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RESERVOIR DESIGN CAPACITIES FOR VARIOUS SEASONAL OPERATIONAL HYDROLOGY MODELS

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1. INTRODUCTION

The importance of operational hydrology in the planning and design phases of water resource development has been recognized in recent years, and has led to rapid advancement of different stochastic runoff generators. Newly developed models, in general, will be of use to water resource system designers who are increasingly recognizing the limitations of conventional design and management techniques but who are confused by the myriad of stochastic generation schemes discussed in the literature.

It is unlikely that the true stochastic nature of streamflow at a particular location will be known. Furthermore, nonstationary features (apart from readily identified seasonality) usually cannot be identified from most hydrologic records which comprise statistically small samples. This implies that only high frequency phenomena can be identified from typical hydrologic time series, yet the presence of low frequency effects, while possibly not detectable, can have significant impacts on design. Many analysts, recognizing the difficulty involved in locating low frequency effects, settle for use of some lumped low frequency effect. The Hurst phenomena, characterized by the statistic H (or K)[21], are the index used for lumped low frequency effects.

There are three principal issues that the designers must consider when deciding to use one or more stochastic streamflow models. The first is: should high or low frequency persistence (or both) be modeled? Secondly: should two or more moments be used to represent estimated marginal distributions of the assumed stochastic process? Finally the issue of parameter identification arises. How should model parameters be estimated? Would it be better to have a simpler model with fewer parameters that could be identified more precisely? Small sample sizes make the above issues difficult to resolve. Thus, it is often necessary to effect designs using several models which have vastly different structures and to resolve differences in a decision making mode.

The pertinent issues of model choice are annual or seasonal (monthly) streamflow generation models, and short-term or long-term persistence models. Annual models are appropriate when modeling over-year storage effects. These effects dominate when reservoir capacities are large—usually greater than the mean annual flow of the river—demands are high, Hurst coefficient is large and the coefficient of variation is relatively large (greater than 0.3–0.5). Seasonal flow generation models are of importance to major water resource facility design and operation, especially for facilities in which storage is provided primarily for within-year smoothing of streamflows. Most existing reservoirs are designed for within-year flow buffering. Choice of marginal distributions to describe seasonal and annual flow volumes is restricted by the available flow data. Regional information is sometimes useful for determining the form of the marginal distribution in data scarse situations. Parameter estimation is influenced by the available sample size, model form, and forms of marginal distributions modeled.

There are two principal objectives addressed in this paper. The first objective is to compare two monthly flow generation models in a reservoir design application. These models are the well known Thomas-Fiering monthly model[7] and a model which disaggregates annually generated streamflow volumes to monthly volumes.[9,17,19,20] The second objective is to determine the importance of including long-term flow persistence in models used to determine storage requirements

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for reservoirs which are used for within-year flow buffering. Issues related to accomodating "outliers" in the data are discussed. Use is made of quantile-quantile (Q–Q) graphical plots[25] to smooth outliers as well as to determine how well theoretical descriptions agree with their corresponding empirical cumulative distributions.

Flow persistence plays an important role in determining the size of storage reservoir[5,12]. Alternatively, for a reservoir of known capacity, the reliability with which a given demand schedule can be satisfied is influenced by flow persistence. Long-term persistence usually requires large storage acpacities than needed when only short-term persistence is present with all other flow characteristics the same. As demand levels increase towards the mean flow, storage requirements become larger than the mean annual flow of the river at reliabilities society expects.[3,5,12] It should be kept in mind, however, that storages of size greater than the mean annual flow volume of most Japanese rivers are physically infeasible because of topological limitations among others. With increasing demands upon water resources high levels of development are to be expected. Consequently, it is important to know the reliability with which an existing facility could satisfy a different future demand schedule.

When investigating the first objective an annual Markov model (AR(1)[2]) was disaggregated to yield monthly flow volumes and the resulting design compared to that obtained using the Thomas-Fiering algorithm. The second objective was addressed using disaggregation models only. High frequency effects were modeled with an AR(1) process while low frequency effects were modeled with an autoregressive moving average (ARMA(1, 1)) process[2].

Numerous marginal distributions could be fitted to observed data and remain statistically indistinguishable. Hence, choice of an appropriate marginal distribution is often influenced by convenience of use. If the coefficient of variation (CV) is less than about 0.5 and the data are approximately symmetrical then a normal distribution might be appropriate (The limit on CV is to restrict the number of negatively generated flows to a small number). When unimodal skewed distributions are encountered it is convenient to use general gamma or three-parameter log-normal (3PLN)[4,7] distributions. Often, however, computational considerations imposed by model choice preclude use of the gamma model[9,13]. Moreover, it has been found that 3PLN distributions give good approximations for most hydrologic situations of interest[13]. Therefore, 3PLN distributions are used here to model skewed marginal distributions. Sample skews, after smoothing via use of Q–Q plots, were corrected for small sample bias[1,24]. The importance of including skewed distributions in the flow generation models is shown.

Model choice and sample size influence parameter estimation. In an infinite sequence (population) estimates of all moments are the population parameters regardless of the presence of any persistence. In finite samples, estimates of the variance, skew, and lag-one correlation coefficient are influenced by the form of the correlation structure present. The impact of model persistence on parameter estimation is fully discussed.

The remaining issue is that of actual design comparisons. To fully assess the sensitivity of required reservoir storage to model and distribution choice, an economic analysis ought to be employed. Even a simple economic model[10] would be expensive. As a preliminary screening tool, some surrogate measure utilizing physical relationships will be of use to distinguish between alternative models and methods.[12,22] In the work reported herein flow-demand coupling was explored using synthetically generated monthly flows and a monthly Thomas-Fiering "Sequent Peak Algorithm" (SPA)[6]. Empirical probability distributions of storage needed to fully satisfy demand were obtained. The impacts of model and distribution choice on reservoir size determination could be conveniently observed by comparing the resulting cumulative distributions of storage for the cases examined.

All comparisons were made using flow statistics estimated from a 22-year record (1951–1973) of monthly and annual flows at Hashimoto in the Ishikari River catchment (5 460 km^2). Numerical experiments were carried out via a CDC 6400 digital computer at the Academic Computer Center, University of Washington.

