PROC. OF JSCE
No. 292, December 1979

BASIC STUDIES ON SCALE-UP OF SPIRAL FLOW AERATION TANKS

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SYNOPSIS

The treatment of sewage and industrial wastes that makes use of a spiral type of flow aeration tank usually involves the following physical operations, viz., the supply of oxygen, mixing of liquid and the stripping of dissolved gaseous substances by bubbling through compressed air. Various types of air diffusion devices and diffuser location patterns have been developed to improve the efficiency of aeration1)~7). Many studies on the optimum operational procedures of these devices are well documented. But very little information is known about the scale-up design procedure of aeration tanks8). In bubble aeration systems, bubble rise only due to the difference between liquid density and bubble density. The supply of oxygen to the liquid, mixing of liquid and the circulation rate of liquid are all determined by the work of the bubbles rising through the liquid from the diffuser device to the surface of the liquid.

In this study, we attributed the bubble-work hypothesis to the over-all oxygen transfer coefficient and derived a scale-up equation for a spiral flow aeration tank using this over-all oxygen transfer coefficient.

1. THEORY

In the development of this theory, the following assumptions were made to simplify matters;

(1) The air bubble is an ideal sphere.

(2) Air bubbles are not structurally transformed, broken up or combined mutually.

The frictional force evident during the ascension of a single bubble is likely equal to the force imbued in ascending bubbles. Thus,

$$\text{frictional force}=\text{ascending bubble force}$$

$$=C_D\cdot(\rho U^2/2)\cdot(\pi d^2/4) \quad \cdots\cdot(1)$$

where

d=diameter of a bubble (cm)

U=rising velocity of a bubble (cm/sec)

C_D=resistance coefficient

ρ=liquid density (g/cm^3)

The work done by a single bubble ascending from the diffuser device to the surface of the liquid is given by the following equation.

$$[W_b]_s=\frac{C_D\cdot\pi\cdot\rho\cdot U^2\cdot d^2\cdot(H-H_0)}{8\,g_c} \quad \cdots\cdots\cdots(2)$$

where

H=depth of water (cm)

H_0=height of diffuser from the bottom (cm)

g_c=conversion factor (g-cm/gr-sec^2)

$[W_b]_s$=the "bubble-work" of a single bubble (g-cm)

The bubble-work, W_b (g-cm/sec) in the case of air flow rate of G_s (cm^3/sec) is

$$W_b=\frac{3}{4}\cdot C_D\frac{\rho U^2}{g_c}\cdot\frac{H-H_0}{d}\cdot G_s \quad \cdots\cdots\cdots\cdots\cdots(3)$$

In Eq. (3), $(H-H_0)$ represents a diffuser submergence. If the diffuser device is placed at the bottom, H_0 in Eq. (3) would be negligible. It is shown by Eq. (3) that the bubble-work depends on the diameter and rising velocity of bubbles, depth of water, diffuser submergence and air flow rates.

There are several studies on bubble rising velocity through the stagnant liquid by using various sizes of bubbles9)~12). Also we can find some studies on the behavior of either a single bubble in fluidized column or a group of bubbles in the bubble column13)~15). To relate a bubble diameter and rising velocity in fluidized column, Toei et al13) derived the following equation;

$$U=0.98\sqrt{gd} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(4)$$

While Eq. (4) was derived for solid-gas system, the fluidized solid-gas mixture in a fluidized column sometimes may be assumed as a moving liquid like mixed liquor of activated sludge. Wallis, G.B.16) classified the bubble rising velocity in the stagnant liquid in five regions depending on the bubble Reynolds number. According to Wallis' classifica-

