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SYSTEM APPROACH TO URBAN RIVER DRAINAGE FOR THE WATER QUALITY PREDICTION

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1. INTRODUCTION

Since the last decade, the sewerage construction has been accelarated in the name of pollution abatement. Despite its heavy investment, the improvement of water quality has been rather slight, and in some cases water quality has even deteriorated in these couple of years. There are still some fundamental arguments about the effectiveness and fairness of the present sewerage system. Moreover as some small streams have dried out as a result of diverting wastewater inflows, a serious question has arisen regarding the psychic significance of urban rivers. It is slightly astonishing that there is no reliable assessment methodology for the change in urban rivers.

Conventional approaches involving organic pollution aimed at formulating the relationship between biochemical oxygen demand (BOD) and dissolved oxygen (DO). Most of these models are essentially based on the theory of Streeter and Phelps$^{1)}$. They have modified suitably to include terms to account for photosynthesis, sedimentation, local runoff, respiration, and benthal demands. A summary of the mathematical models advanced is given in **Table 1.**

In these theories single stream is modelled and analysed while the other are defined as inputs. These approaches have two major defects. First, it fails to express properly the system of the streams involving stems and branches, which is the general form of the river watershed. Second, since less is known about the tributary input in connection with the urban activity in the tributary catchment, it is virtually impossible to predict the water quality when the city is changing or when the drainage form is changing.

In view of these disadvantages of the direct application of these models, we chose a simplified systematic approach. A river is connected to its water sources, either industries, residences, or natural springs. An expression is derived to describe these connection, then the river organic pollution level is described directly in terms of the polluters and its drainage paths. The water quality in rivers are greatly affected by weather, especially by storm runoff. However, as the principal interest of this investigation is on the relationship between the urban activity and the water quality in rivers, it was assumed that the rivers is in steady-state, in other words this study was confiend to dry weather.

2. PRINCIPLES OF DRAINAGE SYSTEM

(1) Independence Theorem of Pollutant Transport

Assume that the system is in steady-state, and the removal of a pollutant, by both decomposition, microbial uptake, and sedimentation, is a first order reaction, then the equation for the pollutant is given by

$$0=\frac{d}{dx}\left(AD_L\frac{dC}{dx}\right)-\frac{d}{dx}(QC)-kAC \quad \cdots\cdots\cdots(1)$$

in which

C: concentration of a pollutant species, ML^{-3}
D_L: longitudinal dispersion coefficient, L^2T^{-1}
Q: flow discharge, L^3T^{-1}
k: total removal rate, or self-purification coefficient, T^{-1}
A: cross-sectional area, L^2

This equation is defined along the stretch which does not involve any joint with other streams. Assuming that the fluid is incompressible and the flow rate is uniform along the stretch, Eq. (1) is simplified to

$$0=D_L\frac{d^2x}{dx^2}-u\frac{dC}{dx}-kC \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

in which

u: flow velocity, LT^{-1}

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Table 1 Models Advanced for BOD and DO Predictions.

Investigators	BOD equation	DO equation
Streeter and Phelps1)	$\frac{dL}{d\tau}=-k_1L$	$\frac{dC}{d\tau}=-k_1L+k_2(C_s-C)$
Camp2)	$\frac{dL}{d\tau}=-k_1L-k_3L+L_a$	$\frac{dC}{d\tau}=-k_1L+k_2(C_s-C)+P$
Dobbins3)	$D_L\frac{d^2L}{dx^2}-u\frac{dL}{dx}-k_1L-k_3L+L_a=0$	$D_L\frac{d^2C}{dx^2}-u\frac{dC}{dx}-k_1L+k_2(C_s-C)-D_B=0$
O'Connor4)	$L_x'=L_0\exp\{-j,\phi(x)\},\quad u_0=\frac{Q_0}{A_0}$ $N_x=N_0\exp\{-j_n\phi(x)\},\quad j=\frac{K}{u_0}$	$\frac{dC}{\partial t}=-\frac{Q}{A_x}\frac{\partial C}{\partial x}+k_2(C_s-C)+P(x,t)$ $-k_1'L_x'-k_1''N_x-(Rx,t)-D_B(x,t)$
Thomann5)	$\frac{\partial L}{\partial t}=-\frac{1}{A}\frac{\partial}{\partial x}(QL)+\frac{1}{A}\frac{\partial}{\partial x}\left(D_L\,A\frac{\partial L}{\partial x}\right)\pm\Sigma_L$	$\frac{\partial C}{\partial t}=-\frac{1}{A}\frac{\partial}{\partial x}(QC)+\frac{1}{A}\frac{\partial}{\partial x}\left(D_L\,A\frac{\partial C}{\partial x}\right)\pm\Sigma_C$
Bella and Dobbins6)	$\frac{\partial(AL)}{\partial t}=\frac{\partial\left(AD_L\frac{\partial L}{\partial x}\right)}{\partial x}-\frac{\partial(uAL)}{\partial x}$ $-(k_1+k_3)AL+AL_a$	$\frac{\partial(AC)}{\partial t}=\frac{\partial\left(AD_L\frac{\partial C}{\partial t}\right)}{\partial x}-\frac{\partial(uAC)}{\partial x}$ $-k_1AL-k_2A(C_s-C)-AD_B$
Ichikawa7)	$AdX\frac{\partial c}{\partial t}=\frac{\partial\{AD_L(\partial c/\partial x)\}}{\partial x}dX-\frac{\partial(AV_c)}{\partial x}dX+(AdX)\frac{dc}{dt}+S$	

A: cross-sectioned area;
c: concentration of a pollutant species;
C: concentration of dissolved oxygen:
C_s: saturation value of dissolved oxygen concentration;
D_B: rate of DO consumption by benthal demand;
D_L: longitudinal dispersion coefficient;
k_1: BOD decay rate constant;
k_2: reaeration rate constant;
k_3: sedimentation rate constant;
k_1': carbonaceous BOD decay rate constant;
k_1'': nitrogeneous BOD decay rate constant;
L: concentration of BOD;
L_a: BOD addition due to local runoff;
L_x': carbonaceous BOD concentration;
N_x: nitrogenous BOD concentration;
$P(x,t)$: oxygen addition by photosynthesis;
Q: net river flowrate;
$R(x,t)$: oxygen consumption by respiration;
S: rate of addition or sedimentary removal;
u: flow velocity;
x: distance along stream;
Σ_C: sources and sinks of DO;
Σ_L: sources and sinks of BOD;
τ: time of travel;
ϕ: collective set of θ and p.

Eq. (2) may be solved, subject to the boundary condition that $C=C^0$ at $x=0$, and $C=0$ at $x=\infty$. The solution is given by

$$C=C^0\exp(\lambda x) \quad\cdots\cdots(3)$$

in which

$$\lambda=\frac{u}{2D_L}\left(1-\sqrt{1+\frac{4kD_L}{u^2}}\right) \quad\cdots\cdots(4)$$

and

C^0: concentration of a pollutant species at $x=0$.

