

INFLUENCE OF RESIDUAL WELDING STRESS ON FATIGUE CRACK GROWTH RATE

By Chitoshi MIKI*, Fumio NISHINO**,
Yasuaki HIRABAYASHI*** and Koei TAKENA****

1. INTRODUCTION

Fatigue life in structural members can be subdivided into the life until fatigue cracks initiate (crack initiation life: N_c) and life until these fatigue cracks propagate to the failure of member (crack propagation life: N_p). In welded joints, it was said that fatigue cracks initiated at the early period of the life, and that has been confirmed experimentally in actual weldments by recent studies^{1),2),3)}. Accordingly, accurate evaluation of N_p is very important in avoiding the fatigue failure of existing structures and in performing fatigue design for new structures.

For the evaluation of N_p , it is valid to apply the fracture mechanics concepts of fatigue crack propagation using the relation between stress intensity factor range (ΔK) and fatigue crack propagation rate (da/dN). But fatigue crack propagation rate is affected by many factors such as mechanical properties and microstructure of steel³⁾⁻⁹⁾. Further, in welded joints, it is also necessary to consider the difference in composition such as weld metal, base metal and heat affected zone, and particularly the influence of residual welding stress.

Horikawa et al.⁹⁾ indicated that fatigue crack propagation rate is accelerated by tensile residual stress and explained this phenomenon by using the behavior of fatigue crack closure. Ohta et al.¹⁰⁾ examined the propagation of fatigue cracks in butt weld joints and showed that due to the

tensile residual stress by welding, propagation rate in butt weld joints is higher than in base metal. The authors et al.^{3),11),12)} clarified that the reduction in fatigue strength due to residual welding stress is substantial in partially penetrated longitudinal welds.

The main objective of this paper is to clarify the influence of residual welding stress on the fatigue crack propagation rate. With actual structures, there are many cases that most of the life is spent while propagation occurred in a region of small ΔK ^{3),11),19)}. Therefore, fatigue crack propagation tests are carried out in the wide region of ΔK including threshold stress intensity factor range (ΔK_{th}). The change of residual stress with fatigue crack propagation is also examined.

2. METHOD OF TESTING

(1) Specimens

The steel for testing is 800 N/mm² class quenched and tempered high tensile steel of 16 mm in thickness. The mechanical properties and chemical compositions of the steel in the mill sheet are shown in Table 1.

The configurations and dimensions of specimens are shown in Fig. 1. The specimens consist of two types, type I in which the test section (fatigue crack propagation section) is base metal and type II in which the test section is welded metal. In B and C type specimens, residual stress is introduced by placing one or two longitudinal weld-beads on the surface of specimens.

Butt weld of type II specimens and longitudinal bead were done by submerged arc welding. The groove shapes of the specimens are shown in Fig. 2. In butt welding, at first the groove face was welded, and another face was welded after gouging. The conditions of each welding are indicated in Table 2.

(2) Fatigue Crack Propagation Tests

Fatigue crack propagation tests were performed by using a hydraulic servo-controlled

* Member of JSCE, Dr. of Eng., Associate Professor, Dept. of Civil Engineering, Tokyo Institute of Technology

** Member of JSCE, Ph. D., Dr. of Eng., Professor, Dept. of Civil Engineering, University of Tokyo

*** Member of JSCE, M. of Eng., Metropolitan Expressway Public Corporation, (formerly Graduate Student of University of Tokyo)

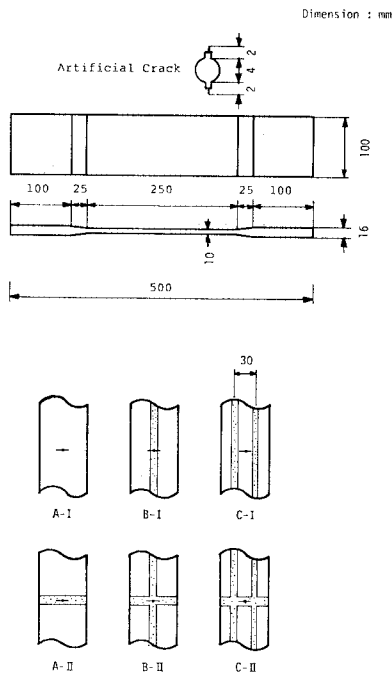
**** Member of JSCE, Honshu-Shikoku Bridge Authority

Table 1 Mechanical Properties and Chemical Compositions of the Steel (Mill Sheet Value).

