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SIMULATION METHODS OF MULTI-DIMENSIONAL NONSTATIONARY STOCHASTIC PROCESSES BY TIME DOMAIN MODELS

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1. INTRODUCTION

A simulation method of multi-dimensional nonstationary stochastic processes, $x_i(t)$; $i=1, 2, \cdots, m$, with zero mean and with nonstationary characteristics of amplitude and frequency contents has already been proposed by one of the authors[1),2)]. This model is composed of a trigonometric series, and is based on a nonstationary cross spectral matrix. Therefore, the spectral characteristics are naturally described in the frequency domain. Since earthquake motions have such nonstationarities, it is shown in references 3) and 4) that the nonstationary model is applicable to the analysis of three dimensional earthquake ground motions, and also used for the generation of multi-dimensional input waves to the response analysis. In the application of this model, for example, in a case of a single process, one first estimates the nonstationary cross spectrum of a time series record which is considered as a sample of a stochastic process, and then generates the time series which satisfies the above nonstationary cross spectrum. Therefore, the theory of this model serves as a mathematical tool to the identification and simulation of a system in frequency domain.

On the other hand, Akaike[5)], Hino[6)], Box and Jenkins[7)], et al. discussed the same topics with time domain models, namely, autoregressive (AR) model, moving average (MA) model and the mixed (AR-MA) model. In their studies, these models were used for the analysis of the identification, estimation and control problems of a system. Recently, Hino[8)] applied these models by combining Kalman control filter[9)] to the appropriate estimation in the fields of hydrology.

The many methods of analysis with time domain models have been developed as mentioned above, and it seems that the multi-dimensional cases as well as one dimensional cases have been fully established for the stationary stochastic processes. These time domain models, however, have not been in suitable forms directly applicable to the stochastic processes with the nonstationarity in the amplitude and frequency domains, especially for the multi-dimensional cases. Recently, Hussain and Rao[10)] have discussed on the identification of a time domain model of a one dimensional nonstationary stochastic process by appling the evolutionary power spectrum proposed by Priestly[11)]. But the corresponding multi-dimensional nonstationary stochastic process model in time domain has not been developed yet.

This paper proposes time domain models for multi-dimensional nonstationary stochastic processes[12)] which are observed in the characteristics of earthquake ground motions, and then develops the theory of identification and simulation of the system. This paper also discusses the relationships between frequency and time domain models. Finally, numerical simulations of earthquake ground motions are presented.

Although both models, either in time domain or in frequency domain, have their own useful advantages for estimation problems of many engineering systems, time domain models are obviously more useful, because of their simple forms and of the forms directly applicable to such as control problems.

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2. AUTOREGRESSIVE MODEL (AR MODEL)

(1) AR Model for Multi-dimensional Nonstationary Stochastic Processes

An autoregressive model for multi-dimensional nonstationary stochastic processes, $x_i(t)$; $i=1, 2, \cdots, m$, with zero mean is given by Eq. (1), in which $\sum_{p=1}^{i}$ is analogous to a multi-dimensional frequency model[1)].

$$x_i(j)=\sum_{p=1}^{i}\sum_{k=1}^{M(j)} b_{ip}(k, j)x_p(j-k)+\varepsilon_i(j) ;\quad i=1, 2, \cdots, m \cdots\cdots\cdots\cdots(1)$$

where, j is an index of discrete time t, that is, $t=j\Delta t$; $j=1, 2, \cdots, N$, and Δt=equal time interval of the given time series. $M(j)$ is a positive integer. $\varepsilon_i(j)$ is an error function and its expectation $E[\varepsilon_i(j)]$ equals zero from Eq. (1), because of $E[x_i(j)]=0$. This error function $\varepsilon_i(j)$ is discussed later again.

Eq. (1) is a basic equation for discrete time series $x_i(j)$ which has nonstationary characteristics of amplitude and frequency contents, since the unknown coefficient $b_{ip}(k, j)$ is a function of discrete time j. In this model given by Eq. (1), the most suitable coefficients $b_{ip}(k, j)$ and the error $\varepsilon_i(j)$ are determined under the assumption that the value of $x_i(j)$ at time $t=j\Delta t$ can be represented by the linear summation of past $M(j)$ values of $x_p(j-1)$, $x_p(j-2)$, $\cdots$, $x_p(j-M(j))$ with the unknown coefficients $b_{ip}(k, j)$. The procedures of determination of $b_{ip}(k, j)$ and $\varepsilon_i(j)$ are discussed hereafter.

The coefficients $b_{ip}(k, j)$ are chosen in such a way that the mean square error $\sum_{i=1}^{m} E[\varepsilon_i^2(j)]$ should be minimum under the condition that the original data $x_i(t)$ satisfy the model given by Eq. (1). Therefore, in order to identify the coefficient $b_{ip}(k, j)$ at time j, the index j is fixed first and then letting the partial derivative of $\sum_{i=1}^{m} E[\varepsilon_i^2(j)]$ with respect to $b_{nq}(l, j)$ be equal to zero, we get

$$\frac{\partial \sum_{i=1}^{m} E[\varepsilon_i^2(j)]}{\partial b_{nq}(l, j)} = \sum_{i=1}^{m} E\Big[2\Big\{x_i(j) - \sum_{p=1}^{i}\sum_{k=1}^{M(j)} b_{ip}(k, j)x_p(j-k)\Big\} \cdot\Big\{-\sum_{p=1}^{i}\sum_{k=1}^{M(j)} \frac{\partial b_{ip}(k, j)}{\partial b_{nq}(l, j)} x_p(j-k)\Big\}\Big]$$
$$= \sum_{n=1}^{m} E\Big[2\sum_{q=1}^{n}\sum_{l=1}^{M(j)}\Big\{-x_n(j)x_q(j-l) + \sum_{p=1}^{n}\sum_{k=1}^{M(j)} b_{np}(k, j)x_p(j-k)x_q(j-l)\Big\}\Big]=0 \quad \cdots\cdots (2)$$

From the above Eq. (2), we can get

$$E[x_n(j)x_q(j-l)] = \sum_{p=1}^{n}\sum_{k=1}^{M(j)} b_{np}(k, j)\, E[x_p(j-k)x_q(j-l)] \quad \cdots\cdots (3)$$

When a set of data $x_n(j)$; $n=1, 2, \cdots, m$ is given, the crosscorrelation function in Eq. (3) is a function of two variables in the nonstationary cases. Therefore, using the data chosen from the neighborhood of time $j\Delta t$[13] and assuming that the chosen data consist of a stationary gaussian process, the crosscorrelation function can be assumed as follows.

$$E[x_n(j)x_q(j-l)] \cong \frac{1}{2N'}\sum_{s=j-N'}^{j+N'} x_n(s)x_q(s-l) \quad \cdots\cdots (4)$$

and

$$E[x_p(j-k)\cdot x_q(j-l)] \cong \frac{1}{2N'}\sum_{s=j-N'}^{j+N'} x_p(s-k)x_q(s-l) \quad \cdots\cdots (5)$$

where, N' is a positive value and $N'<N$. From Eqs. (3), (4) and (5), we get

$$\sum_{s=j-N'}^{j+N'} x_n(s)x_q(s-l) = \sum_{p=1}^{n}\sum_{k=1}^{M(j)} b_{np}(k, j) \sum_{s=j-N'}^{j+N'} x_p(s-k)x_q(s-l) \quad \cdots\cdots (6)$$

where, $j=1, 2, \cdots, N$, $n=1, 2, \cdots, m$, $q=1, 2, \cdots, n$, $l=1, 2, \cdots, M(j)$.