In most rivers, the dimensionless parameter $4kD_L/u^2$ is far less than 1. Hence Eq. (4) is simplified by Tayler's expansion, and by neglecting the terms higher than second order with respect to $4kD_L/u^2$, we get

$$\lambda=\frac{u}{2D_L}\left[1-\left(1+\frac{2kD_L}{u^2}\right)\right]=-\frac{k}{u} \quad\cdots\cdots(5)$$

Eq. (5) indicates that the effect of the longitudinal dispersion is negligible in steady-state. It will be assumed in the subsequent analysis that λ is defined by Eq. (5).

ONE SOURCE

Imagine the simplest case that there exists only one water discharger at the upstream end of the stream, as seen in **Fig. 1** (a). The concentration at the other end will easily be given by

$$C^*=C_1^0\exp(\lambda_1x_1) \quad\cdots\cdots(6)$$

$$Q_1C^*=Q_1C_1^0\exp(\lambda_1x_1) \quad\cdots\cdots(7)$$

in which

C^*: concentration of the pollutant at the downstream end
Q_1, C_1^0: discharge and initial concentration, respectively, of the polluter 1
x_1, λ_1: length of the link and λ defined on the link 1

TWO SOURCES

Suppose that in the second system there are two sources in the watershed. In this system there occurs generally one confluence, which divides the stream into three links, as illustrated in **Fig. 1** (b). Let us assume for the time being that the mixing of two kinds of water occurs

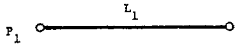

Fig. 1(a) Drainage system with one source.

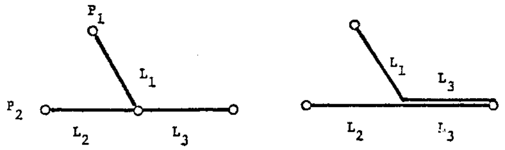

Fig. 1(b) Drainage system with two sources (Figure shows two equivalent stream formations).

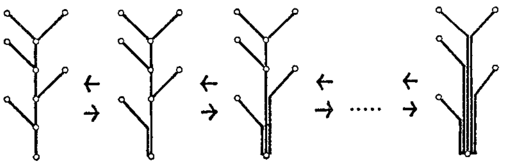

Fig. 1(c) Drainage system with number of sources (Figure shows unfold of the drainage system into individual strands).

instantaneously at the point of confluence, in other words, that the concentration after the confluence is given by lateral average over the cross-section of stream. Thus the concentration of downstream end is given by

$$C^*=\frac{1}{Q_1+Q_2}[Q_1C_1^0\exp(\lambda_1x_1)+Q_2C_2^0\exp(\lambda_2x_2)]\exp(\lambda_3x_3)\quad\cdots\cdots\cdots(8)$$

$$=\frac{1}{Q_1+Q_2}[Q_1C_1^0\exp(\lambda_1x_1+\lambda_3x_3)+Q_2C_2^0\exp(\lambda_2x_2+\lambda_3x_3)]\quad\cdots\cdots\cdots\cdots\cdots\cdots(9)$$

$$(Q_1+Q_2)C^*=Q_1C_1^0\exp(\lambda_1x_1+\lambda_3x_3)+Q_2C_2\exp(\lambda_2x_2+\lambda_3x_3)\quad\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

Comparison of Eqs. (8) and (9) suggests that the terms concerning each sources are independent regardless of the mixing of two kinds of water. This indicates that the system involving the confluence of two streams (**Fig. 1**(b) left) is identical with respect to its water quality to the conceptual system in which two kinds of water go paralelly and seperately without mingling (**Fig. 1** (b) right). This relative independence of the pollutant transport may be extended to the following general case.

MULTIPLE SOURCES

In the system where n sources exist, $(n-1)$ confluences and $(2n-1)$ links are involved at the utmost when every confluence takes place between two streams. Every source in the system has its unique drainge path to the downstream end, which is expressed by a sequence of links involved in the path. The concentration at downstream end is given by

$$C^*\sum_i Q_i=\sum_i Q_iC_i^0\exp\left(\sum_p \lambda_px_p\right)\quad\cdots\cdots\cdots\cdots(11)$$

in which subscript i denotes i'th source and subscript p denotes the links which are involved in the drainge path of water from the source i. This equation also indicates that the total pollutant transport is expressed as a summation of individual transports from the sources. Let us define a strand as a sequence of links involved in the drainage path for an individual source. Then a drainage system may conceptually be reduced to a bundle of independent strands with respect to the water quality, as illustrated in **Fig. 1**(c). In this expression three parameters should be known, i.e., (1) water quantity and quality of each source in the system, (2) composition of links in the strand, (3) rate parameters of the links involving flow rate and total removal rate.

(INTRODUCTION OF THE FIXED TERM)

Another modelling approach may include the fixed term to express photosynthesis or sedimentation which take place regardless of the concentration of the pollutant. By this approach, the basic equation is given by

$$0=\frac{d}{dx}\left(AD_L\frac{dC}{dx}\right)-\frac{d}{dx}(QC)-kAC-AS\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(12)$$

in which

S: fixed rate of addition or removal, either positive or negative, MT^{-1}

The solution is obtained by similar method to that stated previously.

$$C=\frac{S}{k}+\left(C^0-\frac{S}{k}\right)\exp(\lambda x)\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots(13)$$

ONE SOURCE

$$C^*Q_1=Q_1\left[\frac{S}{k}+\left(C_1^0-\frac{S}{k}\right)\exp(\lambda_1x_1)\right]\quad\cdots\cdots(14)$$

TWO SOURCES

$$C^*(Q_1+Q_2)=Q_1\left[\frac{S}{k}+\left(C_1^0-\frac{S}{k}\right)\exp(\lambda_1x_1+\lambda_3x_3)\right]+Q_2\left[\frac{S}{k}+\left(C_2^0-\frac{S}{k}\right)\exp(\lambda_2x_2+\lambda_3x_3)\right]\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(15)$$

MULTIPLE SOURCES

$$C^*\sum_i Q_i=\sum_i Q_i\left[\frac{S}{k}+\left(C_i^0-\frac{S}{k}\right)\exp\sum_p \lambda_px_p\right]\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(16)$$

Despite the slight complexity of the solutions, Eqs. (14) through (16) indicate that the independence of the pollutant transport still holds regardless of the introduction of the fixed term.

(2) Consideration about Filters

The water is sometimes brought away from the drainage system for various utility or by the penetration on to stream bed. As these diversions of the stream flow bring both water and its contaminants, they may be treated as the cuts or the filters. The filters are defined as the point or the section of the stream where some portion of water disappear from the system. Thus defined, there are two types of the filters. Discrete filter stands for the outlet by various utility and continuous filter represents the underflow.

Assume that the concentration is uniform transversely, then the concentration of the filtered and the residual portion of the water are the same, i.e.,

$$C_F^+=C_F^-$$

in which $C_F^{+,-}$ stands for the concentration at either downstream or upstream neighbour of the filter. Hence it may be seen that the existence of the filter never affects the continuity of longitudinal concentration profile along the stretch.