C	Si	Mn	P	S	Cu	Cr	Mo	V	B	Yield Strength N/mm ² (kg/mm ²)	Tensile Strength N/mm ² (kg/mm ²)	Elongation (%)
0.11	0.25	0.88	0.015	0.005	0.26	0.76	0.40	0.04	0.008	823 (84)	853 (87)	33

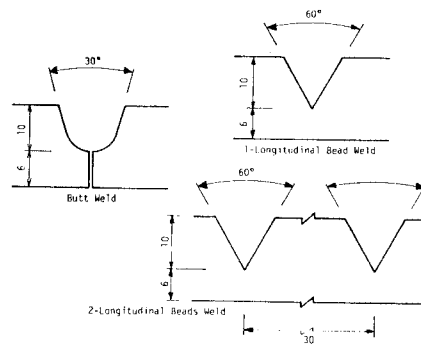
Table 2 Welding Conditions.

	Weld Type	Pass	Current (A)	Voltage (V)	Speed (cm/min)	Heat Input (KJ/cm)	Pre Heat (°C)
Butt Welding	Groove Side	1st	550	32	35	30	150
		2nd	650	32	30	42	150
	Back Side	1st	550	32	35	30	150
		2nd	680	32	25	52	150
Longitudinal Bead Welding	Type B		600	32	25	42	150
	Type C		550	32	25	46	150

**Fig. 1** Configurations and Dimensions of Specimens.

fatigue testing machine with dynamic capacity of 490 kN (50 t). The minimum stress in the fatigue tests is 0, the wave form of loading is a sine wave, and repeating rate is 4 Hz~30 Hz.

Fatigue crack propagation tests consist of three kinds; ① $\Delta\sigma$ constant test in which stress range is kept constant (mainly $\Delta K \geq 500$ N/mm^{3/2}), ② ΔK decreasing test in which ΔK is decreased by stages considering crack propagation rate

**Fig. 2** Weld Details.

(performed in order to obtain ΔK_{th}) and ③ ΔK increasing test performed after ΔK decreasing test in which stress range is increased step by step. In ΔK decreasing tests, in order to avoid the influence of decreasing of ΔK on the propagation of cracks, decreasing ratio of ΔK to crack growth length $\Delta(\Delta K)/\Delta a$ was set as under -49 N/mm^{5/2}. ASTM E-54 committee²¹⁾ recommended that normalized K -gradient ($C = 1/K \cdot dK/da$) is equal to or less than 0.08 mm⁻¹ in K decreasing test. However Ohta et al.¹⁵⁾ indicated that the requirement in the decreasing ratio $\Delta(\Delta K)/\Delta a$ is equal to or less than -98 N/mm^{5/2}. Consulting to these references and considering the efficiency of tests, the decreasing ratio in this study was determined.

The measurement of fatigue crack propagation rate were started after fatigue cracks were propagated to the length of 2 mm ($2a=12$ mm) on both sides of an artificial crack under the condition that stress intensity factor range (ΔK) is approximately 500 N/mm^{3/2}. The measurement

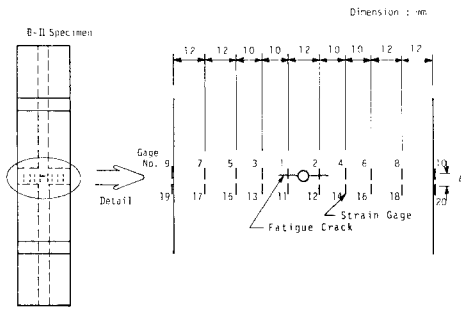


Fig. 3 Location of Strain Gages.

of the length of cracks was done by crack gage in the above stated (a) tests, and in the tests (b) and (c) by taking replicas from the surface of the specimen and observing these by an optical microscope.

(3) Measurement of Residual Welding Stress

Because of the propagation of fatigue cracks, the redistribution of residual welding stress occurs. Here what interests us is the change of residual welding stress with the propagation of fatigue cracks and the distribution of residual welding stress when fatigue cracks exist.

With B-II specimen, changes of residual stress with the propagation of fatigue cracks were measured. Fig. 3 shows the location of strain gages. Strain gages No. 1~No. 10 are for measuring the change of residual stress and No. 11~No. 22 are for measuring initial residual stress on the cross section (only an artificial crack exists). Measurements of strains were done under unloading condition.

