PROPOSED DESIGN EQUATION FOR SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS WITHOUT WEB REINFORCEMENT

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ABSTRACT

A design equation for shear strength is proposed upon investigation of published test results on shear strengths of reinforced concrete beams without web reinforcement. The proposed equation is deduced based on experiments of beams failing in a diagonal tension mode and subjected to no axial force, and basically, this is also the range of application of the equation.

The equation has enough simplicity expressed by concrete strength, reinforcement ratio and effective depth in a cumulative form. The accuracy of the equation is acceptable since it is better than that of any other equations.

1. INTRODUCTION

Many experimental research works have been reported[1] in regard to shear strength of reinforced concrete members. Regretably, however, a generally acceptable theory has not yet been established. After the development of flexural cracks shear force acting on a cracked section is carried by concrete in the compression zone, interlocking action of aggregates, and dowel action of longitudinal bars. The proportions shared by these actions have been clarified to a certain extent by experimental and analytical studies recently conducted[2],[3], but these results are not yet such as to be applicable to design use.

Nominal shear stress is generally used at present in design of reinforced concrete members without web reinforcement. Many test results have shown that the nominal shear stress at a critical diagonal cracking is principally dependent on concrete strength, reinforcement ratio, effective depth of cross section, shear span-depth ratio, and applied axial force. And a part or all of these factors are included in design codes in some way. However, the effects of these factors are different according to the code or investigator.

Based on investigations of published test results, the authors have deduced a simple design equation for reinforced concrete members without web reinforcement. This equation includes the effects of all of the factors mentioned above, is supported by many test results, and is simple enough for ordinary design work.

2. FACTORS USED IN THE PROPOSED EQUATION AND THEIR FUNCTIONAL EXPRESSIONS

(1) Study on the principal factors

The magnitude of axial force and shear span-depth ratio are also the principal factors influencing shear strength of reinforced concrete members, and it is well-recognized that shear strength increases considerably when the axial compressive force is applied or the shear span-depth ratio is small. However, this paper deals only with normal cases, that is, cases of shear span-depth ratios greater than about 3 and without axial forces at the first step.

The failure mode of a test beam without web reinforcement is mainly related to the shear span-depth ratio, and when the ratio is larger than about 3, which is dealt with in this paper, the failure mode is usually so-called diagonal tension failure; a flexural crack in the middle of the shear span suddenly develops into a critical diagonal crack and shear failure occurs almost simultaneously with the development of this critical crack. The principal factors resisting development of the critical diagnoal crack are (1) tensile stress of concrete in the web, (2) interlocking action of aggregates across flexural cracks, (3) dowel action of longitudinal bars, and (4) shear stress of concrete in the compression zone. These factors are related to the strengths of the materials used and the cross-sectional dimensions of the member.

a) Strength of concrete

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It is quite natural for diagonal cracking strength to be closely related to the principal tensile stress of concrete in the web and then to the tensile strength of concrete, and the shear strength without web reinforcement specified in the various codes is considered to be approximately proportional to the tensile strength of concrete. However, shear strength is usually expressed as a function of compressive strength of the concrete since it is not proper to use the tensile strength directly. **Fig. 1** shows the relationships between shear strengths and compressive strengths in recent codes.[4]~[6] The compressive strengths in the figure are cylinder strengths with cub strength used in BSI CP110 multiplied by 0.85.

The tensile strength of concrete is often considered to be proportional to the square root of the compressive strength. However, it is more proper to assume it to be proportional to the two thirds power of the compressive strength. The following equation was obtained from the results of tests carried out by the Cement Association of Japan.[7]

$$f_t = 0.27 f_c'^{2/3} \qquad (f_t \text{ and } f_c' \text{ in MPa})$$

............................(1)

Fig. 2 is derived from **Fig. 1** by using Eq. (1). This figure appears to show that in these codes shear strength is approximately proportional to tensile strength. Although tensile strength of concrete prominently affects shear strength, this does not always mean that the tensile strength should be directly used in the equation for calculation of the shear strength. This is due to the fact that interlocking action of aggregates, dowel action of longitudinal bars, and shear resistance of concrete in the compression zone are closely related to the properties of concrete as described below. In fact, statistical studies[8],[9] shows that

shear strength is proportional to 1/2–1/3 power of compressive strength.

b) Interlocking action of aggregates

It has already been clarified that in the case of a beam without web reinforcement the shear force carried by the interlocking action of aggregates across flexural cracks is large enough that it cannot be neglected. According to Fenwick and Paulay,[2] the interlocking action becomes larger as crack width is smaller or concrete strength is higher. The ratio of stress of tension reinforcement to the nominal shear stress is approximately constant if the position of applied load and the section of the beam are given, and this ratio is considered to be approximately inversely proportional to the reinforcement ratio. As the flexural crack width is approximately proportional to the stress of tension reinforcement, the crack width at the same stress level becomes smaller as the reinforcement ratio is increased. Moreover, it is known that crack width becomes smaller at the same stress level of reinforcement when the amount of tension reinforcement is relatively large compared with the area of concrete in the tension zone. These facts indicate that the interlocking action will increase when the reinforcement ratio is large. And it is naturally expected that the interlocking action will increase when the strength of the mortar, which supports the coarse aggregates, is high.

