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AN ACTIVITY ANALYSIS MODEL IN A METROPOLITAN AREA

Kozo AMANO, Toichi KIMURA**, and Asao ANDO****

1. INTRODUCTION

Urban problems of today have appeared as the results of complicated interactions, both direct and indirect, of various urban activities. That is why a precise description of these connections becomes necessary for urban analyses. As well as these interactions among activities, an inter-regional aspect also should be considered, since effects of each individual activity held in a city do not end in such a restricted territory, but spread over the border and up to the metropolitan area as a whole.

Judging from this point of view, a practical urban analysis scheme should pay respects to inter-sectoral and inter-regional relations among activities explicitly, and should be able to focus on each subdivided district besides the whole object region. To develop such a global metropolitan simulation system is the ultimate purpose of this study.

The model which is applicable to long-run forecasts must not fail to consider the dynamic aspects of activities. In this study, the total system will be composed as a recursive dynamic model, which consists of three sub-models. The first one is called "Activity sub-model", which is to analyze the flow aspects of activities. The second one: "Location sub-model" is to analyze the stock aspects of them. As shown in **Fig. 1**, these two sub-models are executed by turns under the control of the third one: "Control sub-model", which estimates various characteristics of economy held in the object region during each time period.

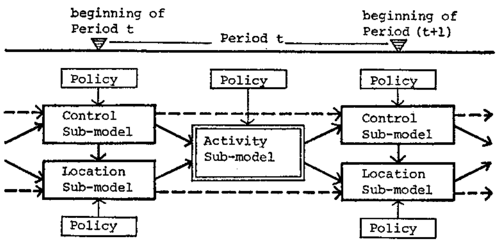

Fig. 1 An Outline of the Recursive Urban Simulation Model.

* Professor, Dept. of Transportation Eng., Kyoto University
** Nomura Research Institute
*** Graduate Student, Dept. of Transportation Eng., Kyoto University

The purpose of this article is to formulate "Activity sub-model", focusing on the flow aspects. The major inputs to this sub-model such as final demands will be provided by "Control sub-model".

2. HIERARCHICAL STRUCTURE OF REGIONS AND ACTIVITIES

To analyze various activities indiscriminately for many subdivided districts would crucially lose operationability which is required in such a practical model. It might be said that to classify these activities into few groups in accordance with their characteristics and to handle each group adequately are reasonable, at least from the viewpoint of operationability. And in this sense, too, it might be useful to classify activities in terms of market ranges where their supplies meet their demands, as Christaller did[1].

In this study, an attempt is made to introduce hierarchical concepts into urban analysis through such a classification.

(1) Activity Analysis in Hierarchical Structure

An Input-Output analysis approach is employed (For convenience sake "Input-Output" will be abbreviated as "I-O" hereafter.) because it can describe multiple interactions among activities clearly, and has a certificated theoretic basis supplemented with increasing stores of data. In addition, regional or inter-regional concepts can easily be introduced into I-O analysis, such as Isard-type or Moses-type inter-regional I-O models[2].

In analyzing a metropolitan area, where each region corresponds to a subdivided district such as a few cities or counties, differences in quality or of technology are very slight throughout the object region. Moreover, such differences can hardly be measured, and if measured, they might be unstable with time. Therefore, it would be justified in a practical sense, to apply a common set of input coefficients to the whole object region. From this point of view, Leontief's "Balanced I-O Model"[3)4)], which has been found to analyze American economy in regional context, is remarkable. This model is based on the following assumptions.

Assumptions

a) All the activities are classified into two groups, i.e., National industries and Local industries. Products of the former meet their demands in the entire nation or still wider, while those of the latter are supposed to balance within each region.
b) The regional distribution of production is exogenously given to the model.
c) Input coefficients are the same throughout all regions, and they coincide with national ones.

The model is composed of two stages, and introduces hierarchical structure of regions and industries into activity analysis explicitly. Although the former two models, of Isard- and Moses-types, obtain regional productions from inter-regional trade, it is almost impossible to investigate such inter-regional trade among subdivided districts in a metropolitan area. Therefore, Leontief's model, which analyzes regional productions without handling inter-regional trade, would be useful. So we shall proceed to discuss the aforementioned assumptions.

Validity of Assumption c) seems to be crucial. However, when discussing a relatively small region such as a metropolitan area, validity of this assumption would be sufficiently secured for the following reasons. In agricultural and manufacturing industries, there are no large technical differences among districts except that of input ratio for transportation. However, that difference is found to be negligible in this case, because the magnitude of travelling costs is relatively smaller than that of terminal costs. On the other hand, in activities belonging to the tertiary industry, factor inputs have a large magnitude relatively. In that case, the input coefficients may be different, e.g. between C.B.D.-locating and suburban-locating activities, even producing the same goods in quality. Since each of them is considered to belong to a different class of activities, it is justified to use the same input coefficients in each district, if all activities are classified adequately. In addition, this assumption guarantees consistency of the Balanced I-O Model.

Assumption a) is to distinguish goods or services for their market ranges, which are assumed to have a hierarchical structure, and conforms to Christaller's theory.

Finally, regional distribution coefficients of assumption b) cannot be considered to be constant with time, except for a non-realistic case where the location patterns of activities never varies. If such a model that can estimate a transition of these location patterns is attached to the Balanced I-O Model, this method would be applicable to the analysis of flow aspects of urban activities.

Therefore, the Balanced I-O Model is employed as a basic method in this study. Actual formulations of the model will be shown in section 3.

(2) The Classification of Regions and Activities

In the previous section, the concept of hierarchical structure was introduced into urban analyses, and this concept was found to be useful. In this section, the definition of hierarchies of regions and activities will be given, and the classification of activities will also be shown.

The breakdown of the activities in a larger number of levels would lead the analysis to be more exact. However, it might not be desirable, since it makes operationability decrease. This is particularly the case with such a model that treats a macroscopic description of a metropolitan area. The minimal unit of space handled in the model should be small enough to be distinguished by its properties, while the largest one corresponds to the object region as a whole. It is proper to regard each prefecture, which possesses rather closed functions, as the medium unit for data bases as well. The defined hierarchy is of three levels. (See **Fig. 2**)

a) Region: the largest unit of space, i.e., the whole object region.
b) Locale: the medium unit assumed as a prefecture.

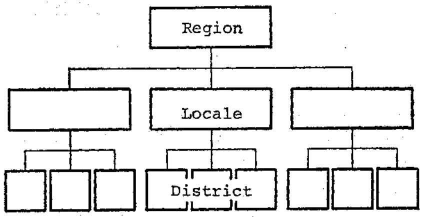

Fig. 2 Hierarchy of Regions.

Table 1 The Classification of Sectors.