With all types of the specimens, prescribed length slits which simulate fatigue cracks were made using a saw. The width of slit is about 0.5 mm. The distribution of residual stress was measured under this condition. Strain gages used in these tests were KFC-2-CI-11 (Kyowa Dengyo), 2 mm in gage length and 2 mm in width.

3. PROPERTIES OF RESIDUAL WELDING STRESS

Fig. 4 shows the change of residual stresses with the propagation of fatigue cracks on each measuring point indicated in Fig. 3. In this figure, the ordinate indicates residual stress and the abscissa indicates the distance from the center of the specimen to the tip of the fatigue crack (1/2 of the length of the crack). At the points of measurement No. 3, 4, 5 and 6, compressive residual stress existed at the early period, but it gradually changed with the propagation

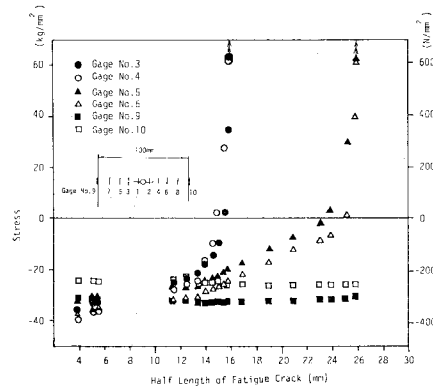


Fig. 4 Change of Residual Stresses with Crack Propagation.

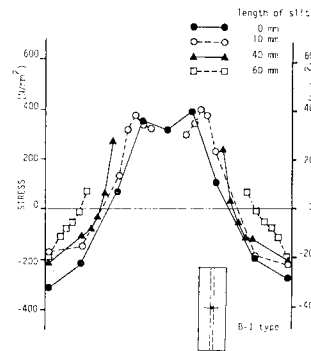


Fig. 5 Welding Residual Stress Distributions of B-I Specimen.

of the fatigue crack, it turned to tensile residual stress. When the crack propagated closer to the strain gage, sudden increasing of strain was measured by this strain gage due to the yielding at this portion. This yielding is caused by the stress concentration of crack to residual stress and stress according to the fatigue test. The result here indicates that the residual stress near the tip of fatigue crack is tension, even when the fatigue crack propagates and enters the area where compressive residual stress existed originally. This behavior is due to the redistribution of residual stresses with the growth of fatigue crack.

Fig. 5 shows the results of measurement of residual welding stress when no crack exists and the length of the slit is 10 mm, 40 mm and 60 mm respectively. By one longitudinal bead, 400—500 MPa tensile residual stress in maximum is introduced originally. The slit of 40 mm in length approximately cuts the area of original tensile residual stress, but in the actual measurement, tensile residual stress exists on the tip of the slit. Even when the length of the slit is 60 mm, tensile residual stress still exists. Because of the

high stress concentration by the slit, the measured value of stress at the closest point to the slit tip is not so accurate, however these trends coincide with the results in Fig. 4.

Fig. 6 shows the similar results of measurement in C-I specimen. On account of two longitudinal beads, tensile residual stress exists in the wide area. The properties of the change of residual stress when the slit is cut are same as those in Fig. 5.

Fig. 7 shows the results of measurement of residual stress when the slit of 40 mm in length exists in the specimens A-II, B-II and C-II. In A-II of butt welding, approximately 100 N/mm² tensile residual stress exists in spite of no longitudinal bead. The distribution of residual stress in the specimens B-II and C-II does not differ much from that in the specimens B-I and C-I considering the accuracy of the measurement.

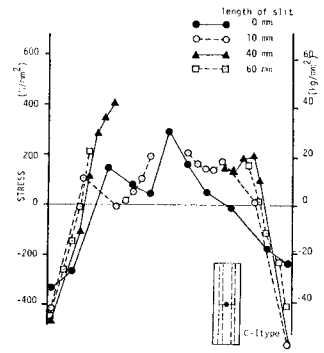


Fig. 6 Welding Residual Stress Distributions of C-I Specimen.