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tion, the rising velocity for bubbles with diameter larger than 0.45 cm was assigned to region 4 and 5. In region 4, the bubble rising velocity is independent on the bubble diameter. In region 5, the bubble rising velocity depends on both accelaration of gravity and bubble diameter as follows;

$$U \propto \sqrt{gd} \quad \cdots\cdots\cdots\cdots (5)$$

Since it is reasonable to assume that the bubble diameter in a spiral flow aeration tank is in the ranges of more than 0.45 cm, Eq. (5) can be used to describe its rising velocity. And for slightly smaller bubbles, Eq. (5) can be extrapolated to cover region 4 with little deviation. When we compare Eq. (4) with Eq. (5), it can be said that Eq. (5) approximates Eq. (4). It is consequently suggested by this approximation that Eq. (4) can be assumed as the equation for the bubble rising velocity in a spiral flow aeration tank. When Eq. (4) was substituted in Eq. (3), the following equation follows;

$$W_b = 0.72\, C_D \cdot \rho \cdot (H - H_0) \cdot G_s \quad \cdots\cdots\cdots\cdots (6)$$

Eq. (6) shows that the bubble-work is directly dependent on the diffuser submergence (or water depth) and air flow rate. Resistance coefficient, C_D depends on the diameter and rising velocity of the bubble because of the function of Reynolds number[9),10)].

Since the aeration tank used in this study was not transparent nor equipped with peepholes, both bubble diameter and bubble rising velocity could not be measured. We were then unable to determine an accurate W_b in each experiment. In place thereof, we examined whether the theoretical horsepower requirement of a compressor could be used as a substitute for the bubble-work. The theoretical horsepower requirement of a compressor is given by the following equation[17)];

$$P = \left(\frac{r}{r-1}\right) p_0 \cdot G_s \left[\left(\frac{p}{p_0}\right)^{\frac{r-1}{r}} - 1\right] \quad \cdots\cdots\cdots\cdots (7)$$

where

P=theoretical horsepower requirement of a compressor (g-cm/sec)

r=adiabatic compression coefficient (r=1.4 in air)

p_0=pressure of the incoming air (atmospheric pressure=1 034 g/cm^2)

p=pressure of the compressed air emitted (g/cm^2)

The pressure of the air emitted is approximately described by[17)];

$$\left.\begin{array}{l} p = p_0 + 1.25\ (H - H_0) \\ p = p_0 + 1.25\, H \end{array}\right\} \quad \cdots\cdots\cdots\cdots (8)$$

(if H_0 is negligible)

When Eq. (8) was substituted in Eq. (7), the following equation would be derived;

$$P = \left(\frac{r}{r-1}\right) p_0 \cdot G_s \left[\left\{1 + \frac{1.25(H - H_0)}{p_0}\right\}^{\frac{r-1}{r}} - 1\right] \quad \cdots\cdots\cdots\cdots (9)$$

Since we usually use a spiral flow aeration tank whose water depth is less thah 5 m, it can be assumed that;

$$\frac{1.25(H - H_0)}{p_0} < 1$$

Eq. (9) can be rewritten as follows;

$$\begin{aligned} P &\fallingdotseq \frac{r}{r-1} p_0 \cdot G_s \left\{1 + \frac{r-1}{r} \cdot \frac{1.25(H - H_0)}{p_0} - 1\right\} \\ &\fallingdotseq 1.25(H - H_0) \cdot G_s \\ &\propto (H - H_0) \cdot G_s \quad \cdots\cdots\cdots\cdots (10) \end{aligned}$$

Comparing Eq. (10) with Eq. (6), it can be seen that the theoretical horsepower requirement of a compressor, P, correlates well with the bubble-work, W_b. The relationship between P and W_b will be verified in the latter part of this study.

Dimension analysis of mass transfer shows[18)];

$$\mathrm{Sh} = c(\mathrm{Re})^m (\mathrm{Sc})^n \quad \cdots\cdots\cdots\cdots (11)$$

where

Sh=Sherwood number ($=K_L d/D$)

Re=Reynolds number ($=dU/\nu$)

Sc=Schmidt number ($=\nu/D$)

ν=kinematic viscosity coefficient (cm^2/sec)

D=diffusion coefficient (cm^2/sec)

Using Eq. (3), the above equation can be rearranged as follows;

$$K_L \frac{A}{V} = K_L a = c' \frac{(d \cdot U)^{m-1}}{C_D U^2} \cdot \frac{W_b}{V} \quad \cdots\cdots\cdots\cdots (12)$$

where $c' = 8\, cD^{1-n} \nu^{n-m} g_c{}^{\rho^{-1}}$. Usually air flow rate is 1～2 m^3 of air/m^3 of liquid/hr in a spiral flow aeration tank and much more in a bubble column. So we can consider that the hold up of a spiral flow aeration tank is quite small.