2. MATHEMATICAL DEVELOPMENT

Unless stated otherwise, monthly and annual flows, and associated random variables at a single site are standardized with zero mean and unit variance. Flows of interest in season i are obtained by multiplying the standardized variable by S_i (standard deviation) and adding the mean flow $\bar{X}_i$.

(1) Stochastic Monthly Flow Model

Two monthly flow generators were used in reservoir storage design applications. The first is the widely used generation model known as a Thomas-Fiering (TF) seasonal flow generator[7]. The second is a disaggregation model originally proposed by Valencia and Schaake[20].

The Thomas-Fiering model does not necessarily guarantee preservation of the first two moments at the annual level[7]; the disaggregation technique can preserve relationships between monthly and annual flows in terms of the first two moments. This is readily seen from the Ishikari River data. For example, if the Thomas-Fiering model uses the monthly variances and month-to-month correlations the resulting theoretical annual standard deviation and lag-one (ρ_1) correlation coefficient are 207.2 mm and 0.011 respectively. The historical annual quantities are 258.9 mm and 0.426 respectively. The inability of the assumed TF model to replicate annual statistics has important consequences in design.

The TF model is defined as

$$Y_i=\rho_i Y_{i-1}+\sqrt{1-\rho_i^2}\,\varepsilon_i \quad \cdots\cdots(1\,\mathrm{a})$$

$$q_i=\bar{X}_i+S_i Y_i \quad \cdots\cdots(1\,\mathrm{b})$$

where q_i is flow in month i, ρ_i is the correlation coefficient between months $i-1$ and i, and ϵ_i is a random component (the Y_i and ϵ_i terms have zero mean and unit variance).

A disaggregation model, developed by the first author, was used to disaggregate annual to monthly flows. This particular model simultaneously disaggregates flow in consecutive years to ensure that over-year covariances are preserved[9]. This model differs from the scheme proposed by Mejia and Rousselle[17] to preserve monthly correlations, and correlations between months of consecutive water years. The disaggregation model for the mth and $(m+1)$th year monthly flows is defined by

$$Y_m=AX_m+B_mV \quad \cdots\cdots(2)$$

and

$$Y_{m+1}=AX_{m+1}+C_{m+1}W \quad \cdots\cdots(3)$$

where Y_m is a (12×1) vector of monthly flows in the mth year; X_m is a scalar of annual flow in the mth year; V and W are (12×1) vectors of random components; A is a (12×1) vector of coefficients; B_m and C_{m+1} are (12×12) matrices of coefficients. Coefficients matrices are estimated by a least-squares technique as follows:

$$A=E(Y_mX_m^T) \quad \cdots\cdots(4)$$

$$B_mB_m^T=E(Y_mY_m^T)-AA^T \quad \cdots\cdots(5)$$

$$C_{m+1}=[E(Y_{m+1}Y_m^T)-\rho_1AA^T][B_m^{-1}]^T \cdots(6)$$

where $E(\cdot)$ denotes the mathematical expectation operator; F^T denotes the transpose and F^{-1} denotes the inverse of a matrix F; $E(Y_mX_m^T)$ is a (12×1) vector of correlation coefficients between monthly and annual flows; $E(Y_mY_m^T)$ is a (12×12) matrix of within-year correlation coefficients between monthly flows; $E(Y_{m+1}Y_m^T)$ is a (12×12) matrix of over-year correlation coefficients between monthly flows; and ρ_1 is a lag-one serial correlation of annual flow.

The matrix $B_mB_m^T$ of eq. (5) is symmetric; hence B_m was solved via a principal components technique. A brief comment on parameter identification of the matrix B_m is appropriate. The disaggregation model has 11 degrees of freedom. Therefore when the first two moments and the correlation coefficients between annual and 11 monthly flows are given, the first two moments of the remaining monthly flow are uniquely determined. This characteristic of the disaggregation technique results in a singular matrix of $B_mB_m^T$ if 12 monthly flows are involved in the specification of eq. (2). Consequently, one of the column vectors of B_m becomes a null vector and the inverse of B_m does not exist. To avoid the linear dependence of $B_mB_m^T$, model building and associated parameter identification were performed in two stages. First, one of 12 monthly flows was deleted in the specification of eq. (2); An (11×11) matrix, $B_1B_1^T$, similar to eq. (5), can be solved because it is nonsingular. Coefficients for the monthly flow deleted in the first stage were determined from the correlation coefficients between the deleted flow and 11 monthly flows and the variance of the deleted flow itself in the second stage. A minor change in model definition described above, can completely overcome computational difficulty with respect to parameter identification. Furthermore, the above computational procedure guarantees the existence of B_m^{-1}.

(2) Skewed Monthly Flows

The above monthly flow generation schemes will only preserve the observed first two moments as well as the month-to-month correlations in the TF model, and all monthly correlations in the disaggregation model. In most practical situations monthly flows have nonsymmetric marginal distributions. Monthly marginal distributions often can be conveniently approximated with a 3PLN distribution. Details of this distribution and its use in hydrology are well documented in the literature[4,15,16]. Other distributions might be used, for example, the generalized gamma distribution. However, from an operational point of view the 3PLN distribution is quite satisfactory[13] and is used herein.

The basic approach involves first of all determining the transform mean (μ_{Z_i}), standard deviation (σ_{Z_i}) and third parameter (a_i) for month i. Next, within-year and over-year correlation coefficients between monthly flows are transformed to account for generation in the transformed (normal) domain; these transformed correlations are used in eqs. (1), (4), (5), and (6). The monthly flows in the transformed domain are then inverse transformed to yield desired monthly flows. Typically for the ith month

$$Z_i=\sigma_{Z_i}Y_i+\mu_{Z_i} \quad \cdots\cdots (7)$$

and

$$q_i=a_i+\exp(Z_i) \quad \cdots\cdots (8)$$

where Y_i results from eq. (1a) (TF model), or eqs. (2) and (3) (disaggregation model); Z_i is a scaled quantity (normally distributed); and q_i is the desired 3PLN distributed flow in the ith month.

