PROC. OF JSCE,
No. 287, JULY 1979
[Technical Note]

SOME NUMERICAL ESTIMATIONS OF ULTIMATE IN-PLANE STRENGTH OF TWO-HINGED STEEL ARCHES

By Shigeru KURANISHI and Tetsuya YABUKI***

1. INTRODUCTION

Several researches of the ultimate in-plane strength of steel arches considering finite deflections, yielding of the material, and its spread into the cross sections and to the longitudinal direction were already reported[1~3]. Owing to those investigations, the ultimate behavior of steel arches has become obvious to a considerable extent.

In the previous works, however, the following problems are not sufficiently discussed: (1) problem concerning iterative approach treating nonlinearities; (2) influence of distributed pattern and magnitude of residual stresses; (3) proportion of cross sections; (4) concentrated loads applied through the posts; (5) ultimate strength based on the strain of cover plate buckling. The first item is an important factor to specify the accuracy of numerical analysis adopted. Considerations of (2) and (3) are always required for formulating the ultimate strength. For the actual arch bridges, loads are applied on arch ribs through the posts which are supporting the deck system. Accordingly, the influence of (4) should be taken into consideration in order to estimate the ultimate strength of the actual arch bridge. Furthermore, considering that the plates composing the cross sections adopted to the actual arch bridges are not proportioned so as to be able to resist the sufficiently large strain within the plastic range, we must pay attention to the problem of (5).

We carry out a series of numerical studies with respect to these problems here in order to get a clue to practical design criteria. Numerical analyses were chiefly performed using the procedure emplyed in Ref. 3).

* Dr. Eng., Professor, Dept. of Civil Engineering, Tohoku University
** Research Instructor, Dept. of Civil Engineering, Tohoku University

2. CONSIDERATION OF RESULTS

The arches subjected are confined to two-hinged palabolic steel ones and a general view of them is shown in the inset of Fig. 3.

Shinke, Zui and Namita made a series of precious tests concerning steel arches having rectangular cross sections with practical slenderness ratios and reported instructive results in Ref. 2). We shall refer to their experimental results in order to examine the accuracy of the analytical method employed herein. Fig. 1 shows a comparison between the experimental results transcribed from Ref. 2) and the numerical ones computed by the method employed in this study. The notation is as follows; P=total magnitude of applied distributed load referred to Ref. 2); w, p=uniform load over the entire span and additional uniform load on one half of the span, respectively; λ=slenderness ratio; V_{max}=maximum vertical displacement. Judging from the results shown in Fig. 1, the numerical analysis adopted here presents a considerably good agreement with the experiment. Herein, the basic state is defined as so-called initial stress state for determining the subsequent state in an iterative procedure. In this analysis, the basic state transfers according to the deformation and

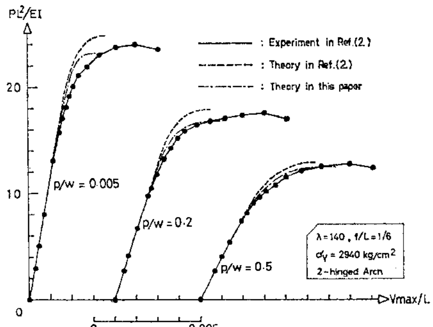


Fig. 1 A comparison between experimental results and theoretical ones.

Table 1 Relationships between results by integration approach and by this approach.

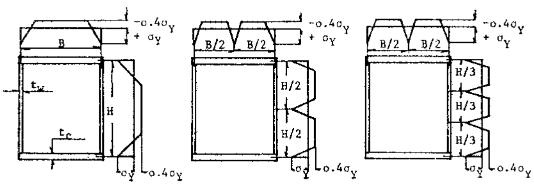
Load ratio \ Approach	This approach	Ref. 1)	Improved Ref. 1)
0.25	0.363	0.364	0.356
0.5	0.446	0.418	0.445
0.75	0.573	0.499	0.570
0.99	0.954	0.805	0.962
N.B.	Steel arch with rectangular section $f/L=0.15$, $\lambda=150$, $\sigma_Y=2\,400\ \mathrm{kg/cm^2}$		

