

## BEHAVIORS OF REINFORCED CONCRETE BEAMS WITH STIRRUPS FAILING IN SHEAR UNDER FATIGUE LOADING

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### SYNOPSIS

In order to investigate the fatigue behaviors of beams with web reinforcement, three series of fatigue tests were carried out. Behaviors of 17 beams, especially changes in strains of stirrups due to repeated loading, were carefully observed. Based on the test results, a new equation for predicting stirrup strain under repeated loading was proposed and a tentative design proposal for stirrups in a beam subjected to fatigue loading was made.

### 1. INTRODUCTION

Lately, design methods for shear have been developed, and stirrups required are apt to be reduced in some cases. This, however, is based on test results under static loading, and it has not yet been clarified whether such methods are applicable to members subjected to fatigue loading, since there is only little information available regarding beams with web reinforcement failing in shear under fatigue loading. Hawkins<sup>1)</sup> reported that under repeated loading failure accompanied by fractures of stirrups could occur at a capacity as little as 21 percent of the theoretical static one. All stirrup fractures occurred where the bars were bent around longitudinal bars. After confirming that the bending operation reduced the stress range at a given fatigue life for a straight bar by 50 percent, Hawkins concluded that the capacity of a beam with stirrups failing in shear under fatigue loading could be predicted by calculating stress range

of stirrup according to classical truss analogy without considering any contribution of concrete. Fractures of stirrups in prestressed concrete beams have also been reported. Price and Edwards<sup>2)</sup> observed that fractures of stirrups took place at some distances from diagonal cracks and the fractures occurred at bends even in the compression flange, about 200 mm away from the diagonal cracks. Hanson et al.<sup>3)</sup> also obtained low fatigue strengths of stirrups in prestressed concrete I-beams, and stated that fractures of stirrups could be attributed to the horizontal openings of the inclined cracks: thus, the stirrups were subjected to transverse shear which caused flexural stresses of sufficient magnitudes to produce fatigue fractures of stirrups.

In order to analyze fatigue fractures of stirrups, a knowledge of strains of stirrups in a beam subjected to repeated loading is essentially needed, but only a few observations have been reported. Kaar and Mattock<sup>4)</sup> observed that strains at the maximum load increased approximately 40 percent with 1-million cycle loading at working stress level. Hawkins<sup>1)</sup> also reported increases of stresses in stirrups from 50 MPa to the yield stress with 100 000 cycles, but he did not give any details. On the other hand, Price and Edwards<sup>2)</sup> reported that in their prestressed concrete beams little changes in strains were found under 1 000 cycles of loading in spite of the considerable increases in diagonal crack widths. It can be considered that no systematic observation on strains of stirrups under fatigue loading has been reported.

In order to investigate the fatigue behaviors of beams with web reinforcement, three series of fatigue tests were carried out. Behaviors of 17 beams, especially changes in strains of stirrups due to repeated loading, were carefully observed. Based on the test results, a new equation for predicting stirrup strain under

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repeated loading was proposed and a tentative design proposal for stirrups in a beam subjected to fatigue loading was made.

## 2. TEST PROGRAM

All seventeen beams had the same rectangular cross section, 250 mm in height and 300 mm in width, and were tested at a span of 1320 mm. The shear span effective depth ratio was kept constant at 2.5, which meant that any stirrup provided would be in a region where the favorable effect from a support or loading point exists. Vertical stirrups were used as shear reinforcement. The spacing of stirrups was not constant in one series of beams as shown in Fig. 1, considering the effects of the supports or loading points<sup>5)</sup>.

The main parameters were the longitudinal reinforcement ratio or amount of stirrups and the pin diameter in bending of stirrups. The outline of the test program is shown in Table 1. Tests were divided into three series. Each series had five or six beams, which were similar except in differences in concrete strengths. Only the

applied maximum load was varied within a series.

All bars in the tests had two longitudinal ribs and parallel transverse lugs perpendicular to bar axis, with the transverse lugs joining the longitudinal ribs. From results of tensile tests, the Young's moduli and yield points of these bars were obtained as shown in Table 1 using the nominal cross-sectional areas.

Three batches of ready mixed concrete were used: one for the first three beams in Series I and II, one for the remaining beams in the two series and one for Series III. The maximum size of coarse aggregate was 20 mm. The strengths of the concretes at the age when the tests of the corresponding beams were started are indicated in Table 1. The ages were between 60 and 120 days.

The apparatus for the fatigue tests consisted of a steel frame and a 50 ton hydraulic jack connected to a pulsator. The minimum load in the tests was kept constant at 39 kN, which corresponded to the nominal shear stress of 0.30 MPa. The maximum load was varied in a manner to get an appropriate S-N curve for each

Table 1 Outline of test program.

Series	Specimen		Concrete $f_c'$ MPa	Stirrup					Longitudinal Bar			
	Fatigue	Static		Pin Diameter mm	$\phi$ mm	$A_w$ mm <sup>2</sup>	$f_{wy}$ MPa	$E_s$ 10 <sup>8</sup> MPa	$\phi$ mm	$A_s$ mm <sup>2</sup>	$p$ %	$f_y$ MPa
I	25F1-3	25S1	18.6	65	13	253	324	176	25	2027	3.07	373
	25F4-6	25S2	35.2									
II	19F1-3	19S1	18.6	25	10	143	379	170	19	1146	1.73	377
	19F4-6	19S2	35.2									
III	19F5-11	—	30.5	50								

$f_c'$ : cylinder strength,  $\phi$ : diameter,  $A_w$  or  $A_s$ : cross-sectional area,  $f_{wy}$  or  $f_y$ : yield strength,  $E_s$ : Young's modulus,  $p$ : steel percentage

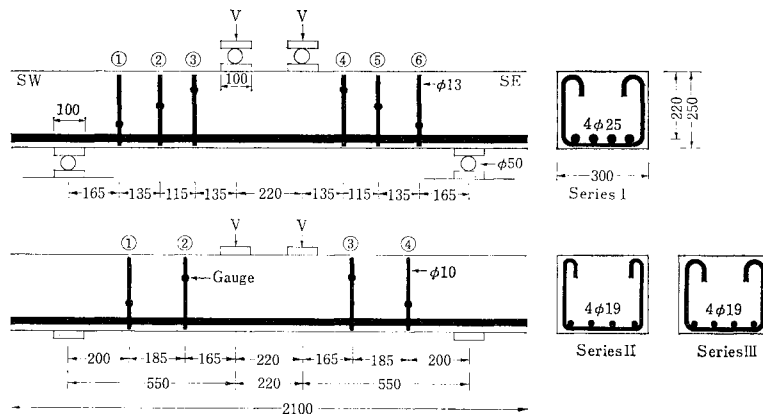


Fig. 1 Test beams and gauge arrangement.

series. The tests were carried out in the following order.

(1) A beam was loaded statically to the maximum load.

(2) The beam was subjected to 999 additional cycles of statically applied loading between the minimum load and the maximum. This was done to get the exact numbers of loading cycles for the corresponding strains of stirrups. At suitable intervals gauge readings and crack propagations were recorded.

(3) Repeated loading of the same magnitude was applied at the rate of 200 cycles per minute. The repeated loading was stopped periodically to measure gauge readings at the maximum and minimum loads and to record crack propagations.