A problem resulting from use of the 3PLN distribution in the disaggregation scheme is that the exact linear relationship cannot be maintained between successive levels of aggregation. For instance, the annual value is equal to the sum of monthly values; however, because of the nonlinearity of transformation, this relationship cannot be fully satisfied. Limited numerical experiments did, however, indicate that the degree of distortion is not severe from an operational standpoint.

(3) Stochastic Annual Flow Model

The disaggregation model requires independently generated synthetic annual flows which follow some generating mechanism. Short-term persistence effects were modeled using an AR(1) process. The annual AR(1) model is given by

$$X_i=\rho_1X_{i-1}+\sqrt{1-\rho_1^2}\,e_i \quad \cdots\cdots (9)$$

where X_i is a sequence of annual flows; e_i is a random component drawn from a normal distribution having zero mean and unit variance; and ρ_1 is a lag-one serial correlation coefficient.

Long-term persistence effects were modeled using an autoregressive moving average (ARMA (1,1)) process. The choice of an ARMA (1,1) model used herein is based on experiments conducted by Lettenmaier and Burges[12] pointing out that the ARMA model proposed by O'Connell[18] was found to give essentially identical storage distributions to Fast Fractional Gaussian Noise (FFGN) generators[14] for Hurst coefficients less than about 0.8 and for short economic life, say 40 years. The primary advantage of the ARMA model is its computational simplicity and economy relative to FFGN. Care should be exercised when initialing ARMA sequences[12]. The ARMA(1,1) process[2] is given by

$$X_i=\phi X_{i-1}+\varepsilon_i-\theta\varepsilon_{i-1} \quad \cdots\cdots (10)$$

where ϕ and θ are parameters and ε is a zero mean normal random deviate with variance

$$Var(\varepsilon)=(1-\phi^2)(1-2\phi\theta+\theta^2)^{-1} \quad \cdots\cdots (11)$$

The correlation structure of the ARMA process is given by

$$\rho_1=(1-\phi\theta)(\phi-\theta)(1-2\phi\theta+\theta^2)^{-1} \quad \cdots\cdots (12)$$

$$\rho_k=\phi^{k-1}\rho_1 \quad (k\geq 2) \quad \cdots\cdots (13)$$

where ρ_k is the lag k serial correlation coefficient. Unfortunately, the ARMA (1,1) model has no population value of the Hurst coefficient. O'Connell[18] has developed a moderate sample size expectation of the Hurst coefficient and lag-one correlation coefficient as a function of parameters, ϕ and θ based on Monte Carlo experiments. The proper choice of ϕ and θ inevitably requires interpolation in Table 3.1 (Ref. 18) to approximately match the observed Hurst coefficient and lag-one correlation. To avoid these parameter estimation problems, Lettenmaier and Burges[12] developed an alternative computationally economical model, which has a distinct population value of the Hurst coefficient. It was not necessary to use their model here.

In the case of 3PLN distributed sequences generated with AR(1) and ARMA(1,1) processes, the lag-one correlation coefficient, and the parameters ϕ and θ were transformed to the logarithmic normal domain. The transformation procedure is given by Matalas[15] and Lettenmaier and Burges[13].

(4) Bias Correction Considerations

The small sample properties of sequences generated from particular models are known to be biased. The bias results from using the sample size, N, when computing summary statistics rather than effective sample size. Thus if the annual model is assumed to be AR(1) the raw sample statistics must be modified to reflect this. Similarly a different adjustment must be made if the data are assumed to be drawn from an ARMA(1,1) population. Because small sample biases are particularly severe for estimates of the variance and lag-one serial correlation of annual flow, appropriate bias correction factors should be administered to ensure consistent results from different models as well as to maintain statistical resemblance between historical and synthetic sequences. Kendall[11] showed that estimated sample and population lag-one correlation coefficients (r_1 and ρ_1 respectively) are related as

$$E(r_1)=\rho_1-\frac{1}{N}(1+4\rho_1) \quad \cdots\cdots(14)$$

where N is the length of record. The population value, ρ_1, is needed for synthetic generation, hence, it is estimated from eq. (14). Wallis and O'Connell[23] pointed out that the bias correction of eq. (14) is appropriate for the lag-one Markov process. When $E(r_1)$ is replaced by its sample estimate, an unbiased estimate of ρ_1 is obtained. For the Ishikari River data, $N=22$, $r_1=0.426$, hence $\rho_1=0.576$ in an untransformed domain. When annual flow was modeled by a 3PLN distribution, ρ_1 became 0.589. Equation (14) was also used to estimate ρ_1 when the generating mechanism was assumed to be an ARMA (1,1) process.

The magnitude of annual flow variance has been shown to appreciably influence storage requirements[5,12]. Therefore it is most important to correct sample variance bias to ensure generation is effected with an appropriately scaled variance component. The sample variance underestimates the population variance for a correlated time series. The sample variance of annual flow for a sequence of N events is given by

$$S^2=\frac{1}{N-1}\sum_{i=1}^{N}X_i^2-\frac{N}{N-1}(\bar{X})^2 \quad \cdots\cdots(15)$$

$$=\frac{1}{N-1}\sum_{i=1}^{N}X_i^2-\frac{N}{N-1}\left(\frac{1}{N^2}\sum_{i=1}^{N}X_i^2+\frac{2}{N^2}\sum_{\substack{i=1\\(i\neq j)}}^{N}\sum_{j=1}^{N}X_iX_j\right) \quad \cdots\cdots(16)$$

where $\bar{X}$ is a sample mean and S^2 is the sample variance. For a stationary process with zero mean, the autocorrelation function is given by

$$\left.\begin{aligned}&E(X_i^2)=\sigma^2\\&E(X_iX_j)=\sigma^2\rho_{|i-j|}=\sigma^2\rho_k\end{aligned}\right\} \quad \cdots\cdots(17)$$

where σ^2 is a population variance and ρ_k is the population serial correlation coefficient at lag k. Manipulation of eqs. (16) and (17) yields

$$E(S^2)=\sigma^2\left\{1-\frac{2}{N(N-1)}\sum_{k=1}^{N-1}(N-k)\rho_k\right\} \cdots(18)$$

It is clear that $E(S^2)=\sigma^2$ for a pure random process.