The interfacial area between liquid and bubbles per unit volume of liquid, a, is expressed by the following equation.

$$a = \frac{A}{V} = \frac{6\, G_s (H - H_0)}{d \cdot U \cdot V} \quad \cdots\cdots\cdots\cdots (13)$$

In Eq. (11), Eckenfelder[19),20)] represented the exponent, m, as a unit. If m is assumed as a unit, Eq. (12) can be simplified as follows;

$$K_L a = c' \frac{1}{C_D \cdot U^2} \cdot \frac{W_b}{V} \quad \cdots\cdots\cdots\cdots (14)$$

Combining Eq. (14) with Eq. (4), the following equation results;

$$K_L a = c'' \frac{1}{C_D \cdot d} \cdot \frac{W_b}{V} \quad \cdots\cdots\cdots\cdots (15)$$

where $c'' = 8\, cD^{1-n} \nu^{n-1} \rho^{-1} g_c g^{-1}$.

Eq. (12) or (15) can be used as the scale-up equation of a spiral flow aeration tank which is equipped with a diffusion device that could release bubbles of uniform determinable diameter.

2. MATERIALS AND METHODS

The aeration tanks used in this study are shown in Fig. 1, Fig. 2 and Fig. 3.

(1) Cylindrical experimental aeration tank : This is a 14 cm diameter ×40 cm height tank. Working volume is 4.5 *l* and water depth is 30 cm. Aeration was performed by a porous diffuser. Air flow rate was 0.5 *l*/min, 1.5 *l*/min or 3.0 *l*/min, and K_La was computed by the moment method[21]. Dissolved oxygen (DO) concentration was measured by a DO meter (see Fig. 1).

(2) Cubical experimental aeration tank : This is a 47 cm long ×47 cm wide ×74 cm high tank and its capacity is 100 *l*. Water depth is 47 cm. Aeration device was a porous plate diffuser placed at the wall of the tank bottom. Air flow rate varied from 5 *l*/min to 30 *l*/min. K_La was computed by the moment method and DO concentration was measured by a DO meter (see Fig. 2).

(3) Pilot scale aeration tank : This tank is 50 cm long ×287 cm wide ×200 cm high with a capacity of 2.68 m^3. And its water depth is 187 cm. Sparger and plastic plate diffuser were used as diffuser devices and were placed either at the center of the tank or at the wall of the tank. Diffuser submergences were varied in each experiment. Sparger's submergences were 48 cm, 93 cm, 137 cm and 173 cm reckoned from the surface of the water for both diffuser locations i.e., at the center and at the wall.

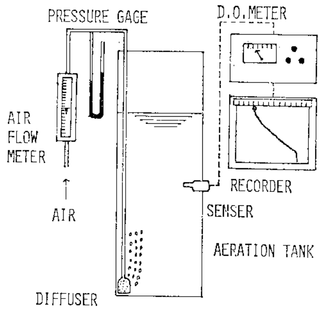


Fig. 1 Schematic diagram of a cylindrical experimental aeration tank (Working volume is 4.5 *l* and water depth is 30 cm).

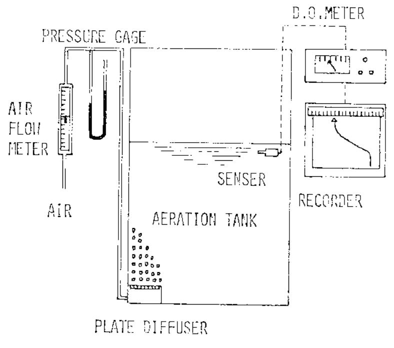


Fig. 2 Schematic diagram of a cubical experimental aeration tank (Working volume is 100 *l* and water depth is 47 cm).