(4) Beams which endured two million cycles of loading were loaded statically to failure (final static test).

Electrical-resistance strain gauges 5 mm in length were attached to all stirrups on the surfaces of the south legs at the positions of the predicted diagonal cracking lines as shown in Fig. 1. 'South' indicates the test position of a beam shown in the figure. Strain gauges were also attached to all stirrups just at the two ends of bends around longitudinal bars.

### 3. BEHAVIORS OF BEAMS UNDER FATIGUE LOADING

In order to provide a better understanding of the general behavior of beams under fatigue loading, details of behaviors of typical beams are presented in the first place. Fig. 2 shows the strain measurements on the south leg of each stirrup together with the crack patterns on the south surfaces for the beams 25F4, 25F5 and 25F6, which comprise the last half of Series I.

#### Beam 25F4

At the first cycle, one major inclined crack had developed in each shear span, which caused 2 to  $3 \times 10^{-4}$  of strain in Stirrups 2, 5 and 6. However, these cracks had not developed enough to cause some strain in the other stirrups.

Until some ten thousand cycles, the strains of Stirrups 2 and 6 increased approximately in proportion to the logarithms of loading cycles,  $\log N$ , while the strain of Stirrup 5 was more or less constant and strains of other stirrups remained small. This coincided with the propagation of inclined cracks. Vertical tensile stresses due to the propagation of the main diagonal cracks were mainly absorbed by Stirrups 2 and 6, which were intersected by these cracks. After ten thousand cycles, increasing of strains in these stirrups stopped and in return strains of Stirrups 1, 3 and 4 increased at higher rates.

This also coincided with the propagation of diagonal cracks crossing these stirrups.

Although the applied maximum shear was only 44 percent of the static strength, a fracture of a stirrup was found at the 300 000 cycle or at  $\log N$  of 5.52. The fracture occurred at the start of the bend of the south leg of Stirrup 6, the strain of which was the highest in the beam. This fracture caused further extension of the main diagonal crack, formed a new inclined crack outside the main one, and increased the strain of Stirrup 5 significantly. This stirrup fractured at  $\log N$  of 6.13. The position of the fracture was the start of the south hook where the newly formed crack intersected.

In the west shear span, a fracture of a stirrup was found at  $\log N$  of 5.73. The fracture occurred at the start of the bend of the north leg of Stirrup 2, which showed the highest strain in this shear span. This fracture caused a rapid increase in strain of the south leg. This indicated that the south leg acted effectively even after the fracture of the north leg, since the relatively wide width of the beam made it possible to maintain anchorage of the remaining leg. Loss of the force due to fracture of one leg seemed to be absorbed by the other leg, since strains of other stirrups and diagonal cracks on the south surface were not affected by the fracture.

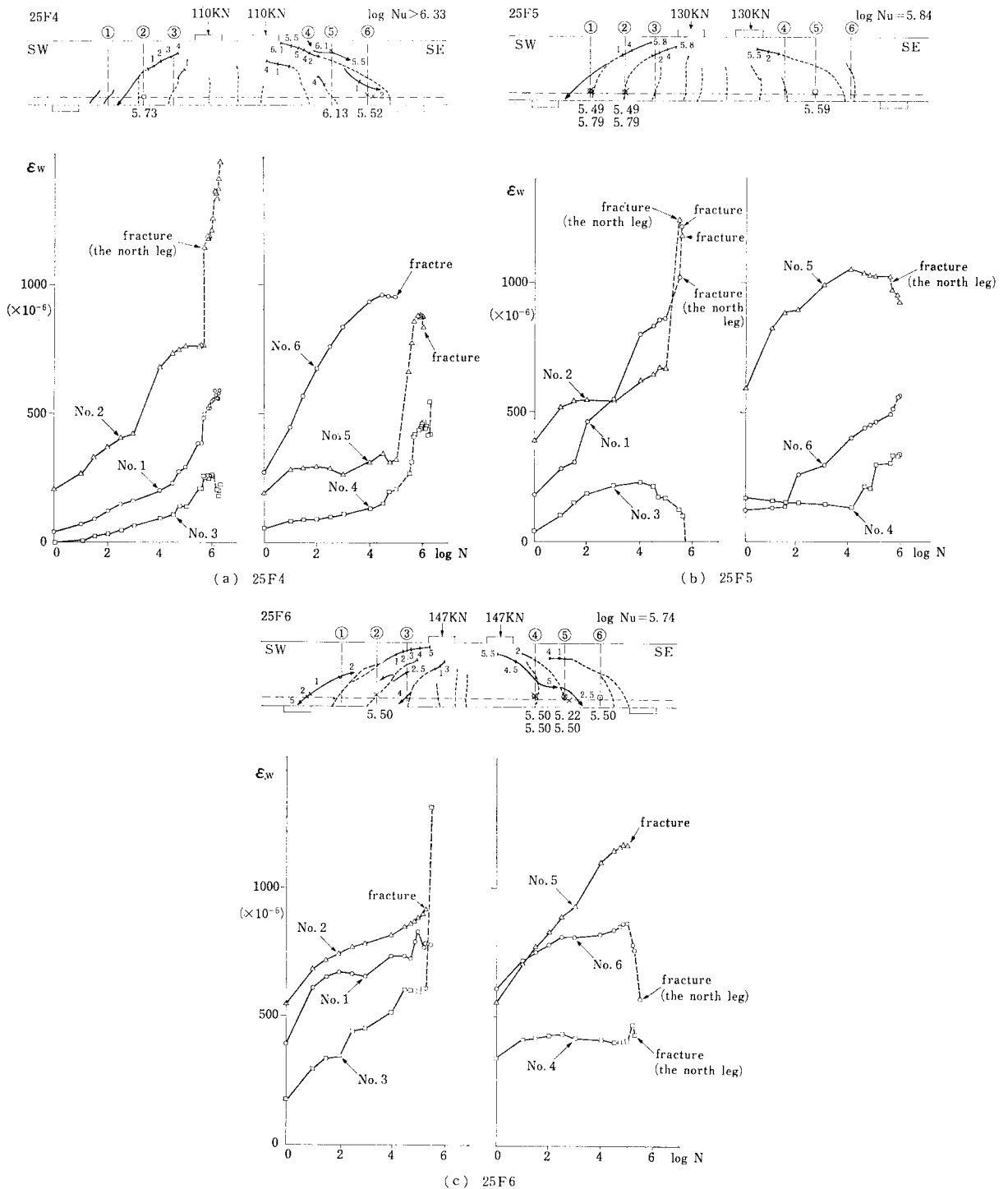
A fracture of a longitudinal bar adjacent to the first fractured stirrup was also found when concrete was removed after the end of the test as shown in Fig. 3. In this figure fractures of Stirrups 5 and 6 can also be seen.

In spite of the fractures of one leg each of two stirrups out of three and a longitudinal bar in the same shear span, this beam endured 2.14 million cycles of loading or  $\log N$  of 6.33 without failure, and under the final static loading it failed in the shear span at the shear force of 186 kN. This was about 1.7 times the applied maximum shear and 74 percent of the static strength.