The lag-one Markov process has autocorrelation function $\rho_k=\rho_1^k$. For this process, eq. (18) reduces to

$$E(S^2)=\sigma^2\left[1-\frac{2\rho_1}{(N-1)N}\left\{\frac{N(1-\rho_1)-(1-\rho_1^N)}{(1-\rho_1)^2}\right\}\right] \quad \cdots\cdots(19)$$

The ARMA (1,1) process has autocorrelation functions given by eqs. (12) and (13). Hence, eq. (18) for this process reduces to

$$E(S^2)=\sigma^2\left[1-\frac{2\rho_1}{(N-1)N}\left\{\frac{N(1-\phi)-(1-\phi^N)}{(1-\phi)^2}\right\}\right] \quad \cdots\cdots(20)$$

The magnitudes of the corrections needed for estimated variance of the Ishikari River annual flows were appreciable. The standard deviation computed from eq. (15) was 258.9 mm. The corrected values for use with AR(1) and ARMA (1,1) generators became 275.4 and 314.9 mm respectively (for 3PLN distributed flows, the standard deviation in the normal domain was changed from 0.290 to 0.309 and 0.355, respectively). Based on work reported by Burges and Lettenmaier[5,12] neglecting to make these corrections would have a substantial impact on the comparisons between models given later in this paper.

3. DATA ANALYSIS

When fitting three-parameter density functions to empirical data, the fit is quite sensitive to estimated skew coefficients. A few "outliers" in the data can have a substantial effect. When the 3PLN distribution is used as a curve fitting distribution the simplest way to test if a sample is describable by this model is to compute the skew of the transformed data; if the effects of outliers have been appropriately removed the skew of $\ln(q-a)$ is close to zero.

A powerful diagnostic check on the presence and impact of outliers is given by use of a quantile-quantile (Q–Q) plot[25]. If the test of normality is of primary concern, the Q–Q plot is a graphical representation of ordered observations against quantiles of a normal distribution. A distinguishing feature of the Q–Q plot is that for marginal probability distributions over an unbounded range, the plot is more sensitive to differences in the tails than to differences in the middle. This characteristic is extremely useful for showing the effect of outliers on the assumed distribution of the data. An advantage of the Q–Q plot is that it does not require a special probability paper with distorted scales, ordinary linear graph paper is used.

Fig. 1 shows an example of the Q–Q plot in

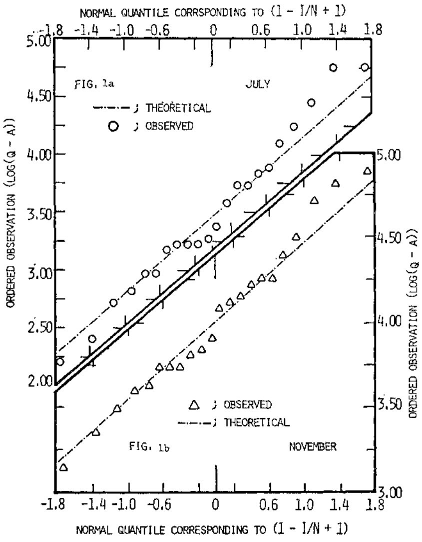

Fig. 1 Normal quantile-quantile plot.

two months where all ordered observations of ln(q–a) (q; flow in millimeter and a; third parameter in millimeter) are plotted against quantiles of a standard normal distribution. An empirical cumulative distribution was determined by the Weibull plotting formula; quantiles of a normal distribution corresponding to these cumulative probability levels were computed after Hastings[26]. The straight lines in Fig. 1 are defined by the first two theoretical moments in the transformed (normal) domain. This example indicates that even though the computed skews of the two ln (q–a) sequences are approximately close to zero (Table 2), the upper tails of the distributions are still divergent from the theoretical lines.

Impacts of outliers on sample skew were particularly obvious from the Q–Q plots for months December, March, July, and October. The question of how to smooth such outliers depends on the purpose of the study. In this study the outliers of the upper tails were smoothed to the extent which the basic structure of the original data was not sacrificed appreciably. The threshold values of $\bar{X}_i + 2S_i$ ($\bar{X}_i$; mean and S_i; standard deviation) were assigned, beyond which outliers were replaced by this specified value in month i.

Table 1 shows the number of smoothed data and associated changes in the first three moments resulting from the above modifications; the enormous impact of a few events on the computed skew coefficient is readily apparent. Three recorded values of annual flow were modified to maintain a linear relationship between annual and monthly flows in the years where changes were made to recorded monthly flows. All subsequent work was done using these modified monthly flow sequences.

The 3PLN model is normally used to model positively skewed data with the third parameter, a, a lower bound. Negatively skewed data pose some difficulties. If the magnitude of negative skew is small these data are approximated by normal distributions. Consequently, flows in February, May, and June (Column 2, Table 5 shows the skew coefficients) were modeled as normal distributions. It is important to examine the magnitude of CV when making this approximation. For these three particular months CV was less than 0.35 (Fig. 3), therefore only a very small fraction (less than 0.1%) of negative flows would be generated. For computational convenience these normal distributions were approximated by 3PLN distributions as described below.

The approximation procedure was effected as follows: when the third parameter a in the 3PLN distribution approaches negative infinity, the 3PLN distribution is identical with the normal distribution; therefore a fairly large negative value can be assigned to the third parameter to force the 3PLN to approximate a normal distribution. In the present study, a negative value five

Table 1 Effect of outliers on statistical parameters.

Para. / Mon.	Raw Data			Smoothed Data			
	Mean (mm)	St. De. (mm)	Skew	Mean (mm)	St. De. (mm)	Skew	Number of Corrected Data
Dec.	65.8	18.3	1.215	65.0	16.0	0.627	1
Mar.	65.9	23.3	1.275	64.8	20.1	0.585	1
Jul.	77.3	36.2	1.731	75.2	30.6	1.366	2
Oct.	94.4	40.3	1.344	92.8	36.1	0.967	1
Ann.	1 336.8	271.8	0.824	1 331.2	258.9	0.644	3

Table 2 Theoretical mean and standard deviation in the normal domain and the first three moments of ln $(q-a)$ sequences (q and a are in millimeter).