keeps always the nearly true stress state corresponding to the appropriate strain caused by the deformation. The numerical results in Ref. 2) are also redrawn in Fig. 1. These results also agree fairly well with the experimental results, showing somewhat higher ultimate loads, especially, for the nearly symmetrical loading pattern. However, in some cases, the variance between them becomes by up to 8%. Table 1 shows a comparison between the results obtained from the approach in this paper and from the integration approach in Ref. 1). Here and hereafter, the applied loads are expressed non-dimensionally with the ratio q/q_p, in which q_p is the nodal load (covering the entire arch) that would cause the arch to yield by axial thrust at the springing[3]. In the computation, the arch axis is divided into thirty member elements and the number of divisions gives practically enough accuracy[3]. The accuracy aimed at in this numerical iteration is the same one as in Ref. 3). For the integration approach in Table 1, the results of Ref. 1) and those of improved computation are shown. The difference between the maximum load intensities of Ref. 1) and of this paper becomes significant as the load ratio s increases. In Ref. 1), the shear force in relationship between axial thrust and horizontal force is given approximately and a deflection slope of an arch axis is taken into account by an inclination of a line which joints adjacent two nodal points in an arch axis. Herein, the deflection slope form is improved so as to be computed using similar procedure to the algebraical expression of the displacement to vertical or horizontal direction in Ref. 1) and the effects of 2nd order values of the shear force and the slope on the relationship between the axial thrust and horizontal one are more accurately considered. Furthermore, the convergence accuracy of the numerical computation is improved. The results are shown in Table 1 as improved Ref. 1). The maximum variance between both results of improved Ref. 1) and this analysis carried out in Ref. 3) becomes about 2%. The numerical analyses adopted in Refs. 1) and 2) were treating that the aforesaid basic state in the iterative approach is usually fixed to the original configuration of arches. Such procedure may be simple for usual analysis of non-linear problems. But, the examination obtained here shows that we must formulate the each compatibility condition more appropriately through out those nonlinear aspects.

We examine the effects of the cross sectional configurations and distribution patterns and magnitudes of the residual stresses on the ultimate strength of arches. As a fundamental loading, $s=0.99$ is adopted for uniformly distributed load over the whole span length in which 1% is unsymmetrized expecting similar effects to the imperfections of arches and $s=0.5$ is adopted for unsymmetrical loading which produce severer effect than standardized loadings of practical bridges. The effect of distribution patterns of residual stresses on the ultimate strength is shown in Table 2 in which the patterns were varied as illustrated in Fig. 2. The arches have TYPE-B cross-section which is proportioned by considering the parameters so that the radius of gyration r/H and the core radius k/H lie about in the average magnitudes between these of sandwich sections and rectangular ones as shown in Table 3. This section is treated as a standard box section herein. For computation in Table 2, $\alpha=0.4$ is employed, in which $\alpha=$ratio of maximum compression residual stress to yield stress, considering the practical distribution[3]. From the results in Table 2, it can be seen that the ultimate strength reduction by the residual stresses reaches as much as 20% and the maximum variance of the ultimate strength among

Table 2 Effect of residual stress pattern on maximum load intensity.

Slenderness ratio	Load ratio	No residual stress	PATTERN-A	PATTERN-B	PATTERN-C
100	0.5	0.601	0.606	0.568	0.565
	0.99	0.974	0.912	0.867	0.840
150	0.5	0.409	0.405	0.377	0.376
	0.99	0.888	0.701	0.668	0.672
200	0.5	0.293	0.279	0.264	0.263
	0.99	0.589	0.521	0.510	0.504
250	0.5	0.220	0.201	0.194	0.193
	0.99	0.386	0.370	0.368	0.365
N.B.	Steel arch with box cross section; TYPE-B ($\alpha=0.4$) $f/L=0.15$, $\sigma_Y=3\,600\ \mathrm{kg/cm^2}$				



(1) PATTERN-A (2) PATTERN-B (3) PATTERN-C

Fig. 2 Idealized patterns of residual stress distribution.