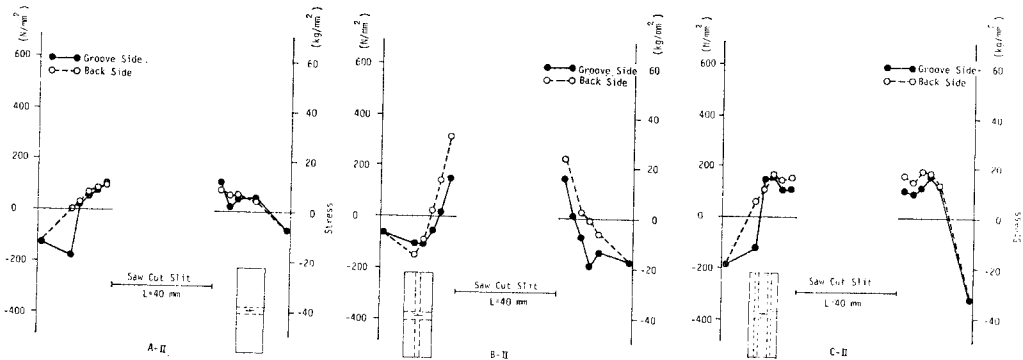


Fig. 7 Welding Residual Stress Distributions of A-II, B-II, C-II Specimens.

4. FATIGUE CRACK PROPAGATION RATE

Fig. 8 shows the relation between fatigue crack propagation rate (da/dN) and the range of stress intensity factor (ΔK) obtained by fatigue crack propagation tests of the specimens. Two each of types of specimen were tested. Results of all the tests are summarized in Fig. 9. It is well known that da/dN and ΔK is linear on a log scale⁴⁾. In order to express the relationship da/dN and ΔK in the region of slow crack growth, Eq. (1) is available^{4), 15)}. The applicability of this equation was confirmed and the adoption of this equation is in general²²⁾.

$$da/dN = C(\Delta K)^m - C(\Delta K_{th})^m \dots (1)$$

In A-I type specimens (base metal without longitudinal bead in Fig. 8 (a)) the relation between da/dN and ΔK is approximately linear in the area where ΔK is above 400 N/mm^{3/2}. When ΔK is under 300 N/mm^{3/2}, fatigue crack propaga-

tion rate drastically reduces, and ΔK_{th} is 255 N/mm^{3/2} (26 kg/mm^{3/2}). C and m in Equation (1) was calculated by the least square method using the data, $\Delta K \geq 400$ N/mm^{3/2}. Then Eq. (1) is written as follows:

$$da/dN = 6.54 \times 10^{-13} (\Delta K)^{2.75} - 2.71 \times 10^{-6} \dots (2)$$

(ΔK : N/mm^{3/2}, da/dN : mm/cycle)

In A-II, B-I, B-II, C-I and C-II type specimens, The relations between da/dN and ΔK are approximately linear in the region where ΔK is above 150 N/mm^{3/2}. In C-I type specimen, fatigue crack did not grow under $\Delta K = 78$ N/mm^{3/2}. For most materials it is practical define ΔK_{th} as ΔK which corresponds to fatigue growth rate of 10⁻⁷ mm/cycle²¹⁾. According to this definition, the value of ΔK_{th} of each type specimen is about 90 N/mm^{3/2} for A-II and 78 N/mm^{3/2} for B-II, C-I, C-II.

Fig. 9 shows that in the specimens A-II, B-I, B-II, C-I and C-II types the relations between da/dN and ΔK does not differ much. Consequently,