The above discussion is based on the assumption that the ratio between the dimension of aggregates and those of the cross section is fixed. The relative relation between the maximum size of aggregates and the dimensions of the cross section affects the contribution of the interlocking action. Taylor[10] has shown based on his experiments in which depths of beams and the sizes of aggregates were changed that the main reason for high shear strength of a small
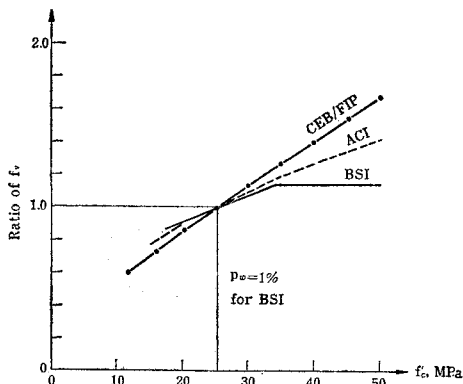

Fig. 1 The relationships between shear strengths f_c and compressive strengths f_c' of concrete in recent codes[4]~[6].
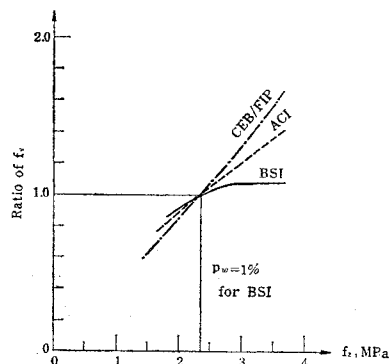

Fig. 2 The relationships between shear strengths f_c and tensile strengths f_t of concrete in recent codes derived from Fig. 1.

specimen is the relatively large aggregate size
for the depth of the beam and the consequently
large interlocking action in the small specimen,
and that the strength of the small specimen
would not be very different from that of a large
specimen if the aggregate size were to be scaled
down properly.   Since in ordinary reinforced
concrete members the maximum size of aggregates
is usually not changed much even when the dimen-
sions of the members differs markedly, the shear
strengths of members with large depths might
be reduced, and this effect should be taken into
consideration in design.

   c)   Dowel action of longitudinal bars

   A part of the shear force can be transfered by
the dowel action of longitudinal bars. The main
factors influencing this action are flexural
rigidities of the bars, and strength and rigidity of
the surrounding concrete.   In more concrete
terms, area, diameter, number and arrangement
of the bars, spacing of flexural cracks, and tensile
strength and elastic modulus of concrete are
considered to be the main factors.   However,
the contribution of each of these factors has not
yet been separately expressed.   Moreover, it is
not practical to include all these factors in the
design equation.   Under present circumstances
it is more realistic to judge the magnitude of
dowel action by using the reinforcement ratio
and tensile strength or compressive strength of
concrete as the main factors.

   d)   Shear resistance of concrete in the com-
        pression zone

   After the development of flexural cracks, a part
of shear is carried by the concrete in the com-
pression zone, and this is closely related to the
area of compression zone.   As the position of
the neutral axis after flexural cracking depends
mainly upon the elastic modulus of concrete and
the reinforcement ratio, shear carried by the
concrete in the compression zone is considered
to be defined by the reinforcement ratio and the
strength of concrete since the elastic modulus is
to be the function of concrete strength.

(2)   Functional expression of the principal factors

   As discussed in section 2.(1), it is considered
that (1) aggregate interlocking action is affected
by compressive strength of concrete, reinforce-
ment ratio, effective depth of cross section, and
ratio of moment to shear, (2) dowel action is
affected by concrete strength, reinforcement
ratio, and ratio of moment to shear, and (3)
shear resistance of concrete in the compression
zone is affected by concrete strength and reinforce-
ment ratio.

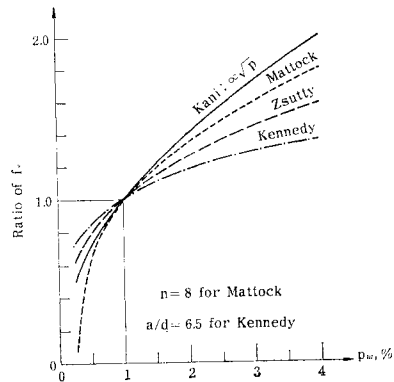   **Fig. 3** shows recent test results[8],[9],[11],[12] con-

Fig. 3   The relationships between shear strengths
f_v and reinforcement ratio p_w in recent invest-
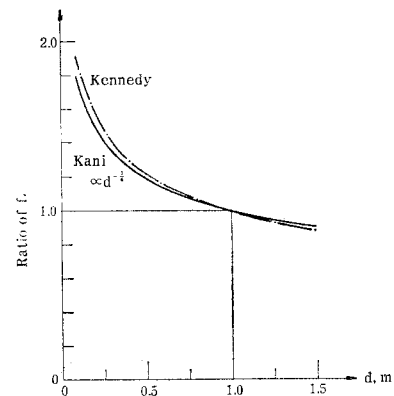igations[8],[9],[11],[12].

Fig. 4   The relationships between shear strengths
f_v and effective depth d in recent investiga-
tions[9],[11].

cerning the relationship between shear strength
and reinforcement ratio.   The simple expression
may be Kani's one in which shear strength is
proportional to the square root of the reinforce-
ment ratio.

   As the effect of the absolute value of effective
depth on shear strength is considerably great
as shown in **Fig. 4**[9],[11], this should be taken into
account.   The most simple expression is the one
proposed by Kani, that is, shear strength is
proportional to one-fourth power of effective
depth, and this is considered to be appropriate.