Eq. (6) forms the $(n\cdot n\cdot M(j)\cdot N)-th$ order simultaneous equation. Therefore, Eq. (6) can be represented by the matrix form as follows.

$$\sum_{p=1}^{n} \boldsymbol{X}_{pq}(j)\boldsymbol{B}_{np}(j) = \boldsymbol{F}_{nq}(j) \quad \cdots\cdots (7)$$

where,

$$\boldsymbol{X}_{pq}(j) = \sum_{s=j-N'}^{j+N'} \begin{bmatrix} x_p(s-1)x_q(s-1), & x_p(s-2)x_q(s-1), & \cdots, & x_p(s-M)x_q(s-1) \\ x_p(s-1)x_q(s-2), & x_p(s-2)x_q(s-2), & \cdots, & x_p(s-M)x_q(s-2) \\ \vdots & \vdots & & \vdots \\ x_p(s-1)x_q(s-M), & x_p(s-2)x_q(s-M), & \cdots, & x_p(s-M)x_q(s-M) \end{bmatrix} \quad \cdots\cdots (8)$$

$$\boldsymbol{B}_{np}(j) = \begin{bmatrix} b_{np}(1, j) \\ b_{np}(2, j) \\ \vdots \\ b_{np}(M, j) \end{bmatrix} \quad \cdots\cdots (9)$$

$$\boldsymbol{F}_{nq}(j) = \sum_{s=j-N'}^{j+N'} \begin{bmatrix} x_n(s)x_q(s-1) \\ x_n(s)x_q(s-2) \\ \vdots \\ x_n(s)x_q(s-M) \end{bmatrix} \quad \cdots\cdots (10)$$

$M=M(j)$ in Eqs. (8), (9) and (10).

Expanding Eq. (7) with respect to p and calculating at $q=1, 2, \cdots, n$, we get n-th order simultaneous matrix equation as follows.

$$\begin{bmatrix} \boldsymbol{X}_{11}(j), & \boldsymbol{X}_{21}(j), & \cdots, & \boldsymbol{X}_{n1}(j) \\ \boldsymbol{X}_{12}(j), & \boldsymbol{X}_{22}(j), & \cdots, & \boldsymbol{X}_{n2}(j) \\ \vdots & \vdots & & \vdots \\ \boldsymbol{X}_{1n}(j), & \boldsymbol{X}_{2n}(j), & \cdots, & \boldsymbol{X}_{nn}(j) \end{bmatrix} \begin{bmatrix} \boldsymbol{B}_{n1}(j) \\ \boldsymbol{B}_{n2}(j) \\ \vdots \\ \boldsymbol{B}_{nn}(j) \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{n1}(j) \\ \boldsymbol{F}_{n2}(j) \\ \vdots \\ \boldsymbol{F}_{nn}(j) \end{bmatrix} \quad \cdots\cdots (11)$$

Therefore, we get

$$\begin{bmatrix} \boldsymbol{B}_{n1}(j) \\ \boldsymbol{B}_{n2}(j) \\ \vdots \\ \boldsymbol{B}_{nn}(j) \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_{11}(j), & \boldsymbol{X}_{21}(j), & \cdots, & \boldsymbol{X}_{n1}(j) \\ \boldsymbol{X}_{12}(j), & \boldsymbol{X}_{22}(j), & \cdots, & \boldsymbol{X}_{n2}(j) \\ \vdots & \vdots & & \vdots \\ \boldsymbol{X}_{1n}(j), & \boldsymbol{X}_{2n}(j), & \cdots, & \boldsymbol{X}_{nn}(j) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \boldsymbol{F}_{n1}(j) \\ \boldsymbol{F}_{n2}(j) \\ \vdots \\ \boldsymbol{F}_{nn}(j) \end{bmatrix}; \; j=1, 2, \cdots, N \quad \cdots\cdots (12)$$

From Eq. (12), we can identify coefficients $b_{np}(k, j)$ for $n=1, 2, \cdots, m$ as follows. For $n=1$, determining $\boldsymbol{X}_{11}^{-1}(j)$ and $\boldsymbol{F}_{11}(j)$ from $j=1$ to $j=N$ by using

Eqs. (8) and (10), we can identify $\boldsymbol{B}_{11}(j)$ from Eq. (12).

$$\boldsymbol{B}_{11}(j)=\begin{bmatrix} b_{11}(1,\ j) \\ b_{11}(2,\ j) \\ \vdots \\ b_{11}(M,\ j) \end{bmatrix}=\boldsymbol{X}_{11}^{-1}(j)\boldsymbol{F}_{11}(j);$$

$$j=1,\ 2,\ \cdots,\ N, \qquad M=M(j) \quad \cdots\cdots (13)$$

At the next step, letting $n=2$, we get the coefficients $\boldsymbol{B}_{21}(j)$ and $\boldsymbol{B}_{22}(j)$. Since $\boldsymbol{X}_{11}(j)$; $j=1, 2, \cdots, N$ has already been calculated, determining $\boldsymbol{X}_{21}(j)$, $\boldsymbol{X}_{12}(j)$, $\boldsymbol{X}_{22}(j)$, $\boldsymbol{F}_{21}(j)$ and $\boldsymbol{F}_{22}(j)$ at $j=1, 2, \cdots, N$ from Eqs. (8) and (10), the coefficients $\boldsymbol{B}_{21}(j)$ and $\boldsymbol{B}_{22}(j)$ can be identified from Eq. (12) as follows.

$$\begin{bmatrix} \boldsymbol{B}_{21}(j) \\ \boldsymbol{B}_{22}(j) \end{bmatrix}=\begin{bmatrix} \begin{bmatrix} b_{21}(1,\ j) \\ b_{21}(2,\ j) \\ \vdots \\ b_{21}(M,\ j) \end{bmatrix} \\ \begin{bmatrix} b_{22}(1,\ j) \\ b_{22}(2,\ j) \\ \vdots \\ b_{22}(M,\ j) \end{bmatrix} \end{bmatrix}=\begin{bmatrix} \boldsymbol{X}_{11}(j) & \boldsymbol{X}_{21}(j) \\ \boldsymbol{X}_{12}(j) & \boldsymbol{X}_{22}(j) \end{bmatrix}^{-1}$$

$$\cdot\begin{bmatrix} \boldsymbol{F}_{21}(j) \\ \boldsymbol{F}_{22}(j) \end{bmatrix};\ j=1,\ 2,\ \cdots,\ N, \qquad M=M(j) \quad \cdots\cdots (14)$$

Repeating the similar calculations from $n=3$ to $n=m$, all $\boldsymbol{B}_{np}(j)$ are identified by using Eq. (12).

However, if these calculations are repeated step by step from $j=1$ to $j=N$, the computational time will become seriously long. But, if this AR model is applied to such as an earthquake accelerogram, $\boldsymbol{B}_{np}(j)$ can be assumed to be nearly equal to $\boldsymbol{B}_{np}(u)$ when j and u are close, since the nonstationarity of the earthquake accelerogram changes gradually in the vicinity of time $j\Delta t$[3),4),14)]. Therefore, if 30 seconds discrete data with $\Delta t=0.05$ sec. is used, for example, total number of data N is equal to 600. Instead of calculating $\boldsymbol{B}_{np}(j)$ for all time $j=1, 2, \cdots, 600$, $\boldsymbol{B}_{np}(j)$ are determined at the specified time with a constant interval ΔT which is bigger than the interval of given data Δt, that is, $\Delta T=a\Delta t$; $a=$ integer and $a\gg 1$. And then, coefficient values between $j\Delta t-\frac{\Delta T}{2}$ and $j\Delta t+\frac{\Delta T}{2}$ are considered that they have the same constant value of $\boldsymbol{B}_{np}(j)$, or are determined by interpolation. Using the above procedure, the computational time can be greatly reduced.