DISCRETE FILTER

Imagine that a link has a discrete filter which divides the flowing water by the proportion r_F toward the outside of the stream and $(1-r_F)$ toward the downstream side, as illustrated in **Fig.** 2(a). Because

$$C_B=C_A\exp(\lambda_p x_p) \quad \cdots\cdots(17)$$

in which the subscripts A and B denotes the upstream end and downstream end of the link respectively, so

$$Q_B=(1-r_F)Q_A \quad \cdots\cdots(18)$$

$$Q_BC_B=Q_AC_A(1-r_F)\exp(\lambda_p x_p) \quad \cdots\cdots(19)$$

in which

r_F: diversion ratio of the discrete filter, dimensionless

Hence, if the drainage system has more than one discrete filter, Eq. (11) are revised into

$$C^*\sum_i Q_i\left[\prod_q(1-r_{Fq})\right]=\sum_i Q_iC_i\left[\prod_q(1-r_{Fq})\right]\exp\sum_p(\lambda_p x_p) \quad \cdots\cdots(20)$$

in which subscript q represents the discrete filters which exist in the path of water drainage from source i.

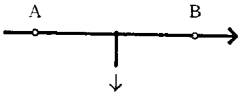

Fig. 2(a) Discrete Filter.

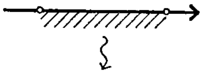

Fig. 2(b) Continuous Filter.

CONTINUOUS FILTER

Gravelly beds of the rivers sometimes absorb flowing waters and form underflows as interstitial waters. From the stand point of water balance, these underflows consist of two types, namely, temporal underflow which may return to the surface flow downstream, and penetrative underflow which penetrates even deeper toward free groundwater or artesian groundwater. As the temporal underflow does not vanish from the system, the continuous filter is confined to the penetrative underflow.

Assume that the penetrative underflow occurs in a link, as illustrated in **Fig.** 2(b), and the reduction of the discharge is given by

$$\frac{dQ}{dx}=-\mu Q \quad \cdots\cdots(21)$$

in which

μ: coefficient of penetration, L^{-1}

Since the solution of Eq. (21) is given by

$$Q=Q^0\exp(-\mu x) \quad \cdots\cdots(22)$$

then, in the link in which penetrative underflow occurs,

$$Q_B=Q_A\exp(-\mu x) \quad \cdots\cdots(23)$$

Therefore, in the drainage system which involves more than one link,

$$C^*\sum_i Q_i\exp\left(-\sum_p \mu_p x_p\right)=\sum_i Q_iC_i^0\exp\left[\sum_p(\lambda_p x_p-\mu_p x_p)\right] \quad \cdots\cdots(24)$$

In inpervious links μ is defined to be zero.

Finally, combination of Eq. (20) and Eq. (24) yields

$$C^*\sum_i Q_i\left[\prod_q(1-r_{Fq})\right]\exp\left(-\sum_p \mu_p x_p\right)=\sum_i Q_iC_i^0\left[\prod_q(1-r_{Fq})\right]\exp\sum_p(\lambda_p x_p-\mu_p x_p) \quad \cdots\cdots(25)$$

Eq. (25) is the general expression of the pollutant transport.

3. DRAINAGE PATHS OF WASTEWATER AND NATURAL WATER

(1) Hierarchy of Drainage Network

Though the rate parameters and the length are defined for each link, which is the minimum unit of the stream, this unit is rather unpractical for its smallness. But as the drainage system of the river is essentially hierarchical, there are some categories among these links, and four categories could be found in urban drainage system. They are primary drain, secondary drain, tributary, and main stream, as illustrated in **Fig. 3**. Primary drain is often referred to as the gutter or the street gutter which is constructed on both sides of the street. Gutters drain both stormwater and wastewater in un-sewered district. The gutter is further divided into the U-gutter and the L-gutter according to its shape. Primary drain also includes the conduit which is laid under the streets. For most minor wastewater sources, the gutter is the first stage of the drainage. The secondary drain is wider than the gutter and often independent of the streets. It may be an ancient canal, 'subtributary', or of sorts. Since it is big enough and independent, most of the secondary drain is distinguishable on a map with a scale finer than 1 : 10 000. The third class is the tributary. Tributaries are considered to be a transient form between wastewater drains and the main stream for they have characteristics of both. The final drainage stage is the mainstream. The difference of the drainage system may be thus reduced to be the difference in the composition of these four drainage categories.

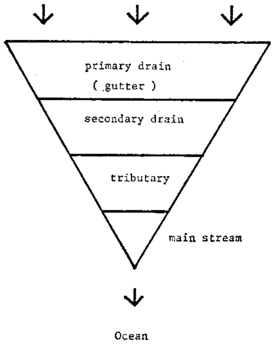

Fig. 3 Hierarchy of Drainage Network.

Table 2 Criteria for Drain Classification.

Class	Distinguishing Criteria
Primary drain Secondary drain	readability by 1 : 10 000 topographic map
Tributary	10 meters of width
Main stream	100 meters of width

Specific criteria for classification as shown in **Table 2** was used in the subsequent analysis of the Tama River.

(2) Sampled Study of Drainage Network

Since conventional approach to river pollution has been concentrated on main streams, little is known about the smaller streams both at the administrative and research levels. The size, location or the length of the primary or secondary drains are not registered in many administra-

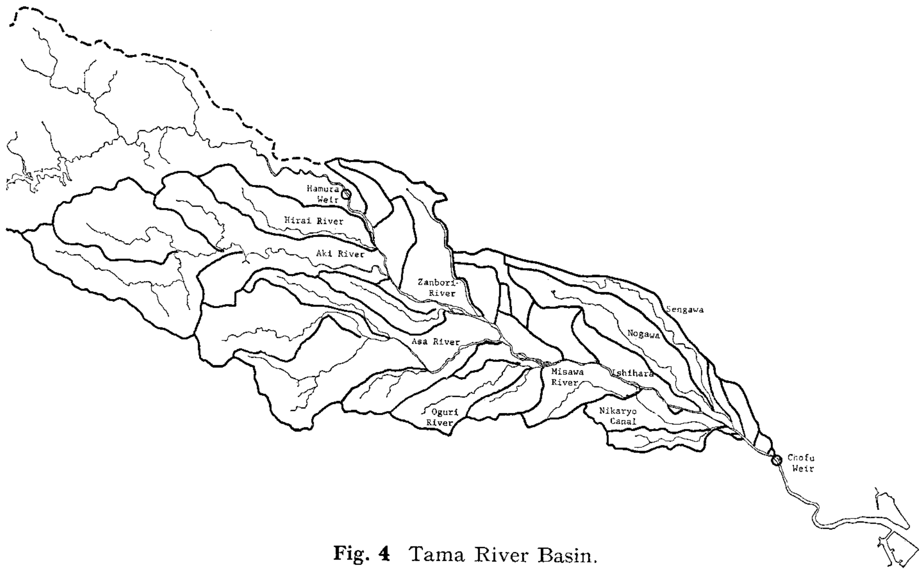

Fig. 4 Tama River Basin.

Fig. 5 Yorozu Cho.

tions. So it is necessary for us to identify the drains through a field study first.

As a typical example, the results of Yorozu-cho, Hachioji-city is shown in following figures. Hachioji-city is in the Asa River Subbasin of the Tama River Basin. This district is so near to the Hachioji central district that the land is covered densely by both residential and business buildings as shown in **Fig. 5**. There is a moderate hill to the west of the district. But another part is comparatively flat, inclined gently to the east. The density of land use result in a stuffed network of drains, as shown in **Fig. 6**. These drains are connected to Yamada River, which is classified secondary drain terminating at Asa River. **Fig. 7** shows the distribution of drain path length including primary and secondary drains. This figure indicates that the length of paths varies from 2.2 to 3.0 km with the mean value 2.6 km.