   The effect of concrete strength is rather com-
plicated.   If the effect of tensile strength is con-
sidered to be dominant, shear strength may be
proportional to two-thirds power of the com-
pressive strength of concrete.   On the other hand,
one-third power may be usable if the effect of
concrete strength on the interlocking action of
aggregates is emphasized as proposed by Placas
and Regan[13].

3. PROPOSED EQUATION AND ITS EVALUATION

(1) Expression of the proposed equation

Many of the equations proposed for estimating shear strength have been as products of main factors such as concrete strength and reinforcement ratio. However, product form equations will give extreme values of shear strength if more than two variables were to take extreme values. The authors believe that such tendencies are not found in real cases. Therefore, it was decided to use a sum form in principle. According to the discussions in section 2, reinforcement ratio p, effective depth d and compressive strength of concrete f_c' are used as the variables in the proposed equation to estimate the nominal shear strength f_v of reinforced concrete members without web reinforcement.

$$f_v/f_t = \beta_0(1+\beta_p+\beta_d) \quad\cdots\cdots\cdots\cdots(2)$$

$$\beta_p = \sqrt{p_w}-1 \quad (p_w \text{ in } \%) \quad\cdots\cdots\cdots(3)$$

$$\beta_d = 1/\sqrt[4]{d}-1 \quad (d \text{ in m}) \quad\cdots\cdots\cdots(4)$$

β_0: constant
f_v: nominal shear strength of beam without web reinforcement
   $= V_c/(b_w d)$, MPa
f_t: tensile strength of concrete $= 0.27 f_c'^{2/3}$, MPa
p_w: reinforcement ratio $= 100 A_s/(b_w d)$, %
d: effective depth, m
b_w: width of web
V_c: shear force at failure of beam

f_c': compressive strength of concrete, MPa
A_s: area of tension reinforcement

The reasons why ratio of moment to shear or shear span-depth ratio is not included as a variable are that the effect is not great in the range treated in this paper, and it is not always appropriate to use the ratio in the practical design equations.[14]

(2) Determination of β_0

In order to determine the constant β_0 in Eq. (2), the authors used the published test data listed in **Table 1**[11),12),15)~29)]. These data were selected considering the following:

(1) More than five data with shear span-depth ratio greater than 3 or showing failure in the diagonal tension mode are to be included.

(2) More than two factors among reinforcement ratio, effective depth, shear span-depth ratio and compressive strength of concrete are more or less constant.

The investigation based on these data indicated that the values of calculated by Eq. (5) was nearly constant for the wide range of p_w and d.

$$\beta_0 = (f_v/f_t)/(1+\beta_p+\beta_d) \quad\cdots\cdots\cdots\cdots(5)$$

However, a clear trend was recognized concerning the compressive strength of concrete. The calculated values of β_0 for small f_c' were obviously larger compared with those for large f_c' if β_0 is treated as constant. This was considered to be due to the fact that tensile strength, which was applied to take into account the effect of concrete, could not properly cover the total contribution of concrete strength. Therefore, in

Table 1 Outline of data used; range (and average) of parameters.