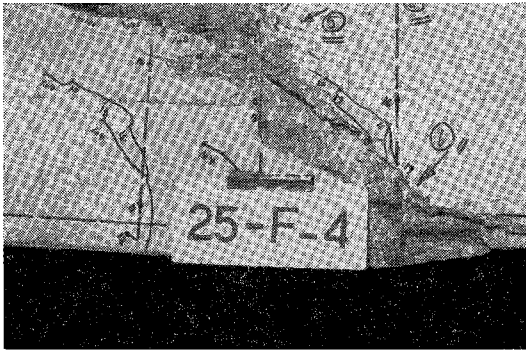
#### Beam 25F5

At the first cycle two inclined cracks were developed in the west shear span and one in the east. The development of these cracks caused the strain in Stirrup 2 or 5 to reach 4 or  $6 \times 10^{-4}$ , while those in others, however, remained below  $2 \times 10^{-4}$  at the first cycle.

Due to repeated loading, the strains of Stirrup 1 increased with extension of the outer diagonal crack at a higher rate approximately in proportion to  $\log N$  until the fracture of its north leg occurred. On the contrary, the strains in Stirrups 2 and 3 increased at a lower rate as the inner diagonal crack was extended slightly. In the east shear span the strain of Stirrup 5 in-



**Fig. 2** Crack propagation and strains of each stirrup  $\epsilon_w$  observed during fatigue loading (dotted line indicates the crack developed at the first cycle, solid line shows the crack propagation due to repeated loading with the number indicating the logarithms of applied loading cycles, broken line represents the center line of reinforcing bars, and circle or cross on the line indicates the fracture position with the logarithm number at which the fracture was recorded and the circle means north and the cross means south).



**Fig. 3** Fatigue fractures of stirrups and a longitudinal bar (25F4).

creased gradually until 10 000 cycles due to the slight extension of the diagonal crack, while the strain of Stirrup 4 remained nearly constant. After 10 000 cycles the strain in Stirrup 4 began to increase as the extension of the diagonal crack started to affect this stirrup, and in return the strain in Stirrup 5 began to decrease slightly. Thus, even without any fracture of bars, redistribution of stresses among the stirrups could occur owing to the extension of diagonal cracks under repeated loading. This kind of redistribution was also seen in the previous beam 25F4.

In the east shear span a fracture was found at  $\log N$  of 5.59. The fracture occurred in the north leg of Stirrup 5, which showed the highest strain in the beam. This fracture did not cause any significant increases of strains in the south legs of other stirrups and there were no significant extensions of cracks on the south surface and no more fractures of stirrups in this shear span were found until failure of the beam.

In the west shear span two fractures in north legs were found at  $\log N$  of 5.49. These fractures in Stirrups 1 and 2 caused rapid increases in strains of their south legs, but affected neither the strain of Stirrup 3, nor the crack propagation of the south surface as seen in beam 25F4. Fractures in the south legs of these stirrups were found at  $\log N$  of 5.79, 300 thousand cycles after the fractures of the north legs. These fractures caused significant extensioning of the diagonal cracks on the south surface as clearly seen in Fig. 2(b).

In the shear span on this side, only one stirrup out of three was effective. Yet this beam was able to sustain an additional 85 thousand cycles and failed at  $\log N$  of 5.84 due to the penetration of the outer diagonal crack in this shear span through the compression zone.

### Beam 25F6

At the first cycle inclined cracks in this beam developed enough to cause strains more than about  $2 \times 10^{-4}$  in four of the stirrups, and the differences among the four were not significant while strains of Stirrups 3 and 4 were smaller. The maximum applied shear of this beam was the highest in the group. This high shear seemed to have been capable of producing three inclined cracks in each shear span.

Due to repeated loading the strains of Stirrups 1, 2, 3 and 6 increased at approximately the same rate, but the strain of Stirrup 5 increased at a higher rate and the first fracture was in the south leg of this stirrup at  $\log N$  of 5.22. After an additional 151 thousand cycles fractures of four legs of three stirrups were found. Nevertheless, this beam was able to sustain an additional 234 thousand cycles of loading and failed at  $\log N$  of 5.74. In this beam a longitudinal bar was also fractured just beside the first fractured stirrup as in Beam 25F4. After the fracture of the south leg of Stirrup 5, no changes occurred in the strains of the south legs of other stirrups. However, increases of strains should have occurred in the north legs of Stirrups 4 and 6, judging from the fact that these legs were fractured later. After the fracture of the north leg of Stirrup 6, the strain of the south leg did not show any increase but decreased. This might indicate that the fracture of one leg could reduce the efficiency of the other leg due to redistribution of stresses induced by the fracture.

Fracturing of Stirrup 4 occurred at both legs in spite of the rather low strain. The reason was not clear, but one possibility was that in this stirrup the strain at the position of fracture, which was the lower bend, might have been larger than at the measured position which was distant from it as shown in Fig. 1. Another possibility was that the strain of the north leg was larger so that it might have failed first, but then the strain of the south leg increased very rapidly before the fracture was found.

The general behaviors of all the test beams can be summarized as follows.

(1) At the first cycle of loading inclined cracks were generally recognized. When the applied maximum shear was relatively low, only one or two stirrups in a beam developed some amounts of strain but others did not. In these cases development of such inclined cracks was not prominent at the first cycle. When the applied maximum shear was higher, the number of stirrups which developed some amounts of strain increased, and the differences in strains among stirrups became relatively smaller. These

beams tended to have more inclined cracks developed to greater extents as seen in Beam 25F6.

(2) Under the effect of repeated loading the inclined cracks formed at the first cycle increased gradually in width and extended upward or downward. But, new diagonal cracks were seldom formed in shear spans where main diagonal cracks had already been formed. With increases of the inclined cracks in widths and/or lengths, strains of stirrups intersected by the cracks increased approximately in proportion to  $\log N$ . The rate of increase, however, was not always the same among the stirrups, and the strains of some stirrups remained almost constant if the inclined crack intersected did not propagate. From this observation, it can be said that the strain of each stirrup depends mainly upon the development of diagonal cracks intersected. Any increase of strain affects the crack propagation again, and thus a close relation exists between crack propagation and increase of stirrup strain during fatigue loading.

(3) After a considerable number of cycles a stirrup often started to increase its strain affected by extension of some diagonal cracks. In these cases at least one of the other stirrups always stopped increasing in strain. Thus, even without any fracture of a stirrup, redistribution of stresses among stirrups occurred, and the total strains or the average strain in a beam continued to increase at an almost constant rate against the logarithms of loading cycles, although individual stirrups showed great deviations.

(4) Due to repeated loading fractures of stirrups occurred in all test beams without exception even in beams where strains of stirrups were very low and below  $2 \times 10^{-4}$  at the first cycle. In all, forty two legs were fractured, and 41 legs were fractured at the same position where the legs started to be bent. One exception was at the start of a hook in Beam 25F4. In these tests the applied maximum shear ranged between 44 and 62 percent of the static strengths, which were lower than the tested static strengths. This indicated that fracturing of stirrups should be considered in design of beams subjected to severe fatigue loading.