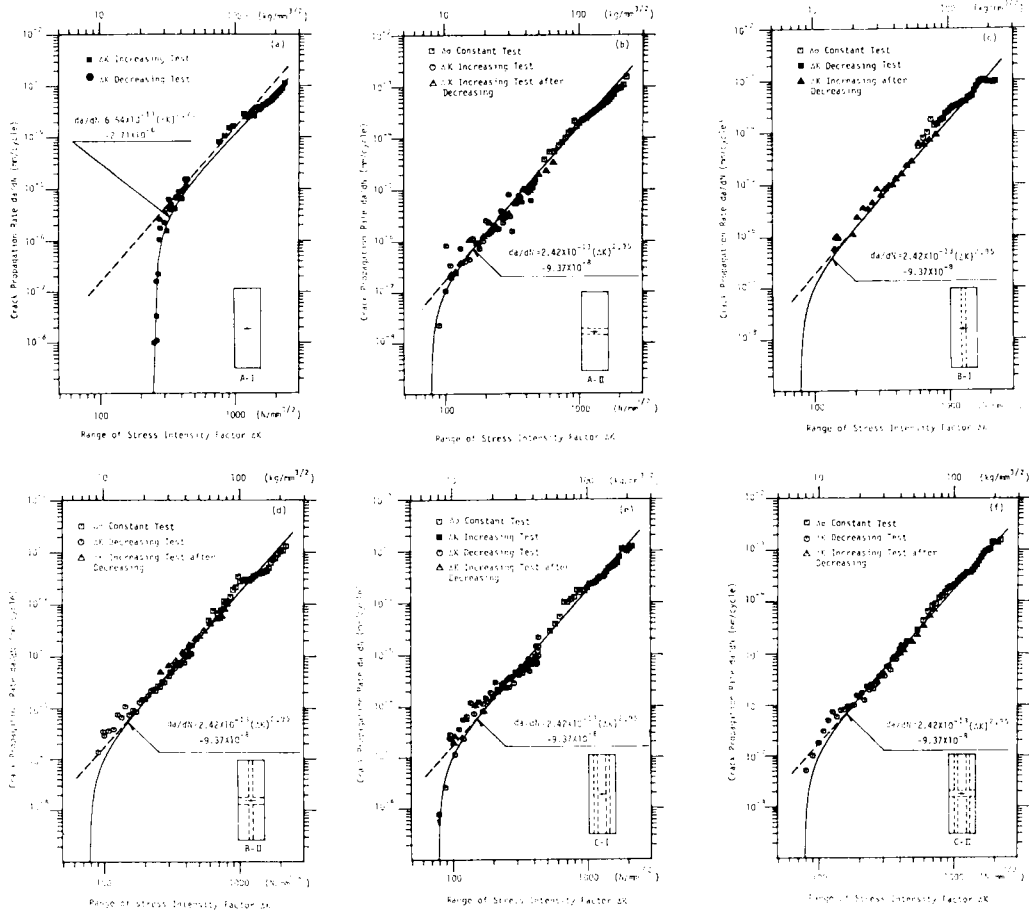


Fig. 8 Results of Fatigue Crack Propagation Tests.

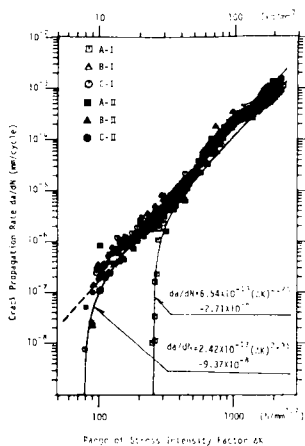


Fig. 9 Summary of All Tests Results.

with the results of these specimens, the relations between da/dN and ΔK were calculated making no distinction among them. As the relation be-

tween da/dN and ΔK is linear when ΔK is above $150 \text{ N}/\text{mm}^{3/2}$, C and m were calculated by the least square method using these data. With ΔK_{th} , there exist only few data, but it is about $78 \text{ N}/\text{mm}^{3/2}$ ($3 \text{ kg}/\text{mm}^{3/2}$). Then Eq. (1) is written as follows:

$$da/dN = 3.82 \times 10^{-18} (\Delta K)^{2.95} - 9.37 \times 10^{-8} \dots (3)$$

In (b)~(f) of Fig. 8, the experiment data of each type of the specimens are compared with Equation (3). The equation shows good coincidence with the data of all types of the specimens. Accordingly, it can be said that in the sphere of this study the relation between da/dN and ΔK does not differ from each other as to whether fatigue cracks propagate in base metal or weld metal. Furthermore there were remarkable difference in the maximum tensile residual stress between A-II ($100 \text{ N}/\text{mm}^2$) and B-I ($400 \sim 500 \text{ N}/\text{mm}^2$) type specimen, the influence of this difference is almost imperceptible.

The broken line in each figure is the $da/dN-\Delta K$

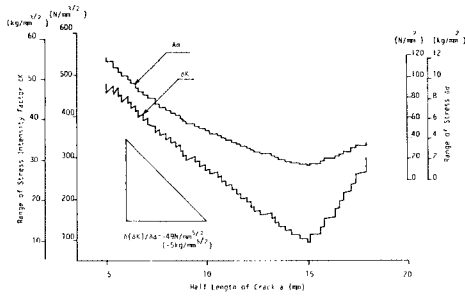


Fig. 10 Stress History in ΔK Decreasing Test on A-II Specimen.

relation obtained by Okumura et al²⁰. This line is very close to the Eq. (2) in Fig. 8 (a), and almost coincide with Eq. (3) in Fig. 8 (b)~(f) in the linear $da/dN-\Delta K$ region.