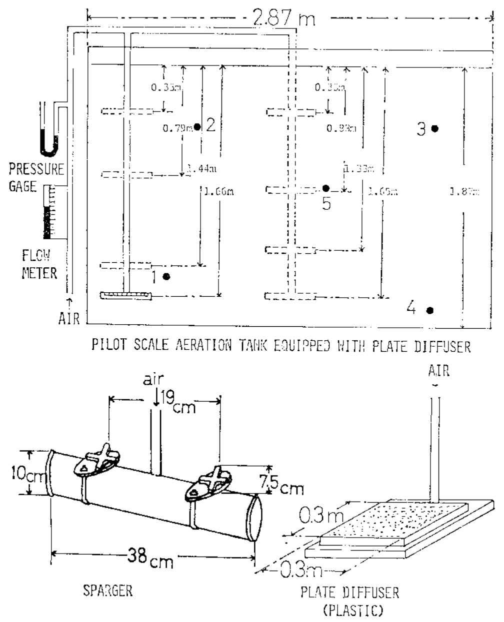


Fig. 3 Schematic diagrams of a pilot scale aeration tank and diffusion devices (Number 1 to 5 is the sampling position, working volume is 2.68 m^3 and water depth is 1.87 m).

The plate diffuser's submergences are shown in Fig. 3. The air flow rate varied from 3 m^3/hr to 11 m^3/hr, and K_La was computed by the moment method. DO concentration was measured by the Winkler's method[22].

Tap water was used in all experiments and K_La at the room temperature was converted to K_La of 20°C.

3. RESULTS AND DISCUSSIONS

(1) Relationship between bubble-work or the theoretical horsepower requirement and K_La in various spiral flow aeration tanks.

Using the pilot scale aeration tank in Fig. 3, the bubble-work, W_b, calculated by Eq. (6) was compared with the theoretical horsepower requirement of a compressor calculated by Eq. (7). When we calculated the bubble-work, W_b, we assumed that the diameter of a bubble was 0.45 cm in spite of air flow rate and the bubble rising velocity was 28 cm/sec as reported by Kashiwaya et al[23], who measured the bubble diameter and the bubble rising velocity by using a ceramic plate diffuser in a pilot scale aeration tank. The value of $Re \cdot M^{0.23}$ was first calculated as described by Harberman·Morton[9]. And then, using the relationship between $Re \cdot M^{0.23}$ and the resistance coefficient, C_D shown by Tadaki·

Maeda[10], the resistance coefficient was estimated and the bubble-work, W_b was calculated. The pressure of the compressed air emitted, p, was measured directly and the theoretical horsepower requirement of a compressor, P, was evaluated. The relationship between W_b and P is shown in Fig. 4. This was done purportedly to circumvent the difficulty of directly measuring the diameters and rising velocities of the bubbles, from which we were to calculate the bubble-work. We used instead the theoretical horsepower requirement of a compressor, P, as a substitute for W_b in as much as we have theoretically and experimentally shown that they were well intercorrelated.

The relationship between K_La and P/V using various experimental aeration tanks (Fig. 1, 2 and 3) is shown in Fig. 5. In Fig. 5, the correlation lines between K_La and P/V derived by Teraoka et al[24], Morgan·Bewtra[3] and King[1] are simultaneously shown. Table 1 shows tank dimensions and kinds of diffusers presented in Fig. 5.

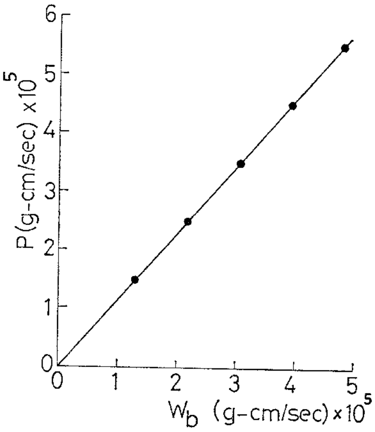
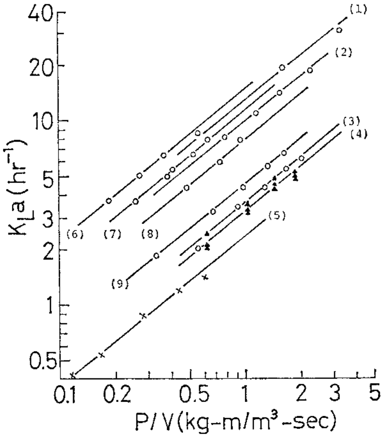