(5) The first fracture of a stirrup in each beam was found between the 6 300 and 2 140 000 cycles (see **Table 3**). The fracture was usually noticed from strain measurements on stirrups. The first fractured stirrup was in general the one that developed the highest strain in the beam. The first fractured stirrups in Series I were the outer stirrups (No. 1 or 6) in five beams and the middle stirrup in one beam. There was no first fracture at an inner stirrup where the strain was

considerably lower than in other stirrups. In Series II and III, the first fractures occurred in outer stirrups in all eleven beams, and in one beam fracture of an inner stirrup was found at the same time. Stresses of the outer stirrups were usually greatest. This could of course be the reason why the first fractures occurred at the outer stirrups. However, the main diagonal crack usually intersected the outer stirrup at a position very near to the fractured position. Therefore, the stress at the fractured position can be considered to be the largest one in the stirrup. This could be another reason.

(6) As the fracture of a leg imposed a further burden on the beam, failure could occur unless the applied shear force was small enough for the beam to sustain. Three beams, 25F3, 19F1 and 19F5 failed just after the fracture of one leg. These beams were the ones where the applied shear was the highest in their groups. In each of the other beams, the remaining capacity could sustain the force induced by the redistribution occurring after the fracture of one leg, and the beam could endure further cycles of loading. There were six beams where fractures of stirrups occurred later in the other shear spans. These were also in the category of the first fractured stirrup in the shear span. In ten beams, however, fractures did not occur in the other shear spans. In other words, considerable deviations in fracturing of stirrups were recognized even within the same beam. This was partially because the height of beam was relatively small and  $a/d$  ratio was small, and thus the spacing of stirrups became larger compared with that of diagonal cracks. Considerable difference in the stress acting on each stirrup was recognized according to the relative relation of the position of diagonal cracks and the stirrup.

(7) When a fracture of one leg occurred, the strain of the other leg or the leg of the adjacent stirrup increased exceedingly, and the next fracture usually occurred in either of the legs after some thousands of cycles (see **Table 3**). The beam would fail if with the remaining legs it became unable to sustain the applied maximum shear. A considerable number of loading cycles was usually required for the failure of a beam after the last fracture of a stirrup. After the fracture of one leg, Beams 19F8 and 19F9 endured 1 600 cycles and 681 thousand cycles, respectively. Beam 19F2 endured an additional 150 thousand cycles after the loss of two legs in the same shear span. Beams 19F7 and 19F6 also endured 1 600 cycles and 77 thousand cycles, respectively, even after the loss of three legs out of four, and Beams 25F5 and 25F6 were able to endure 85 thousand and 234 thousand cycles

more after the loss of four legs and five legs out of six in one shear span, respectively. Seven beams endured two million cycles in spite of the fracture of stirrups. One of the reasons, why the test beams could endure many cycles of loading after losing some legs, may be the fact that a considerable part of the applied shear could be transmitted by the so called arch action because of the small  $a/d$  ratio of 2.5. Actually, the final static tests of seven beams showed that three beams in Series I with fracturing of two legs had capacities more than 77 percent of the flexural capacities and two beams in Series II had capacities more than the calculated flexural strengths even with the loss of three legs out of four in the same shear spans (See **Table 2** and **3**).

(8) Fracturing of longitudinal bars was also found in three beams. The fracture always occurred just beside the first fractured stirrup leg. In Beam 19F6, two outer longitudinal bars were fractured on fracturing of both legs of Stirrup 4. Fracturing of the stirrup caused the longitudinal bars to be subjected to greater dowel actions in addition to the axial tensile stresses.

#### 4. STRAIN OF STIRRUP UNDER FATIGUE LOADING

Since fracturing of stirrups due to fatigue loading occurred even in beams where strains were only  $2 \times 10^{-4}$  at the first cycle, the strain or stress increase during fatigue loading was carefully examined. Based on an analysis of strain measurements in detail, a new method for predicting the average strain or stress of stirrups under fatigue loading was proposed.

##### (1) Strain of stirrup at the maximum load

Although the behavior of each stirrup under fatigue loading was fairly complicated as described in Section 3, the strain usually tended to increase with the number of loading cycles. There are many causes for increase of stirrup strain and extension of diagonal cracks. Deterioration of aggregate interlocking action along a crack should increase the strain of a stirrup. Deterioration of bond between a stirrup and concrete will increase the total elongation of the stirrup and this will produce extension of diagonal cracks and then increase the strains of stirrups. Deterioration of the bond will increase the strain at the gauge point if the maximum strain along the stirrup is obtained at a different point. Creep of concrete during a test may increase the strains of stirrups. **Fig. 4** shows the relationships between the average strain  $\epsilon_w$  of all stirrups in a beam at the maximum load and the logarithms of loading cycles,  $\log N$ . The average strains in

**Table 2** Properties of specimens.

Series	Specimen		(1)	(2)	(3)	(4)	(5)
	Static	Fatigue	$V_c$ kN	$V_y$ kN	$V_f$ kN	$V_u$ kN	$\frac{V_u}{V_f}$
I	25 S1	25 F1-3	91.2	276	191	221	1.16
	25 S2	25 F4-6	93.9	278	243	252	1.04
II	19 S1	19 F1-3	77.3	159	136	162	1.19
	19 S2	19 F4-6	84.4	167	154	157	1.02
III	—	19 F7-11	78.8	161	151	—	—

(1)  $V_c$  denotes tested value derived from  $V-\epsilon_w$  relation of static specimen and fatigue specimens at the first cycle.

(2)  $V_y = V_c + A_w f_{wy}(z/s)/\beta_x$

(3)  $V_f = A_s f_y d(1 - 0.6 p f_y / f_c') / a$

(4)  $V_u$  denotes tested value.

Series I apparently increase approximately in proportion to  $\log N$  as shown in **Fig. 4** (a), (b), although each stirrup shows complicated behavior depending upon the development of diagonal cracks intersected. It is also clearly recognized from these figures that the rate of increase in the strain of a stirrup is not dependent on the strain at the first cycle, but is almost the same in spite of differences in the applied maximum shear forces. This seems strange but can be explained by the following idea.

Since it may be reasonably considered from Reference 5) that the total shear force  $V$  will be carried by  $V_s$  and  $V_c$ , where  $V_s$  is the shear force carried by the assumed truss with  $45^\circ$  diagonals and  $V_c$  is the one carried by other means than the truss, the following equation is obtained under the maximum shear force  $V_{max}$  in the fatigue test.

$$V_s = V_{max} - V_c \quad \dots\dots\dots(1)$$

The shear force  $V_c$  can be obtained by extending the shear-stirrup stress line toward the shear axis. The measured values of  $V_c$  are shown in **Table 2**. They could be very near to the shear when a critical diagonal crack is initially developed. Equation (1) is, however, for the part of a beam where the influences of the supports or loading points are negligibly small. For the part where these influences do exist, the shear force  $V_s$  carried by the truss is lightened as follows<sup>5)</sup>.

$$V_s = \beta_x (V_{max} - V_c) \quad \dots\dots\dots(2)$$

where  $\beta_x$  is a coefficient determined by the relative positions of the stirrup to the supports or loading points, and

$$\left. \begin{aligned} \beta_x &= x/(1.5d) & : & x < 1.5d \\ &= 1 & : & x \geq 1.5d \end{aligned} \right\} \quad \dots\dots\dots(3)$$