In the calculation of Eq. (12), the values of $x_i(j)$ at negative time in Eqs. (8) and (10) are needed when time j stays in early stages. In this case, proper dummy values of $x_i(j)$, e.g. 0.0, can be used for the calculation.

The unknown coefficients $b_{ip}(k, j)$ have been identified from Eq. (12) as mentioned above. Then, the error function $\varepsilon_i(j)$ in Eq. (1) must be considered. From Akaike's procedure,

$$E[\varepsilon_i(j)x_q(j-l)]$$
$$\cong\frac{1}{2N'}\sum_{s=j-N'}^{j+N'}\varepsilon_i(s)x_q(s-l)$$
$$=\frac{1}{2N'}\sum_{s=j-N'}^{j+N'}\left\{x_i(s)-\sum_{p=1}^{i}\sum_{k=1}^{M(j)}b_{ip}(k,\ s)\cdot x_p(s-k)\right\}x_q(s-l)$$
$$=\frac{1}{2N'}\sum_{s=j-N'}^{j+N'}x_i(s)x_q(s-l)$$
$$-\frac{1}{2N'}\sum_{s=j-N'}^{j+N'}\sum_{p=1}^{i}\sum_{k=1}^{M(j)}b_{ip}(k,\ s)\cdot x_p(s-k)x_q(s-l);\ l=1,\ 2,\ \cdots,\ M(j) \quad \cdots\cdots (15)$$

If Eq. (6) is satisfied, the second term in Eq. (15) is equal to the first term. Therefore, we get

$$E[\varepsilon_i(j)x_q(j-l)]=0;\ l=1,\ 2,\ \cdots,\ M(j) \quad \cdots\cdots (16)$$

Hence, $\varepsilon_i(j)$ and $x_q(j-l)$ are mutually independent for $l=1, 2, \cdots, M(j)$. But, if $x_i(j)$ are applied to the engineering fields, for example; earthquake motions, etc., it is natural that the autocorrelation decreases rapidly when the time lag increases. Therefore, if it is assumed that $x_q(j-l)$ and $x_q(j-r)$: $r>M(j)$ are approximately independent, Eq. (16) can be expanded as follows.

$$E[\varepsilon_i(j)x_q(j-l)]=0;\ l=1,\ 2,\ \cdots,\ M(j),\ M(j)+1,\ \cdots,\ \infty \quad \cdots\cdots (17)$$

Since $\varepsilon_q(j-v)$ is given by the linear summation of $x_q(j-v)$, $x_p(j-v-1), \cdots, x_p(j-v-M(j))$; $v\geqq 1$, as shown in Eq. (1), and also satisfies the relation of Eq. (17), it is seen that $\varepsilon_i(j)$ and $\varepsilon_q(j-v)$; $v\geqq 1$, are mutually independent. Therefore, we get

$$E[\varepsilon_i(j)\varepsilon_q(j-v)]=0;\ v\geqq 1,\ \text{any } i \text{ and } q \quad \cdots\cdots (18)$$

From Eq. (18), it can be seen that $\varepsilon_i(j)$; $i=1, 2, \cdots, m$ is the band limited white noise with zero mean. Exactly speaking, $\varepsilon_i(j)$ is a nonstationary white noise, that is, a shot noise, since the variance changes depending on time j. And $\varepsilon_i(j)$ and $\varepsilon_q(j-v)$ are mutually independent for $i\neq q$ and $v\geqq 1$.

The crosscovariance of $\varepsilon_i(j)$ can be determined from Eq. (1) as follows.

$$\sigma_{iq}^2(j)=E[\varepsilon_i(j)\varepsilon_q(j)]$$
$$\cong\frac{1}{2N'}\sum_{s=j-N'}^{j+N'}\varepsilon_i(s)\varepsilon_q(s)$$
$$=\frac{1}{2N'}\sum_{s=j-N'}^{j+N'}\left\{x_i(s)-\sum_{p=1}^{i}\sum_{k=1}^{M(j)}b_{ip}(k,\ s)\cdot x_p(s-k)\right\}\left\{x_q(s)-\sum_{p=1}^{q}\sum_{k=1}^{M(j)}b_{qp}(k,\ s)\cdot x_p(s-k)\right\} \quad \cdots\cdots (19)$$

In order to generate the error function $\varepsilon_i(j)$, the following crosscorrelation matrix can be used.

$$\boldsymbol{\sigma}^2(j)=\begin{bmatrix} \sigma_{11}^2(j) & \sigma_{12}^2(j) & \cdots & \sigma_{1m}^2(j) \\ \sigma_{21}^2(j) & \sigma_{22}^2(j) & \cdots & \sigma_{2m}^2(j) \\ \vdots & \vdots & & \vdots \\ \sigma_{m1}^2(j) & \sigma_{m2}^2(j) & \cdots & \sigma_{mm}^2(j) \end{bmatrix} \quad \cdots\cdots (20)$$

Now, let us introduce the following matrix equation

$$\boldsymbol{\varepsilon}(j)=\begin{bmatrix}\varepsilon_1(j)\\ \varepsilon_2(j)\\ \vdots\\ \varepsilon_m(j)\end{bmatrix}=\begin{bmatrix}c_{11}(j) & & & 0\\ c_{21}(j) & c_{22}(j) & & \\ \vdots & \vdots & \ddots & \\ c_{m1}(j) & c_{m2}(j) & \cdots & c_{mm}(j)\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\\ \vdots\\ \xi_m\end{bmatrix}=\boldsymbol{C}\boldsymbol{\xi} \quad \cdots (21)$$

where, the matrix $\boldsymbol{C}$ is a lower triangular linear transformation matrix, and ξ_i; $i=1, 2, \cdots, m$ are mutually independent random variables with zero mean and its variance is unity. From Eq. (21), we get

$$\boldsymbol{\varepsilon}(j)\boldsymbol{\varepsilon}^T(j)=\boldsymbol{C}(j)\boldsymbol{\xi}\boldsymbol{\xi}^T\boldsymbol{C}^T(j) \quad \cdots (22)$$

Taking the expectation of both sides in Eq. (22), we get

$$E[\boldsymbol{\varepsilon}(j)\boldsymbol{\varepsilon}^T(j)]=\boldsymbol{C}(j)\boldsymbol{C}^T(j) \quad \cdots (23)$$

because

$$E[\boldsymbol{\xi}\boldsymbol{\xi}^T]=\boldsymbol{I}=\begin{bmatrix}1 & & \\ & 1 & \\ & & \ddots \\ & & & 1\end{bmatrix}$$

Therefore, from Eqs. (19), (20) and (23), we get

$$\boldsymbol{\sigma}^2(j)=\boldsymbol{C}(j)\boldsymbol{C}^T(j) \quad \cdots (24)$$

From Eqs. (20), (21) and (23), $\varepsilon_i(j)$ can be generated as follows. First of all, decompose the cross-correlation matrix $\boldsymbol{\sigma}^2(j)$ given by Eq. (19) to the products of two triangular matrices $\boldsymbol{C}(j)$ and $\boldsymbol{C}^T(j)$ by utilizing Eq. (24). And then, generate the error term $\boldsymbol{\varepsilon}_i(j)$ from $\boldsymbol{C}(j)$ and Eq. (21) at every time j independently.

Using the unknown coefficients $b_{ip}(k, j)$ and the error term $\varepsilon_i(j)$ determined by the above procedure, the multi-dimensional nonstationary stochastic processes can be simulated from Eq. (1).