Although this initial investigation selected only a few samples which were not enough to cover the whole basin, the investigation, nevertheless, generated sufficient information concerning the drainage network to draw the following conclusions.

That primary drains which involves gutters and conduits may be estimated by street patterns since no drains pass through private sites except in a few minor cases.

(3) Basinwide Estimation of the Drainage Paths

Based on the above kinds of knowledge, the drainage path lengths in a whole basin can be estimated. In the Tama River case, where the unit of the investigation is cho-chome, the procedure was as follows.

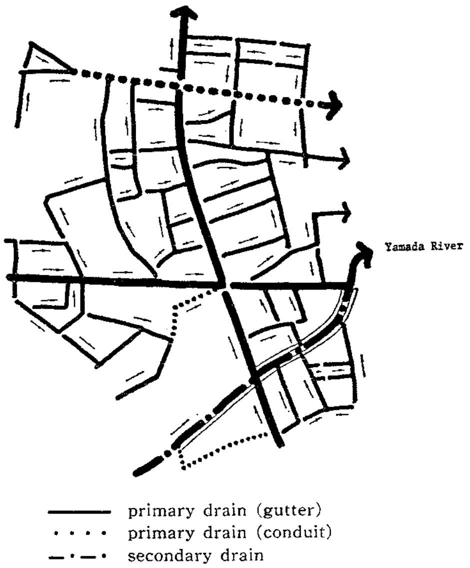

Fig. 6 Drainage Network of Yorozucho.

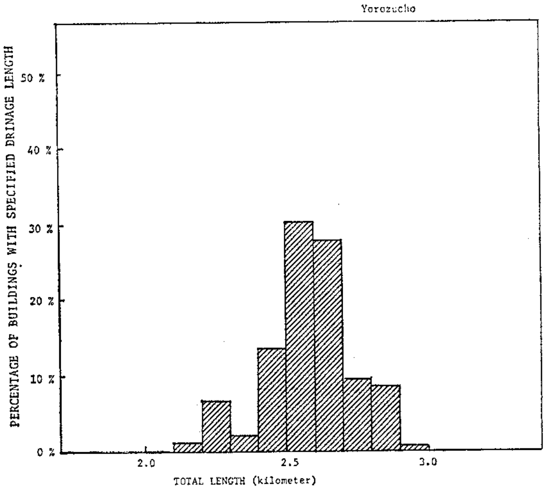

Fig. 7 Length of Primary and Secondary Drains.

(1) First, main stream, tributaries, and secondary drains were distinguished, classified and marked on a 1 : 10 000 scale topographic map. Then boundaries were drawn according to contours and the stream directions to seperate subbasins.

(2) Every district, cho-chome, was assumed to be a plate, and the slope of the plate was determined by the topographic map. In this stage, it is already possible to find the drainage direction and destination. Three points in the district were selected to represent the nearest (toward downstream), the farthest, and the central points.

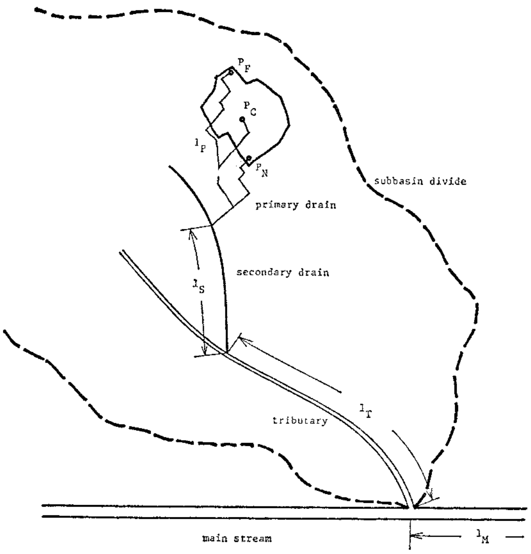

Fig. 8 Conceptual Display of the Estimation Procedure of the Drainage Paths.

(3) For these three points, the drainage paths to the secondary drains were estimated and drawn along the streets as illustrated in **Fig. 8.**

(4) Finally the lengths of each of these three paths were measured by curvimeter. The representative path lengths were determind as the mean of these three.

Fig. 9 indicates the total length of the drainage paths to the Tama River Mainstream, involving primary drains, secondary drains, and tributaries. The right side of Tama River Basin has a complex and irregular drainage pattern, in contrast to the left side which is comparatively regular. The Nogawa has such a long and slender shape, that there are distant and remote districts from the main stream. It should be noted that these drainage paths were determined topographically disregarding any sewerages and occasional penetration disposal of the wastewater that might exist in the watershed.

4. PREDICTION OF WATER QUALITY LEVEL

There are three important points to be clarified before bringing the independence theorem into practice. These concern

- Identification of the drainage paths and their components.
- Determination of rate constants on the drainage paths.

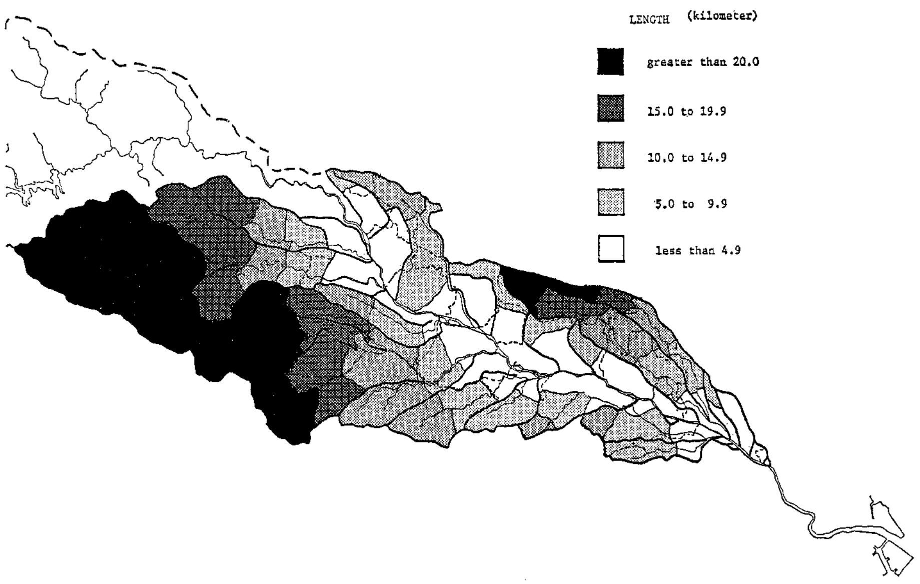

Fig. 9 Total Drainage Path Length to the Tama River Main Stream (the total length involves the path lengths of primary drains, secondary drains and tributaries).

Estimation of the wastewater discharge and pollutant loadings.

The first point was discussed in the previous section. The essential features of the latter two points are covered here before the computation of the pollution level prediction.

The study area is the Tama River Basin. As almost the entire amount of the upstream water is taken out at Hamura Weir, and as Chofu Weir seperates the tidal section and the regular current, the study is focused on the stretch between Hamura Weir and Chofu Weir.