|	Number	f_c' (MPa)	p (%)	d (m)	a/d
Kani[11]	31	25–31 (27.0)	2.6–2.9 (2.73)	0.13–1.10 (0.44)	2.6–8.0 (4.71)
Morrow[15]	11	15–46 (29.9)	1.2–3.8 (2.26)	0.34–0.36 (0.35)	3.8–7.9 (4.70)
Mathey[16]	7	24–31 (26.2)	0.5–0.9 (0.75)	0.403	2.8–3.8 (3.37)
Kani[17]	13	18–35 (27.1)	0.5–0.8 (0.71)	0.27–0.27 (0.27)	3.0–5.1 (3.51)
Rajagopalan[18]	10	25–37 (29.8)	0.3–1.7 (0.75)	0.26–0.27 (0.27)	3.9–4.3 (4.14)
Diaz de Cossio[19]	22	14–29 (23.8)	1.9–2.9 (2.18)	0.08–0.17 (0.11)	4.00
Krefeld[20]	53	12–39 (24.6)	1.1–4.5 (3.06)	0.24–0.48 (0.26)	3.6–8.5 (5.28)
Moody[21]	24	12–41 (26.0)	0.8–2.4 (1.90)	0.26–0.27 (0.27)	2.9–3.4 (3.26)
Chang[22]	12	31–39 (33.0)	1.9–2.9 (2.38)	0.137	3.72
Mattock[12]	6	16–47 (24.3)	1.0 3.1 (2.24)	0.254	3.0–5.4 (4.20)
Van den Berg[23]	34	15–66 (34.5)	4.35	0.359	3.5 4.9 (3.78)
Leonhardt[24]	6	30–31 (30.6)	2.0–2.1 (2.04)	0.27–0.28 (0.28)	3.0–5.9 (4.76)
Leonhardt[25]	12	35–39 (37.9)	1.3–1.7 (1.55)	0.07–0.60 (0.24)	3.00
Taylor[26]	23	19–37 (26.3)	0.9–2.3 (1.70)	0.222	4.11
Taylor[27]	12	24–32 (27.3)	1.2–1.9 (1.43)	0.22–0.22 (0.22)	3.8–3.8 (3.78)
Aster[28]	5	28–28 (27.9)	0.4–0.9 (0.62)	0.50–0.75 (0.45)	3.70
Higai[29]	7	31–37 (32.9)	2.39	0.160	3.5–6.5 (4.36)
Total	288	12–66	0.3–4.5	0.07–1.10	2.6–8.5
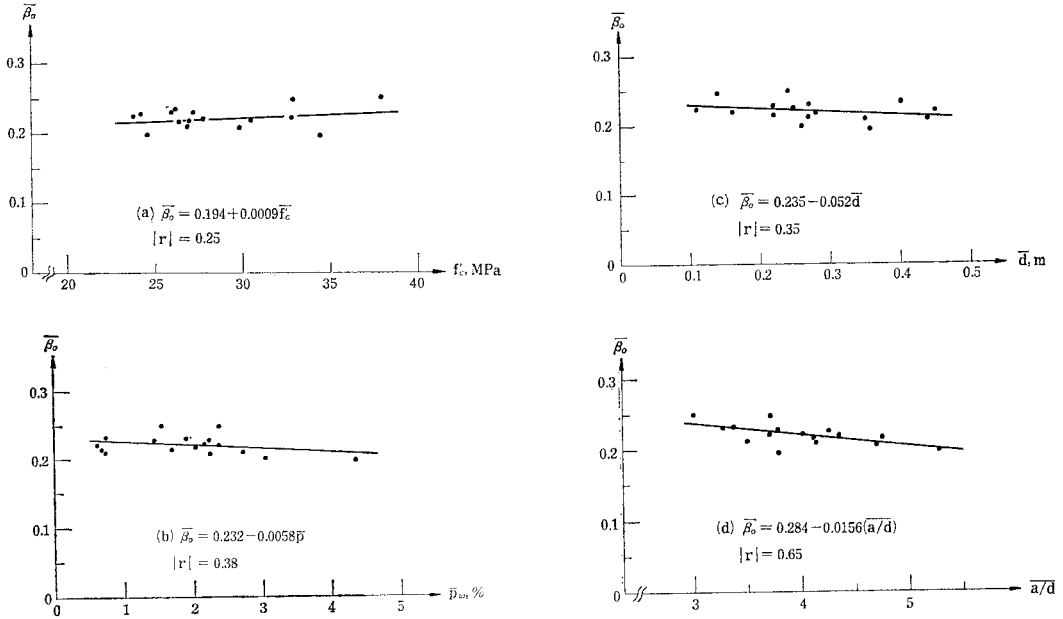
Fig. 5 The relationships between $\bar{\beta}_0$ calculated by Eq. (5) and each parameter ($\bar{\beta}_0$, $\bar{f}_c{}'$, $\bar{p}_w$, $\bar{d}$ and $\overline{a/d}$ are averages in Table 1).

the next step Eq. (6) was used to express the contribution of concrete strength, and an investigation based on Eq. (7) was carried out.

$$f_0 = f_c{}'^{1/3} \qquad (f_0 \text{ and } f_c{}' \text{ in MPa}) \cdots\cdots\cdots(6)$$

$$f_v/f_0 = \beta_0(1+\beta_p+\beta_d) \cdots\cdots\cdots\cdots\cdots(7)$$

Where, f_0 is not any kind of strength of concrete, but an interpretation of all the contributions by the quality of concrete, which includes tensile strength and influences of aggregate interlocking and dowel action.

The coefficient of variation of β_0 calculated by Eq. (7) concerning the data in **Table 1** is 12.5% with an average of 0.216. This means that the coefficient of variation for the ratios of the shear strengths tested to that calculated with β_0 of 0.216 is also 12.5% and the accuracy of this equation may be considered to be satisfactory.

Detailed discussions on the influences of the individual variables are as follows. The ranges of the variables in each data group are shown in **Table 1** together with the averages. **Fig. 5** shows the relationship between the averages of the individual variable within each group and the calculated average β_0 for each group. No special trend of average is seen about $f_c{}'$ or d, and p_w unless p_w is larger than 3%. Some of the groups with p_w larger than 3% show tendencies for smaller β_0. Concerning shear span-depth ratio, **Fig. 5** (d) shows that the larger the ratio the smaller the β_0 value. Therefore, p_w is limited to under 3% in the first place, and the authors

recalculated β_0 by using $p_w = 3\%$ if p_w was larger than 3%. The coefficient of variation of β_0 was 11.3% and this was about 1% less than that without limitation of p_w. This limitation of p_w was considered effective.

In the next place, data with wide range of shear span-depth ratios such as Kani[11], Morrow[15], Krefeld[20], Mattock[12], Leonhardt[24],[25] and Higai[29] were investigated. The effect of the ratio in these data was recognized to have two different treads. One was the trend seen in the data by Morrow, Krefeld, Mattock, and a part of the data by Kani, and β_0 is clearly reduced as the ratio is increased as shown in **Fig. 6** (a) which gives a part of Kani's data. On the other hand, test data obtained by Leonhardt and Higai, and a part of the data by Kani show that β_0 is nearly constant for the wide range of shear span-depth ratio as indicated in **Fig. 6** (b). Since the reason for these differences was not clarified, it is rather difficult to obtain a modified equation which gives a better result for all the data. Therefore, for the time being, the relation between β_0 and shear span-depth ratio was assumed to be expressed by the following equation.