Fig. 4 Relationship between the bubble-work, W_b and the theoretical horsepower requirement of a compressor, P (Calculated for a pilot scale aeration tank shown in Fig. 3).

The slopes of all lines in Fig. 5 are nearly equal to 0.8. This empirical finding suggested that Eq. (15) might be applicable to various scales of aeration tanks. The reason why these slopes are less than one is believed to be due to the increase of bubble diameter as the air flow rate is increased, such increase which consequently promoted the profuse coalescence of bubbles[2].

From a practical point of view, we plotted the K_La value at $P/V=1$ (kg-m/m^3-sec) against water depth. We thus obtained Fig. 6 to explain the two types of linear lines. King's data made up one line and Teraoka et al's and Morgan·Bewtra's date composed the other line with our data. These two lines vary due to the different bubble sizes produced by



(1) authors' result, $H=0.30$ m, $V=4.5\,l$
(2) authors' result, $H=0.47$ m, $V=100\,l$
(3) authors' result, $H=1.87$ m, $V=2.68$ m^3
(4) Teraoka et al[24], $H=3.00$ m, $V=36$ m^3
(5) Morgan, P.F., Bewtra, J.K.[3], $H=4.57$ m, $V=40.78$ m^3
(6) King, H.R.[1] $H=0.28$ m, $V=40\,l$
(7) King, H.R.[1] $H=0.53$ m, $V=150\,l$
(8) King, H.R.[1] $H=1.0$ m, $V=950\,l$
(9) King, H.R.[1] $H=4.57$ m, Industrial scale

Fig. 5 Relationship between the over-all oxygen transfer coefficient and the theoretical horsepower requirement of a compressor.

Table 1 Description of tank shape and dimension and diffusers employed by other investigators which were utilized in Fig. 5 for comparative purposes.

Reference	Tank Dimensions	Water Depth H (m)	Tank Volume V (m^3)	Diffuser
(1) Authors	14 cm × 40 cm	0.30	0.0045	Porous Stone
(2) Authors	47 cm × 47 cm × 74 cm	0.47	0.100	Ceramic Plate
(3) Authors	50 cm × 287 cm × 200 cm	1.87	2.68	Plastic Plate
(4) Teraoka et al[24]	Pilot Scale	3.00	36	Plastic Tube
(5) Morgan Bewtra[3]	7.3 m × 1.2 m × 5.0 m	4.57	40.78	Saran Tube
(6) King[1]	25.4 cm × 55.9 cm × 33.0 cm	0.28	0.040	Ceramic Plate
(7) King[1]	25.4 cm × 111.8 cm × 58.4 cm	0.53	0.150	Ceramic Plate
(8) King[1]	48.3 cm × 198.1 cm × 104.7 cm	1.00	0.950	Ceramic Plate
(9) King[1]	Industrial Scale	4.57		Ceramic Plate

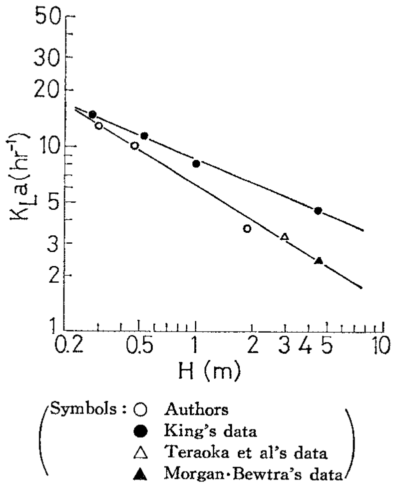


Fig. 6 Effect of water depth on the over-all oxygen transfer coefficient.