In each type of the specimen, the value of experiment when ΔK was under approximately $500 \text{ N/mm}^{3/2}$ was obtained by ΔK decreasing tests. Fig. 10 shows the relation between the length of the crack (a) and range of stress ($\Delta\sigma$) and between the length of the crack (a) and range of stress intensity factor (ΔK) in ΔK decreasing tests on A-II specimen. As in all ΔK decreasing tests, decreasing ratio of ΔK to Δa is fixed, the relation is almost the same in other types of the specimens except A-I. In all types of the specimens except A-I, ΔK_{th} was obtained when "a" was 14—15 mm and $\Delta\sigma$ was 10—12 N/mm^2 .

The influence of residual stress on fatigue strength is sometimes considered as the rise of minimum stress or that of stress ratio^{16),17)}. However, it is difficult to estimate strictly stress ratio or stress intensity factor ratio at the tip of cracks which exist in the area of residual welding stress. In this paper, minimum and/or maximum stress are decided as a resultant obtained by adding nominal minimum and/or maximum stress given by the fatigue testing to the residual stress. When the stress history shown in Fig. 10 is given to the specimen, and when residual stress is 400 N/mm^2 , the stress ratio changes from 0.77 (around $\Delta K = 500 \text{ N/mm}^{3/2}$) to 0.98 (around ΔK_{th}) and when residual stress is 100 N/mm^2 , it changes from 0.45 (around $\Delta K = 500 \text{ N/mm}^{3/2}$) to 0.91 (around ΔK_{th}). However, in the region where Δk is under $300 \text{ N/mm}^{3/2}$, it is 0.95—0.98 in the former and 0.83—0.91 in the latter.

By contrast with the difference of the maximum tensile residual stress, the difference of stress ratio between them is very small. Furthermore, considering stress concentration by the crack, higher tensile residual stress exists on the tip of the crack in all the types of the specimens, and the difference in stress ratio among these speci-

mens becomes smaller. If we consider that the influence of the residual stress on the fatigue crack propagation rate is similar to that of the rise of stress ratio, these are one of the causes that the relations between da/dN and ΔK are almost the same in A-II, B-I, B-II, C-I and C-II type specimens.

In consideration of these results, it can be said that in the data of fatigue crack propagation rate obtained by ΔK decreasing tests includes some effects by the decrease of ΔK affected by residual welding stress. This effect, however, is not the cause of retardation of fatigue crack propagation generally admitted when ΔK is decreased. Consequently, it is considered that if yielding on the tip of the crack by loading stress and the change of residual stress accompanying it can be ignored, the result obtained by $\Delta\sigma$ constant tests (It is very difficult to obtain data around ΔK_{th}) coincides with these results mentioned above.

5. FATIGUE CRACK PROPAGATION LIFE

As is clear in Fig. 9, there exists big difference between Eq. (2) and Eq. (3) in the region where ΔK is small, and they are close to each other in the area where ΔK is above $400 \text{ N/mm}^{3/2}$.

The influence of the difference of these two crack growth curves on the crack growth life of structural members is examined by using the partially penetrated longitudinal joint in Fig. 11 as a model. Fatigue crack growth life is predicted on the following assumptions.

- (i) Fatigue crack originates from a defect at the middle of plate-thickness,
- (ii) The initial defect is regarded as a penny-shaped crack with the radius a_i , and fatigue crack grows keeping this shape. Consequently, stress intensity factor against this crack is calculated by the Eq. (4).

$$\Delta K = \Delta\sigma \cdot \sqrt{\pi a} \cdot \frac{2}{\pi} \sqrt{\sec\left(\frac{\pi a}{T}\right)} \dots\dots\dots (4)$$

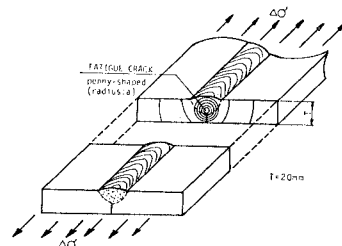


Fig. 11 Model Joint for the Analysis of Fatigue Crack Propagation Life.