Month	Theoretical		ln$(q-a)$ sequences		
	Mean	St. De.	Mean	St. De.	Skew
Dec.	3.98	0.282	3.98	0.284	0.025
Jan.	6.47	0.015	6.47	0.015	0.018
Feb.	5.39	0.046	5.39	0.046	−0.122
Mar.	4.28	0.263	4.28	0.266	−0.027
Apr.	6.71	0.081	6.71	0.081	0.013
May	7.27	0.059	7.27	0.060	−0.242
June	6.38	0.039	6.38	0.040	−0.396
July	3.49	0.666	3.48	0.694	0.206
Aug.	5.66	0.260	5.67	0.257	0.305
Sep.	4.76	0.296	4.77	0.296	0.143
Oct.	4.24	0.448	4.24	0.460	−0.027
Nov.	4.01	0.462	4.01	0.466	0.180

times the mean of each monthly flow was specified which yielded approximately a normal distribution after transformation. Table 2 compares the theoretical mean and standard deviation in the log domain with the first three moments of ln $(q-a)$ sequences. It is of interest to note that the two sets of statistics are almost the same except for the standard deviations in July, August, and October. The transformed data of ln $(q-a)$ were used to compute the required correlation coefficients of eqs. 4 and 5, because the $B_m B_m^T$ matrix resulting from the theoretically transformed correlation[16] did not yield a positive difinite matrix.

Determination of a Hurst coefficient from the small sample of flow data is indeed difficult. This coefficient is required to obtain parameters ϕ and θ for the ARMA (1,1) model. The pox diagram approach[21] was used here because it is the most useful known method for exploring lumped low frequency effects, particularly in small samples. The indicated value was $H=0.804$ as shown in

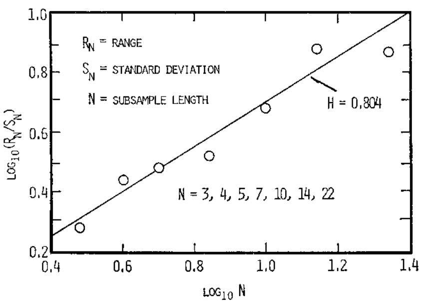

Fig. 2 Pox diagram of R_N/S_N versus N.

Fig. 2. There is no way of knowing if this is correct, hence no effort was made to correct for bias. Table 3.1 of Ref. (18) with $H=0.8$ and $\rho_1=0.576$ yeilds $\phi=0.900$ and $\theta=0.542$. The appropriate values for ϕ and θ when annual flows are taken to be 3PLN distributed are $\phi_Z=0.902$, $\theta_Z=0.539$[13].

4. RESERVOIR SIZE DETERMINATION

The approach used here to determine reservoir size was based on physical flow-demand coupling. The Sequent Peak algorithm (SPA)[6] was used to determine the necessary reservoir size to meet a given demand schedule. Storage probability distributions were determined as follows: 500 synthetic monthly flow traces, each trace having a length of 40 years were generated. Each of these synthetic traces was routed through the SPA to yield a value of the required storage. The mth largest storage value for 500 traces is plotted at cumulative probability level, $1\text{-}m/501$; cumulative storage distributions determined in this way have been shown[3] to follow the Extreme Value Type I distribution (Gumbel[8]). Thus it is convenient to show the resulting distributions as straight lines on Extreme Value probability paper.

Storage reliability curves obtained in the above manner are immediately useful for estimating the size of a storage facility needed to meet a given physical demand schedule at a prescribed reliability level.

Alternatively, the results may be used to determine, for an existing facility, either the reliability with which a given demand level may be met or the demand schedule which may be met at a given reliability.

The SPA is a conservative algorithm in that it finds the amount of storage that would be needed so that if flow from a particular scenario is routed through it then the flow plus water stored would be just sufficient to fully satisfy demand. While satisfying demand in this way may be uneconomical in a particular case the use of the SPA here for generalized comparison purposes provides a useful index for showing differences in flow-demand coupling for the generation schemes examined.

5. DESIGN EXAMPLE

(1) Monthly Flow Generation

Synthetic flows were generated using a monthly time increment. This increment was used because

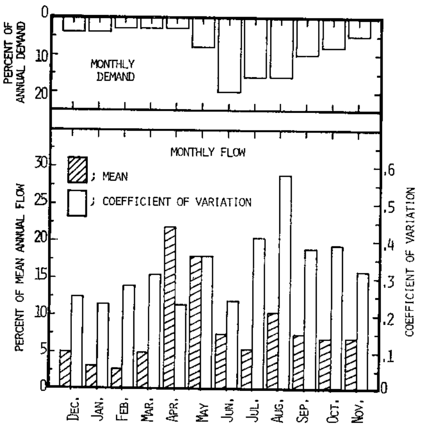

Fig. 3 Monthly demand and flow patterns.

it was appropriate relative to the size and type of reservoir being examined. Storage design was principally directed towards reservoirs that would be used for seasonal flow buffering. Monthly flows were generated by the TF model and the disaggregation model. In the latter case annual flows were generated first by an AR(1) process; the experiments were repeated with annual flows generated by an ARMA (1,1) process. Monthly marginal distributions were first assumed to be normally distributed, i.e., only the first two moments were preserved. A second set of experiments was conducted with the monthly flows modeled by 3PLN distributions which more accurately characterized the nature of skewed flows. These two forms of marginal distributions were used because some workers argue (at least privately) that it is inappropriate to model other than the first two moments. It will be seen below that there is cause to model skewed marginal distributions.

Relevant flow summary information for the first two moments of monthly flow is given in Fig. 3 (approximately 40% of the total annual runoff is yielded by snowmelt, hence the unregulated streamflow reflects the effects of release from a natural storage system). Monthly and annual skew coefficients are given in Tables 5 and 7. Each sample skew coefficient was corrected for small sample bias after Bobée and Robitaille[1].

(2) Storage Distributions

Storage probability distributions were obtained using the monthly distributed demand pattern in Fig. 3 for various annual magnitudes and for each of the generation models described above. Operation life was held fixed at 40 years, the annual demand was constant over this period. Three annual demand levels $D^*=D/U_X$ (D=annual demand (mm), U_X=mean annual flow (mm)) of 0.7, 0.5 and 0.3, reflecting within-year storage situations, were used. Storage distributions are plotted nondimensionally as $S^*=S/U_X$ (S is required storage (mm)) and corresponding cumulative probability.