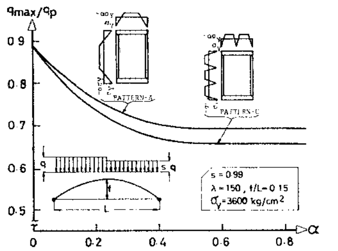


Fig. 3 Relationships between maximum load intensity and maximum compression residual stress.

the patterns is about 9%. Web plates box of sections provide a significant portion of a thrust carrying capacity so that the distribution of residual stress in web becomes an important problem. Fig. 3 presents the influence of the magnitude of the compressive residual stress on the ultimate strength. From Fig. 3, the effect of the magnitude of the compressive residual stress on the ultimate strength becomes certain when the magnitude is larger than $0.4\sigma_Y$. Therefore, formulating the ultimate strength, it may be appropriate that the practical magnitude is assumed to be $0.4\sigma_Y$ or some more.

In order to know the effect of the cross sectional efficiency, the proportions of the box cross section were varied. Three types of box section are adopted as shown in Table 3 in which the width-thickness ratio of the web plates, $H/t_w=190$, is determined by averaging the magnitudes of over ten practical steel arch bridges with span length ranging from 100 m to 300 m. Furthermore, a rather heavy proportion is considered as TYPE-D in which the width-thickness ratio of web plate is 60. Table 4 shows the effects of the cross sectional proportion on the ultimate load. The maximum variance among them is by 6%. Table 4 also shows the computed results for sandwich and rectangular cross sections. The maximum variance between them is about 9%. Therefore, the maximum influence of the cross sectional shape on the ultimate strength may be within $\pm 5\%$ in this computed range.

Considering the practical loading system of arch bridges, the influence of the concentrated loads applied through the posts on the ultimate strength is checked. Example computation is presented in Fig. 4 in which the loads are applied discretely as 7 concentrated ones shown in the inset of Fig. 4. When the slenderness ratio is large, the maximum load intensity for concentrated loading case has similar magnitude to the strength for nearly distributed loading case.

Table 3 Proportion of employed box cross sections.

Dimension \ TYPE	TYPE-A	TYPE-B	TYPE-C	TYPE-D
H/t_w	190	190	190	60
B/H	0.8	0.6	0.4	0.2
A_c/A_w	1.2	0.9	0.6	0.3
r/H	0.414	0.400	0.378	0.338
k/H	0.343	0.320	0.286	0.228
N.B.	A_c: cover plate area, A_w=web plate area			

Table 4 Effect of cross sectional configuration on maximum load intensity.

Slenderness ratio	Load ratio	Idealized cross section		Box cross section (Residual stress: PATTERN-A)			
		Sandwich	Rectangular	TYPE-A	TYPE-B	TYPE-C	TYPE-D
100	0.5	0.592	0.603	0.600	0.606	0.616	0.630
	0.99	0.960	0.976	0.902	0.912	0.926	0.951
150	0.5	0.412	0.400	0.402	0.405	0.408	0.409
	0.99	0.816	0.870	0.690	0.701	0.715	0.746
200	0.5	0.303	0.281	0.279	0.279	0.279	0.276
	0.99	0.572	0.578	0.521	0.521	0.522	0.532
250	0.5	0.229	0.208	0.203	0.201	0.200	0.196
	0.99	0.381	0.381	0.370	0.370	0.370	0.370
N.B.	Steel arch with idealized or box cross section: $f/L=0.15$, $\sigma_Y=3\,600$ kg/cm^2						

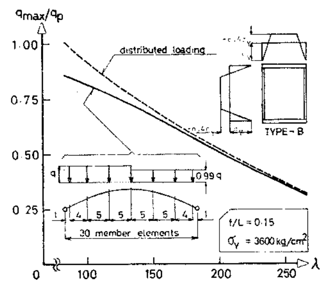


Fig. 4 Influence of loading case applied by posts.