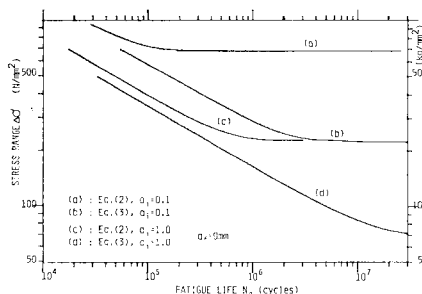


Fig. 12 Predicted Life for Model Joint.

- (iii) The final crack size (a_f) is supposed to be 90% of the distance from the point of crack initiation to the surface of plate.

In this type welded joint, high tensile residual stress remains at weld metal portions and their vicinities. Consequently, Eq. (3) is employed for the prediction of the fatigue crack growth life. In the case that residual stress is released by some process, the fatigue crack growth life of this joint is predicted by employing Eq. (2). By substituting Eq. (4) into Eq. (2) or Eq. (3) and integrating them from the length of the initial crack (a_i) to that of the final crack (a_f), N_p can be obtained.

Fig. 12 shows the relations between stress range ($\Delta\sigma$) and N_p calculated from the Eqs. (2) and (3). The initial crack size (a_i) is supposed to be 0.1 and 1.0 mm. The $\Delta\sigma$ - N_p curves calculated from Eq. (2) and that from Eq. (3) greatly differ, and the difference is remarkable in the case of $a_i = 0.1$ mm when N_p is more than 10^5 cycles and in the case of $a_i = 1$ mm when N_p is more than 10^6 cycles. Most of the fatigue life of structural member is spent while crack propagation occurred in a region of small ΔK ^{(9), (11), (19)}, therefore the accurate estimation of propagation rate in the region of small ΔK including ΔK_{th} is very important for the prediction of fatigue life.

6. CONCLUSION

The main findings of this study are summarized as follows:

(1) The distribution of residual welding stress changes with the propagation of fatigue cracks. Even if the fatigue crack propagates and enters into the area where compressive residual stress existed originally, tensile residual stress exists around the tip of the crack.

(2) The relation between da/dN and ΔK in the specimen, base metal with no longitudinal bead, is roughly linear on a log scale in the region of $\Delta K \geq 400$ N/mm^{3/2}, and ΔK_{th} is 255 N/mm^{3/2}. Equation (2) is obtained from these

results.

(3) There are 400–500 N/mm² of tensile residual stresses in the specimens, base metal with one or two longitudinal bead. In the specimen of weld metal with no longitudinal bead, 100 N/mm² of tensile residual stresses also existed. The relations between da/dN and ΔK are approximately the same in spite of the difference of the maximum tensile residual stress and composition (base metal and weld metal) in these specimens. These data show that the relation is linear on a log scale in the region of $\Delta K \geq 150$ N/mm^{3/2}, and ΔK_{th} is approximately 78 N/mm^{3/2}. Equation (3) can be obtained from these results. There exists big difference between Equation (2) and Equation (3) in the region of small ΔK , and they are close to each other in the region of $\Delta K \geq 400$ N/mm^{3/2}.

(4) The relations between $\Delta\sigma$ and N_p are calculated with one model joint applying fracture mechanics concept. The $\Delta\sigma$ - N_p curves calculated from Eq. (2) and that from Eq. (3) greatly differ especially in the range of long life.

7. ACKNOWLEDGEMENTS

Dr. Kazuyoshi Yamanaka, lecturer of Engineering Research Institute, University of Tokyo, provided facilities for the use of testing machine. The welding of the specimens were done through the kindness of Mr. Yasumasa Nakanishi of Welding Research Institute of Ishikawajima Harima Heavy Industries. And the cooperation of Mr. Masaru Ueno, Research Associate of Department of Civil Engineering, University of Tokyo, made possible the experiments. The authors wish to express their sincerest gratitude to all those mentioned above.

REFERENCES

- 1) Yamada, K., T. Makino and Y. Kikuchi: Fracture Mechanics Analysis of Fatigue Cracks Emanating from Toe of Fillet Weld, Proc. of JSCE, No. 292, pp. 1–12, 1979–12 (in Japanese).
- 2) Yamada, K., T. Makino, C. Baba and Y. Kikuchi: Fatigue Analysis Based on Crack Growth from Toe of Gusset End Weld, Proc. of JSCE, No. 303, pp. 31–41, 1980–11 (in Japanese).
- 3) Miki, Ch., F. Nishino, J. Tajima and Y. Kishimoto: Initiation and Propagation of Fatigue Cracks in Partially-Penetrated Longitudinal Welds, Proc. of JSCE, No. 312, pp. 129–140, 1981–8.
- 4) Paris, P. C. and F. Erdogan: A Critical Analysis of Crack Propagation Laws, Journal of Basic Engineering, Trans. of ASME, 85, No. 3, pp. 528–535, 1963.