Fig. 4 shows the theoretically fitted distributions resulting from the TF model. The information contained in Fig. 4 can be interpreted as follows: If it were practical to construct a facility whose capacity was equal to, say, $S^*=0.35$, then a specified demand $D^*=0.7$, for example, could be supplied with a reliability of 0.84 for the normal generator while the same demand could

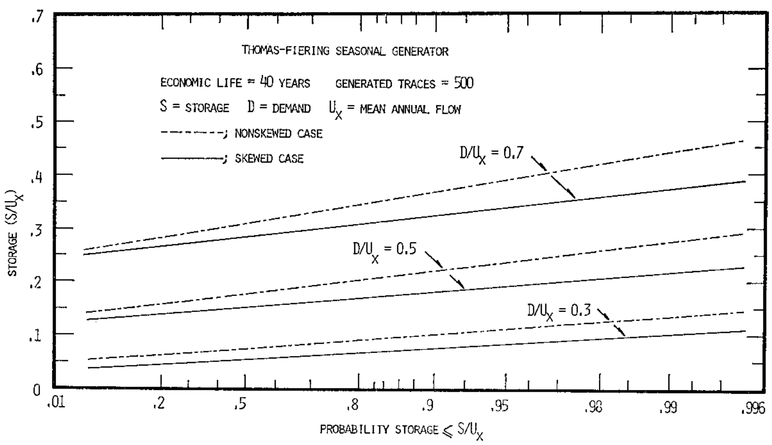

Fig. 4 Storage probability distribution by Thomas-Fiering seasonal model.

be supplied with a much larger reliability (0.97) for the 3PLN generator. Here 0.97 reliability means that of all possible inflow scenarios 97% of the inflow traces would have sufficient flow plus buffer storage to fully satisfy demand. The remaining 3% of all traces would not fully satisfy demand and some supply shortage would occur. Another way of interpreting the curves of Fig. 4 is to consider the development of a water supply whose desired reliability for the proposed operating life is, for example, 98% (this level is often used to represent "firm yield"). To achieve this level, the demand of $D^*=0.7$ would require a storage, $S^*=0.42$ for the normal distribution model while the same demand level would require a storage, $S^*=0.36$ for the 3PLN distribution model, a 17% reduction.

Fig. 5 shows storage probability results when flows were obtained by disaggregating flows generated by AR(1) models. The Extreme Value

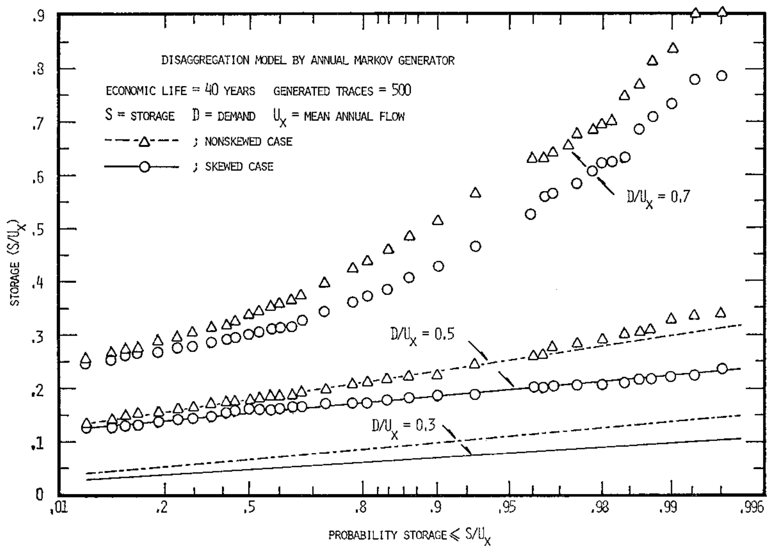

Fig. 5 Storage probability distribution by disaggregation model (annual Markov generator).

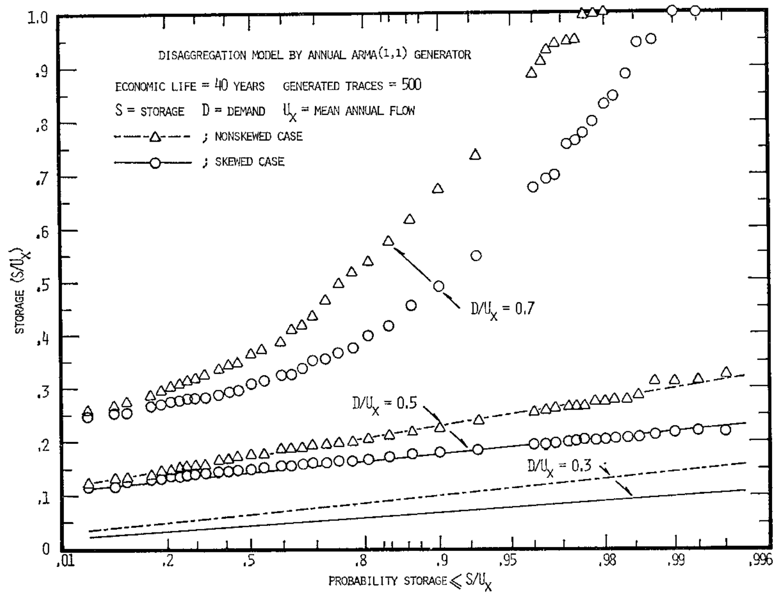

Fig. 6 Storage probability distribution by disaggregation model (annual ARMA generator).

theory adequately described these distributions for demands up to $D^*=0.5$. The empirical cumulative distributions of storage diverge from the theoretical result at a demand of $D^*=0.70$. This is to be expected because at high demand levels both over-year and within-year storages could result from different flow traces; two Extreme Value populations are present. Fig. 6 shows the resulting storage probability storage distribution for the case in which annual ARMA (1,1) sequences were disaggregated to monthly flows. The behavior of storage distributions for the ARMA model bears out a similar conclusion as for the Markov model; the difference between over-year and within-year storage effects particularly for normally distributed flows is more distinct than for the AR(1) generator. Shorter and less severe deficits tend to occur in sequences which are generated with a model which preserves both the first two moments and replicates skewed effects.