$$\beta_0 = A + B(d/a) \cdots\cdots\cdots\cdots\cdots\cdots\cdots(8)$$

a: shear span
A, B: constant

The constant A and B were determined by applying the method of least squares to the data above. A was 0.157 and B was 0.257 with a
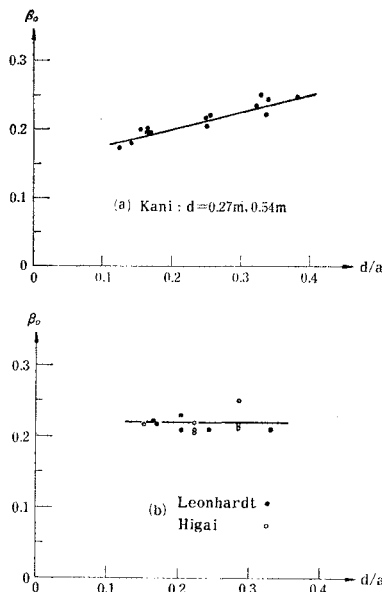
Fig. 6 The relationships between β_0 and a/d in the typical data[11],[24],[29]

coefficient of correlation of 0.721 and the number of data of 114. Considering these results and the average value of β_0 for all of the data used, β_0 was finally determined to be expressed by the following equation.

$$\beta_0 = 0.20(0.75 + 1.4d/a) \quad \cdots\cdots\cdots\cdots (9)$$

When the shear strength was calculated for all of the data ($n = 288$) applying Eq. (9), the average of the ratios of shear strengths tested to that calculated was 1.00 and the coefficient of variation was 9.2%. This variation is equivalent to that of compressive strength or tensile strength of concrete. And this modification has not shown poorer applicability to any group of data as indicated in **Table 2**. The average ratios regarding the data within each paper are between 0.92 and 1.10, and moreover in 15 papers out of 17 the ratios are between 0.97 and 1.03. After these considerations, the authors proposed Eq. (10) to calculate shear strengths of reinforced concrete members without web reinforcement.

$$f_v = f_{v0}(0.75 + 1.4d/a)(1 + \beta_p + \beta_a) \quad \cdots\cdots (10)$$
$$f_{v0} = 0.20f_c'^{1/3} \quad (f_{v0} \text{ and } f_c' \text{ in MPa})$$
$$\beta_p = \sqrt{p_w} - 1 : p_w \leq 3\% \ (p_w \text{ in } \%)$$
$$\beta_a = d^{-1/4} - 1 : d \leq 1.1 \text{ m } (d \text{ in m})$$

where, f_{v0} has a dimension of strength, and is defined as the shear strength of the beam with a/d of 5.6, p_w of 1% and d of 1 m.

(3) Evaluation of the proposed equation

Many equations for calculating shear strengths of reinforced concrete members have been proposed, and they may be classified into three categories:

(1) so-called theoretical equations introduced by assuming the failure mechanism,

(2) so-called empirical equations introduced by finding the main factors from test results

Table 2 Averages and coefficients of variation within each paper of the ratios of shear strengths tested to those calculated by the various equations.