(2) Special Cases

a) One Dimensional Nonstationary Process

For the case of one dimensional nonstationary stochastic process, letting $x_i(j)=x(j)$, $b_{ip}(k, j)=b(k, j)$, $x_p(j-k)=x(j-k)$ and $\varepsilon_i(j)=\varepsilon(j)$ in Eq. (1), the AR model can be given as follows.

$$x(j)=\sum_{k=1}^{M(j)} b(k, j)x(j-k)+\varepsilon(j) \quad \cdots (25)$$

Refferencing Eqs. (8), (9), (10) and (13), the coefficients $b(k, j)$ can be identified easily as follows.

$$\boldsymbol{F}(j)=\sum_{s=j-N'}^{j+N'}\begin{bmatrix}x(s)x(s-1)\\ x(s)x(s-2)\\ \vdots\\ x(s)x(s-M)\end{bmatrix} \quad \cdots (26)$$

$$\boldsymbol{X}(j)=\sum_{s=j-N'}^{j+N'}\begin{bmatrix}x(s-1)x(s-1), & x(s-2)x(s-1), & \cdots, & x(s-M)x(s-1)\\ x(s-1)x(s-2), & x(s-2)x(s-2), & \cdots, & x(s-M)x(s-2)\\ \vdots & & & \\ x(s-1)x(s-M), & x(s-2)x(s-M), & \cdots, & x(s-M)x(s-M)\end{bmatrix} \quad \cdots (27)$$

$$\boldsymbol{B}(j)=\begin{bmatrix}b(1, j)\\ b(2, j)\\ \vdots\\ b(M, j)\end{bmatrix}=\boldsymbol{X}^{-1}(j)\boldsymbol{F}(j) \quad \cdots (28)$$

$$;\ j=1, 2, \cdots, N, \qquad M=M(j)$$

b) Multi-dimensional Stationary Process

For the multi-dimensional stationary stochastic process, since the unknown coefficient $b_{ip}(k, j)$ which must be identified is independent of time j, Eq. (1) can be modified by letting $b_{ip}(k, j)=b_{ip}(k)$ as follows.

$$x_i(j)=\sum_{p=1}^{i}\sum_{k=1}^{M} b_{ip}(k)x_p(j-k)+\varepsilon_i(j) \quad \cdots (29)$$

$$;\ i=1, 2, \cdots, m$$

In this case, the basic equation of identification of $b_{ip}(k)$ is changed by summing up the both sides of Eq. (6) with respect to time j, since $b_{ip}(k)$ is independent of time j.

$$\sum_{s=1}^{N} x_n(s)x_q(s-l)=\sum_{p=1}^{n}\sum_{k=1}^{M} b_{np}(k)\sum_{s=1}^{N} x_p(s-k)x_q(s-l) \quad \cdots (30)$$

Rearranging Eq. (30) with matrix form, we get

$$\boldsymbol{F}_{nq}=\begin{bmatrix}x_q(0), & x_q(1), & \cdots, & x_q(N-1)\\ x_q(-1), & x_q(0), & \cdots, & x_q(N-2)\\ \vdots & \vdots & & \vdots\\ x_q(1-M), & x_q(2-M), & \cdots, & x_q(N-M)\end{bmatrix}\cdot\begin{bmatrix}x_n(1)\\ x_n(2)\\ \vdots\\ x_n(N)\end{bmatrix} \quad \cdots (31)$$

$$\boldsymbol{X}_{pq}=\begin{bmatrix}x_q(0), & x_q(1), & \cdots, & x_q(N-1)\\ x_q(-1), & x_q(0), & \cdots, & x_q(N-2)\\ \vdots & \vdots & & \vdots\\ x_q(1-M), & x_q(2-M), & \cdots, & x_q(N-M)\end{bmatrix}\cdot\begin{bmatrix}x_p(0), & x_p(-1), & \cdots, & x_p(1-M)\\ x_p(1), & x_p(0), & \cdots, & x_p(2-M)\\ \vdots & \vdots & & \vdots\\ x_p(N-1), & x_p(N-2), & \cdots, & x_p(N-M)\end{bmatrix} \quad \cdots (32)$$

$$\begin{bmatrix}\boldsymbol{B}_{n1}\\ \boldsymbol{B}_{n2}\\ \vdots\\ \boldsymbol{B}_{nn}\end{bmatrix}=\begin{bmatrix}\boldsymbol{X}_{11}, & \boldsymbol{X}_{21}, & \cdots, & \boldsymbol{X}_{n1}\\ \boldsymbol{X}_{12}, & \boldsymbol{X}_{22}, & \cdots, & \boldsymbol{X}_{n2}\\ \vdots & \vdots & & \vdots\\ \boldsymbol{X}_{1n}, & \boldsymbol{X}_{2n}, & \cdots, & \boldsymbol{X}_{nn}\end{bmatrix}^{-1}\cdot\begin{bmatrix}\boldsymbol{F}_{n1}\\ \boldsymbol{F}_{n2}\\ \vdots\\ \boldsymbol{F}_{nn}\end{bmatrix} \quad \cdots (33)$$

where,

$$\boldsymbol{B}_{np}=\begin{bmatrix}b_{np}(1)\\ b_{np}(2)\\ \vdots\\ b_{np}(M)\end{bmatrix} \quad \cdots (34)$$

The above models are derived for two particular cases. These are easily induced from Eq. (1) which represents the multi-dimensional nonstationary stochastic AR model.

(3) Relationships between Nonstationary Spectrum and Unknown Coefficient $b_{ip}(k, j)$

The relations between the unknown coefficient $b_{ip}(k, j)$ and the nonstationary cross spectrum of the observed data or $x_i(j)$ determined in section **2. (1)** are discussed hereafter.

Nonstationary cross spectrum is defined as follows[1].

$$S_{ij}(\omega, t)=E\left[\frac{1}{2\pi}X_i(\omega, t)X_j^*(\omega, t)\right] \cdots (35)$$

$$; \ -\infty<\omega<\infty, \ -\infty<t<\infty$$

where,

$$X_i(\omega, t)=\int_{-\infty}^{\infty} W(t-u)x_i(u)e^{-r\omega u}du; \ r^2=-1$$

Although details on the data window $W(t)$ can be seen in references 14) or 15), its characteristic is considered as the weighting function. Therefore, appling the weighted Fourier transformation to the both sides of Eq. (1) for the discrete type $x_i(j)$, we have

$$X_i(\omega, j)=\sum_{l=-\infty}^{\infty} W(j-l)x_i(l)e^{-r\omega l\Delta t}\Delta t$$
$$=\sum_{p=1}^{i}\sum_{k=1}^{M(j)}\sum_{l=-\infty}^{\infty} W(j-l)b_{ip}(k, l)$$
$$\cdot x_p(l-k)e^{-r\omega l\Delta t}\Delta t$$
$$+\sum_{l=-\infty}^{\infty} W(j-l)\varepsilon_i(l)e^{-r\omega l\Delta t}\Delta t \cdots\cdots (36)$$

Assuming that the coefficient $b_{ip}(k, l)$ has a characteristic that its value changes gradually with the, change of time l as shown in references 1) or 14), we get the following approximate equation after replacing $l-k$ by s in the first term of Eq. (36) and taking out the coefficient $b_{ip}(k, s+k)$ from the window function $W(j-s-k)$.

$$X_i(\omega, j)\cong\sum_{p=1}^{i}\sum_{k=1}^{M(j)} b_{ip}(k, j)e^{-r\omega k\Delta t}$$
$$\cdot\sum_{s=-\infty}^{\infty} W(j-s-k)x_p(s)e^{-r\omega s\Delta t}\Delta t$$
$$+\sum_{l=-\infty}^{\infty} W(j-l)\varepsilon_i(l)e^{-r\omega l\Delta t}\Delta t \cdots\cdots (37)$$