Regional activities

01	Agriculture, forestry, and fishery products
02	Mining
03	Foods and drinks
04	Textile products
05	Wood milling and wooden products
06	Pulp, paper, and paper articles
07	Printing and publishing
08	Chemicals
09	Non-metalic mineral products
10	Metal and metal products
11	Machinery
12	Miscellaneous industrial products
13	Electric power and gas supply
14	Non-residential building construction
15	Far-flung transportation
16	Wholesale trade
17	Public construction of R-level
18	Community services of R-level
19	Government service of R-level
20	Clerical business

Local activities

21	Water supply
22	Residential building construction
23	Urban passenger transportation
24	Urban freight transportation
25	Communication
26	Financial, insurance, and real estate business
27	Business services
28	Recreation services
29	Public construction of L-level
30	Community services of L-level
31	Government service of L-level

District activities

32	Retail trade
33	Personal services
34	Public construction of D-level
35	Community services of D-level
36	Government service of D-level
37	Sanitary services

c) District: the smallest unit consisting of several cities, wards, towns and/or villages.

Corresponding to the regional hierarchy, all activities are classified into three levels.

a) Regional activities: activities whose products meet their demands in the whole object region or even a wider area.

b) Local activities: activities whose products balance in each locale.

c) District activities: daily activities whose products are consumed in respective districts.

Products of these levels of activities are called R-goods, L-goods, and D-goods respectively. It is difficult to consider actual activities as completely separable into three groups. Nevertheless, the hierarchical structure of urban activities can be described in this way. The formulations are made to be in conformity with this concept.

In this study, all activities are classified into 37 sectors as shown in **Table 1**.

3. 3-LEVEL INPUT-OUTPUT ANALYSIS MODEL

With the hierarchy defined as 3 levels, the corresponding I-O analysis model also should be composed as a 3-level model. The purpose of this model is to obtain products or activity levels in each district from regional final demands estimated in the control sub-model. The methodology is summarized in **Fig. 3**.

Initially, regional products are obtained from given final demands through "I-O Analysis of R-level". After dividing regional outputs of R-goods into each locale, local outputs of L-goods and D-goods are obtained through "I-O Analysis of L-level". In the same way, district outputs of D-goods are obtained through "I-O Analysis of D-level".

The fundamental assumptions of the model are described as follows.

Assumption 1 Hierarchy of activities is defined as three levels.

Assumption 2 Each good is produced by only one activity. This means that activities and goods are in one-to-one correspondence.

Assumption 3 Input coefficients in each locale or district can be identified, which also coincide with regional ones.

Assumption 4 Trading is competitive, i.e., goods are never identified by the places of production.

(1) I-O Analysis of R-level

The regional input coefficient matrix is divided into 9 minor matrices corresponding to three levels of activities.

$$A=(a_{ij})=\begin{bmatrix} A_{RR} & A_{RL} & A_{RD} \\ A_{LR} & A_{LL} & A_{LD} \\ A_{DR} & A_{DL} & A_{DD} \end{bmatrix} \qquad \cdots\cdots\cdots\cdots(1)$$

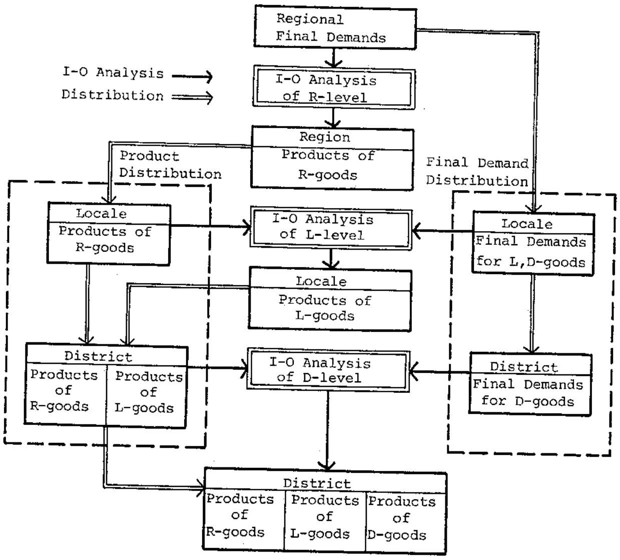

Fig. 3 3-level I-O Analysis Model.

The balance equation for the whole region is

$$\begin{bmatrix} X_R \\ X_L \\ X_D \end{bmatrix} = \begin{bmatrix} A_{RR} & A_{RL} & A_{RD} \\ A_{LR} & A_{LL} & A_{LD} \\ A_{DR} & A_{DL} & A_{DD} \end{bmatrix} \begin{bmatrix} X_R \\ X_L \\ X_D \end{bmatrix} + \begin{bmatrix} \tilde{Y}_R + J_R + F_R - M_R \\ \tilde{Y}_L + J_L \\ \tilde{Y}_D + J_D \end{bmatrix} \quad \cdots\cdots\cdots\cdots(2)$$

where X is the regional output vector, $\tilde{Y}$, the net regional final demand vector, J, the net increase in stocks vector, $F(M)$, the export vector to (import vector from) the outer-object area, with suffixes R, L, D which indicate levels of activities. Besides, F_L, M_L, F_D, and M_D are fixed to $\mathbf{0}$ for Assumption 1.

For specified values assigned for the second term in the right hand side together with unit matrices I_R, I_L, and I_D, regional products are obtained.

$$\begin{bmatrix} X_R \\ X_L \\ X_D \end{bmatrix} = \begin{bmatrix} I_R - A_{RR} & -A_{RL} & -A_{RD} \\ -A_{LR} & I_L - A_{LL} & -A_{LD} \\ -A_{DR} & -A_{DL} & I_D - A_{DD} \end{bmatrix}^{-1} \times \begin{bmatrix} \tilde{Y}_R + J_R + F_R - M_R \\ \tilde{Y}_L + J_L \\ \tilde{Y}_D + J_D \end{bmatrix} \quad \cdots\cdots\cdots\cdots(3)$$

(2) I-O Analysis of L-level

The balance equation for local and district activities in locale l is

$$\begin{bmatrix} X_L^l \\ X_D^l \end{bmatrix} = \begin{bmatrix} A_{LL} & A_{LD} \\ A_{DL} & A_{DD} \end{bmatrix} \begin{bmatrix} X_L^l \\ X_D^l \end{bmatrix} + \begin{bmatrix} A_{LR} X_R^l + \tilde{Y}_L^l + J_L^l \\ A_{DR} X_R^l + \tilde{Y}_D^l + J_D^l \end{bmatrix} \quad \cdots\cdots\cdots\cdots(4)$$

The local output vector of R-goods, X_R^l is derived with a diagonal matrix of local product distribution coefficients, R_R^l which would be given a priori in the model.

$$X_R^l = R_R^l X_R \quad \cdots\cdots\cdots\cdots(5)$$

where $\sum_l R_R^l = I_R$.

In addition, if the local net final demand vectors and net increase vectors in stocks of L- and D-goods, i.e., $(\tilde{Y}_L^l + J_L^l)$ and $(\tilde{Y}_D^l + J_D^l)$ are given, the local products of each level can be obtained.

$$\begin{bmatrix} X_L^l \\ X_D^l \end{bmatrix} = \begin{bmatrix} I_L - A_{LL} & -A_{LD} \\ -A_{DL} & I_D - A_{DD} \end{bmatrix}^{-1} \times \begin{bmatrix} A_{LR} X_R^l + \tilde{Y}_L^l + J_L^l \\ A_{DR} X_R^l + \tilde{Y}_D^l + J_D^l \end{bmatrix} \quad \cdots\cdots\cdots\cdots(6)$$

The property that the sum of X_L^l or X_D^l in eq. (6) with respect to l coincides with X_L or X_D in eq. (3) is sufficiently secured.