The results of the three cases examined for different monthly flow generators show significant differences in the probability distributions of required storage when determined from the normal and 3PLN distribution procedures; the 3PLN distributed monthly sequences yield lesser storage requirements for every case than do normally distributed monthly flows. For a fixed reservoir size, the 3PLN distributed monthly flows yield a much larger reliability level than normally distributed monthly flows, particularly as the demand level is reduced. (At high demand levels $D^*=0.9$, Lettenmaier and Burges[12] have shown for annual flow-demand coupling the impact of skew is not as dramatic as shown here for lesser demand levels.)

Careful examination of Figs. 4~6 shows that the storage distributions obtained are model independent provided $D^*\leq 0.5$. Here, all storage results indicate that only within-year flow buffering is needed. It is clear that it makes little difference what form of model is used at these demand levels. It is important, however, to include skew effects in whatever model is used. Otherwise storage needs will be considerably overstated.

The form of model is most important, however, as the demand level becomes larger. In Figs. 4~6 at $D^*=0.7$ substantially different storage distributions are obtained. (Again, whichever model is used, it is important to include the skew effects of monthly (and annual) flows.) The effects of mixed storage populations, within-year and over-year, are clearly evident in Figs. 5 and 6. For this reason empirically determined storage values are given rather than theoretical descriptions. The theoretical distributions fully described the results obtained for the TF model (Fig. 4); over-year effects were not evident. Differences between Fig. 4 and Figs. 5 and 6 were anticipated because the TF model did not replicate annual variance.

When over-year storage effects are manifest the form of annual flow persistence, given that the form of marginal distribution remains unchanged, governs storage requirements. Therefore, it is understandable that storage results for annual flows generated with the ARMA (1,1) model (Fig. 6) exceed those in Fig. 5 (AR (1)) with $D^*=0.7$. Differences in S^* for three reliability levels and $D^*=0.7$ are given in Table 3 to emphasize the importance of model type and the form of marginal distribution used. In the case investigated here the TF model is clearly inappropriate. When the model was first introduced, it was applied to situations that closely followed the assumed persistence structure and was suitable for its intended use. As shown here, however, unthinking use of the TF model gives rise to serious design error.

(3) Flow Summary Statistics

The three flow generation models were compared on the basis of how well they represented the first three moments for monthly and annual flows and how well they replicated observed correlation structures. The ability to preserve the statistics of interest varies with the assumed

Table 3 Storage needs S^* for $D^*=0.7$ at three reliability levels.

Model / Reliability	Thomas-Fiering Seasonal Model		Disaggregation Model: Annual Markov Generator		Disaggregation Model: Annual ARMA Generator	
	Nonskewed Case	Skewed Case	Nonskewed Case	Skewed Case	Nonskewed Case	Skewed Case
80%	0.35	0.31	0.43	0.37	0.53	0.38
95%	0.40	0.34	0.58	0.50	0.82	0.61
98%	0.42	0.36	0.70	0.62	1.08	0.83

$S^*=S/U_X$ (S; storage, U_X: mean annual flow) $D^*=D/U_X$ (D: annual demand)

Table 4 Mean of standard deviation (mm) for 500 traces, each trace having a length of 40 years (Skewed case)

Month	A*	B**	C+	D++
Dec.	16.0	15.9	16.1	16.1
Jan.	9.6	9.5	9.5	9.6
Feb.	10.2	10.2	10.1	10.3
Mar.	20.1	20.0	19.9	19.9
Apr.	66.2	65.6	66.4	68.9
May	85.7	85.2	85.9	90.2
June	23.2	23.0	23.1	24.1
July	30.6	29.3	29.8	30.1
Aug.	78.9	77.4	78.6	81.8
Sep.	37.0	36.6	36.5	36.5
Oct.	36.1	36.0	36.2	37.6
Nov.	30.0	29.3	30.0	31.6

Table 5 Mean of skewness for 500 traces, each trace having a length of 40 years (Skewed case)

Month	A*	B**	C+	D++
Dec.	0.627	0.753	0.728	0.721
Jan.	0.033	0.019	0.045	0.046
Feb.	−0.067	0.100	0.081	0.137
Mar.	0.585	0.670	0.679	0.677
Apr.	0.178	0.217	0.196	0.212
May	−0.126	0.145	0.168	0.166
June	−0.331	0.085	0.110	0.088
July	1.366	1.688	1.691	1.659
Aug.	0.578	0.648	0.692	0.717
Sep.	0.656	0.757	0.739	0.699
Oct.	0.967	1.171	1.175	1.162
Nov.	0.995	1.161	1.188	1.213

Table 6 Mean of month-to-month correlation for 500 traces, each trace having a length of 40 years (Skewed case)

Month	A*	B**	C+	D++
Dec.#	0.207	0.210	0.244	0.237
Jan.	0.550	0.547	0.497	0.489
Feb.	0.827	0.826	0.820	0.821
Mar.	0.453	0.456	0.444	0.441
Apr.	0.100	0.093	0.087	0.104
May	0.461	0.458	0.465	0.493
June	0.646	0.648	0.646	0.673
July	0.200	0.194	0.170	0.188
Aug.	0.269	0.265	0.056	0.054
Sep.	0.183	0.163	0.220	0.247
Oct.	0.492	0.491	0.459	0.462
Nov.	0.593	0.599	0.622	0.638

* Observed data
** Thomas-Fiering seasonal model
\+ Disaggregation model by annual Markov generator
++ Disaggregation model by annual ARMA generator
\# November-December correlation

generating mechanisms. The TF generator preserves only the first three moments and month-to-month correlations when using 3PLN distributions. In addition to preserving the first three moments, the disaggregation scheme used has the ability to preserve within-year and over-year correlations between monthly flows and the correlations between annual and monthly flows. For comparison purposes, Table 4, 5, and 6 are included to show how well each model preserved standard deviations, skew coefficients and month-to-month correlation coefficients. Summary statistics for seasons modeled under assumed normal distributions are not given. Each summary statistic given in these tables was computed as follows: For each trace of length 40 years, the mean, standard deviation, skew coefficient and month-to-month correlation coefficient were computed. This procedure was repeated for all 500 generated traces and the average of each summary statistic was computed and recorded in Tables 4~6.