| Data form	Average (coefficients of variation, %)				
	Eq. (10)	Eq. (11)	Eq. (12)	Eq. (13)	Eq. (14)
Kani[11]	0.98 (8)	1.03 (15)	1.12 (8)	1.04 (11)	1.06 (10)
Morrow[15]	1.00 (5)	0.99 (4)	1.06 (7)	0.93 (11)	1.06 (8)
Mathey[16]	0.99 (5)	0.90 (4)	0.90 (5)	0.97 (6)	1.17 (6)
Kani[17]	0.92 (10)	0.93 (10)	0.83 (12)	0.91 (21)	1.07 (13)
Rajagopalan[18]	0.97 (14)	0.99 (14)	0.85 (12)	0.82 (12)	1.05 (14)
Diaz de Cossio[19]	1.01 (15)	1.30 (18)	0.99 (12)	1.03 (15)	1.12 (14)
Krefeld[20]	1.01 (6)	1.06 (7)	1.09 (6)	0.99 (12)	1.05 (11)
Moody[21]	0.98 (9)	1.03 (9)	1.03 (9)	0.98 (11)	1.16 (10)
Chang[22]	1.10 (8)	1.32 (7)	1.09 (7)	1.08 (10)	1.24 (7)
Mattock[12]	1.02 (6)	1.10 (5)	1.09 (7)	1.05 (14)	1.14 (15)
Van den Berg[23]	1.03 (6)	0.94 (6)	1.11 (5)	1.12 (7)	1.16 (6)
Leonhardt[24]	1.03 (10)	1.09 (10)	1.06 (9)	0.88 (3)	1.09 (4)
Leonhardt[25]	1.03 (7)	1.15 (19)	0.98 (6)	1.04 (11)	1.25 (7)
Taylor[26]	0.99 (8)	1.07 (8)	0.98 (7)	0.91 (9)	1.08 (8)
Taylor[27]	1.02 (13)	1.10 (12)	0.99 (11)	0.96 (15)	1.14 (13)
Aster[28]	0.98 (6)	0.87 (7)	0.84 (7)	0.88 (5)	1.10 (6)
Higai[29]	1.01 (7)	1.20 (7)	1.02 (7)	0.94 (7)	1.10 (6)
Total	1.00 (9.2)	1.06 (15.5)	1.03 (11.4)	0.99 (13.7)	1.11 (11.3)
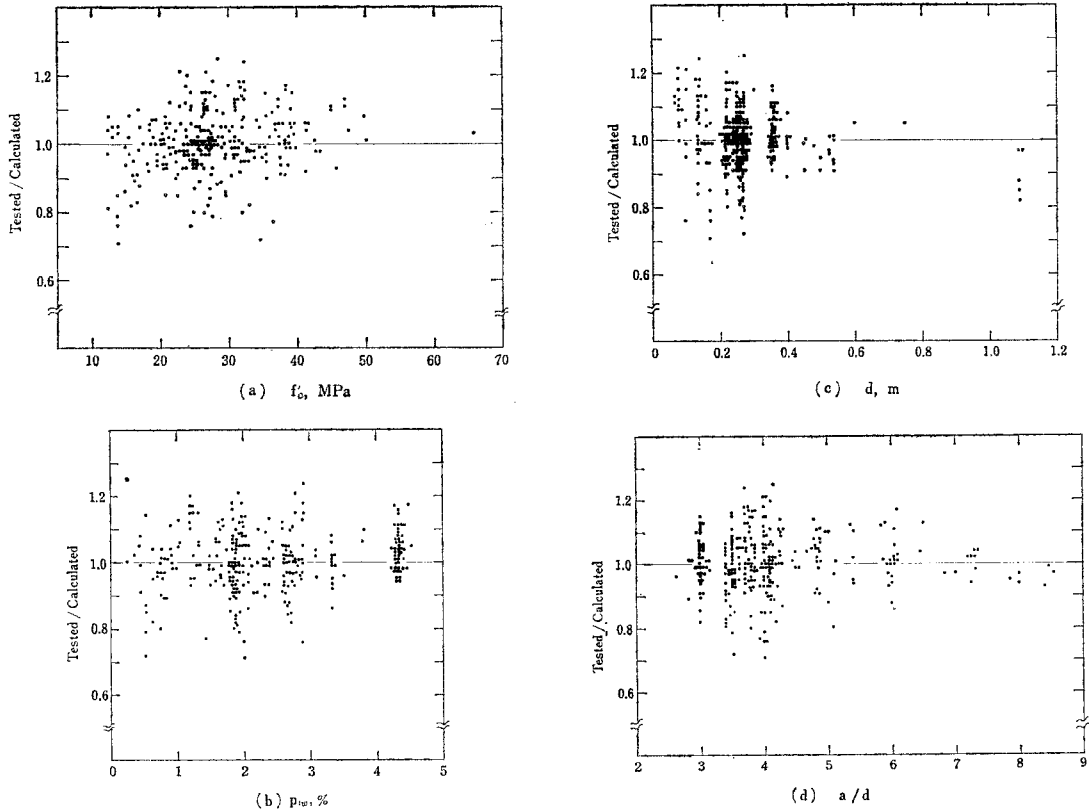
Fig. 7 The relationship between the ratio of shear strength tested to that calculated by Eq. (10) and the value of each variable.

and determining the format of the equation and the constants in the equation from the test results, and

(3) equations introduced by statistical analyses of many published test data.

If the failure mechanism and the properties of materials are well-understood, Category (1) is to be applied for the widest range. However, it is considered that this type of equation is not yet applicable to test data with sufficient accuracy. Equation in Category (2) will naturally agree with test results which comprise the basis of the equation, but they do not agree so well with other test data where the range of variables are different. This is one of the reasons why many empirical equations have been proposed. Therefore, in introducing this kind of equation, selection of the test data to be used is quite important. In Category (3) a statistical method such as multi-regression analysis is applied to analyze the experimental data, and it agrees relatively well with the test results if the format of the equation and the data used are appropriate. However, the meanings of the variables in the equation usually are not clear.

The equation proposed by the authors is considered to belong to Category (2). Therefore, careful selection of data was carried out considering that the variables should cover entire range used in ordinary structures as mentioned in section 3.(2). Finally, 288 data, which cover a sufficient range of parameters such as concrete strength, reinforcement ratio, and shear span-depths ratio were obtained, although the range of depths is not sufficient. The relationship between the ratio of shear strength tested to that calculated by Eq. (10) and the value of each variable used are shown in **Fig. 7**. Compressive strengths of concrete f_c' are in the range from 12 MPa to 66 MPa, and in most data they are between 20 MPa and 40 MPa. No special relation is seen in **Fig. 7** (a) between the ratio and compressive strength of concrete. The reinforcement ratios are between 0.25% and 4.5%. And no special relation is seen within this range. The effective depth of sections range from 0.07 m to 1.1 m, and in most of the data the effective depths are less than 0.35 m. Although the data for large depths is insufficient to show the tendency of the ratio when the effective depth is

Table 3 Average and coefficient of variation of the ratios of shear strengths tested to those calculated by Eq. (10).

	Number	Average	C.V.*
$f_c' < 20$ MPa	40	0.96	10.1
$20 \leq f_c' < 35$	190	1.01	9.2
$f_c' \geq 35$ MPa	58	1.03	7.4
$p < 1.5\%$	59	1.00	10.7
$1.5 \leq p < 3.0$	163	1.00	9.6
$p \geq 3.0\%$	66	1.03	6.1
$d < 0.2$ m	53	1.03	11.7
$0.2 \leq d < 0.4$	207	1.00	8.5
$d \geq 0.4$ m	28	0.97	6.2
$a/d < 3.5$	61	0.98	8.7
$3.5 \leq a/d < 5.5$	190	1.01	9.7
$a/d \geq 5.5$	37	1.00	6.6

* Coefficient of variation

changed, no special trend is recognized in the data used. The range of shear span-depth ratios is between 3 and 8.5, and no significant trend is seen concerning the average and the variation in this range. However, the variation of the ratios are obviously different according to the values of f_c', p_w or d as indicated in **Table 3**. The variations become larger when the concrete strength, reinforcement ratio or effective depth becomes smaller, although the average ratios are more or less same. This result is to be expected when the applicability of Eq. (10) is good. These discussions show that the proposed equation may be considered to be appropriate in the selection of the principal factors and in expression of these factors.