(3) I-O Analyis of D-level

The balance equation of district activities in district d is

$$X_D^d = A_{DD} X_D^d + (A_{DR} X_R^d + A_{DL} X_L^d + \tilde{Y}_D^d + J_D^d) \quad \cdots\cdots\cdots\cdots(7)$$

As done in the I-O analysis of L-level, the district products of regional and local activities are derived with given diagonal matrices of district product distribution coefficients, $R_R{}^d$ and $R_L{}^d$.

$$\left.\begin{array}{l} \boldsymbol{X}_R{}^d = R_R{}^d \boldsymbol{X}_R{}^l = R_R{}^d R_R{}^l \boldsymbol{X}_R \\ \boldsymbol{X}_L{}^d = R_L{}^d \boldsymbol{X}_L{}^l \end{array}\right\} \text{(for } d \in l) \cdots (8)$$

where $\sum_{d\in l} R_R{}^d = I_R$ and $\sum_{d\in l} R_L{}^d = I_L$.

Then if the district final demand vector of D-goods: $(\tilde{\boldsymbol{Y}}_D{}^d + \boldsymbol{J}_D{}^d)$ is given, the district products of district activities are obtained.

$$\boldsymbol{X}_D{}^d = (I_D - A_{DD})^{-1}(A_{DR}\boldsymbol{X}_R{}^d + A_{DL}\boldsymbol{X}_L{}^d + \tilde{\boldsymbol{Y}}_D{}^d + \boldsymbol{J}_D{}^d) \cdots\cdots (9)$$

Consequently, products of all activities: $(\boldsymbol{X}_R{}^d, \boldsymbol{X}_L{}^d, \boldsymbol{X}_D{}^d)$ are obtained in each district.

With the 3-level I-O Analysis Model, calculations are executed from upper to lower levels irreversibly. Since the model's consistency is secured as noted in Section (2), a limited application to a particular district is also possible. The second merit of the model is operationability. Therefore, this model is thought eligible for analyzing many subdivided districts.

4. THE DISTRIBUTION MODELS

The following variables should be predetermined for the 3-level I-O Analysis Model.

a) regional input coefficients; a_{ij}.
b) regional net final demands; $\tilde{Y}_i$.
c) regional net increase in stocks; J_i.
d) regional exports and imports; F_i, M_i.
N.B. From Assumption 1, they are required only in regional activities.

a) are determined from a regional I-O table compiled in the base time period and considered stable. For a long-run application, methods (e.g. RAS) are available for revising them[2].

In this study, every final demand item is to be estimated in the Control sub-model. However, in general, estimations of item bases may be obtained from governmental statistics. To convert them into goods bases, unit converters c_{ij} are used, where subscript i indicates goods and j indicates items. Then the regional net final demands in goods bases; $\tilde{Y}_i$ are obtained from those in item bases; Y_j.

$$\tilde{Y}_i = \sum_j c_{ij} Y_j \cdots\cdots (10)$$

or in a vector form:

$$\tilde{\boldsymbol{Y}} = C\boldsymbol{Y} \cdots\cdots (11)$$

where C is the net converter matrix constituted from c_{ij}.

Concerning unit converters, the following assumption is added.

Assumption 3a Unit converters are the same throughout the Region. Similarly as input coefficients, unit converters can also be revised with time[5]. c) and d) are to be determined for each activity from the Control sub-model.

Using those variables, the I-O analysis of R-level can be executed. However, to execute analyses of lower levels, the following variables should be distributed in advance.

$\boldsymbol{X}_R{}^l$, $\tilde{\boldsymbol{Y}}_L{}^l$, $\tilde{\boldsymbol{Y}}_D{}^l$, $\boldsymbol{J}_L{}^l$, and $\boldsymbol{J}_D{}^l$ for the local level,
$\boldsymbol{X}_R{}^d$, $\boldsymbol{X}_L{}^d$, $\tilde{\boldsymbol{Y}}_D{}^d$, and $\boldsymbol{J}_D{}^d$ for the district level.

Therefore, the first subject is to determine product distribution coefficients $R_R{}^l$, $R_R{}^d$, and $R_L{}^d$. Their values are hardly likely to be stable, since regional distributions of producing factors vary with time. That is why the product distribution model is required.

Although the 3-level I-O Analysis Model just requires $\tilde{\boldsymbol{Y}}_L{}^l$, $\tilde{\boldsymbol{Y}}_D{}^l$, and $\tilde{\boldsymbol{Y}}_D{}^d$, distributions of net final demands for all goods are determined at the same time from eq. (11). Since those distributions are supposed to depend on income distributions, capital formations, etc., which are also unstable, the final demand distribution model becomes necessary.

(1) The Final Demand Distribution Model

In this study, final demands are classified into 8 items as follows.

a) Private consumption expenditures; C1
b) Consumption expenditures outside household; C2
c) National government consumption expenditures; C3
d) Local government consumption expenditures; C4
e) Industrial capital formation; IF
f) Private housing stock formation; IR
g) Public housing stock formation; IGR
h) Government capital formation except housing; IG

Since the Activity sub-model is to be executed with the Location sub-model which focuses on the stock aspects of activities, distributions of investment items e) to h) are to be given in the latter sub-model. Therefore, the distribution models for consumption items are required to be formulated here.

a) Private consumption expenditures; C1

The distribution of C1 is found to be in proportion to personal incomes YB^l, YB^d of the previous period, which are given by the Control sub-model.

$$\left.\begin{array}{l} C1^l(t)=\dfrac{YB^l(t-1)}{YB(t-1)}C1(t) \quad \text{or} \\ C1^d(t)=\dfrac{YB^d(t-1)}{YB^l(t-1)}C1^l(t) \quad (d\in l) \end{array}\right\} \cdots\cdots(12)$$

where (t) indicates time period.

These formulations incorporate a time lag, corresponding to savings behavior.

b) Consumption expenditures outside household; C2

C2 are distributed in proportion to net products YA^l, YA^d of the previous period, which are given by the Control sub-model.

$$\left.\begin{array}{l} C2^l(t)=\dfrac{YA^l(t-1)}{YA(t-1)}C2(t) \quad \text{or} \\ C2^d(t)=\dfrac{YA^d(t-1)}{YA^l(t-1)}C2^l(t) \quad (d\in l) \end{array}\right\} \cdots\cdots(13)$$

c) National government consumption expenditures: C3

Distributions are determined from numbers of employees in Community services of R-level $E^l_{18}(t)$, $E^i_{18}(t)$, and Government service of R-level $E^l_{19}(t)$, $E^i_{19}(t)$, obtained in the Location sub-model.

$$\left.\begin{array}{l} C3^l(t)=\dfrac{k_{31}E^l_{18}(t)+k_{32}E^l_{19}(t)}{k_{31}E_{18}(t)+k_{32}E_{19}(t)}C3(t) \\ C3^d(t)=\dfrac{k_{31}E^d_{18}(t)+k_{32}E^d_{19}(t)}{k_{31}E^l_{18}(t)+k_{32}E^l_{19}(t)}C3^l(t) \quad (d\in l) \end{array}\right\} \cdots\cdots(14)$$

where k_{31} and k_{32} are constants, representing expenditures per employee. The ratio of k_{31} to k_{32} would be comparatively stable.