Since $\varepsilon_i(l)$ is a nonstationary white noise, letting

$$E_i(\omega, j)\equiv\sum_{l=-\infty}^{\infty} W(j-l)\varepsilon_i(l)e^{-r\omega l\Delta t}\Delta t, \cdots (38)$$

we get

$$X_i(\omega, j)\cong\sum_{p=1}^{i}\sum_{k=1}^{M(j)} b_{ip}(k, j)e^{-r\omega k\Delta t}X_p(\omega, j)$$
$$+E_i(\omega, j) \cdots\cdots (39)$$

where,

$$X_p(\omega, j)\cong\sum_{s=-\infty}^{\infty} W(j-s-k)x_p(s)e^{-r\omega s\Delta t}\Delta t \cdots\cdots (40)$$

Rearranging Eq. (39) with matrix form, we get

$$\boldsymbol{X}(\omega, j)=\boldsymbol{B}(\omega, j)\boldsymbol{X}(\omega, j)+\boldsymbol{E}(\omega, j) \cdots\cdots (41)$$

where,

$$\boldsymbol{X}(\omega, j)=[X_1(\omega, j), X_2(\omega, j), \cdots, X_m(\omega, j)]^T \cdots\cdots (42)$$

$$\boldsymbol{E}(\omega, j)=[E_1(\omega, j), E_2(\omega, j), \cdots, E_m(\omega, j)]^T \cdots\cdots (43)$$

Therefore, letting

$$\boldsymbol{I}-\boldsymbol{B}(\omega, j)=\boldsymbol{H}(\omega, j), \cdots\cdots (45)$$

we get

$$\boldsymbol{H}(\omega, j)\boldsymbol{X}(\omega, j)=\boldsymbol{E}(\omega, j) \cdots\cdots (46)$$

From the above Eq. (46), we get

$$\boldsymbol{H}(\omega, j)\boldsymbol{X}(\omega, j)\boldsymbol{X}^{*T}(\omega, j)\boldsymbol{H}^{*T}(\omega, j)$$
$$=\boldsymbol{E}(\omega, j)\boldsymbol{E}^{*T}(\omega, j) \cdots\cdots (47)$$

Taking the expectation of both sides of Eq. (47), and following the definition of nonstationary spectral matrix, we get

$$\boldsymbol{S}_x(\omega, j)=E\left[\frac{1}{2\pi}\boldsymbol{X}(\omega, j)\boldsymbol{X}^{*T}(\omega, j)\right]$$
$$=\boldsymbol{H}^{-1}(\omega, j)\boldsymbol{S}_\varepsilon(\omega, j)\{\boldsymbol{H}^{*T}(\omega, j)\}^{-1} \cdots\cdots (48)$$

where, $\boldsymbol{S}_\varepsilon(\omega, j)$ is a nonstationary cross spectral matrix and given by

$$\boldsymbol{S}_\varepsilon(\omega, j)=\frac{\Delta t}{2\pi}\boldsymbol{\sigma}^2(j)$$
$$=\frac{\Delta t}{2\pi}\begin{bmatrix}\sigma_{11}^2(j), & \sigma_{12}^2(j), & \cdots, & \sigma_{1m}^2(j)\\ \sigma_{21}^2(j), & \sigma_{22}^2(j), & \cdots, & \sigma_{2m}^2(j)\\ \vdots & & & \\ \sigma_{m1}^2(j), & \sigma_{m2}^2(j), & \cdots, & \sigma_{mm}^2(j)\end{bmatrix} \cdots\cdots (49)$$

$$; \ -\frac{\pi}{\Delta t}\leq\omega\leq\frac{\pi}{\Delta t}$$

After identifing $b_{ip}(k, j)$, we can immediately calculate the nonstationary cross spectrum by using Eq. (48). For the one dimensional nonstationary stochastic processes, Eq. (48) becomes as follows.

$$\boldsymbol{S}_x(\omega, j)=\frac{\sigma^2(j)\Delta t}{2\pi}\Bigg/\left|1-\sum_{k=1}^{M(j)} b(k, j)e^{-i\omega k\Delta t}\right|^2 \cdots\cdots (50)$$

$$; \ -\frac{\pi}{\Delta t}\leq\omega\leq\frac{\pi}{\Delta t}$$

For the multi-dimensional stationary stochastic processes, Eq. (29), similar equations of relationship corresponding Eqs. (48) and (49) can be derived easily. But, in this case, error function is a stationary white noise and is independent of time j. Therefore, Eq. (49) has a constant value and is independent of time j.

3. MOVING AVERAGE MODEL (MA MODEL)

(1) MA Model for Multi-dimensional Nonstationary Stochastic Processes

A multi-dimensional MA model in time domain is proposed by the one of the authors in the previous paper[2] as follows.

$$\boldsymbol{B}(\omega, j)=\begin{bmatrix}\sum_{k=1}^{M(j)} b_{11}(k, j)e^{-r\omega k\Delta t} & & & 0\\ \sum_{k=1}^{M(j)} b_{21}(k, j)e^{-r\omega k\Delta t}, & \sum_{k=1}^{M(j)} b_{22}(k, j)e^{-r\omega k\Delta t} & \ddots & \\ \vdots & \vdots & & \\ \sum_{k=1}^{M(j)} b_{m1}(k, j)e^{-r\omega k\Delta t}, & \sum_{k=1}^{M(j)} b_{m2}(k, j)e^{-r\omega k\Delta t}, & \cdots, & \sum_{k=1}^{M(j)} b_{mm}(k, j)e^{-r\omega k\Delta t}\end{bmatrix} \cdots\cdots (44)$$

$$x_i(t)=\sum_{p=1}^{i}\int_{-\infty}^{\infty}h_{ip}(t-\xi,t)a_p(\xi)d\xi$$
$$=\sum_{p=1}^{i}\int_{-\infty}^{\infty}h_{ip}(\xi,t)a_p(t-\xi)d\xi;\ i=1,2,\cdots,m \quad \cdots\cdots(51)$$

Therefore, a multi-dimensional nonstationary MA model, is given by

$$x_i(j)=\sum_{p=1}^{i}\sum_{k=1}^{M(j)}h_{ip}(k,j)a_p(j-k)+\varepsilon_i(j) \quad \cdots\cdots(52)$$
$$;\ i=1,2,\cdots,m$$

where, Δt is included in the expression of $h_{ip}(k,j)$ and symbols j and $M(j)$ have the same meanings in AR model. And $a_p(j)$ are m mutually independent random variables (white noise) with zero mean and its variances $E[a_p^2(j)]$ are equal to σ^2.

The multi-dimensional nonstationary stochastic processes $x_i(j)$; $i=1,2,\cdots,m$ given by Eq. (52) can be considered as the output of the multi-dimensional nonstationary filter $h_{ip}(k,j)$, where the input is the mutually independent white noise $a_p(j)$; $p=1,2,\cdots,m$.