(1) Rate Parameters

The rate parameters to be estimated are flow velocity, u, and total removal rate, k. Several field data which has been reported by previous investigators are available for the parameters of the Tama River Main stream. Adopted parameters in this investigation were those reported by Tokyo Metropolitan Government[8] for flow velocity and k of BOD, and those by Aiba[9] for k of nutrients. They are shown in **Table 4.** On the other hand only little is known about smaller streams, including primary and secondary drains or tributaries. To study the process of decomposition and accumulation on these small streams, a specially built waterway model was used, as shown in **Fig. 10.** The model was made of PVC with inner surface attached by sands, whose size ranged between 0.6 and 1.2 mm. The waterway was linked squarely with a tub at the end, and the water was aerated in the tub to keep the oxygen level. The water was circulated by a rotary pump which may control the flow rate, and at the end of the waterway, a plastic gate was built to control the depth of water. The water which was sampled on the last side of the waterway was analysed for total oxygen demand (TOD), nitrogen and phosphorus.

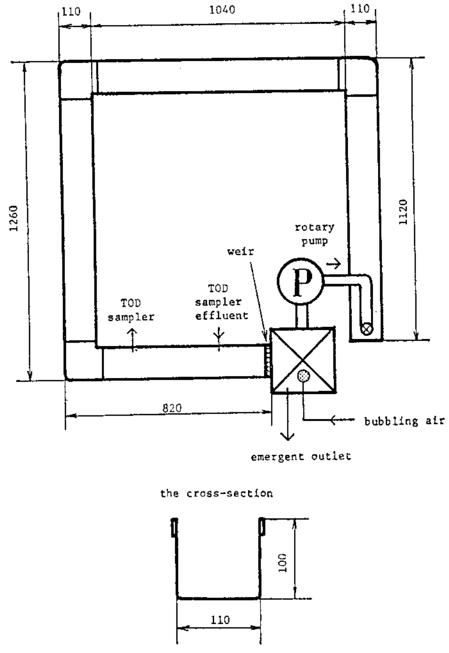

Fig. 10 Experimental Apparatus.

Water temperature ranged from 20°C to 26°C throughout the experiment.

In spite of the advantage that the analysis of TOD is performed continuously with small amount of sample water, it involves both biochemically active and inactive portions in it. In other words, TOD involves soft-TOD and hard-TOD. Soft-TOD is defined as the portion which may be degraded or adsorbed on stream bed, and hard-TOD is also defined as the residue of TOD after sufficient elaption of time. The definition of soft-TOD suggests that it is conceptually the same index as ultimate-BOD. Thus, the subsequent discussion will focus on soft-TOD as a substitute of BOD.

The process dominated by the contact to the stream bed is expressed by

$$\frac{dT_S}{dt}=-\frac{V_W}{V_W+V_T}\frac{d_B}{h_e}T_S \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(26)$$

in which

T_S: soft-TOD, defined as $(T-T_\infty)$, ML^{-3}

T: total oxygen demand, $M\ L^{-3}$

T_∞: hard-TOD, or TOD after sufficient elaption of time, $M\ L^{-3}$

h_e: effective depth, L which is given by

$$h_e=\frac{S}{A}$$

in which A: cross sectional area, L^2

S: wetted perimeter, L

V_W: volume of the water on the waterway, L^3

V_T: volume of the water in the tub, L^3

d_B: rate of TOD removal on stream bed, LT^{-1}

The solution of Eq. (26) is given, subject to the initial condition that $T_S=T_S{}^0$ at $t=0$.

$$\frac{T_S}{T_S{}^0}=\exp\left(-\frac{V_W}{V_W+V_T}\frac{d_B}{h_e}t\right) \quad \cdots\cdots\cdots\cdots\cdots\cdots(27)$$

Fig. 11 shows the relationship between $\ln\left(\frac{T_S}{T_S{}^0}\right)$ on ordinate and $\frac{V_W}{V_W+V_T}\frac{t}{h_e}$ on abscissa, in which the slope indicates $(-d_B)$. The values of d_B in nine experimental run ranged from 8.99 through 13.08 with the average 10.81.

According to the relationship between d_B and the total removal rate, k, that

$$k=\frac{d_B}{h_e}$$

k's of BOD which is assumed to be identical to soft TOD were estimated as shown in **Table 3.** Though nutrients cannot be decomposed biologically, adsorption onto the stream bed resulted in decrease of nutrients as well as TOD. As the change of nutrients occured with constant rate, the total removal rates for nutrients was assumed to be constant among the stream categories, as indicated in **Table 3**.

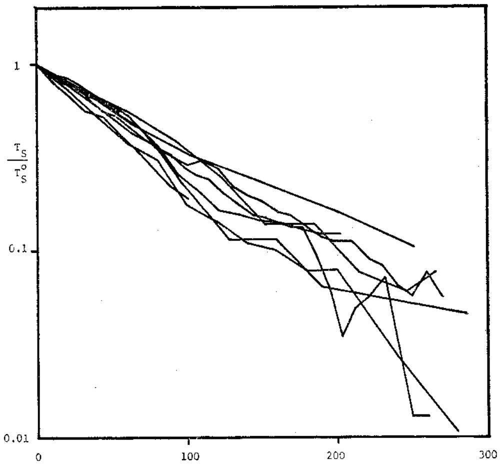

Fig. 11 Relationship between $\frac{V_W}{V_W+V_T}\frac{t}{h_e}$ and $\frac{T_S}{T_S{}^0}$.

Table 3 The Constants of Smaller Streams.

	Flow Velocity m/sec	Effective Depth m	Total Removal Rate BOD	N 1/hr	P
primary drain	0.1	0.02	0.54	0.21	0.094
secondary drain	0.3	0.10	0.108	0.21	0.094
tributary	0.5	0.20	0.054	0.21	0.094

(2) Pollutant Loadings

This study of pollutant loadings has two major steps. In the first step, the pollutant production in cho-chome is estimated for each of the separate source sectors. In the second step, the pollutant loadings to the river are computed taking into account the existence of sewerage. These steps are shown in **Fig. 12.**

There are six sectors of pollutant source in the river basin. They are,

- Natural or geographical yield (forest+spring)
- Industry (notified factory+un-notified factory)
- Household (community plant+individual dwelling)
- Office or commercial
- Livestock
- Water treatment facility

Among these six sectors, the natural sector, the

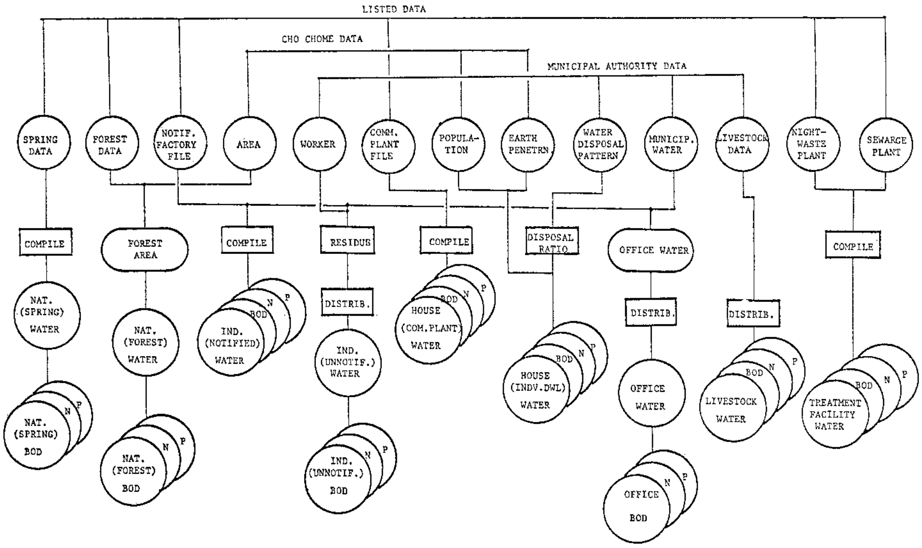

Fig. 12 Estimation of Pollutant Loadings.