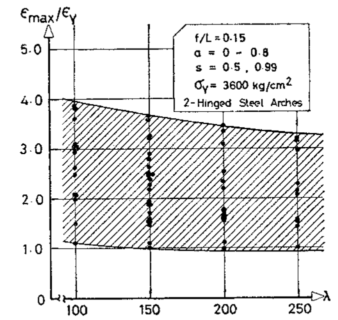


Fig. 5 Range of maximum compressive strains occuring cover plate calculated in this study.

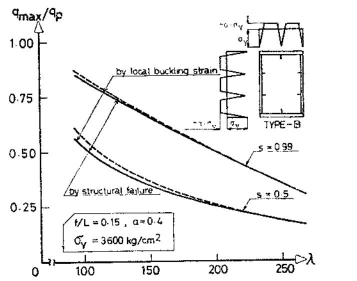


Fig. 6 Ultimate strength results specified by local buckling strain and by structural failure.

However, as the slenderness ratio becomes small, the maximum load intensity for concentrated loading case becomes smaller than nearly uniformly distributed loading case. The variance between them is about 9% when λ is equal to 100. Checking up the failure aspect according to the numerical simulation to the collapse of discretely loaded arches, the arches with the large slenderness ratio collapse as a whole structure. On the other hand, when the slenderness ratio becomes small, a certain amount of large negative moments which locally occur at the arch rib between concentrated loads cause the collapse. From this fact, we should pay attention to this ultimate strength reduction in formulating the design criteria of arch bridges.

The range of the maximum compressive strains occuring in the cover plates of box cross section under the nearly ultimate loads calculated herein is shown in Fig. 5. The load ratio s ranged from 0.99 to 0.5 and the slenderness ratio varied from 100 to 250. The residual stresses and the proportion of the cross sections also varied. The effect of the load pattern factor is not shown to be identified in the figure. But, summarizing the computed strain obtained here, the following remarks can be drawn. The maximum strain occuring at the cross section of the arch subjected to the unsymmetric load is higher than it subjected to nearly symmetric load. The difference between them becomes significant as the slenderness ratio becomes large. The structural parameters of practical steel arch bridges seem to be ranged within the above-mentioned magnitudes and if the arch bridge keeps its load carrying capacity to the ultimate state avoiding local buckling of the cover plate, they are required to endure the strain of $4\epsilon_Y$ by the maximum, in which ϵ_Y=yield strain. However, for practical thin walled box ribs, the cover plates will buckle by the strain of $2\epsilon_Y$ or a some more. Therefore, by assuming that $2\epsilon_Y$ is the critical strain, the strength results specified by the strain are shown in Fig. 6. The ultimate strength reduction appears as the loading pattern becomes unsymmetric and the slenderness ratio becomes small. Accordingly, when we estimate the ultimate strength of steel arches having thin walled box cross section with small slenderness ratio, we also must estimate the strength specified by the strain of the cover plates local buckling.

3. REMARKS

From some numerical analyses of the ultimate inplane strength of the steel arches performed herein, it can be summarized that the practical estimation of the design criteria should be made after summing up the knowledges obtained here, furthermore, making more detailed parametric investigations.

Part of this work was supported by the Funds of Aid for Scientific Researches from the Ministry of Education.

REFERENCE

1) Kuranishi, S. and L. W. Lu: Load carrying capacity of two hinged steel arches, Proc. of JSCE, No. 204, pp. 129–140, Aug., 1972.
2) Shinke, T., H. Zui and Y. Namita: Analysis and experimation on inplane load carrying capacity of arches, Proc. of JSCE, No. 263, pp. 11–24, July, 1977 (in Japanese).
3) Kuranishi, S. and T. Yabuki: In-plane ultimate strength of 2-hinged steel arches subjected to lateral loads, Proc. of JSCE, No. 272, pp. 1–12, April, 1978 (in Japanese).

(*Recevied July 21, 1978*)