Table 4 compares observed and average synthetic monthly standard deviations. The ARMA (1,1) model yielded slightly large standard deviations in May and August. Table 5 shows observed and generated average monthly skew coefficients (flows in Feburary, May, and June had been approximated as normal distributions). There are no glaring differences in this table. Table 6 shows month-to-month correlation coefficients. Major interest is associated with the November-December period used in the disaggregation method. The July–August correlation was not well modeled by the disaggregation scheme. This resulted from approximating a force fitted normal distribution in August with a 3PLN distribution. Tradeoffs between the importance of modeling marginal distributions and exactly preserving the correlation structure had to be made. From a practical standpoint the small discrepancy in the modeled July–August correlation coefficient was unimportant relative to the impact on storage requirements including skewed marginal distributions.

Tests were also made to investigate the extent to which the 3PLN distribution would distort a linear relationship between synthetically generated annual and monthly flows. It is usually theoretically awkward to derive a functional relationship between annual and monthly values when the 3PLN distribution is applied in the disaggregation model because the sum of log-normal variates is not necessarily a log-normal distribution. It is, however, possible to examine the degree of distortion numerically. Therefore, synthetically generated monthly flows were summed to yield annual flows from which essential

Table 7 Summary statistics (mean) of annual flows and percentage of negative monthly flows for 500 traces, each trace having a length of 40 years.

Statistics			Mean (mm)	St. De. (mm)	Skew	Lag-one Correlation	Negative Monthly Flows (%)
Observed Data			1 331.2	258.9	0.644	0.426	0
Thomas-Fiering Model		Nonskewed Case	1 332.3 (33.4)	206.1 (22.9)	0.010 (0.339)	−0.002 (0.166)	0.513
		Skewed Case	1 332.0 (33.1)	204.1 (24.9)	0.199 (0.380)	0.009 (0.173)	0.117
Disaggregation Model	Annual Markov Generator	Nonskewed Case	1 337.3 (81.8)	259.9 (39.0)	0.035 (0.346)	0.494 (0.149)	0.580
		Skewed Case	1 332.9 (80.8)	250.7 (40.5)	0.296 (0.401)	0.485 (0.149)	0.138
	Annual ARMA Generator	Nonskewed Case	1 342.1 (162.6)	275.7 (51.7)	0.049 (0.351)	0.407 (0.217)	0.793
		Skewed Case	1 342.5 (153.2)	269.3 (57.8)	0.335 (0.385)	0.399 (0.205)	0.249

Numbers in parentheses are standard errors.

statistics of interest were calculated. Table 7 shows annual flow summary statistics. Of all generated monthly flows ($=12\times40\times500$), the proportion of generated negative flows was negligibly small and these values were truncated to zero without appreciably affecting the statistical results. The percentages of monthly flows truncated to zero are shown in the last column of Table 7. The standard deviation and lag-one correlation yielded by the Thomas-Fiering model are almost identical with the theoretical values given earlier (207.2 mm and 0.011 respectively).

The annual standard deviations and lag-one correlations computed from annual AR(1) and ARMA(1,1) generated flows were corrected for small sample size as described earlier. The unbiased standard deviation of an ARMA model (314.9 mm) is much larger than the one of an AR model (275.4 mm). As was expected, the results yielded by the ARMA model are more variable than the AR model. It is clear from Table 7 that use of the bias correction factors assuming AR and ARMA persistence structure to obtain estimated population standard deviations was essential to preserve the annual variance of synthetically generated AR and ARMA sequences. Failure to do this would have resulted in underestimating storage requirements particularly for $D^*=0.7$.

The average Hurst coefficient and its standard error for the ARMA model were 0.811 and 0.079, respectively, using a normal distribution, while these values were 0.805 and 0.078, using a 3PLN distribution. The ARMA model is clearly capable of preserving the observed Hurst coefficient.

Annual skew was not well preserved by any of the models as indicated in Table 7. The skews resulting from the disaggregation models are, however, acceptably close to the observed sample skew coefficient which itself has an approximate standard error of 0.49 (based on $N=22$). Within the total model context, failure to perfectly replicate the annual skew coefficient is of small consequence when the first two moments and all correlations are well preserved.

6. CONCLUSIONS

Development and use of a seasonal stochastic streamflow generation model involves many steps. A most important issue is the choice of model to be used. This choice is heavily influenced by the availability of a sufficiently long record of seasonal flow data to determine the form of annual persistence structure and properties of the seasonal and annual flow marginal distributions. The importance of correcting annual variance for small sample bias was clearly demonstrated. The magnitude of bias correction used depends on the identified (or assumed) form of flow persistence. Failure to convert the annual standard deviation (258.9 mm) to the appropriate magnitude for generation (275.4 mm) would have produced AR sequences with annual variance of about 88% of the observed variance. Similarly ARMA sequences would have had annual variance of about 62% of the observed variance.

The storage probability results obtained indicated that it made little difference which model was used, given that the same marginal distributions were used, when determining storage needs provided that the demand level was sufficiently small that within-year flow buffering dominated, for $D^*=0.5$. When demand was increased to $D^*=0.7$, however, the form of the model became critical to reservoir size reliability relationships. The Thomas-Fiering (TF) model was completely unsuitable for the mixed over-year, and within-year storage situations corresponding to $D^*=0.7$. Use of this model will underdesign a storage facility when over-year storage effects are important.

In all cases examined use of skewed marginal distributions for the monthly and annual flows gave rise, for a given flow persistance structure, to smaller required storage than indicated when the flows were improperly modeled by assumed normal distributions.

The disaggregation scheme has an appealing property in that any desired form of annual flow sequences can be readily disaggregated to seasonal flows. In the application used here it was not possible to identify precisely the form of annual flow persistence. Therefore the AR and ARMA models should bracket actual storage needs; the Hurst coefficient was computed from a small sample ($N=22$) so a relatively precise index of lumped low frequency effects was unobtainable.

The authors recommend the use of graphical data display techniques when examining flow time series data. Quantile-quantile plots are extremely useful for determining how well theoretical distributions conform with empirical data. Skewed marginal distributions can be conveniently approximated with 3PLN distributions.

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