$=80\,000$(yen/ton) and $\mu=0.05$ are employed.

The results of the optimum design in Case 1 are substituted into the following equations to obtain $\boldsymbol{\lambda}^0$:

$$\left.\begin{aligned} &\boldsymbol{\lambda}^{0T}\cdot g(\boldsymbol{x}^0, \boldsymbol{y}^0)=0\ , \\ &\partial\phi/\partial\boldsymbol{x}=\partial f/\partial\boldsymbol{x}+\boldsymbol{\lambda}^{0T}\cdot\partial g/\partial\boldsymbol{x}=0\ , \\ &\partial\phi/\partial\boldsymbol{y}=\partial f/\partial\boldsymbol{y}+\boldsymbol{\lambda}^{0T}\cdot\partial g/\partial\boldsymbol{y}=0\ . \end{aligned}\right\} \cdots\cdots\cdots(43)$$

The following values are then calculated:

$$\phi(\boldsymbol{x}^0, \boldsymbol{y}^0, \boldsymbol{\lambda})=3\,618.4\ ,$$
$$\phi(\boldsymbol{x}^0, \boldsymbol{y}^0, \boldsymbol{\lambda}^0)=3\,820.1\ ,$$
$$\phi(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\lambda}^0)=3\,993.1\ ,$$



(4) unit mm

Fig. 3 10-panel truss and notations for truss.

and

$$\boldsymbol{\lambda}^0\geqq 0\ .$$

These values satisfy Eq. (22) and the calculated values of $\boldsymbol{x}^0$ and $\boldsymbol{y}^0$ are the optimum values.

(2) Example II: Ten-panel truss

Second, as another example of an optimum design using the present method, the optimum design of a steel highway bridge of ten-panel truss type with a live load of TL20, as shown in **Fig. 3**, is carried out. Requirements for an optimum design for one example of the bridge, are the same as an example in the reference (11), with the effective span length of 84(m) and the effective width of 8(m) consisting of 0.75(sidewalk)+6.5(roadway)+0.75(m). For other examples of the optimum design of the bridge, 60(m), 80(m) and 100(m) in its effective span length and 10(m) (roadway) in its effective width are treated.

The variables, constraints and objective function of the example II are almost the same as those of the example I, except a new classification of the compressive member section, (4). In the classification (4), S, T_u, T_w and B_u among the design variables are related to $\boldsymbol{x}$. The constraints contain the limit of stress, the limit of deflection, the upper and lower limits of values of the design variables, the limit of ratio of plate width to thickness for prevention from local buckling and $l/r\leqq 120$. The objective function is the same as the two-panel truss. Also, SMH=

Table 3 Optimum values.

Member				Upper chord							End post							Lower		
Case	L (m)	H (cm)	B (cm)	S_5	B_{u5} (cm)	T_{u5} (cm)	B_{w5} (cm)	T_{w5} (cm)	B_{l5} (cm)	T_{l5} (cm)	S_6	B_{u6} (cm)	T_{u6} (cm)	B_{w6} (cm)	T_{w6} (cm)	B_{l6} (cm)	T_{l6} (cm)	S_{11}	B_{u11} (cm)	T_{u11} (cm)
1	84	1 003.7	45.1	5.0	53.1	1.75	47.1	2.05	45.1	2.06	5.0	53.1	0.99	47.1	1.00	45.1	1.16	5.0	53.1	1.45
2	84	900.0	50.0	5.0	58.0	2.20	46.0	1.90	50.0	2.50	4.0	58.0	1.40	45.0	1.40	50.0	1.40	5.0	58.0	1.60
3	60	736.6	36.0	5.0	44.0	1.69	38.0	2.27	36.0	2.07	5.0	44.0	0.91	38.0	1.24	36.0	1.11	5.0	44.0	1.55
4	80	1 020.8	47.8	5.0	55.8	1.79	49.8	2.24	47.8	2.08	5.0	55.8	1.11	49.8	1.12	47.8	1.30	5.0	55.8	1.58
5	100	1 326.4	54.7	6.0	62.7	1.68	56.7	1.70	54.7	1.93	5.0	62.7	1.36	56.7	1.40	54.7	1.56	5.0	62.7	2.03