The authors have proposed an equation in consideration of obtaining an equation of sufficient simplicity and applicability for the experimental results available, and this purpose is considered to have been well-achieved. However, there are many equations already proposed and it is thought necessary for comparisons to be made with these equations. From among the equations in Category (3), the well-known Zsutty's equation[8] and Kennedy's equation[9] deduced by multiple regression analysis, and Hedman-Losberg's equation[30], which is the basis of the equation in the CEB/FIP Model Code, were selected for comparison.

Zsutty:
$$f_v = 0.476(f_c' p_w d/a)^{1/3} \quad \cdots\cdots\cdots\cdots\cdots\cdots (11)$$
Kennedy:
$$f_v = 0.312 f_c'^{0.426} d^{-0.282} \left(1 + \frac{M/Vd}{0.25p}\right)^{-0.244}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots (12)$$

Hedman-Losberg:
$$f_v = 0.09(1.75 - 1.25d)(1 + 50p_w) \sqrt{f_c'} \cdots\cdots (13)$$
$$1.75 - 1.25d \geq 1.0$$
$$p_w \leq 0.02$$

These equations are also expressed in terms of SI units. Eq. (11) or Eq. (13) is the same as that proposed by Zsutty or Hedman-Losberg while Eq. (12) is that for beams without compression reinforcement as proposed by Kennedy.

The ratio of all test values of shear strength in **Table 1** to those calculated by Zsutty's, Kennedy's or Hedman-Losberg's equation are as shown in **Table 2**. The average ratios of shear strengths tested to that calculated are 1.06, 1.03 and 0.99, and the coefficients of variation are 15.5%, 11.4% and 13.7%, respectively. The equation proposed by the authors with coefficient of variation 9.2% is considered to be better than these equations with respect to accuracy and simplicity. Data used in Zsutty's analysis have small range of depth, while he has neglected the effect, which is now known to be one of the main factors. Therefore, his equation estimates the strength to be higher when depth is larger. The relatively high coefficient of variation by Kennedy's equation is due to the fact that the variation in the average for each data group is somewhat large. It may be one of the reason why the coefficient of variation by Hedman's equation is relatively larger that the equation neglects the effect of shear span-depth ratio. However, the coefficient is still larger than that obtained by Eq. (14), which is derived from Eq. (10) by neglecting the effect of shear span-depth ratio.

These discussions are based on the data collected by the authors. Therefore, comparisons based on the data used in original analyses are also made. The data used in Zsutty's work are from the experiments by Diaz de Cossio, Moody, Morrow, Van den Berg and Kani. According to Zsutty's paper, the coefficient of variation of the ratios of shear strengths to the calculated one was 8.6% for the 86 beams with shear span-depth ratios larger than 2.5. The authors, however, obtained 94 data from the reports. As it was not clear which data Zsutty excluded, these 94 data were used in comparisons. The average of the ratios of shear strengths tested to that calculated is 1.00 with the coefficient of variation 13.2% by Zsutty's equation, while the average is 0.99 and the coefficient of variation 10.0% by the authors' proposed equation. With regard to these data Eq. (10) is again considered to be better than Zsutty's equation. In the data collected by Kennedy, simply supported beams with shear span-depth ratios larger than 3 were those from Diaz de Cossio,

Moody, Chang, Hanson, Maclarnon, Gaston, Krefeld, and Bresler. Regarding the above 107 data, the ratio of shear strength tested to that calculated is 1.06 by Kennedy's equation and 1.00 by Eq. (10), and the coefficients of variation are 12.3% and 10.0%, respectively. Hedman & Losberg have deduced their equation based on 255 data obtained from 15 papers, and the co-efficient of variation of the ratio of shear strengths tested to those calculated is 16%. The authors could collect 157 data among them. The coefficients of variation about these 157 data are 13.8% by Hedman's equation and 10.1% by Eq. (10). These comparisons show the same trend as the results given in **Table 2**, and it is substantiated that Eq. (10) even better fits the data which are the bases of other typical equations.

4. DESIGN EQUATION AND ITS APPLICATION

(1) Design equation

It is desirable for a design equation to be as simple as possible so long as it maintains acceptable accuracy. And it should not be one which gives the average strength such as Eq. (10), but should be a more conservative one which takes into account variations. Eq. (10) is relatively simple, but it is difficult to define shear span-depth ratios properly for such as which sustain distributed loads or moving loads, or for statically indeterminate structures such as continuous beams. Shear strength is markedly affected by shear span-depth ratio only under a certain loading condition that the load is acting on the top surface and the reaction is from the bottom surface, and then the vertical compressive stress is developed in the web concrete. Under this condition the shear strength increases significantly if the shear span-depth ratio becomes lower than three. Therefore, a rational design method for such conditions should be developed separately since the authors' proposed euqation does not cover. Except for such conditions the effect of the shear span-depth ratio can be neglected for the design equation without significant loss of accuracy. With these considerations, the authors propose here a design equation for shear strength of reinforced concrete beams without web reinforcement.