different diffusers. From Fig. 5 and Fig. 6, we can get the following equation. Our data containing Teraoka et al's and Morgan·Bewtra's data would result to :

$$K_La=5.9(P/V)^{4/5}(H)^{-2/3} \quad\cdots\cdots\cdots\cdots\cdots\cdots(16)$$

while King'data would be :

$$K_La=8.6(P/V)^{4/5}(H)^{-2/5} \quad\cdots\cdots\cdots\cdots\cdots\cdots(17)$$

where P/V is kg-m/m^3-sec, and H is m.

Exponents and constants in Eq. (16) and (17) are influenced by the dimensions and geometrical shapes of the aeration tanks and diffusion devices. It has been shown that oxygen transfer efficiencies from the water surface decreases with the increase of water depth, so this is considered to be one of the reasons why the exponent of H carries a negative sign. But we will investigate this problem further.

In the scale-up of a spiral flow aeration tank, we have to determine the theoretical horsepower requirement of a compressor to maintain the same $(K_La)_L$ in a practical tank as $(K_La)_S$ in a bench or pilot scale tank. But sometimes it is difficult to measure the $(K_La)_S$ in a bench or pilot scale aeration tank during the treatment of wastewater. In this case, we have to use the following equation derived from Eq. (16) ;

$$\left(\frac{P}{V}\right)_L=\left(\frac{H_L}{H_S}\right)^{5/6}\left(\frac{P}{V}\right)_S \quad\cdots\cdots\cdots\cdots\cdots\cdots(18)$$

where subscript L represents a large tank and subscript S is for a small tank. If we derive an equation from Eq. (17) in the same way, the exponent of (H_L/H_S) in Eq. (18) becomes 1/2. Since the water depth used in Fig. 5 ranged from 0.3 m to 4.6 m, Eq. (16), (17) or (18) would be applicable to the ordinary spiral flow aeration tank. We can thus predict the theoretical horsepower requirement of a spiral flow aeration tank by using either Eq. (16) (or Eq. (17)) or Eq. (18) if we can get either the optimal K_La or the optimal horsepower requirement during the treatment of wastewater in a bench or pilot scale aeration tank.

(2) Effects of diffuser submergences and diffuser locations on scale-up of spiral flow aeration tank.

If diffusers are hung at the middle point of the tank from the surface of the water, the system requires less horsepower for compressing air as compared with the other conventional processes, one of which is called Inka system[25),26)].

In this studies, we set up the diffuser both at the wall and at the center of the aeration tank, and varied the diffuser submergence at each diffuser location. We determined the relationship between K_La and the bubble-work or theoretical horsepower requirement in each case of aeration system. In this experiment, we used a pilot scale aeration tank and adopted a sparger and a plate diffuser as a diffusion device as shown in Fig. 3. The Eq. (3) shows that the bubble-work, W_b, is in proportion to the diffuser submergence, $(H-H_0)$. It was then assumed that K_La in each case of aeration system was correlated to the bubble-work or the theoretical horsepower requirement of a compressor. As it has been mentioned above, the theoretical horsepower, P, could be used as a substitute for the bubble-work, W_b. Hence, we plotted K_La values against P/V calculated from Eq. (7). This result is shown in Fig. 7. This figure shows two linear correlation lines in spite of the different diffuser submergences and locations. The data from the sparger represent one line and the data obtained from the plate diffuser represent the other line. This is due to the difference of the bubble diameter between the sparger

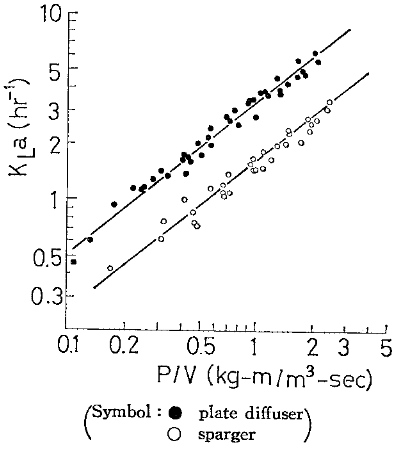


Fig. 7 Relationship between K_La and P/V in a pilot scale aeration tank under a variable diffuser submergence and diffuser location.