d) Local government consumption expenditures; C4

Similarly as C3, the ratio of expenditures per employee of local and district community services to those of local and district government services is supposed to be stable.

$$\left.\begin{array}{l} C4^l(t) \\ =\dfrac{k_{41}(E^l_{30}(t)+E^l_{35}(t))+k_{42}(E^l_{31}(t)+E^l_{36}(t))}{k_{41}(E_{30}(t)+E_{35}(t))+k_{42}(E_{31}(t)+E_{36}(t))}C4(t) \\ C4^d(t) \\ =\dfrac{k_{41}(E^d_{30}(t)+E^d_{35}(t))+k_{42}(E^d_{31}(t)+E^d_{36}(t))}{k_{41}(E^l_{30}(t)+E^l_{35}(t))+k_{42}(E^l_{31}(t)+E^l_{36}(t))}C4^l(t) \\ \qquad (d\in l) \end{array}\right\} \cdots\cdots(15)$$

where E_{30}, E_{35}, E_{31}, and E_{36} are numbers of employees in Community services of L-level, and of D-level, Government service of L-level, and of D-level respectively.

Connecting local and district consumption expenditures formulated above with investment distributions, local and district final demands in goods bases are obtained.

$$\left.\begin{array}{l} {}^t\tilde{Y}^l=(C1^l\cdots C4^l IF^l\cdots IG^l)^t C \quad \text{or} \\ {}^t\tilde{Y}^d=(C1^d\cdots C4^d IF^d\cdots IG^d)^t C \end{array}\right\} \cdots\cdots(16)$$

(2) The Product Distribution Model

The production function of Leontief's type requires constant input coefficients. But this admits substitutions within value added, i.e., factor substitutions. When productivity $S_j^l(t)$ is defined as the maximal products obtainable by the combination of factors, the production function is given as

$$X_j^l=\min\left(\frac{X^l_{1j}}{a_{1j}},\ \cdots\ \frac{X^l_{nj}}{a_{nj}},\ S_j^l(t)\right)=\sigma_j^l(t)S_j^l(t)$$

where $\sigma_j^l(t)\in[0,\ 1]$ is the rate of operation of $S_j^l(t)$. $\sigma_j^l(t)$ would be determined by the industrial fluctuations of the time, which would affect all districts by a similar degree. Then it might be said that $\sigma_j^l(t)$ is constant throughout the region, and a distribution coefficient of products $r_j^l(t)$ is

$$r_j^l(t)=\frac{X_j^l(t)}{\sum_l X_j^l(t)}=\frac{\sigma_j^l(t)S_j^l(t)}{\sum_l \sigma_j^l(t)S_j^l(t)}=\frac{S_j^l(t)}{\sum_l S_j^l(t)}$$

$r_j^l(t)$ is simply the distribution ratio of productivity.

The object is to formulate $S_i^l(t)$ as a function of producing factors, i.e., labor E, capital K, and land L. When $r_i^l(t)$ and $r_i^d(t)$ are determined, local and district products can be obtained as follows.

$$X_i^l(t)=r_i^l(t)X_i(t) \quad \text{or} \quad X_i^d(t)=r_i^d(t)X_i^l(t) \quad (d\in l) \cdots\cdots(17)$$

a) Regional activities

i) Sectors 01 and 02

In agriculture, labor and land are the principal factors. But in suburban agriculture, land would depend on labor. And in mining, labor and capital may be proportional.

$$r_i^l(t)=\frac{E_i^l(t)}{E_i(t)} \quad \text{or} \quad r_i^d(t)=\frac{E_i^d(t)}{E_i^l(t)} \quad (d\in l) \cdots\cdots(18)$$

ii) Sectors 03 to 12

In manufacturing industries, labor and capital are substitutive, but land would be dependent on these two factors. When $S_i^l(t)$ is formulated in a function of Cobb-Douglas' type[6] as

$$S_i^l(t)=\alpha_i(K_i^l(t))^{\beta_i}(E_i^l(t))^{\gamma_i}$$

the consistent condition of productivity $\sum_l S_i^l(t)=$

$S_i(t)$ is not satisfied. Therefore, the following function is adopted here with parameters s_i.

$$\left.\begin{aligned} r_i^l(t) &= \frac{s_{i1}K_i^l(t)+s_{i2}E_i^l(t)}{s_{i1}K_i(t)+s_{i2}E_i(t)} \\ r_i^d(t) &= \frac{s_{i1}K_i^d(t)+s_{i2}E_i^d(t)}{s_{i1}K_i^l(t)+s_{i2}E_i^l(t)} \quad (d \in l) \end{aligned}\right\} \cdots(19)$$

iii) Sectors 13, 15, 16, 18, 19, and 20

These sectors are belonging to the tertiary industry, where labor is almost the only producing factor, and eq. (18) is also held.

iv) Sectors 14 and 17

In these capital formation activities, products inevitably coincide with final demands, and distributions of products are obtained in the Location sub-model. However, Sector 14 contains building maintenance which corresponds to intermediate demands and consumption expenditures. Products corresponding to building maintenance $X_{14}-(C_{14.5}IF(t)+C_{14.8}IG(t))$ are distributed here through eq. (18).

b) Local activities

i) Sectors 21, 23, 24, 25, 26, 27, 28, 30, and 31

All of them are also belonging to the tertiary industry, which can be related to labor.

$$r_i^d(t)=E_i^d(t)/E_i^l(t) \quad (d \in l) \quad \cdots\cdots(20)$$

ii) Sectors 22 and 29

These are capital formation activities, too. So no distribution model is required.

(3) Net Increase in Stocks and Regional Balance

Net increase in stocks is influenced by industrial fluctuations. Since their effects would be equivalent in each district, products are considered to be stocked at a uniform rate throughout the Region.

Assumption 5 Regional distributions of net increase in stocks are in proportion to those of productions.

$$\left.\begin{aligned} J_i^l(t) &= (X_i^l(t)/X_i(t))J_i(t), \\ J_i^d(t) &= (X_i^d(t)/X_i^l(t))J_i^l(t) \quad (d \in l) \end{aligned}\right\} \cdots\cdots(21)$$

Then the I-O analysis of L-level is rewritten as

$$\begin{bmatrix} X_L^l \\ X_D^l \end{bmatrix} = \begin{bmatrix} I_L-A_{LL} & -A_{LD} \\ -A_{DL} & I_D-A_{DD} \end{bmatrix}^{-1} \times \left[\begin{pmatrix} A_{LR}X_R^l+\tilde{Y}_L^l \\ A_{DR}X_R^l+\tilde{Y}_D^l \end{pmatrix} + Z_L \begin{pmatrix} X_L^l \\ X_D^l \end{pmatrix} \right] \cdots\cdots(22)$$

Similarly, the I-O analysis of D-level is

$$X_D^d=(I_D-A_{DD})^{-1}(A_{DR}X_R^d + A_{DL}X_L^d+\tilde{Y}_D^d+Z_DX_D^d) \cdots\cdots(23)$$

where Z_L is a diagonal matrix consisting of J_i/X_i (for L- and D-goods) and Z_d is the one consisting of J_i/X_i (for D-goods). In eqs. (22) and (23), products and net increase in stocks are determined simultaneously.