Table 4 The Constants of Main Stream.

Distance from Chofu Weir km	Flow Velocity m/sec	Total Removal Rate BOD	N 1/hr	P
0. - 4.5	0.344	0.066	0.02	0.02
4.5- 9.2	0.398	0.078		
9.2-12.5	0.300	0.018		
12.5-14.6	0.251	0.024		
14.6-18.1	0.579	0.075		
18.1-21.5	0.306	0.017		
21.5-26.6	0.222	0.039		
26.6-32.9	0.210	0.0063		
32.9-40.5	0.151	0.0063	0.02	0.02

industry sector, and the household sector are each further divided into two subsectors as indicated in the parentheses. The notified factories and the community plants are defined as the factories and the facilities specified in the Water Contamination Prevention Law. Spring subsector is defined as a set of point sources by springs discretely found in urban or suburban area, and forest subsector is defined as non-point source by forestal water yield. The computation for FY 1978 was conducted using the measured data as far as possible by courtesy of the Tokyo Metropolitan Government and Kawasaki City. In total, the water yield in FY 1978 was 1 135 000 m^3/day, and the loadings of BOD, nitrogen, and phosphorus were 47 030, 15 586, and 2 516 kg/day respectively.

(3) The Water Quality Level Prediction

As the conclusion of the analysis of water quality, organic and nutrient concentration is computed according to the formulation described in section 2 with the case of the Tama River in FY 1978.

The input parameters other than that referred earlier were given as follows.

- Downstream discharge at Hamura Weir

The downstream discharge at Hamura Weir is assumed to be 50 000 m^3/day in non-irrigation period, and 200 000 m^3/day in irrigation period.

- Filter parameters

The outlet in the Tama River in 1978 is summarized in **Table 5**. Since the water outlet is larger in irrigation period, the parameter of discrete filter should be determined for irrigation and non-irrigation period separately. Parameters thus determined are listed in the right columns of **Table 5**. On the other hand, little information is available to determine the parameters of continuous filter, i.e. the penetrative underflow. Murakami[15] estimates that most of the underflow in the Tama River returns to surface flow in downstream stretch. According to this result, the continuous filter was assumed to be negligible in this computation.

LONDITUDINAL PROFILE

The continuous predictions for 0.5 km intervals are shown in **Fig. 13** for irrigation period, and **Fig. 14** for non-irrigation period. The actual water quality levels are indicated as the geometric means of the monthly observed data in FY 1978[10] in **Fig. 15**. Mean values were computed by the data including both irrigation and non-irrigation period. The geometric means were used as representatives instead of arithmetic means in this

Table 5 Outlets from the Tama River.

Station	Distance from Chofu Weir (km)	Amount of Outlet (m^3/sec)								r_F	
		May	June	July	Aug.	Sept.	Oct.	Average June-Sept.	Winter	Irrigation Period	Non-Irrigation Period
Showa Canal	34.6	0.65	0.93	0.79	0.66	0.45	0.00	0.71	0.00	0.14	0.00
Hino Canal	32.0	0.73	0.58	1.11	1.03	1.12	0.67	1.04	0.00	0.25	0.00
Fuchu Canal	26.4	0.15	1.56	1.20	1.27	0.86	0.06	1.22	0.00	0.29	0.00
Yotsuya-Honjuku Canal	25.0	0.96	1.15	0.76	1.17	0.73	0.00	0.95	0.00	0.26	0.00
Yotsuya-Shimo Weir	23.7	0.00	—	—	—	—	0.00	0.00	0.00	0.00	0.00
Daimaru Canal	19.2	0.59	1.06	1.12	1.09	0.95	0.59	1.06	0.00	0.15	0.00
Kamigawara Weir	12.5	1.07	1.24	1.60	1.65	1.45	0.95	1.49	1.00	0.21	0.10
Shukugawara Weir	9.2	0.34	0.37	0.37	0.40	0.37	0.39	0.38	0.38	0.06	0.04
Kinuta-Kami Water Plant	8.1	0.79	—	—	—	—	0.79	0.79	0.79	0.13	0.08
Kinuta-Shimo Water Plant	0.3	0.46	—	—	—	—	0.46	0.46	0.46	0.09	0.05

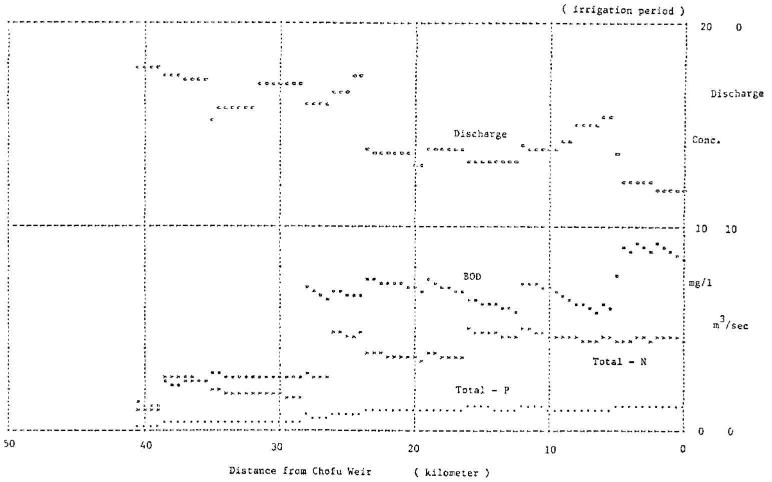

Fig. 13 Computed Water Quality Level Profile in Tama River.

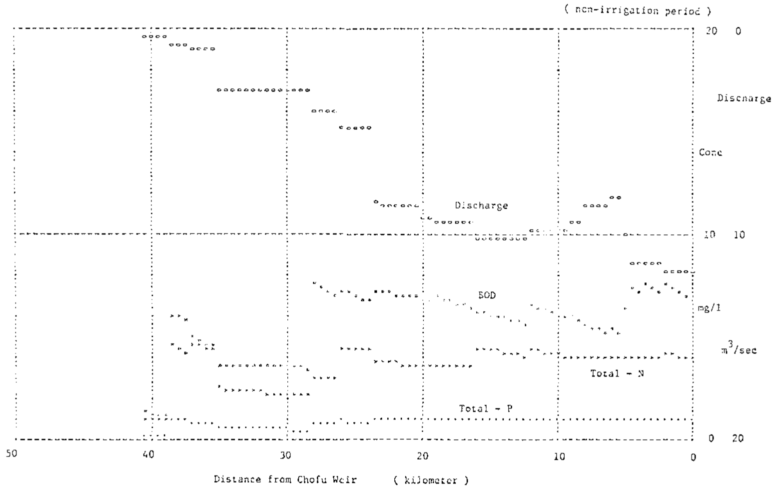

Fig. 14 Computed Water Quality Level Profile in Tama River.