and the plate diffuser. The following equations are derived from Fig. 7;

The plate diffuser :

$$K_La=3.3(P/V)^{4/5} \quad \cdots\cdots(19)$$

The sparger :

$$K_La=1.6(P/V)^{4/5} \quad \cdots\cdots(20)$$

The plate diffuser achieved higher oxygen transfer efficiency gauged against the theoretical horsepower as compared with that of the sparger.

The slopes of the two lines in Fig. 7 are equal to 0.8 which is similar to the result of Fig. 5. According to Eq. (15), the slopes of the lines should equal to 1. The difference is probably due to the ideal conditions assumed for developing Eq. (15). But in this study, we are trying to apply the idea of bubble-work theory to the scale-up of a spiral flow aeration tank. Fig. 7 suggests basically that Eq. (3) and Eq. (6) can be successfully applied to the spiral flow aeration tank even if we vary the depth of diffuser submergence and the diffuser location. In summary, using the bubble-work or the theoretical horsepower requirement of a compressor for practical reason, we can actually scale-up spiral flow aeration tanks for as long as they equipped with a diffusion device which could release bubbles of uniform diameter not withstanding variances in diffuser submergence position, the diffuser location and the water depth.

4. ABSTRACT

(1) We attributed the bubble-work to the over-all oxygen transfer coefficient and derived the scale-up equation of a spiral flow aeration tank by using this over-all oxygen transfer coefficient. It was shown theoretically and experimentally that the theoretical horsepower requirement of a compressor, P, could be used as a substitute for the bubble-work, W_b. By using different scales of aeration tanks, we could correlate the K_La value to P/V and verify the presented scale-up theory.

(2) We could obtain a good correlation equation between K_La, P/V and water depth, H, by using various spiral flow aeration tanks which were equipped with diffusion devices releasing bubbles of uniform diameter.

(3) In spite of the diffuser submergence and location of the pilot scale spiral flow aeration tank, a linear relationship between K_La and P/V was obtained. It is plausible to infer then that the scale-up of the spiral flow aeration tank could be adopted per se in wastewater treatment processes for as long as the release bubbles would be diametrically uniform.

NOMENCLATURE

U=bubble rising velocity (cm/sec)
d=bubble diameter (cm)
ρ=liquid density (g/cm^3)
H=water depth (cm)
H_0=diffuser height from the bottom (cm)
h=diffuser submergence (cm)$=H-H_0$
G_s=air flow rate (cm^3/sec)
W_b=works by rising bubbles (g-cm/sec) =bubble-work
$[W_b]_s$=work by a single rising bubble (g-cm)
g_c=conversion factor (g-cm/gr-sec^2)
C_D=resistance coefficient
g=acceleration of gravity (cm/sec^2)
Sh=Sherwood number ($K_L \cdot d/D$)
Re=Reynolds number ($d \cdot U/\nu$)
Sc=Schmidt number (ν/D)
ν=kinematic viscosity coefficient (cm^2/sec)
D=diffusion coefficient (cm^2/sec)
r=adiabatic compression coefficient ($r=1.4$ in air)
A=surface area of bubbles (cm^2)
V=tank volume (cm^3)
P=theoretical horsepower requirement of a compressor (g-cm/sec)
p=pressure of the compressed air emitted (g/cm^2)
p_0=pressure of the incoming air (atmospheric pressure=1 034 g/cm^2)
c, c', c''=constant
m, n=exponent

ACKNOWLEDGEMENT

The authors want to express their gratitude to Profs. K. Ichikawa, H. Taguchi and S. Aiba of Osaka University for the suggestions and discussions during elaboration of this work.

This work was supported in part by a Grant-in-aid for Scientific Research, 1976 from the Ministry of Education, Japan.

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(Recieved June 29, 1978)