Unit converters are adopted in this study. And this reduces the "degree of freedom" in the original 3-level I-O model. In consequence, local and district imbalances are determined inevitably. Assumption 1 can be rewritten with local and district imbalances; (F^l-M^l) and (F^d-M^d).

Assumption 1a

$$F_L-M_L=0, \quad F_L^l-M_L^l=0$$
$$F_D-M_D=0, \quad F_D^l-M_D^l=0, \quad F_D^d-M_D^d=0$$

The I-O analysis of L-level, eq. (4) can be extended to include R-goods.

$$\begin{bmatrix} I_R-A_{RR} & -A_{RL} & -A_{RD} \\ -A_{LR} & I_L-A_{LL} & -A_{LD} \\ -A_{DR} & -A_{DL} & I_D-A_{DD} \end{bmatrix} \begin{bmatrix} X_R^l \\ X_L^l \\ X_D^l \end{bmatrix} = \begin{bmatrix} \tilde{Y}_R^l+J_R^l+(F_R^l-M_R^l) \\ \tilde{Y}_L^l+J_L^l \\ \tilde{Y}_D^l+J_D^l \end{bmatrix} \cdots\cdots(24)$$

Then the local imbalance of R-goods is

$$(F_R^d-M_R^l)=(I_R-A_{RR})X_R^l-A_{RL}X_L^l - A_{RD}X_D^l-\tilde{Y}_R^l-J_R^l \cdots\cdots(25)$$

Similarly from the extended I-O analysis of D-level, the district imbalances of R-goods and L-goods are obtained respectively.

$$\left.\begin{aligned} (F_R^d-M_R^d) &= (I_R-A_{RR})X_R^d-A_{RL}X_L^d - A_{RD}X_D^d-\tilde{Y}_R^d-J_R^d \\ (F_L^d-M_L^d) &= -A_{LR}X_R^d+(I_L-A_{LL})X_L^d - A_{LD}X_D^d-\tilde{Y}_L^d-J_L^d \end{aligned}\right\} \cdots\cdots(26)$$

The following conditions of consistency are satisfied.

$$\sum_l (F_R^l-M_R^l)=(F_R-M_R)$$
$$\sum_{d \in l}(F_R^d-M_R^d)=(F_R^l-M_R^l), \quad \sum_{d \in l}(F_L^d-M_L^d)=0$$

The total flow-chart of the model is shown in **Fig. 4.**

5. SOME IMPLICATIONS OF INTER-REGIONAL TRADE

Differences between exports and imports are obtained as imbalances in each locale or district in the last section. From these differences, an idealised inter-regional trading pattern can be deduced under the following assumptions which

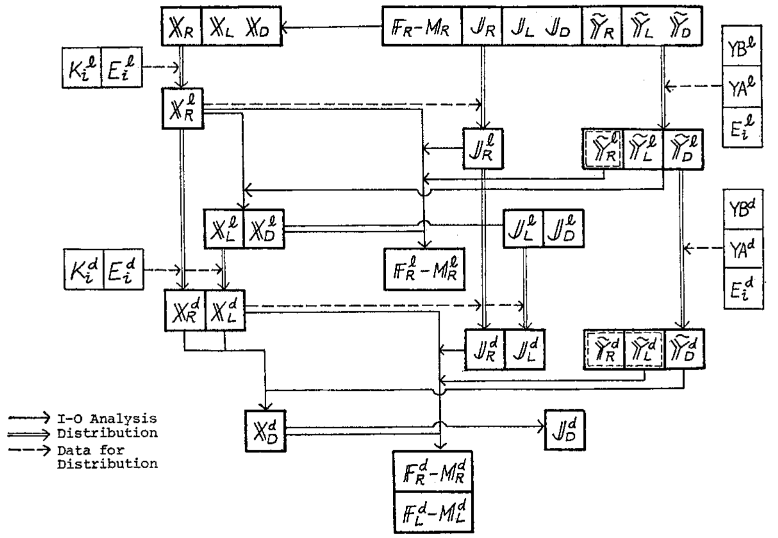

Fig. 4 The Total Flow-Chart of the Activity Sub-model.

are consistent with Leontief's ones.

Assumption 6 Exports to the outer-region from locale l, $F1_i^l$ and exports to the outer-locale from district d, $F1_i^d$, are in proportion to products.

$$\left.\begin{array}{l} F1_i^l=(X_i^l/X_i)F_i\,, \\ F1_i^d=(X_i^d/X_i^l)F_i^l \quad (d\in l) \end{array}\right\} \cdots\cdots\cdots\cdots\cdots(27)$$

Assumption 7 Inter-locale trade of R-goods and inter-district trade of R- and L-goods are in proportion to net demands in the receiving places.

$$\left.\begin{array}{l} t_i^l=\dfrac{\sum_j a_{ij}X_j^l+\tilde{Y}_i^l}{\sum_j a_{ij}X_j+\tilde{Y}_i}\,, \\ t_i^d=\dfrac{\sum_j a_{ij}X_j^d+\tilde{Y}_i^d}{\sum_j a_{ij}X_j^l+\tilde{Y}_i^l} \quad (d\in l) \end{array}\right\} \cdots\cdots\cdots\cdots\cdots(28)$$

where t_i^l and t_i^d are trading pattern coefficients based on the places of production.

Then inter-local or inter-district trade would be given as

$$X_i^{ll'}=t_i^{l'}(X_i^l-F1_i^l), \quad X_i^{dd'}=t_i^{d'}(X_i^d-F1_i^d) \cdots\cdots\cdots\cdots(29)$$

For example, the case of inter-locale trade of R-goods is taken here. When inter-locale exports and imports within the region are indicated by $F2_i^l$ and $M2_i^l$ respectively, locale exports and imports are divided as follows.

$$F_i^l=F1_i^l+F2_i^l, \quad M_i^l=M1_i^l+M2_i^l \cdots\cdots(30)$$

where $M1_i^l$ is locale imports from the outer-region. From eq. (29), inter-locale trade, $F2_i^l$ and $M2_i^l$ are given.

$$F2_i^l=\sum_{l'\neq l}X_i^{ll'}=(1-t_i^l)(X_i^l-F1_i^l) \cdots\cdots\cdots(31)$$

$$M2_i^l=\sum_{l'\neq l}X_i^{l'l}=t_i^l[(X_i-F_i)-(X_i^l-F1_i^l)] \cdots\cdots\cdots\cdots(32)$$

Then $M1_i^l$ is obtained as

$$M1_i^l=M_i^l-M2_i^l=F1_i^l+(F2_i^l-M2_i^l)-(F_i^l-M_i^l) \cdots\cdots\cdots\cdots(33)$$

where $(F_i^l-M_i^l)$ is already given in eq. (25).

The sum of imbalances of inter-locale trade, $\sum_l(F2_i^l-M2_i^l)$ is assured to be closed to 0. And $\sum_l M1_i^l=M_i$ is also satisfied. Therefore, the definition of inter-locale trade in eq. (29) is consistent with the model constructed in the former chapters. In this connection, to execute trade analysis, it is necessary that regional exports and imports are known separately.