- 5) Rolfe, S. T. and J. M. Barsom: Fracture and Fatigue Control in Structures, Prentices-Hall, Inc. pp. 224-236, 1979.
- 6) Maddox, S. J.: Assessing the Significance of Flaws in Welds Subject to Fatigue, Metal Construction (B.W.J.), pp. 4019-4099, Sept. 1974.
- 7) Gurney, R. R.: Fatigue of Welded Structures, Second Edition, Cambridge University Press, pp. 53-64, 1979.
- 8) Kitagawa, H.: Fracture Mechanics Analysis of Fatigue Crack Growth, (Part 1), Journal of the Society of Material Science Japan, Vol. 26, No. 284, pp. 88-99, 1975-5 (in Japanese).
- 9) Horikawa, K., S. Watari and S. Fukuda: Preprints of the National Meeting of JWS, No. 25 (Autumn 1979), pp. 35-42 (in Japanese).
- 10) Ohta, A., E. Sasaki, M. Kamakura, M. Nihei, M. Kosuge, M. Kanao and M. Inagaki: Effect of Residual Tensile Stresses on Threshold Level for Fatigue Crack Propagation in Welded Joints of SM50B Steel, Journal of JWS, Vol. 50, No. 2, pp. 161-168, 1981-2 (in Japanese)
- 11) Miki, Ch., J. Tajima, K. Ashahi and H. Takenouchi: Fatigue of Large-Size Partially Penetrated Longitudinal Butt Welds, Proc. of JSCE, No. 322, pp. 143-156, June 1982.
- 12) Nishimura, T., J. Tajima, A. Okukawa and Ch. Miki: Fatigue Strength of Longitudinal Single-Bevel-Groove Welded Joints, Proc. of JSCE, No. 291, pp. 27-40, Oct. 1979 (in Japanese).
- 13) Fujimoto, T.: A Method for Analysis of Residual Welding Stresses and Deformations Based on the Inherent Strain, Journal of JWS, Vol. 39, No. 4, pp. 236-252 (in Japanese).
- 14) Klesnil, M. and P. Lukas: Effect of Stress Cycles Asymmetry on Fatigue Crack Growth, Materials Science and Engineering, pp. 231-240, 1972.
- 15) Ohta, A., E. Sasaki and M. Kosuga: Effect of Stress Ratios on the Fatigue Crack Propagation Rate, Trans. of JSME, Vol. 43, No. 373, pp. 3176-3191, 1977-9 (in Japanese).
- 16) Kodama, S.: Residual Stress and Fatigue Strength, Journal of the JSME, Vol. 75, No. 642, pp. 1026-1033, 1972 (in Japanese).
- 17) Nishioka, N. and K. Hirakawa: Residual Stress and Fatigue Strength, The Society of Materials Science, Proc. of Sympo. on Strength of Material, pp. 47-54, 1972 (in Japanese).
- 18) Lawrence, F. V. and W. H. Munse: Fatigue Crack Propagation in Butt Welds Containing Joint Penetration Defect, Welding Research Supplement, Welding Journal, pp. 2215-2325, May 1973.
- 19) Hirt, M. A. and J. W. Fisher: Fatigue Crack Growth in Welded Beams, Engineering Fracture Mechanics., Vol. 5, pp. 415-429, 1973.
- 20) Okumura, T., T. Nishimura, Ch. Miki and K. Hasegawa: Fatigue Crack Growth Rates in Structural Steels, Proc. of JSCE., No. 322, pp. 175-178, June 1982.
- 21) Proposed ASTM Test Method for Measurement of Fatigue Crack Growth Rates, ASTM STP 738, pp. 340-356, 1981.
- 22) Tanaka, K.: Fatigue Crack Growth Property of Steel with Particular Reference to Threshold Stress Intensity Range ΔK_{th} , Iron and Steel, Vol. 67, No. 2, pp. 21-37, 1981 (in Japanese).

(Received November, 1981)