	chord				Compressive diagonal					Tensile diagonal							
Case	B_{w11} (cm)	T_{w11} (cm)	B_{l11} (cm)	T_{l11} (cm)	S_{12}	B_{f12} (cm)	T_{f12} (cm)	B_{w12} (cm)	T_{w12} (cm)	S_{17}	B_{f17} (cm)	T_{f17} (cm)	B_{w17} (cm)	T_{w17} (cm)	Z (¥1000)	UC (¥1000/m^2)	UW (kg/m^2)
1	45.1	1.56	45.1	1.70	5.0	47.1	0.90	43.1	0.90	5.0	40.2	1.26	42.4	1.26	12 785.8	78.4	167.9
2	42.0	1.90	50.0	1.60	4.0	40.0	1.00	47.8	1.30	5.0	40.0	1.60	46.6	1.10	13 559.7	80.7	186.6
3	36.0	1.70	36.0	1.74	4.0	49.2	0.92	33.9	1.25	5.0	39.8	1.24	33.3	1.24	10 696.8	71.3	113.8
4	47.8	1.70	47.8	1.85	5.0	52.0	1.01	45.6	0.95	5.0	44.6	1.39	44.8	1.39	13 274.0	66.4	151.5
5	54.7	1.89	54.7	2.03	5.0	63.5	1.19	52.1	1.09	5.0	50.6	1.58	51.4	1.58	16 748.0	67.0	190.9

4 000(yen/hour), CM=80 000(yen/ton) and μ= 0.05 are applied to this example. The number of the variables is 66 and the number of the constraints is 342.

The results of the optimum design are given in **Table 3**, Case 1 shows the case of span length of 84(m), and Case 2 shows the case of an example in the reference (11) for comparison with Case 1. Case 3, Case 4 and Case 5 show the case of the span length of 60(m), 80(m) and 100(m), respectively, with the effective width of 10(m).

In **Table 3**, the optimum values of the design variables $\boldsymbol{x}$ and $\boldsymbol{y}$, the objective function Z, the unit cost UC per a bridge deck area and the unit steel weight UW per a bridge deck area are shown for the five cases, where

L: the effective span length,
UC: the unit cost of the main truss per its bridge deck area,
UW: the unit steel weight of the main truss per its bridge deck area.

The suffix of the variables shows the number of the members.

The value of the truss chord width B becomes $0.547L \sim 0.600L$ (the unit of L is shown in meters). When the values of the design variables are given in mm for a unit of plate thickness and in cm for a unit of plate width by rounding up to the next integer, the value of the cost increases only by 1.4 percent. And the ratio of the span length L to the truss height H becomes about 8.2~7.5 for the span lengths L=60~100 (m). In comparison of Case 1 with Case 2, the cost for the optimum design decreases by 5.7 percent. When optimum designs of structures for large scale bridges are carried out, it is possible to design the structures economically. Although the optimum values of the variables are in general not integers, the value of an objective function is not increased so much by rounding up to the next integer except the variables of the steel material grade S. In Case 3, Case 4 and Case 5, the standard examples for the optimum design of the bridges are shown, and the values of the most economical truss type bridge with a span length ranging from 60(m) to 100(m) are obtained.

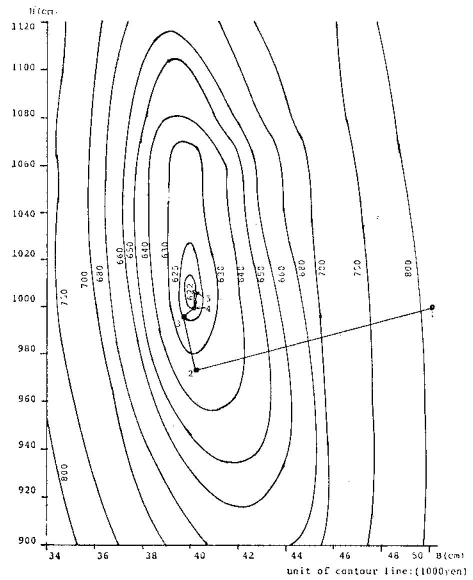


Fig. 4 Contour lines for B and H.

A contour line diagram for B and H in Case 1 for $R_k=1.0$ is shown in **Fig. 4**. In this figure, the points 1, 2, 3, 4 and 5 show the positions of convergence at each step, and Point 5 shows the optimum position. According to this figure, it seems that the optimum solutions converge to the global minimum values. At the optimum state, the working stresses reach the full allowable limit, but for diagonal members near the center of the effective span length, the sectional dimensions are determined by the limit of the slenderness ratio.

4. CONCLUSION

As the number of member elements becomes greater and the configuration of structures becomes more complicated, the number of design variables and constraints increase, and when the cost is chosen as an objective function, it is impossible to omit some of the design variables. By the present method the number of the design variables and constraints can be decreased, and it is easy for the objective function to converge to the optimum value. Even though the number of the design variables is quite large, if the number of the variables of sections common to the whole structure is few, it is possible to effectively carry out an optimum design, and this method is useful for bridge designs. According to Kuhn-Tucker's theorem, it is possible to prove that the results of an optimum design using the present method are mathematically appropriate.

In this study, the examples of the optimum design of statically indeterminate trusses using the present method were not shown. The studies on the optimum design of statically indeterminate structures using a better method than the present one will however, be carried out in the near future.

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