$x$ : distance from support or loading point

Table 3 Results

Series	Specimen	Maximum Load				First Fractured Stirrup					
		$V_{max}$ kN	$\frac{V_{max}}{V_{co}}$	$\frac{V_{max}}{V_u}$	$\frac{V_{max}}{V_y}$	Stirrup (1)	$N_f$		$\sigma_{wr}$		(4) (5)
							(2) $\times 10^4$	(3) $\times 10^4$	(4) MPa	(5) MPa	
I	25F1	98.0	1.07	0.45	0.36	1S/1N	192.5	214.0	86.8	65.9	1.32
	25F2	122.5	1.34	0.57	0.44	1N	124.0	133.0			
	25F3	137.2	1.50	0.62	0.50						
	25F4	110.3	1.17	0.44	0.40	2N	51.5	54.0	110.6	77.9	1.42
	25F5	129.9	1.38	0.52	0.47	1N/2N	9.0	31.0	104.6	102.2	1.02
	25F6	147.0	1.57	0.58	0.53	2S	20.0	31.7	93.8	133.9	0.70
II	19F1	98.0	1.27	0.60	0.62	1N	0.1	0.63	(8)	136.2	
	19F2	83.3	1.08	0.51	0.52						
	19F3	83.3	1.08	0.51	0.52	1S	160.0	183.2	(8)	139.6	
	19F4	83.3	0.99	0.53	0.50						
	19F5	93.1	1.10	0.59	0.56	1S	1.0	2.5	209.8	117.8	1.78
	19F6	88.2	1.05	0.56	0.53	1N	20.5	49.0	47.0	126.2	0.37
III	19F7	98.0	1.24	—	0.61	1S/1N	1.36	2.2	155.8	155.0	1.01
	19F8	88.2	1.12	—	0.55	1S	3.86	4.65	156.8	124.5	1.26
	19F9	78.4	0.99	—	0.49	1S	2.5	4.6	142.0	83.9	1.69
	19F10	68.6	0.87	—	0.43						
	19F11	68.6	0.87	—	0.43						

- (0)  $V_{min}=19.6$  kN
- (1) S and N denote south and north legs, respectively.
- (2) Number of cycles at time of previous inspection
- (3) Number of cycles at which fracture found
- (4) Measured value
- (5) Calculated value

The  $V_c'$  expressed as in Equation (4) is the shear force transferred to the support or the loading point directly. That is,  $V_c'$  is transferred without a chord of truss mechanism, and this should be distinguished from the  $V_c$ .

$$V_c' = (1 - \beta_x)(V_{max} - V_c) \dots\dots\dots(4)$$

Then the following equation is finally derived.

$$V_{max} = V_s + V_c + V_c' \dots\dots\dots(5)$$

where,

- $V_s = A_w E_s \epsilon_w z / s$  for vertical stirrups
- $A_w$ : area of pair of stirrups
- $E_s$ : Young's modulus of steel
- $\epsilon_w$ : strain of stirrup
- $z$ : distance from centroid of compression to tensile force
- $s$ : spacing of stirrups

Under fatigue loading the value of  $V_c$  in Equation (5) is not considered as constant but should be decreased with the increase of loading cycles. When the decrease of  $V_c$  is assumed as proportional to  $\log N$  as indicated by this experiment, the following equation is obtained.

$$\left. \begin{aligned} V_c &= V_{co} - V_{co} k \log N \\ &= V_{co}(1 - k \log N) \end{aligned} \right\} \dots\dots\dots(6)$$

where,  $V_{co}$  is the value of  $V_c$  at  $\log N$  of 0 or  $N$  of

1, and  $k$  is a constant which indicates the rate of decrease of  $V_c$ . When the maximum shear force  $V_{max}$  is constant, the value of  $V_s$  should increase with decrease in the value of  $V_c$  as indicated in Equation (2). Finally, the strain of the stirrup at maximum load is expressed by the following equation.

$$\epsilon_w = \frac{\beta_x \{V_{max} - V_{co}(1 - k \log N)\}}{A_w E_s z / s} \dots\dots\dots(7)$$

The values calculated by this equation are also shown in **Fig. 4**. In this calculation  $k$  is taken to be constant at 0.07, which is consistent with the results of fatigue tests on beams without web reinforcement<sup>6)</sup>, and the measured values of  $V_c$  indicated in **Table 2** are used. The value of  $\beta_x$  is assumed to be the same as under the static loading indicated by Equation (3). In spite of the bold assumptions the calculated values seem to be able to follow the measured values fairly well.

Beams in Series II and III were divided into two groups according to the level of the maximum load applied (see **Table 3**). When the value of  $V_{max}$  was larger than that of  $V_{co}$ , the increase in stirrup strain due to repeated loading was similar to that in Series I, although the rate of increase was apparently higher as shown in **Fig. 4** (c), (d), (e). This higher rate can be ex-



of fatigue tests.

First Fractured Stirrup						Others			$N_u$ (6) $\times 10^4$	$V_u$ (7) kN
East Span						Stirrup (1)	$N_f$			
Stirrup (1)	$N_f$		$\sigma_{wr}$		(4) (5)		(2) $\times 10^4$	(3) $\times 10^4$		
	(2) $\times 10^4$	(3) $\times 10^4$	(4) MPa	(5) MPa						
6S	69.1	74.0	93.1	101.2	0.92	5S	124.0	133.0	274.3	196.0
6S	10.0	21.12	128.6	117.3	1.10				214.0	191.1
6S	10.0	33.0	114.1	71.1	1.61	5S(10)	114.0	135.0	21.12	
5N	31.0	39.0	124.7	107.5	1.16	1S/2S	39.0	61.0	214.0	186.2
5S	10.0	16.6	118.0	130.9	0.90	4S/4N/5N/6N	20.0	31.7	69.5	
									55.1	
4S/4N	10.0	31.0	161.9	(9)					0.63	
3N/4S/4N	183.2	213.2	130.7	140.7	0.93				46.0	
3N/4S	155.8	210.7	135.6	129.1	1.05				213.2	151.9
									210.7	161.7
3N	10.0	49.0	145.8	96.8	1.51	4S/4N	90.0	163.0	2.5	
									123.0	
						2N	2.2	2.85	3.01	
4S	4.6	10.0	124.1	89.0	1.39				4.81	
4S	84.0	94.4	114.8	107.7	1.07				78.1	
4S	64.8	85.4	87.6	105.6	0.83				235.0	132.3
									205.0	129.9

- (6) Failure life of beam
- (7) Final static test
- (8) Unmeasurable case
- (9) Uncalculated case
- (10) Fracture at the start of its hook

plained by Equation (7). According to the equation, the ratio of  $V_{co}/V_s$  is the main factor influencing the rate of increase in stirrup strain, and the ratio in these series is larger because the web reinforcement ratio is smaller. The calculated values according to Equation (7) seem to coincide with the measured ones in these series also as shown in Fig. 4 (c), (d), (e), although the divergences are large.