The unknown coefficients $h_{ip}(k,j)$ are identified in order that the observed earthquake data satisfies Eq. (52). Following the criterion of minimizing the mean square error $\sum_{i=1}^{m}E[\varepsilon_i^2(j)]$, and pursuing the same derivation in AR model, we get

$$\sum_{s=j-N'}^{j+N'}x_n(s)a_q(s-l)$$
$$=\sum_{p=1}^{n}\sum_{k=1}^{M(j)}h_{np}(k,j)\sum_{s=j-N'}^{j+N'}a_p(s-k)a_q(s-l) \quad \cdots\cdots(53)$$

However, since $a_p(j)$ and $a_q(u)$ are mutually independent if $p\neq q$, we have

$$\sum_{s=j-N'}^{j+N'}a_p(s-k)a_q(s-l)=\begin{cases}2N'\sigma^2; & p=q\\ 0 ; & p\neq q\end{cases} \quad \cdots\cdots(54)$$

Therefore, Eq. (53) can be written as follows.

$$\sum_{s=j-N'}^{j+N'}x_n(s)a_q(s-l)=\sum_{k=1}^{M(j)}h_{nq}(k,j)\cdot 2N'\sigma^2 \quad \cdots\cdots(55)$$
$$j=1,2,\cdots,N \qquad n=1,2,\cdots,m$$
$$q=1,2,\cdots,n \qquad l=1,2,\cdots,M(j)$$

Rearranging Eq. (55) into the matrix form, we get

$$\boldsymbol{H}_{nq}(j)=\boldsymbol{A}_{qq}^{-1}\boldsymbol{F}_{nq}(j) \quad \cdots\cdots(56)$$
$$n=1,2,\cdots,m \qquad q=1,2,\cdots,n$$

where,

$$\boldsymbol{H}_{nq}(j)=\begin{bmatrix}h_{nq}(1,j)\\ h_{nq}(2,j)\\ \vdots\\ h_{nq}(M,j)\end{bmatrix} \quad \cdots\cdots(57)$$

$$\boldsymbol{A}_{qq}=2N'\sigma^2\boldsymbol{I} \quad \cdots\cdots(58)$$

$$\boldsymbol{F}_{nq}(j)=\sum_{s=j-N'}^{j+N'}\begin{bmatrix}x_n(s)a_q(s-1)\\ x_n(s)a_q(s-2)\\ \vdots\\ x_n(s)a_q(s-M)\end{bmatrix} \quad \cdots\cdots(59)$$

$$M=M(j)$$

Hence, $h_{ip}(k,j)$ can be identified from Eq. (56).

(2) Special Cases

For one dimensional nonstationary stochastic processes, the equation for an identification of $h(k,j)$ can be derived by the same procedure as mentioned in the previous section.

For multi-dimensional stationary stochastic processes, since the unknown coefficient $h_{ip}(k,j)$ is independent of time j, Eq. (52) is modified as follows,

$$x_i(j)=\sum_{p=1}^{i}\sum_{k=1}^{M}h_{ip}(k)a_p(j-k)+\varepsilon_i(j) \quad \cdots\cdots(60)$$
$$;\ i=1,2,\cdots,m$$

Instead of Eq. (54), all data are used for the identification of $h_{ip}(k)$ as follows.

$$\boldsymbol{F}_{nq}=\sum_{s=1}^{N}\boldsymbol{F}_{nq}(s)$$
$$=\begin{bmatrix}a_q(0), & a_q(1), & \cdots, & a_q(N-1)\\ a_q(-1), & a_q(0), & \cdots, & a_q(N-2)\\ \vdots & \vdots & & \vdots\\ a_q(1-M), & a_q(2-M), & \cdots, & a_q(N-M)\end{bmatrix}\cdot\begin{bmatrix}x_n(1)\\ x_n(2)\\ \vdots\\ x_n(N)\end{bmatrix} \quad \cdots\cdots(61)$$

And instead of Eq. (58),

$$\boldsymbol{A}_{qq}=N\sigma^2\boldsymbol{I} \quad \cdots\cdots(62)$$

Therefore, we get

$$\boldsymbol{H}_{nq}=\begin{bmatrix}h_{nq}(1)\\ h_{nq}(2)\\ \vdots\\ h_{nq}(M)\end{bmatrix}=\boldsymbol{A}_{qq}^{-1}\boldsymbol{F}_{nq}$$
$$=\frac{1}{N\sigma^2}\begin{bmatrix}a_q(0), & a_q(1), & \cdots, & a_q(N-1)\\ a_q(-1), & a_q(0), & \cdots, & a_q(N-2)\\ \vdots & & & \\ a_q(1-M), & a_q(2-M), & \cdots, & a_q(N-M)\end{bmatrix}\cdot\begin{bmatrix}x_n(1)\\ x_n(2)\\ \vdots\\ x_n(N)\end{bmatrix} \quad \cdots\cdots(63)$$
$$;\ n=1,2,\cdots,m \qquad q=1,2,\cdots,m$$

$\varepsilon_i(j)$ can be determined as in the case of AR model, and it is a nonstationary white noise for the nonstationary process and is a stationary white noise for the stationary process.

(3) Relationships between Nonstationary Spectrum and Unknown Coefficient $h_{ip}(k,j)$

The multi-dimensional nonstationary stochastic process of Eq. (52) is considered again hereafter. The relationships between the nonstationary cross spectral matrix $\boldsymbol{S}_x(\omega,j)$ and the identified $h_{ip}(k,j)$ can be derived by assuming the same approximation in the previous section as follows.

$$\boldsymbol{X}(\omega,j)=\boldsymbol{H}(\omega,j)\boldsymbol{A}(\omega)+\boldsymbol{E}(\omega,t) \quad \cdots\cdots(64)$$

where,

$$\boldsymbol{X}(\omega,j)=[X_1(\omega,j),X_2(\omega,j),\cdots,X_m(\omega,j)]^T \quad \cdots\cdots(65)$$

$$A(\omega)=[A_1(\omega),\ A_2(\omega),\ \cdots,\ A_m(\omega)]^T \cdots\cdots\cdots\cdots\cdots (66)$$

$$H(\omega,\ j)=\sum_{k=1}^{M(j)}\begin{bmatrix} h_{11}(k,j)e^{-r\omega k\Delta t} & & & 0 \\ h_{21}(k,j)e^{-r\omega k\Delta t}, & h_{22}(k,j)e^{-r\omega k\Delta t} & & \\ \vdots & \vdots & \ddots & \\ h_{m1}(k,j)e^{-r\omega k\Delta t}, & h_{m2}(k,j)e^{-r\omega k\Delta t}, & \cdots, & h_{mm}(k,j)e^{-r\omega k\Delta t} \end{bmatrix} \cdots\cdots (67)$$

From Eq. (64), we get

$$\begin{aligned} S(\omega,\ j) &= \frac{1}{2\pi}E[X(\omega,\ j)X^{*T}(\omega,\ j)] \\ &= H(\omega,\ j)S_a(\omega)H^{*T}(\omega,\ j)+S_\varepsilon(\omega,\ j) \\ &= \frac{\sigma^2\Delta t}{2\pi}H(\omega,\ j)H^{*T}(\omega,\ j)+S_\varepsilon(\omega,\ j) \end{aligned} \cdots\cdots (68)$$

$$;\ -\frac{\pi}{\Delta t}\leq\omega\leq\frac{\pi}{\Delta t}$$

Comparing Eqs. (51) and (52), and considering the results of Ref. 2), it is obvious that $S_\varepsilon(\omega,\ j)\rightarrow 0$ when $M\rightarrow\infty$.

4. NUMERICAL CALCULATIONS AND DISCUSSIONS

In this section, an appropriateness of the simulation model proposed by Eq. (1) is discussed. The earthquake acceleration records used for the numerical calculations are three components of San Fernando Earthquake in Feb. 9, 1971[16] which were observed at a basement of Millikan Library, Calf. Inst. of Tech. The informations of these records are listed in **Table 1**. These records which are digitized with equal time interval of $\Delta t=0.04$ sec. are also illustrated in **Fig. 1** and their physical spectra are shown in **Figs. 2** to **4**.

In order to determine the nonstationarity in time domain, the acceleration record is first divided into small groups with equal time length of $2N'\Delta t$ sec. by shifting the central time $j\Delta t$ with constant interval $\Delta T=10\Delta t$ sec. as mentioned in section **2. (1)**. Then $M(j)$ values, if assumed not to be constant, and coefficients $B_{np}(j)$ in Eq. (1) are determined at the specified time $j\Delta t$.