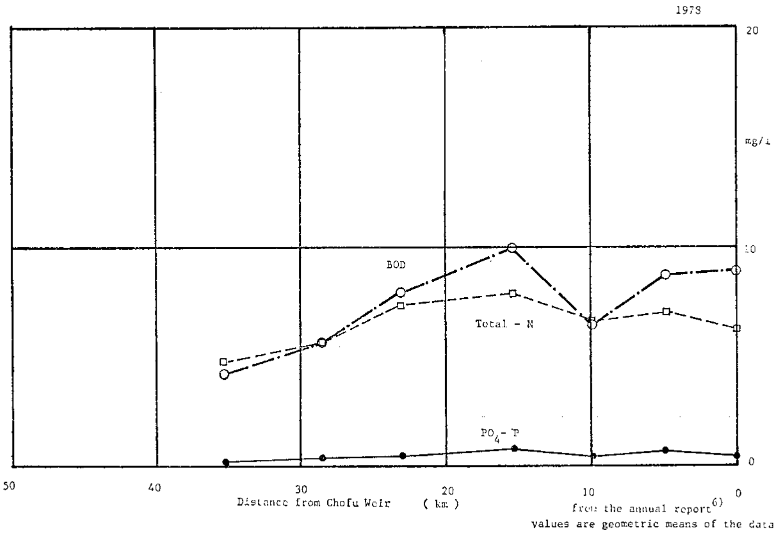

Fig. 15 Observed Water Quality Level Profile in Tama River.

study, because the water quality data in many urban rivers fit log-normal distribution better. The major characteristic of the longitudinal profiles in the middle Tama River is that the water quality levels are generally constant. The pollutant loadings from the tributaries are balanced with the stream self-purification process in the stretch downstream from the junction with Asa River. This characteristic is reproduced by the computation. A closer look shows that BOD agrees well with the real longitudinal profile except in the upstream section, where the water quality is highly dependent on the downstream discharge of Hamura Weir. For nutrients, nitrogen is computed less than the observed level, while phosphorus is larger. The observed phosphorus being only in the phosphate form, the real prediction error might be negligible if expressed in terms of total phosphorous. The discrepancy in nitrogen may be caused by the rate constants, which might be too large. But as the constants were obtained by experimental procedure, they were not corrected, in view of the deterministic stand-point.

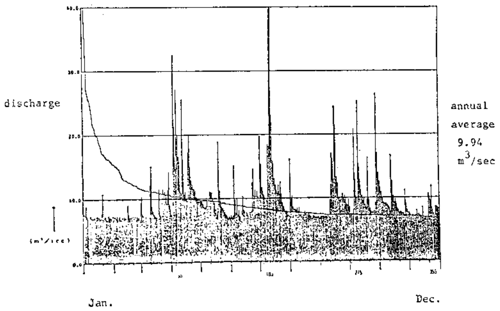

Fig. 16 Flow Discharge at Ishihara, Tama River (from ANNUAL REPORT5)) (17.6 upstream from Chofu Weir).

The discharge of middle Tama River is measured only at Ishihara, 17.6 km from Chofu Weir11). The result is shown in **Fig. 16**. The figure shows a peculiarity that the discharges have a minimum level or a bottom. This bottom indicates the level of upstream water yields, principally by the steady discharge of the urban wastewater. The level of the bottom was between 7.1 and 7.3 m^3/sec. This level lies in the middle of the predicted irrigation and non-irrigation periods. There remains a question why the minimum level of the discharge is constant throughout the year regardless of irrigation. This may be attributed to the complex mechanism of the underflow and the return of irrigated water beneath the stream bed. At Chofu Weir, the downstream end of the middle Tama River, the discharges had been measured up to 1972. The normal flow, which is the 185'the day in the order of a year, was 9.51 m^2/sec. This level also falls between 11.8 and 8.2, respectively the predicted discharge of non-irrigation and irrigation period.

TRIBUTARY INFLOW

In the same computation procedure as the longitudinal prediction, the quality and the quantity of the tributary input, including the secondary drain input, are figured out. The result of the computation is displayed in **Table 6**, while the corresponding observation data is sum-

Table 6 Computed Water Quality of Tributaries and Drians.

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-------------------------------------------------------------------------------------------------
                              LOAD (GRAM OR M**3/DAY)               CONCENTRATION (MG/L)
                       --------------------------------------    ------------------------------
      SUBBASIN              Q        BOD         N         P         BOD          N         P
-------------------------------------------------------------------------------------------------
  KOSUGI DRAIN              0.        0.        0.        0.         0.0        0.0       0.0
  KAMINUMABE DRAIN          0.        0.        0.        0.         0.0        0.0       0.0
  TODOROKI DRAIN            0.        0.        0.        0.         0.0        0.0       0.0
  YAZAWA DRAIN          10044.   192994.    38041.    23020.        19.215      3.787     2.292
  MIYAUCHI DRAIN        10777.   377644.   213090.    40023.        35.763     11.348     2.483
  SHIMONOGE DRAIN           0.        0.        0.        0.         0.0        0.0       0.0
  SUWA DRAIN            13317.   648070.   161845.    36490.        48.665     12.153     2.740
  FUTAKO DRAIN              0.        0.        0.        0.         0.0        0.0       0.0
  NOGAWA RIVER         111841.  1875175.   494395.   161831.        16.766      4.421     1.447
  HIRASE RIVER         155199.  1955469.   672211.   202396.        12.600      4.331     1.304
  SEKI DRAIN             7286.   238647.    98672.    17484.        32.754     13.543     2.400
  UNANE DRAIN            1887.    47112.    17814.     3138.        24.967      9.440     1.663
  IKATA DRAIN             150.      224.     1355.      472.         1.494      9.031     3.144
  NOBORITO DRAIN            0.        0.        0.        0.         0.0        0.0       0.0
  ROKUGO DRAIN            227.     3618.     1884.      682.        15.940      8.300     3.005
  CHOFU DRAIN            3302.    64830.    16540.     6279.        19.634      5.009     1.902
  MISAWA RIVER          59430.  1168593.   572459.   123976.        19.663      9.632     2.086
  YANOKUCHI DRAIN           0.        0.        0.        0.         0.0        0.0       0.0
  KOREMASA AKUSUI       66017.   357993.  1012690.   135363.         5.423     15.340     2.050
  YATO RIVER                0.        0.        0.        0.         0.0        0.0       0.0
  YAZAKICHO DRAIN           0.        0.        0.        0.         0.0        0.0       0.0
  KOREMASA DRAIN        17804.   438885.   188012.    44064.        24.651     10.560     2.475
  OOGURI RIVER          54228.   351472.   164743.    44296.         6.481      3.038     0.817
  KUNITACHI DRAIN        7354.   135224.    47774.    17630.        18.388      6.496     2.397
  HODOKUBO RIVER            0.        0.        0.        0.         0.0        0.0       0.0
  ASAKAWA RIVER        296041.  2292429.   903971.   306156.         7.744      3.054     1.034
  HINO DRAIN            21394.   239461.    49110.    28020.        11.193      2.296     1.310
  KUNITACHI YAHO DRAIN   2933.    38236.    52090.    11984.        13.037     17.760     4.086
  MIDORI RIVER           1714.    53490.    10548.     3114.        31.208      6.154     1.817
  NEGAWA RIVER          53023.   513540.   807393.    94207.         9.685     15.227     1.777
  ZANBORI RIVER         49709.  1364735.   420043.   120434.        27.454      8.450     2.423
  YAJI RIVER            37202.   447825.   104974.    26744.        12.038      2.822     0.719
  TAMA JORYU DRAIN          0.        0.        0.        0.         0.0        0.0       0.0
  MIYAGAWA DRAIN            0.        0.        0.        0.         0.0        0.0       0.0
  FUSSA DRAIN            8239.   339846.    92795.    26116.        41.249     11.263     3.170
  AKI RIVER            164524.   192845.   139002.    30754.         1.172      0.845     0.187
  HIRAI RIVER           27957.   201155.    40851.    13414.         7.195      1.461     0.480
  HAMURA TOSHIGESUIRO   25024.   295016.   408837.    63289.        11.789     16.338     2.529
  HAMURA WEIR          200000.   242000.   188000.    10200.         1.210      0.940     0.051
-------------------------------------------------------------------------------------------------
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Table 7 Observed Water Quality of Tributaries and Drains.