Similarly, inter-district trade within a locale can be obtained. In addition, all trades within the region can be investigated for each district.

$$F1_i^d=\tilde{F}1_i^d+\sum_{l'\neq l}X_i^{dl'} \quad (d\in l) \cdots\cdots\cdots\cdots(34)$$

$$\tilde{F}1_i^d=(X_i^d/X_i^l)F1_i^l\,, \quad X_i^{dl'}=(X_i^d/X_i^l)X_i^{ll'} \cdots\cdots\cdots\cdots(35)$$

$$X_i^{dd''}=t_i^{d''}X_i^{dl'} \quad (d\in l,\ d''\in l',\ l'\neq l) \cdots\cdots(36)$$

where $\tilde{F}1_i^d$ means outer-regional exports from district d.

As a matter of fact, trades obtained in this way are idealised ones under Assumptions 6 and 7. However, when representing a comparatively small metropolitan area, this could describe actual trades to some extent, especially in relative comparison.

6. AN EXTENSION CONTAINING ENVIRONMENTAL ANALYSIS

Sanitary services are handled in this study as Sector 37. However, this is an independent sector of public utilities, which eliminates pollutions caused by final demand consumption. Therefore, pollutions caused by industries should be considered anyway. Since activity levels have been estimated only from their products, a scheme which makes it possible to estimate non-productive activities, such as pollution eliminating activities within firms, should be developed.

Leontief's model[7] is an early work of this kind, where all activity levels including pollution eliminating ones are determined simultaneously from environmental standards exogenously given. On the other hand, the model adopted by the Ministry of International Trade and Industry of Japan[8] is to estimate products of goods, quantities of pollutants, pollutants eliminated, and pollutants discharged under the given self-eliminating rates. Since pollutant elimination much depends on environmental policies, the latter model is adopted here for the extension.

The extension is summarized as follows.

a) Pollution handled here is what originated from fixed sources and measured in quantity.

b) Eliminating activities within firms are the object of the analysis. Regardless of diffusion, these activities are completed in each district.

c) Sanitary services as public utilities are handled in Sector 37.

The following assumptions are added.

Assumption 3b Polluting coefficients, a_{pj} and input coefficients to anti-pollution activities, a_{ip} are the same throughout the Region, where $a_{pj}=X_{pj}/X_j$ and $a_{ip}=X_{ip}/X_p$.

Assumption 8 Self-eliminating rates, q_p are the same throughout the Region and common to all activities.

where $q_p=X_p/U_p$.

Note that X_p is pollutant p eliminated and U_p is quantity of pollutant p before elimination $(U_p=\sum_j X_{pj})$.

Assumption 9 There is no secondary pollution caused by pollution eliminating activities, i.e., $X_{pp}=0$.

The balance eq. (2) is extended as follows.

$$\begin{bmatrix} \boldsymbol{X}_R \\ \boldsymbol{X}_L \\ \boldsymbol{X}_D \\ \boldsymbol{X}_P \end{bmatrix} = \begin{bmatrix} A_{RR} & A_{RL} & A_{RD} & A_{RP} \\ A_{LR} & A_{LL} & A_{LD} & A_{LP} \\ A_{DR} & A_{DL} & A_{DD} & A_{DP} \\ A_{PR} & A_{PL} & A_{PD} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_R \\ \boldsymbol{X}_L \\ \boldsymbol{X}_D \\ \boldsymbol{X}_P \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{Y}}_R+\boldsymbol{J}_R+\boldsymbol{F}_R-\boldsymbol{M}_R \\ \tilde{\boldsymbol{Y}}_L+\boldsymbol{J}_L \\ \tilde{\boldsymbol{Y}}_D+\boldsymbol{J}_D \\ -\boldsymbol{Y}_P \end{bmatrix} \quad \cdots\cdots\cdots\cdots(37)$$

where $\boldsymbol{Y}_p$ is a vector consists of pollutants discharged.

The bottom line of eq. (37) is rewritten as

$$\boldsymbol{X}_P=Q_P(A_{PR}\ A_{PL}\ A_{PD})\begin{bmatrix} \boldsymbol{X}_R \\ \boldsymbol{X}_L \\ \boldsymbol{X}_D \end{bmatrix} \quad \cdots\cdots\cdots\cdots(38)$$

where Q_p is a diagonal matrix consisting of q_p.

When the minor matrices of eq. (1) are substituted by

$$\left.\begin{aligned} A^*_{RR}&=A_{RR}+A_{RP}Q_PA_{PR} \\ A^*_{RL}&=A_{RL}+A_{RP}Q_PA_{PL} \\ A^*_{RD}&=A_{RD}+A_{RP}Q_PA_{PD} \\ A^*_{LR}&=A_{LR}+A_{LP}Q_PA_{PR} \\ A^*_{LL}&=A_{LL}+A_{LP}Q_PA_{PL} \\ A^*_{LD}&=A_{LD}+A_{LP}Q_PA_{PD} \\ A^*_{DR}&=A_{DR}+A_{DP}Q_PA_{PR} \\ A^*_{DL}&=A_{DL}+A_{DP}Q_PA_{PL} \\ A^*_{DD}&=A_{DD}+A_{DP}Q_PA_{PD} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(39)$$

eq. (37) can be simplified.

$$\begin{bmatrix} \boldsymbol{X}_R \\ \boldsymbol{X}_L \\ \boldsymbol{X}_D \end{bmatrix} = \begin{bmatrix} A^*_{RR} & A^*_{RL} & A^*_{RD} \\ A^*_{LR} & A^*_{LL} & A^*_{LD} \\ A^*_{DR} & A^*_{DL} & A^*_{DD} \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_R \\ \boldsymbol{X}_L \\ \boldsymbol{X}_D \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{Y}}_R+\boldsymbol{J}_R+\boldsymbol{F}_R-\boldsymbol{M}_R \\ \tilde{\boldsymbol{Y}}_L+\boldsymbol{J}_L \\ \tilde{\boldsymbol{Y}}_D+\boldsymbol{J}_D \end{bmatrix} \quad \cdots\cdots\cdots\cdots(40)$$

This is the same in form as eq. (2), where input coefficients are substituted. When this equation is adopted in the I-O analysis of R-level, 3-level I-O Analysis including environmental effects can be executed in the same way as stated in section 3.

From $\boldsymbol{X}_R{}^d$, $\boldsymbol{X}_L{}^d$, and $\boldsymbol{X}_D{}^d$ obtained in the I-O analysis of D-level, gross quantities of pollutants, pollutants eliminated, and pollutants discharged are obtained in each district.

$$U_P{}^d = (A_{PR}\ A_{PL}\ A_{PD}) \begin{bmatrix} X_R{}^d \\ X_L{}^d \\ X_D{}^d \end{bmatrix} \quad \cdots\cdots\cdots\cdots (41)$$

$$X_P{}^d = Q_P U_P{}^d \quad \cdots\cdots\cdots\cdots (42)$$

$$Y_P{}^d = U_P{}^d - X_P{}^d \quad \cdots\cdots\cdots\cdots (43)$$

The framework of the 3-level I-O Analysis has been shown to be unchanged whether or not environmental analysis is incorporated into it. The size of the input coefficient matrix is also the same on account of Assumption 9. Therefore, the extension is made without reducing operationability of the model.