$M(j)$ value at time $j\Delta t$ is determined by appling the Final Prediction Error (FPE) method proposed by Akaike[5] under the assumption of partial stationarity of the processes for each small group. And then, the values of $M(u)$ where $j\Delta t-\frac{\Delta T}{2}\leq u\leq j\Delta t+\frac{\Delta T}{2}$ are considered to be the same constant value of $M(j)$ at time $j\Delta t$.

$B_{np}(j)$ values at time $j\Delta t$ are determined by Eq. (12) and the values of $B_{np}(u)$ where

Table 1 Reference of Earthquake Acceleration.

DATA	Component	Max. Acc. (gal)	E.D. (km)	Location
Millikan Library	*E-W*	178.2	40	Basement
	N-S	206.9		
	U-D	97.0		

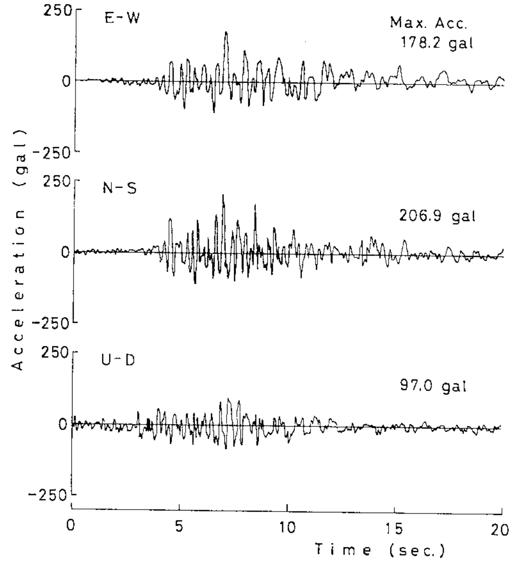

Fig. 1 Acceleration Records of Millikan Library.

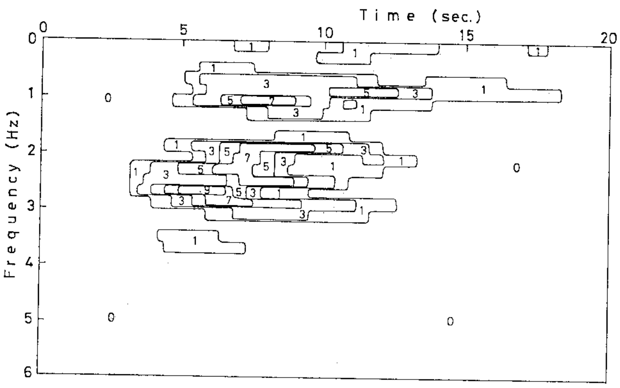

Fig. 2 Physical Spectrum of Millikan Library (*E-W* Component).

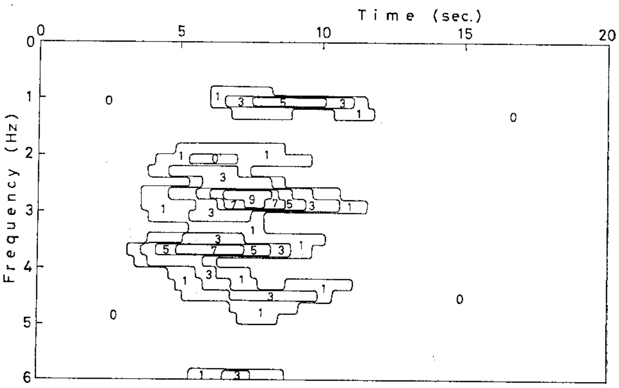

Fig. 3 Physical Spectrum of Millikan Library (*N-S* Component).

$j\Delta t \leq u < j\Delta t + \Delta T$ are interpolated by second order polynomials.

The effects of N' values in Eq. (4) or (8) are first examined for one dimensional model, that is, $m=1$ in Eq. (1). In this case, $M(j)$ values are determined by FPE method for each small group which has a $2N'\Delta t$ second time duration. Simulated waves with different N' values are shown in **Fig. 5.** From **Fig. 5** and the physical spectra of these simulated waves (not shown), we can see that the each simulated wave contains higher frequency content waves than the original wave does. And it can be seen that the physical spectra of the simulated waves are seemingly slightly flatter than that of the original wave and the appearance time of predominant frequency are delayed slightly. However, it seems that there is no significant difference between the simulated waves and the original wave. Therefore, we can assume that the suitable value of N' is about 30.

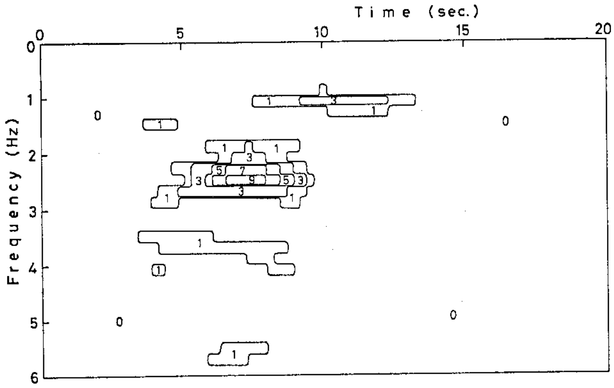

Fig. 4 Physical Spectrum of Millikan Library (*U-D* Component).

Next, five sample waves are generated for two cases with $N'=25$ and 30 by changing the random variables ξ in Eq. (21). Though the sample waves are not shown here, the average value of maximum acceleration of five simulated waves is 197 gal (max.; 255 gal, min.; 166 gal) for $N'=25$, and is 186 gal (max.; 221 gal, min.; 166 gal) for $N'=30$. Since the original wave has the maximum value of 178 gal, $N'=25$ to 30 may be accepted in the simulation.

$M(j)$ values vs. time determined by FPE method are shown in **Figs. 6** and **7**. The transition of $M(j)$ values on E-W component shows particular shape like an envelope function of an earthquake acceleration wave,

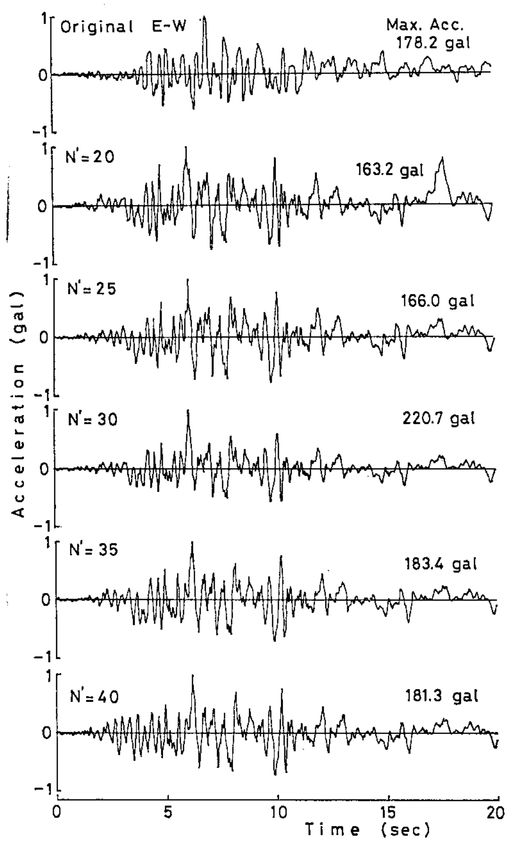

Fig. 5 Simulated Waves with Different N' Values.