$$f_v = f_{v0}(1 + \beta_p + \beta_d) \quad \cdots\cdots\cdots\cdots\cdots (14)$$

Eq. (14) is given by substituting 5.6 for a/d in Eq. (10). Therefore, this equation gives a more conservative value in case of a/d smaller than 5.6. Actually, the average of the ratios of shear strengths with a/d smaller than 5.6 to that calculated by Eq. (14) is 1.13 with coefficient of
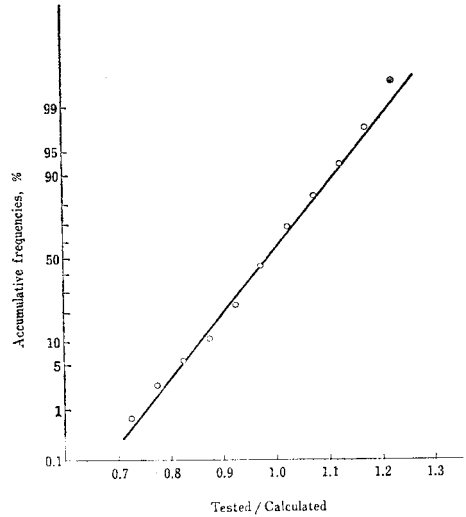
Fig. 8 Accumulative frequencies of the ratio of shear strength tested to that calculated by Eq. (10).
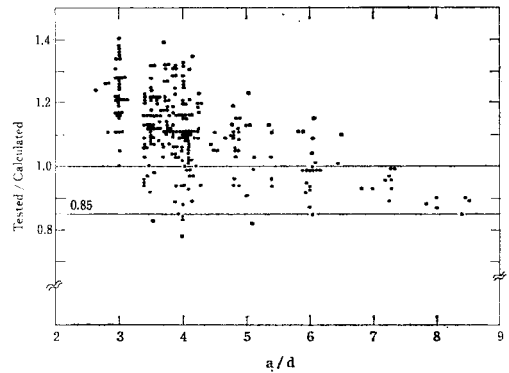
Fig. 9 The relationships between the ratio of shear strength tested to that calculated by Eq. (14).

variation 10.4%. Although it will give a slightly dangerous value in case a/d were to be larger than 5.6, the effect of a/d will be very smaller in this range, since the average ratios for this range is 0.97 with coefficient of variation 7.5%. Furthermore, in such a slender beam there will be very little possibility of failure in shear. Consequently, it is considered that Eq. (14) can safely be used in design. **Fig. 8** shows the accumulative frequencies of ratios of shear strengths tested to that calculated by Eq. (10) on a normal probability paper. It may well be assumed that the distribution of the ratios is normal. If the characteristic shear strength is defined as the one with 95% reliability, 85% of f_v or f_{v0} in Eq. (14) is to be used for f_{vk} or f_{v0k}, since the

coefficient of variation of the ratios of shear strengths tested to that calculated by Eq. (10) is 9.2% with the average of 1.00. Actually, among 288 data listed in **Table 1**, there are only 16 cases ($=5.6\%$) in which the shear strengths tested are less than 85 % of the calculated one. **Fig. 9** shows that only 5 cases out of 288 are below the characteristic strength f_{vk} calculated by Eq. (15) even when the actual concrete strengths are used instead of the characteristic strengths. Therefore, the following equation is proposed for design purposes.

$$f_{nk}=f_{v0k}(1+\beta_p+\beta_d) \quad\cdots\cdots\cdots\cdots\cdots (15)$$

$$f_{v0k}=0.85 f_{v0}$$

$$=0.17 f_{ck}'^{1/3} \quad \text{(in MPa)}$$

(2) Examples of application for specified structures

Although Eq. (15) is simple enough for practical use, more simplification is possible if structures to which it is applied are limited. For example, applications to T-girder highway bridges and floor slabs of the bridges are described in the section below[31].

In simply-supported T-girder highway bridges maximum spans are about 20 m and are usually less than 15 m, and the span depth ratios are about 10. Therefore, the effective depths are usually less than 1.5 m. Reinforcement ratios of web ($p_w = A_s/b_w d$, b_w: width of web) at the span centers are usually higher than about 1.5%, and at the critical sections against shear they are considered to be about 1% at least. Therefore, for T-girder highway bridges, it is usually safe if Eq. (15) is applied with d equal 1.5 m and p_w equal 1.0%. Similarly, Eq. (15) can be safely applied with d equal to 0.2 m and p_w equal to 1% for the floor slabs of the bridges in case the slabs are designed as beams against shear.

5. CONCLUSIONS

A design equation for shear strength is proposed upon investigation of published test results on shear strengths of reinforced concrete beams without web reinforcement. The proposed equation is deduced based on experiments of beams failing in a diagonal tension mode and subjected to no axial force, and basically, this is also the range of application of the equation.

(1) Shear strengths of reinforced concrete members with shear span-depth ratio larger than 3 and without web reinforcement are estimated well by Eq. (10) which includes concrete strength, reinforcement ratio, effective depth, and shear span-depth ratio in a cumulative form.

(2) For design purposes Eq. (15), which is derived from Eq. (10) by ignoring the effect of shear span-depth ratio, is usually adequate.

(3) If the structures to which the equation is applied are limited, shear strength can be safely expressed by the function of compressive strength of concrete only since the reinforcement ratio and the effective depth are generally within certain limits.

ACKNOWLEDGEMENT

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