7. SOME EMPIRICAL INVESTIGATIONS USING THE MODEL

In this section, some results of empirical experiments are shown. The object region is settled on Kanto Region of Japan, which consists of Tokyo Metropolis and 6 surrounding prefectures. Each of them is taken for a locale.

(1) Some Investigations on Assumptions

Assumption 1 requires that both L- and D-goods should balance in each locale while R-goods need not. The absolute sum of local unbalances divided by regional products is given as

$$b_i' = \sum_l |F_i{}^l - M_i{}^l| / X_i \quad \cdots\cdots\cdots\cdots (44)$$

This is employed to investigate this assumption. **Table 2** shows b_i' accompanied with regional imbalance ratio, $b_i = (F_i - M_i)/X_i$ and local one, $b_i{}^l = (F_i{}^l - M_i{}^l)/X_i{}^l$ in Tokyo Metropolis. (Sectors not mentioned there have no imbalance except Sector 14 where only little trade occurs in building maintenance.) In R-goods, the maximum of b_i' is 4.9991 with an average of 0.6945. In L- and D-goods, however, the maxima are 0.4799 and 0.3645, and the averages are 0.1543 and 0.1199 respectively. Therefore, it might be said that R-goods are more widely traded than L- and D-goods. This tendency is better clarified in connection with regional imbalances. In general, the values of $|b_i|$ in L- and D-goods are rather smaller than those of R-goods except for Sectors 24, 25, and 37. On the contrary, it must be observed that some R-goods are never traded in nature. This is derived from the definition of hierarchy of activities adopted in this study. For example, regional public facilities are utilized by anyone in the Region and their influences are diffuse, but construction activities are confined within their locations and never cause actual trade in money terms. From this

Table 2 Regional and Local Imbalances.

Sectors	Regional Imbalances $\frac{F_i - M_i}{X_i}$	Absolute Sum of Local Imbalances $\frac{\sum_l \lvert F_i^l - M_i^l \rvert}{X_i}$	Local Imbalances in Tokyo Metropolis $\frac{F_i^l - M_i^l}{X_i^l}$
01	-0.5399	0.9793	-4.9506
02	-4.6166	4.9991	-12.6782
03	-0.5194	0.5435	-1.2636
04	-1.0053	1.1211	-2.0879
05	-1.1042	1.1521	-1.5795
06	-0.7018	0.7986	-1.5556
07	0.3165	0.6019	0.5095
08	0.0117	0.5704	-0.7758
09	-0.3744	0.3863	-1.6180
10	-0.1930	0.2890	-0.4260
11	0.1729	0.2622	-0.0460
12	-0.0118	0.2065	-0.0106
13	-0.0324	0.1471	0.0904
15	0.1905	0.2899	0.2765
16	0.1551	0.4909	0.4730
18	0.0931	0.4062	0.3698
19	—	0.0364	0.0102
20	—	0.5490	0.3602
21	0.0704	0.2787	-0.2525
23	0.1138	0.2111	0.2876
24	0.1539	0.2123	0.2698
25	0.0624	0.2215	0.2179
26	0.0831	0.2631	0.2837
27	0.1688	0.4799	0.4674
28	0.0858	0.1778	0.1882
30	0.0931	0.1155	0.1107
32	0.0624	0.0767	-0.0094
33	0.0230	0.1689	0.1594
35	0.0931	0.1095	0.0576
37	0.1719	0.3645	0.5101

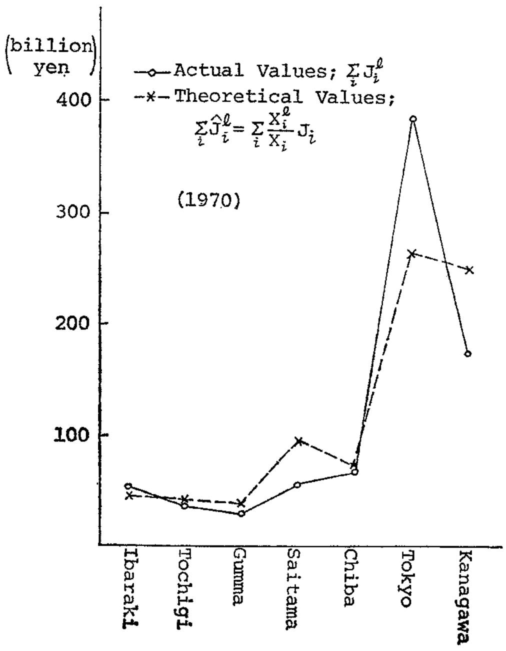

Fig. 5 An Examination of Assumption 5.

viewpoint, the classification of activities might be quite reasonable in general, except for some minor problems.

The next subject discussed here is Assumption 5, which requires that net increases in stocks are in proportion to products. **Fig. 5** shows the sum of theoretical net increase in stocks obtained from eq. (21), $\sum_i \hat{J}_i^l$, in comparison with actual ones, J^l, in 1970. Observe that both values coincide with reasonable accuracy except in Tokyo and in Kanagawa. In Tokyo, the theoretical value is larger than the actual one, and in Kanagawa smaller to the same degree. These differences are derived from Assumption 5 that net increases are stored only in places of production. They are stored in places of consumption as well in practice. Since these differences are adjusted with local imbalances in eq. (25), they never affect the ultimate results of the model.

(2) Some Investigations on Distribution Models

Some results of final demand distribution models are illustrated here. **Fig. 6**a) shows local private consumption expenditures obtained from eq. (12) accompanied with the actual values in 1970. There is much correlation among these values with 0.9996 for the correlation coefficient. The local distribution of national government consumption expenditures by eq. (14) is displayed in **Fig. 6**b), where the value for Tokyo is underestimated. This fact might be because the consumption expenditures per employee in Tokyo are larger than those in the other prefectures.

Finally, product distributions of manufacturing industries are mentioned. Sectors 04 and 11 are taken as examples, and results of local and district distributions of these activities in 1970 are shown in **Fig. 7**a)~d). **Fig. 7**a) and b) show theoretical distributions obtained from eq. (19) and those from the Cobb-Douglas' formulations for the purpose of reference. The correlation coefficients of the former distributions are 0.9873 (Sector 04) and 0.9967 (Sector 11), while the latter ones give 0.9884 and 0.9909 respectively. Therefore, it might be concluded that the linear formulations adopted here have the same accuracy as the traditional Cobb-Douglas ones.