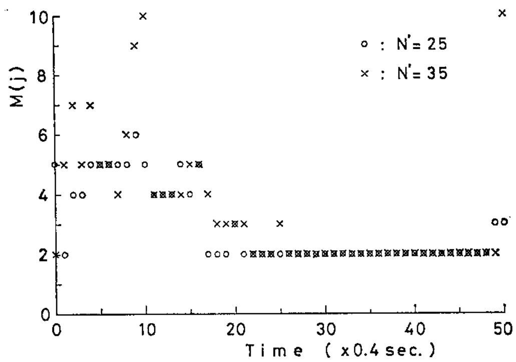

Fig. 6 $M(j)$ Values Determined by FPE (E-W Component).

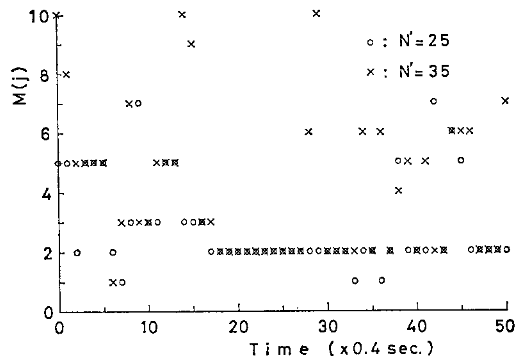

Fig. 7 $M(j)$ Values Determined by FPE (N-S Component).

whereas the other two components (*N-S*, *U-D*) do not have any particular characteristic.

In accordance to the above results, the simulation was carried out with constant M value for $M=3$, 4, 5, 6 and 7 and with $N'=25$. Though these figures of simulated waves are omitted, the maximum accelerations are 183 to 191 gal for each M value and the shapes of these waves are almost similar to that of the original wave. Therefore, we may decide from the above discussions that $N'=25$ and $M=4$ are appropriate values. Another five sample waves are also generated by changing random variables ξ in Eq. (21) with $N'=25$ and $M(j)=4$, and it is observed that each sample wave is generated fairly well.

From the above discussions, we chose the values of N' and M are 25 and 4 respectively in the simulation of three dimensional waves based on Eq. (1). Then, the simulations are carried out four times with different random variables ξ in Eq. (21). One of these simulated waves is shown in **Fig. 8** and the physical spectra of its each component are shown in **Figs. 9** to **11**. As can be seen from **Figs. 8** to **11**, it seems that each simulated wave has higher frequency content waves than the original waves do and the maximum acceleration of the *U-D* component of the simulated wave becomes higher than that of the original *U-D* component.

In conclusions, despite of some defects observed in the above discussions, it is totally considered that the AR model proposed in this paper, Eq. (1), is reasonable.

5. CONCLUSIONS

The multi-dimensional nonstationary stochastic process models in time domain (AR and MA models) were proposed in this paper. These models can be applicable to the identification and simulation problems of many engineering systems.

The following conclusions can be made from the study.

(1) The identification and simulation method by multi-dimensional nonstationary autoregressive model (AR model) was led.

(2) The identification and simulation method by multi-dimensional nonstationary moving average model (MA model) was led.

(3) Though a multi-dimensional nonstationary mixed AR-MA model was not discussed in this paper, it is obvious that the mixed AR-MA model can be given by the following equation.

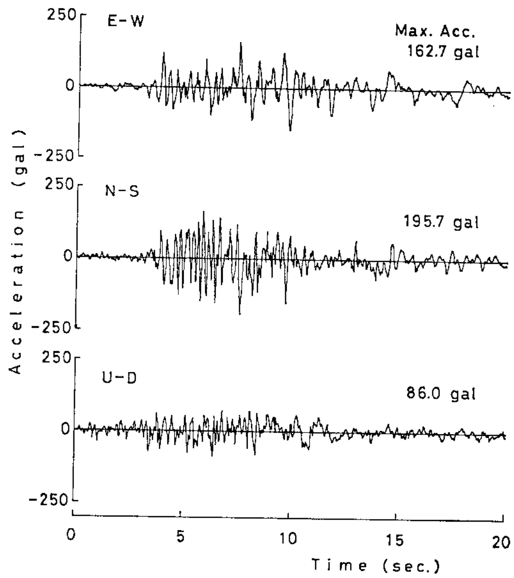

Fig. 8 Simulated 3-dimensional Waves (Sample No. 3).

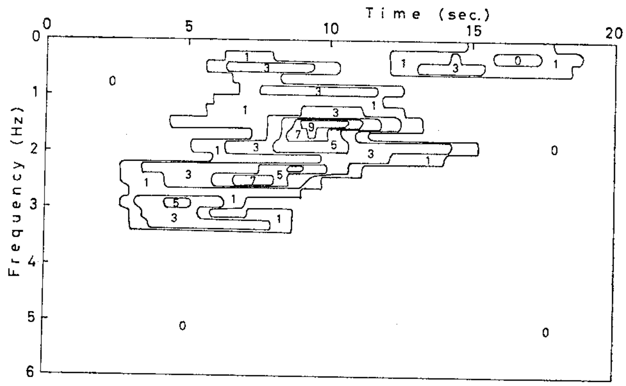

Fig. 9 Physical Spectrum of Simulated Wave (*E-W* Component, Sample No. 3).

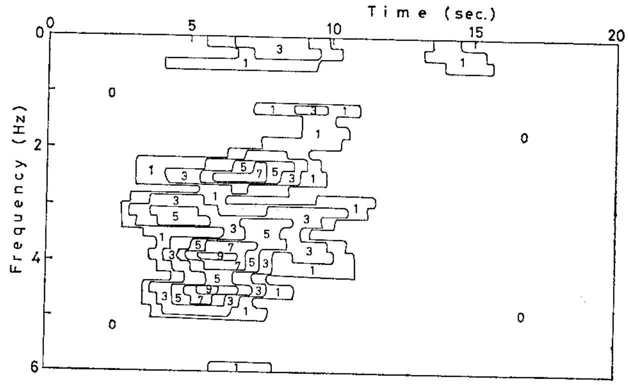

Fig. 10 Physical Spectrum of Simulated Wave (*N-S* Component, Sample No. 3).

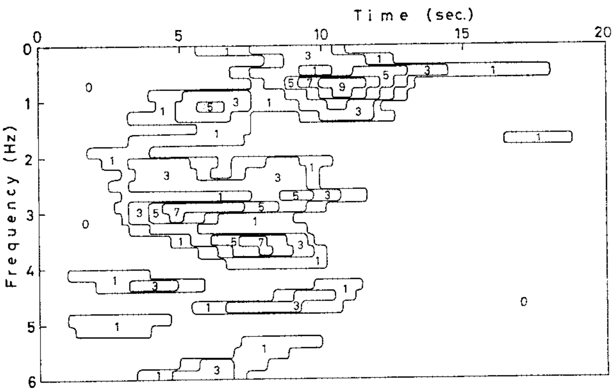

Fig. 11 Physical Spectrum of Simulated Wave (*U-D* Component, Sample No. 3).

$$x_i(j)=\sum_{p=1}^{i}\sum_{k=1}^{M(j)}b_{ip}(k,\ j)x_p(j-k)$$
$$+\sum_{p=1}^{i}\sum_{k=1}^{L(j)}h_{ip}(k,\ j)a_p(j-k)+\varepsilon_i(j)$$
$$;\ i=1,\ 2,\ \cdots,\ m$$

(**4**) The multi-dimensional AR and MA models were led for the phenomena that the nonstationarity changes gradually along time axis. The stationary or one dimensional model was induced easily from the above multi-dimensional model as a special case.

(**5**) The appropriateness of the multi-dimensional nonstationary AR model was demonstrated by the simulation of the observed three dimensional earthquake records.

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