When the value of  $V_{max}$  was less than  $V_{co}$ , the increase of stirrup strain was very small until a certain number of loading cycles, and then a rapid increase was recognized as shown in Fig. 4 (f), (g). In such cases, Equation (7) cannot be applied but should be modified. The number of cycles  $N_c$  when the rapid increase in stirrup strain starts might be the one when a critical diagonal crack appears. This is supported by observations. And at  $N_c$ , the shear force when typical cracking appears is equal to the applied maximum shear force  $V_{max}$ . If the value of  $V_c$  when the number of loading cycles reaches to  $N_c$  is assumed to be equal to  $V_{max}$ , and the rate of decrease of  $V_c$  after the diagonal crack appears is assumed to be the same as in Equation (6), the following equation is obtained.

$$V_c = V_{max} - V_{co} k \log(N - N_c) \dots \dots \dots (8)$$

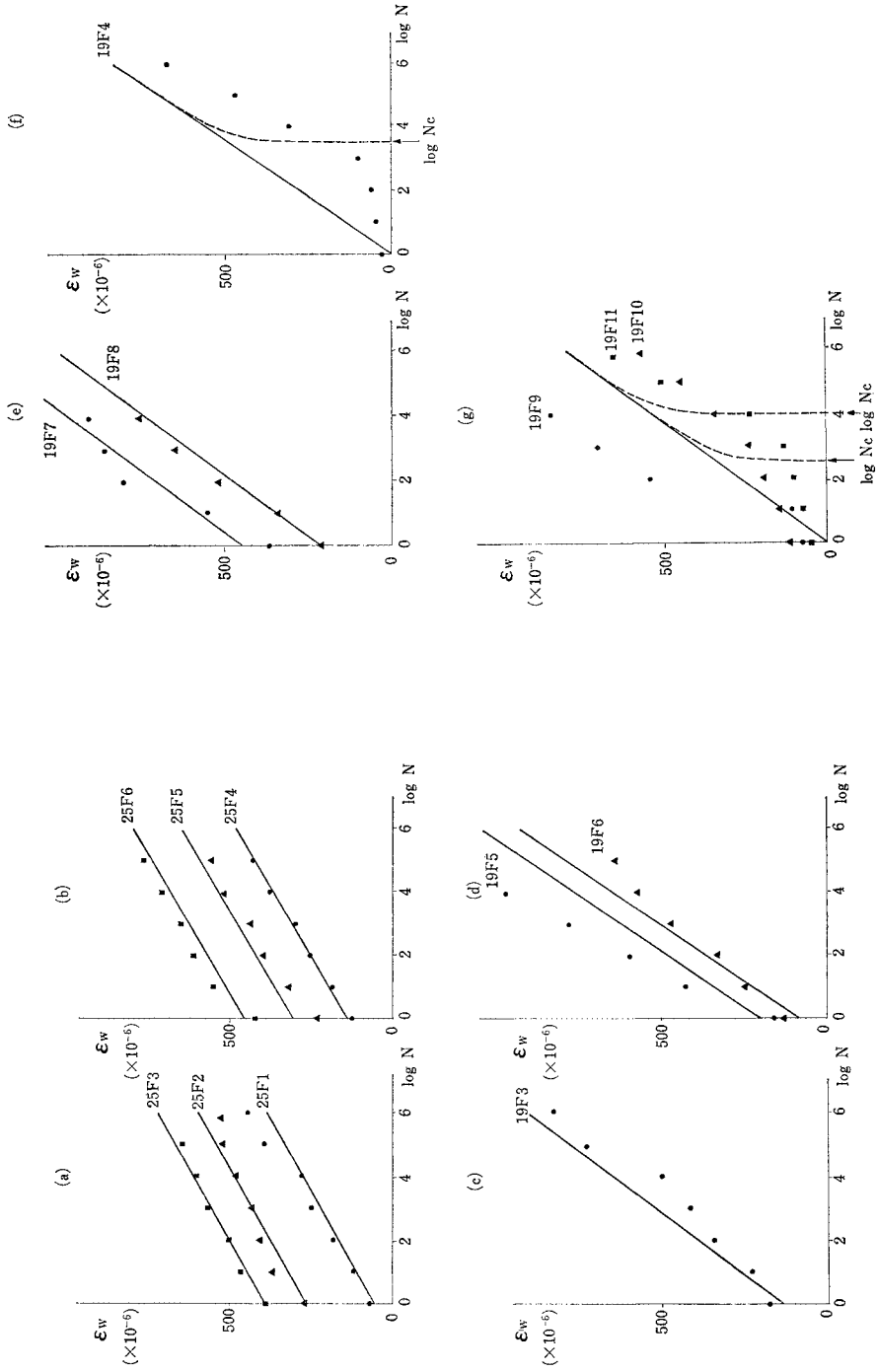
That is, in Equations (6) and (8), the amounts of decrease of  $V_c$  are proportional to logarithms of the numbers of loading cycles after diagonal cracking. From Equations (2) and (8), stirrup strain is expressed as follows.

$$\epsilon_w = \frac{\beta_x V_{co} k \log(N - N_c)}{A_w E_s z / s} \dots \dots \dots (9)$$

The values calculated by this equation, using the observed value of  $N_c$ , are shown with broken lines in Fig. 4 (f), (g). This equation can follow the observed rapid increase of stirrup strain after the loading cycles of  $N_c$ , but the value of  $N_c$  cannot be accurately estimated at the moment. For practical purpose  $\log(N - N_c)$  can be replaced with  $\log N$ , because designing fatigue lives of stirrups usually larger than  $10^6$  cycles and in such a case the value of  $\log(N - N_c)$  becomes very near to the value of  $\log N$  when  $N$  becomes larger than two or three times of  $N_c$ . In spite of lack of experimental data, the rate of decrease of  $V_c$  can be expected to be not more than  $V_{co} k \log N$ . Therefore, the following equation can be used at the moment for design of fatigue failure of stirrups.

$$\epsilon_w = \frac{\beta_x V_{co} k \log N}{A_w E_s z / s} \dots \dots \dots (10)$$

The lines calculated by this equation are shown



**Fig. 4** Measured stirrup strain  $\epsilon_w$  at maximum load under fatigue loading—straight lines indicate values calculated by Equation (7) or (10) and broken lines indicate values calculated by Equation (9).

with solid lines in Fig. 4 (f), (g).

(2) Stress range of stirrups

As the stress range, the difference between the maximum and minimum stresses, is the dominant factor influencing the fatigue lives of reinforcing bars, it is necessary to derive a proper method to calculate the minimum stress, since the maximum stress can be calculated according to Section 4 (1).