Since the distribution models correspond to production functions, the parameters of district distributions are expected to coincide with those of local distributions. **Fig. 7**c) and d) show district distributions with parameters determined at local distributions and with the locally determined ones. Observe that both results have the same degree of accuracy to the actual ones. The correlation coefficients of the former results

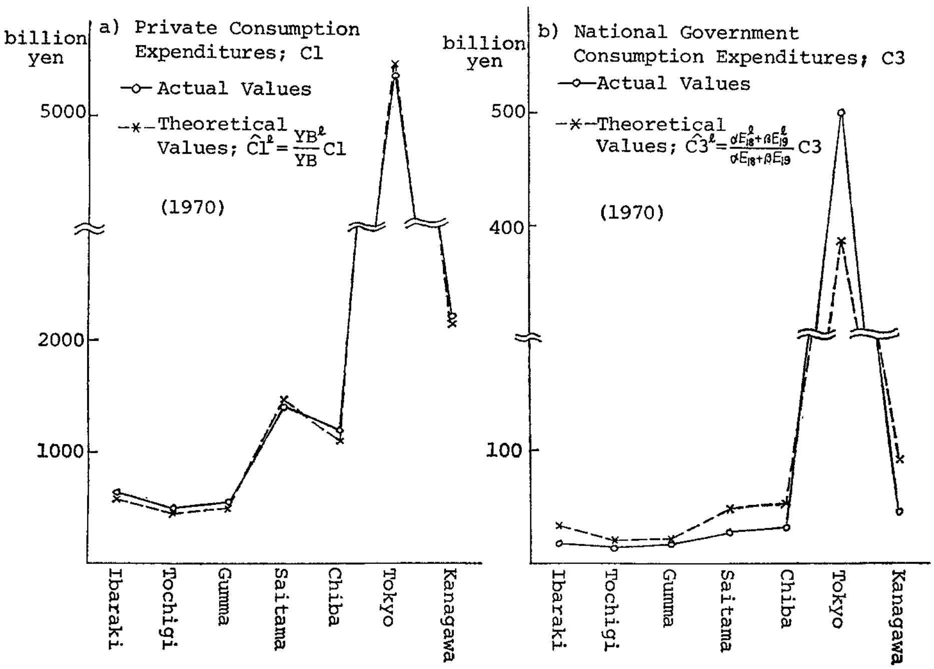

Fig. 6 Examples of Final Demand Distributions.

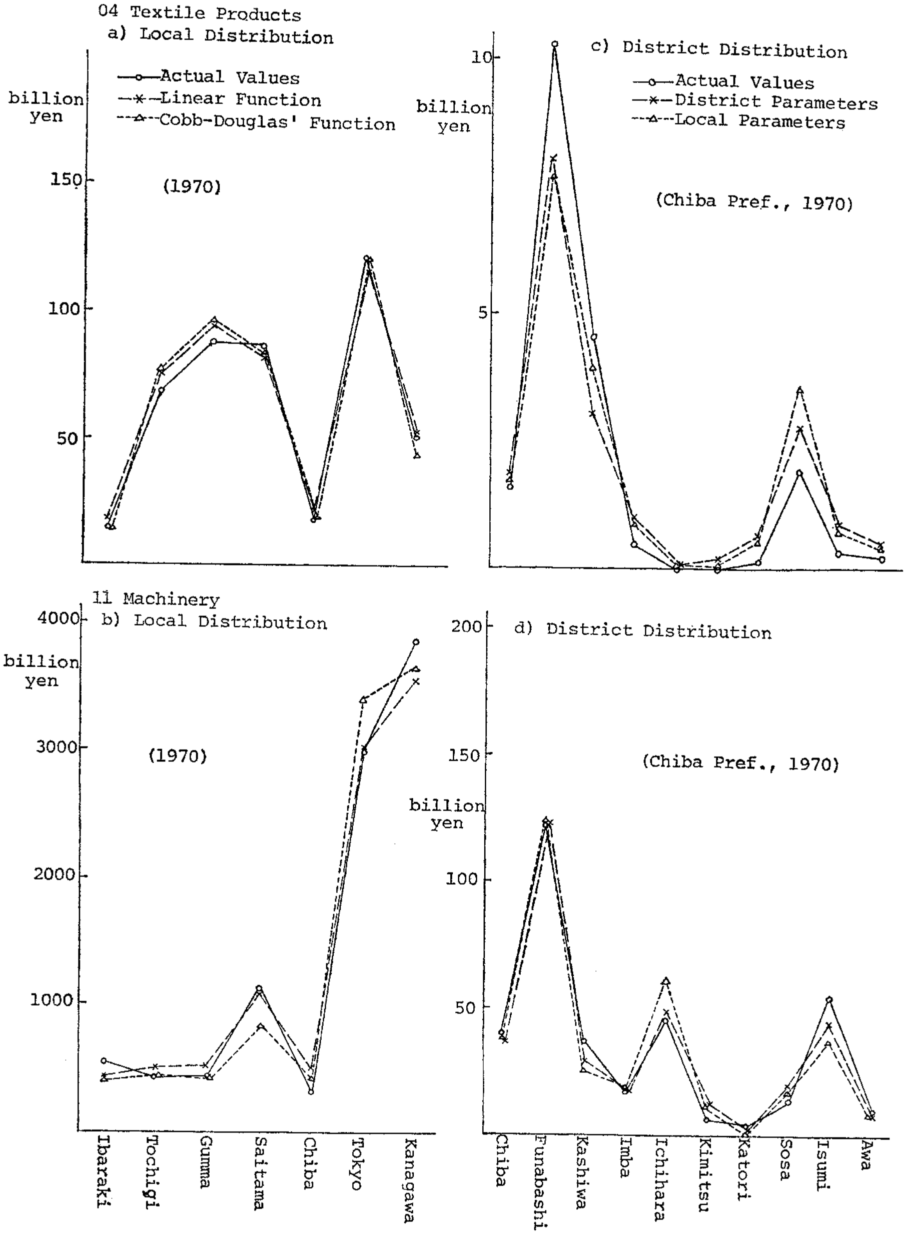

Fig. 7 Examples of Product Distributions.

are, in fact, less than those of the latter results, i.e., the former ones are 0.9359 (Sector 04) and 0.8991 (Sector 11), while the latter ones are 0.9412 and 0.9010 respectively. This means that the assignment of the same values for the parameters of both local and district distributions is found to be practically justified, assuring the required accuracy in executing 3-level I-O Analysis.

8. CONCLUSION

In this article, a flow analyzing model has been composed to form a part of a global metropolitan simulation system. The features of the model are summarized as follows.

a) The model can analyze each subdivided district as well as the metropolitan area as a whole.

b) Activities which characterize urbanization—such as city services or retail trade—are handled positively in addition to productive industries.

c) Sectoiral and spatial interactions among activities are considered explicitly. Pollution and inter-regional trades which are derived from activities can also be analyzed within the model.

d) The model would be a large-scale one in nature. In this study, an attempt to make the model to be compatible with operationability is made through introducing hierarchical concepts into the model.

Some investigations on the model were made in section 7. Another requirement is to combine the flow analyzing model composed here with an appropriate stock analyzing model, and to complete the model as a global metropolitan simulation system.

Using the complete system, various policies can be evaluated totally. For example, investments in the transportation facilities have two major effects, viz. investmental effects and transportation cost effects. In this model, the latter are to be considered through locational adjustments of activities, and this is why the simple trading model is adopted here. These effects will be reflected in the analyses of following periods with locational feedbacks. However with only the flow analyzing model, some meaningful policy evaluations can be performed. In this case, the policy impact will be given in terms of changes in investments or their regional distributions. The inter-regional and inter-sectoral effects can be observed as the distributions of productions and inter-regional trades. And if a more sophisticated trading model is adopted, even transportation cost effects can be evaluated to some extent.

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