Subbasin	No. Samples	Discharge(m^3/sec)	BOD (mg/l)	Total-N (mg/l)	Total-P (mg/l)
1. Kosugi Drain					
2. Kaminumabe Drain					
3. Todoroki Drain					
4. Yazawa Drain	1 (5)‡	0.19	27.4	8.5	2.64
5. Miyauchi Drain	1 (7)¶	0.17	36.9	7.7	2.32
6. Shimonoge Drain	1 (5)‡	0.04	31.7	15.2	3.37
7. Suwa Drain	1 (7)¶	0.29	61.6	6.6	1.70
8. Futako Drain					
9. Nogawa River	24/6*	1.57(+0.78/−0.52)	13.2(+8.5/−5.2)	10.7 (+2.4/−2.0)	0.96(+0.42/−0.29)
10. Hirase River	1 (7)¶	3.03	17,2	6.8	0.99
11. Seki Drain	1**	0.18	42.6		
12. Unane Drain					
13. Ikata Drain					
14. Noborito Drain	1 (7)¶	0.16	31.3	6.9	1.53
15. Rokugo Drain	1 (5)‡	0.06	15.9	24.9	3.50
16. Chofu Drain	12/6*	0.14(+0.20/−0.18)	32.8(+37.8/−17.4)	13.3 (+13.6/−6.7)	1.56(+1.43/−0.74)
17. Misawa River	1 (7)¶	0.56	22.5	5.9	1.35
18. Yanokuchi Drain					
19. Koremasa Akusui	12/6*	0.52(+0.54/−0.26)	14.3(+10.7/−6.7)	8.26 (+2.19/−1.73)	0.94(+0.30/−0.23)
20. Yata River					
21. Yazakicho Drain					
22. Koremasa Drain	1**	0.23	30.9	5.6	6.3
23. Ooguri River	36/6*	0.66(+0.70/−0.34)	6.9(+2.5/−1.8)	4.10(+0.74/−0.63)	0.36(+0.17/−0.14)
24. Kunitachi Drain	1 (5)‡	0.10	65.1	25.4	4.92
25. Hodokubo River	24/6*	0.31(+0.24/−0.14)	8.2(+5.6/−8.4)	6.33(+1.66/−1.32)	0.76(+0.33/−0.22)
26. Asakawa River	56/36*	2.70(+1.98/−1.14)	11.0(+6.0/−3.9)	8.80(+2.19/−1.75)	0.71(+0.31/−0.22)
27. Hino Drain	12/6*	0.61(+0.33/−0.21)	9.7(+6.8/−4.0)	11.40(+4.08/−3.00)	1.13(+0.44/−0.31)
28. Kunitachi Yaho Drain	1 (5)‡	0.10	32.2	35.0	1.94
29. Midori River					
30. Negawa River					
31. Zanbori River	24/6*	0.90(+0.41/−0.28)	18.1(+15.9/−8.5)	8.52(+1.99/−1.61)	1.21(+0.20/−0.17)
32. Yaji River	24/6*	0.85(+0.23/−0.18)	8.2(+8.7/−4.2)		0.37(+0.63/−0.23)
33. Tamajoryu Drain	1**	0.36	11		0.37(+0.63/−0.23)
34. Miyagawa Drain					
35. Fussa Drain	1**	0.44	10		
36. Aki River	24/6*	2.14(+1.02/−0.72)	1.05(+0.54/−0.36)	1.23(+0.27/−0.22)	0.023(+0.005/−0.004)
37. Hirai River	24/6*	0.31(+0.61/−0.20)	1.23(+0.86/−0.51)	2.77(+0.52/−0.45)	0.045(+0.010/−0.008)
38. Hamura Toshigesuiro	12/6*	0.45(+0.14/−0.11)	37.3 (+23.4/−16.1)	26.3 (+6.9/−5.4)	2.48 (+0.45/−0.38)

reference
* Tokyo Metropolitan Government (1978)[10]
¶ Kawasaki City (1977)[12]
‡ Tokyo Metropolitan Government (1974)[8]
** Ichikawa (1977)[13]

Note: No. of samples, N_1/N_2 (N_3)
N_1: BOD, N_2: nutrients, N_3: samples used to compute dayly average
: Values in parenthesis indicate standard deviation range after logarithmically transformed

marized in **Table 7**. In **Table 6**, Hodokubo River is displayed to be null, because the point of conjuction is the same as Asa River. The data for Hodokubo River were summed and displayed with that for Asa River. The comparison of the two suggests that the computation of some tributaries agree well with the measured data, while some others do not. The larger tributaries, e.g., the Nogawa and the Asa River, show appreciable agreement. Since measurements for the smaller tributaries are conducted only scarcely, a little more amount of data is needed for detailed discussion.

5. SUMMARY

An expression is derived for the water quality of an urban drainage system which involves network of streams. This approach utilizes a theorem that total pollutant transport is reduced to a summation of unit transports from individual sources. Three parameters should be known before its application, i.e., (1) water quantity and quality of each source, (2) drainage paths from the sources, (3) rate parameters including flow rate and removal rate. There are two types of leakage in a drainage system, outlets by various utility and penetrative underflow. These leakages may be introduced to the expression as filters. Streams in a drainage system have hierarchical structure with four components, which are primary drain, secondary drain, tributary and main stream. Latter three are distinguished by topographic maps, and primary drains including gutters and conduits may be estimated by street pattern, for they seldom pass through private sites. Thus theoretically derived organic and nutrient concentration agreed favorably with field data in the Tama River.

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