Fig. 5 shows an example of the relationship between the applied shear force and the measured stirrup strain under the final static test together with the one at the first cycle. These data are obtained from the shear span where no stirrup fractures have been found after 2 million cycles of fatigue loading. Point A indicates the average strain of stirrups at the maximum load of the first cycle, and Point B shows the one after 2 million cycles. The wave line between the two shows the increase of stirrup strain during the fatigue test. Line (1) shows the relation between  $V_{max}$  and  $\epsilon_w$  derived from Equation (7), taking  $N$  as 1, and the broken line (2) derived from the equation, taking  $N$  as 2 million. This means that Line (1) shows the relation between  $V_{max}$  and  $\epsilon_w$  at the first cycle, and the broken line (2) shows how much  $\epsilon_w$  increases during 2 million cycles of loading. Line (3) is the straight line between Points B and D, which indicates the residual strain. According to the final test the measured strains of stirrups are at the vicinity of Line DB except near Point D, and are on Line (1) from Point C, where Line DB crosses Line (1). The fact that the strains of stirrups are on Line (1) shows that the behavior of stirrup strain is the same as at the first cycle, so after 2 million cycles of loading the behavior of stirrups is to be the same as at the first cycle above a certain load level. Although the explanation in Section 4 (1), that the loss of  $V_c$  under fatigue loading corresponds with the increase of stirrup stresses, seems to be contradictory to the

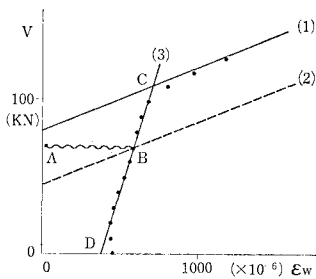


Fig. 5 An example of relationship between shear force  $V$  and measured stirrup strain  $\epsilon_w$  at final static test.

above observation, it can be explained clearly. Line (3) is considered to correspond to a certain situation of a diagonal cracking propagation, and the stirrup strain should be on the line in spite of the magnitude of load. Therefore, the points B, C and D indicate the same condition of diagonal crack propagation, and diagonal cracks are not extended until Point C is attained. Above Point C, diagonal cracks are extended newly. In Section 4(1) the term 'loss of  $V_c$ ' was used, but actually, 'a decrease in percentage of  $V_c$  shared' occurs. In this sense, under static loading the percentage of  $V_c$  shared is continuously decreasing with increase in load, and essentially, there can be the same phenomenon under fatigue loading.

From the observation of Fig. 5 the minimum strain of a stirrup under fatigue loading can be assumed roughly to be on Line (3). The strain, however, does not seem to decrease near Point D. This is one of the problems which this paper cannot deal with. Another problem is the calculation of Point D which indicates residual strain. For the moment, it is boldly assumed that Point D is in accordance with the original point, that is, the residual strain is zero. It is easily understood that with this assumption the minimum strain is evaluated lower, and the stress range is on the safer side. Fig. 6 is an idealization of Fig. 5 using this assumption. Based on this assumption the following equation is derived.

$$\sigma_r = \sigma_{max} - \sigma_{min} = \frac{V_{max} - V_{min}}{V_{max}} E_s \epsilon_w \dots\dots(11)$$

where,  $\epsilon_w$  is obtained from Equation (7) or (9). Fig. 7 shows comparisons of calculated strain ranges with the measured ones. As they are considered based on the above assumption the calculated values are generally lower than the measured ones. The measured values in the figure are the average values of a shear span of a specimen.

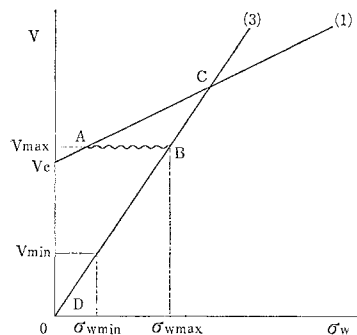


Fig. 6 Idealized relationship between shear force  $V$  and stirrup stress  $\sigma_w$  under the first and the final static test.

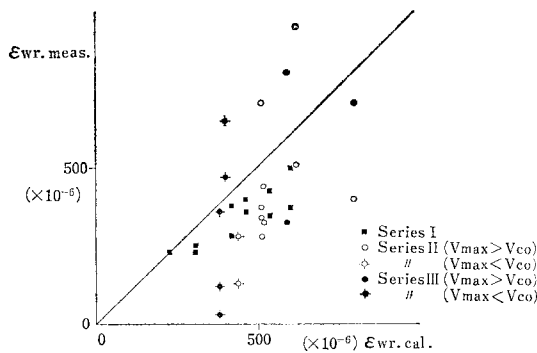


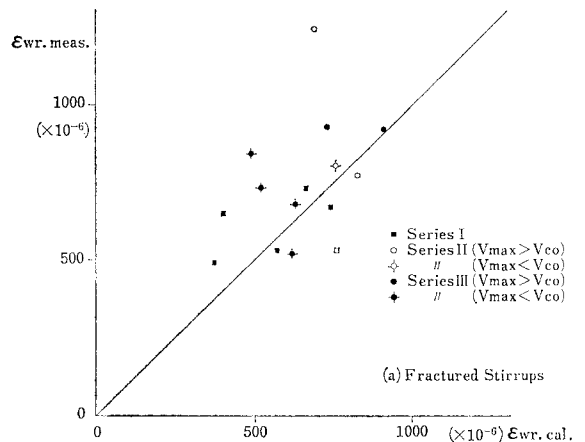
Fig. 7 Comparison between measured and calculated stirrup strain ranges  $\epsilon_{wr.meas.}$  and  $\epsilon_{wr.cal.}$  at  $N=10^4$ .

## 5. FATIGUE STRENGTHS OF STIRRUPS

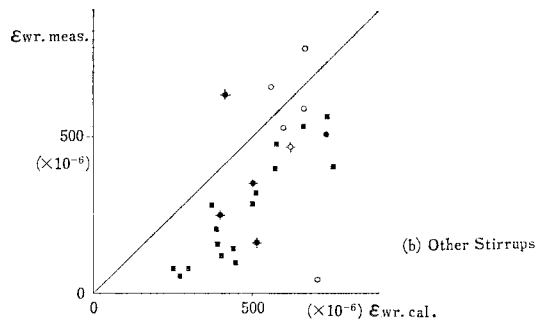
As the fracture of a stirrup due to fatigue loading has a great influence on the behavior of a beam, it is necessary to take into account the fatigue fracturing of stirrups for one of the ultimate limit states in the design of a beam to be subjected to fatigue loading of shear forces. This is clearly indicated by the results where fatigue fracturing of stirrups was found in all tested beams having shear capacities larger than the calculated flexural capacities under static loading.

### (1) Test results

Table 3 summarizes all the test results concerning fatigue fractures of stirrups. Fatigue fracturing of stirrups occurred in all test beams without exception even in the beam where the maximum shear was only 44 percent of the static strength. In three beams, 25F3, 19F1 and 19F5, shear failures of the beams occurred just after the first fractures of stirrups. In the other 14 beams, the remaining capacities after the first fractures of stirrups were capable of sustaining the forces induced by the redistributions occurring after the first fractures of stirrups and could endure further cycles of loading. There were seven beams in which fractures of stirrups occurred later in the other shear spans. These were also in the category of the first fractured stirrup in the shear span. It is difficult to determine the exact number of loading cycles at which the fracture of a stirrup occurred as strain measurements were taken at intervals. Therefore, in the table the numbers of cycles at which the fractures were found and those of the previous inspections are shown. The first fractured stirrup was generally the one that developed the highest



(a) Fractured Stirrups



(b) Other Stirrups

Fig. 8 Comparison between measured and calculated stirrup strain ranges  $\epsilon_{wr.meas.}$  and  $\epsilon_{wr.cal.}$  at the first fracture of stirrup. (a) fractured stirrups. (b) non fractured stirrups.

strain in the beam. Regarding the ratios of measured stress ranges to calculated ones, the average of the first fractured stirrups was 1.17, although the ratios were usually lower than 1 in other stirrups as shown in Fig. 8. After the failure of a beam the concrete cover around the bars was removed to check the positions of the stirrup fractures and to examine their failure surfaces. In all forty two legs had been fractured, and 41 legs were fractured at the same position, where the legs started to be bent around longitudinal bars, and fatigue cracks were initiated at the bases of lugs inside of curvatures and extended into the centers of the cross sections (see Fig. 9).

### (2) Fatigue strengths of straight bars

Fatigue stress ranges of the deformed bars used for stirrups were determined from fatigue tests of beams having these bars as longitudinal reinforcement. Excluding the data of the beam

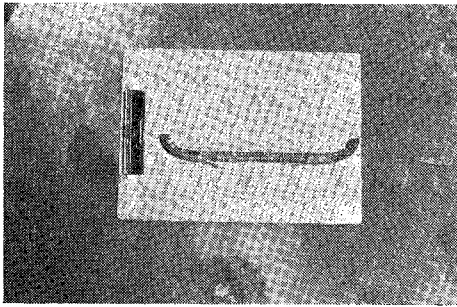


Fig. 9 Stirrup fractures at lower bends.

which failed due to shear force, the following regression line is obtained.

$$\log \sigma_{ro} = -0.0885 \log N + 2.86 \quad \dots \dots \dots (12)$$

Since the results of fatigue tests of both 10-mm bars and 13-mm bars can be considered to be equal, the results are expressed as one equation.

### (3) Fatigue strengths of stirrups

Fig. 10 shows the measured stress ranges of fatigue fractured stirrups and the number of cycles to fracture on log-log scaled paper. Although the stress range of stirrup increases with the number of cycles, it increases greatly during the first several thousand cycles and slightly after that if it is shown in the normal scale. Therefore, the measured stress ranges in Fig. 10 are those just prior to failure without any correction. This figure shows that the fatigue strengths of stirrups are about half of those of straight bars. The fracture of a stirrup always started at the inside of the bend where the actual stress range was larger than that of the average stress range of the section. The measured strains of the outsides of the bar-bends were more or less equal to zero. This seems to mean that the stress range of the inside of the bar-bend might be two times the average stress range of the bar. This may be the reason why the fatigue strengths of stirrups were about half of those of straight bars. One of the factors influencing the actual stress range of the inside of a bar-bend may be the radius of the bend, which is one of the parameters in this test. From the test results, a tendency of fatigue strength being slightly larger for smaller radius of bend was seen. This, however, cannot be definitely stated since the data were not adequate. The 50-percent decrease in fatigue strength of the bent region of a stirrup is consistent with previous reports. Hawkins reported that the fatigue strengths of 10-mm bars bent through 180 degrees around 29-mm bars were reduced to 50 percent of those

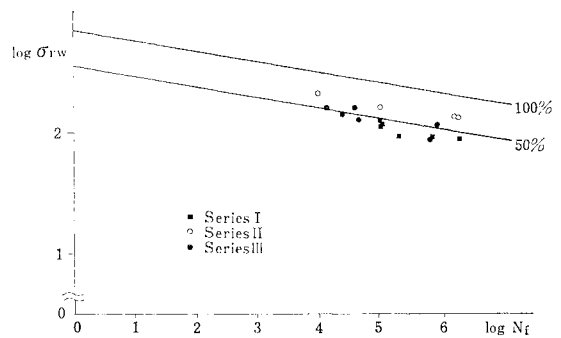


Fig. 10 Relationships between measured stress range of stirrup  $\sigma_{rw}$  and number of cycles at the first fracture of stirrup  $N_f$ .

of straight bars<sup>1)</sup>. Many reports also indicate that fatigue strengths of bars are reduced to about half of those of straight bars<sup>7-9)</sup>.

Although all stirrups fractured at the bar-bend in our tests, it is possible that fracture occurs at the straight part in such beams with large height where the stress distribution in a stirrup is less uniform, and the stress at the inside of the bar-bend is not always critical. Since it is not clarified yet, the fatigue life of a stirrup had better to be calculated as follows at the moment, assuming conservatively that the fracture occurs at the bend.

(1) The stress range and maximum stress of fatigue loading are calculated by Equation (11) and (7).

(2) The fatigue stress ranges of stirrups are obtained by dividing those of straight bars by two.

(3) The fatigue life of the stirrup is obtained from (1) and (2) above.

## 6. CONCLUSIONS

(1) Under the effect of repeated loading the inclined cracks formed at the first cycle increased gradually in width and extended upward or downward. But, new diagonal cracks were seldom formed in shear spans where main diagonal cracks had already been formed. With increases of the inclined cracks in widths and/or lengths, strains of stirrups intersected by the cracks increased. The rate of increase, however, was not always the same among the stirrups, and the strains of some stirrups remained almost constant if the inclined crack intersected did not propagate.

After a considerable number of cycles a stirrup often started to increase its strain affected

by extension of some diagonal cracks. In these cases at least one of the other stirrups always stopped increasing in strain. Thus, even without any fracture of a stirrup, redistribution of stresses among stirrups occurred, and the total strains or the average strain in a beam continued to increase at an almost constant rate against the logarithms of loading cycles, although individual stirrups showed great deviations.

(2) The rate of increase in the average strain of stirrups is not dependent on the applied maximum shear forces but on the amount of stirrups, and the strain at the maximum shear forces under repeated loading can be calculated by the proposed Equation (7).

$$\epsilon_w = \frac{\beta_x \{V_{max} - V_{co}(1 - k \log N)\}}{A_w E_s z / s} \dots\dots\dots (7)$$

When the applied maximum shear forces was very small and less than  $V_{co}$ , the increase of stirrup strain was very small until a certain number of loading cycles, and then a critical diagonal crack appeared and a rapid increase was recognized. In such cases, Equation (9) derived from Equation (7) can be used.

The average of the stress range of stirrups can be safely calculated on the assumption that the residual strain is zero.

(3) Due to repeated loading fractures of stirrups occurred in all beams even in the beam where the maximum applied shear was only 44 percent of the static strength. In all, forty two legs were fractured, and 41 legs were fractured at the same position where the legs started to be bent. The fatigue life of a stirrup fractured first in each beam can be obtained as follows: (i) The stress range and maximum stress of fatigue loading are calculated by Equations (11) and (7), (ii) The fatigue stress ranges of stirrups are obtained by dividing those of straight bars by two. (iii) The fatigue life of the stirrup is obtained from (i) and (ii) above.

(4) When a fracture of one leg occurred, the strain of the other leg or the leg of the adjacent stirrup increased exceedingly, and the next fracture usually occurred in either of the legs after some thousands of cycles. The beam would fail if with the remaining legs it became unable to sustain the applied maximum shear.

(5) Fracturing of longitudinal bars was also found in three beams. The fracture always occurred just beside the first fractured stirrup leg. In Beam 19F6, two outer longitudinal bars were

fractured on fracturing of both legs of Stirrup 4. Fracturing of the stirrup caused the longitudinal bars to be subjected to greater dowel actions in addition to the axial tensile stresses due to bending moment.

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