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NONLINEAR GOAL PROGRAMMING APPROACH TO INTER-BASIN, MULTI-MODAL WATER ASSIGNMENT PROBLEM

By Kazuhiro YOSHIKAWA* *and Norio* OKADA**

1. INTRODUCTION

Practitioners and researchers in the art of water management have increasingly come to the recognition that allocation and control of this resource in the densely populated areas where the demand for water is growing rapidly, requires considerably greater skills than have been utilized in the past if effective management is to be attained. Furthermore with the increasing awareness of the importance of environmental quality, planning and construction agencies are asked to broaden environmental considerations. Thus, the principal goals for the low-flow management are now identified in general terms as (i) to secure as much water as required by the water users, (ii) to attain economic efficiency as much as possible, and (iii) to alleviate water quality deterioration as much as possible.

With these goals identified as such, much work has been done to present effective strategies for the low-flow management. Many researchers have already noted that in the near future a system including the freshwater development by impounding water in dams and the wastewater reclamation by recycling the wastewater through the tertiary treatment process, would be preferred over a system depending exclusively on the freshwater development or the wastewater reclamation. Among this line of attack are the studies by N. Okada and K. Yoshikawa[9),12)], where the primary concern was with the efficient blending of the two different systems, freshwater development and wastewater reclamation.

It is worth noting that in modeling the problem the cost is set as the "primary" (explicit) goals, being treated as the objective function, where as the other goals as identified above are considered "incidental" (implicit) goals, being checked with the incorporated constraints.

Another line of attack, where the goals are equally weighted, consists in treating all of them in explicit terms by framing the problem in a class of multi-objective programming. This kind of problem seems to be frequently encountered in the practical field of water resource management where the practical interest of the planner is not to find the optimal policy in terms of a single objective, but to choose such an alternative for low-flow management and presents a goal programming approach for modeling and analyzing this problem. Thereby the following are given further consideration to it.

(1) The freshwater development system under study means the "inter-basin" system whose sources of water are the impounded water in dams to be constructed. The term "inter-basin" is used to refer to the different types of transporting water from one basin to another, i.e., ① inter-basin streamflow diversion and ② inter-zonal purified water distribution; the difference being that the former refers to the conveyance of "streamflow" prior to the treatment, and latter to that of "purified" water posterior to the treatment.

(2) The inter-basin, multi-modal system is a system consisting of two subsystems, i.e., ① the freshwater development system and ② the wastewater reclamation system.

(3) It is often the case that the above-defined goals are competing each other—a matter called "trade-offs" between goals. Thereby the efficient solution is among the Pareto optima and any additional procedure is required to single out the solution. The explicit formulation of the problem is accessible by use of goal programming, which will be discussed in the preceding section.

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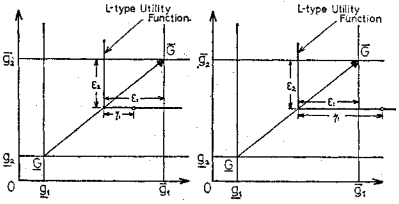
* Dr. Eng., Professor, Department of Civil Engrg., Faculty of Engrg., Kyoto University.
** Dr. Eng., Assistant Professor, Department of Civil Engrg., Faculty of Engrg., Tottori University.

2. GOAL PROGRAMMING

(1) Preliminary Discussion

Goal Programming is a special extension of linear programming. Though the interest for it is rather new, the literature[1),2),3),4)] is already copious. Recently, T. Fushimi and T. Yamaguchi[5)] demonstrated that the introduction of the *L*-type utility function into the formulation of the model as a mathematical representation of the trade-off relations between the concerned goals, leads to the problem of finding such a solution which can be thought of as well-balanced attainments of those goals (see Fig. 1). We shall make use of this approach developed by T. Fushimi and T. Yamaguchi, primarily because our main concern is to find the alternative which assures well-balanced attainments of our goals.



($\varepsilon_1 > \eta_1$) ($\varepsilon_1 < \eta_1$)

Fig. 1 *L*-type Utility Function.

In this regard it seems to be of vital importance that their approach bases its frame of modeling on linear programming and in this context it is not applicable to those cases where the goals to be set takes a mold of nonlinearity, as is exactly the case with our model. (This point will be explicitly described later.)

In light of these considerations our secondary concern is placed on exploring Fushimi-Yamaguchi's approach and developing a new type of goal programming which is applicable to the problem with nonlinear goal constraints. As will be seen, the authors present a modified goal programming approach based on the cutting plane technique developed by Kelley[6)] and will demonstrate its applicability to the nonlinear-type problems.

(2) Permitted-level and Satisfied-level of a Goal

To bigin with, let us introduce the notions of "the permitted-level" and "the satisfied-level" with reference to a given goal. The term "the permitted level" is defined as the level such that the planner is determined to accept any alternative that assures the attainment of the concerned goals to the extent equal to, or larger than it, but otherwise he would never accept that alternative. The term "the satisfied level" is used to mean that after taking account of various conditions, such as the case where one specified goal would be attempted to reach its full attainment by confining the remaining goals to be achieved at their permitted-levels, the planner becomes willing to accept that level of alternative as a satisfactory one, if not optimal.

(3) Specification of the Method

The special form of goal programming based on the incorporated *L*-type utility function between the attainment of different goals comprises the following three parts, i.e., ① physical and technical constraints, ② goal constraints and ③ objective function. Let the problem be formulated as follows.

(1) physical and technical constraints

$$\sum_{j=1}^{n} a_{ij}x_j \leqq b_i \quad (i=1, \cdots, m) \quad \cdots\cdots\cdots\cdots\cdots\cdots (2\text{-}1)$$

$$(x_j \geqq 0)$$

(2) goal constraints

$$\left\{\begin{array}{ll} \sum_{j=1}^{n} h_{ij}x_j + \varepsilon_i - \eta_i = \bar{g}_i & \cdots\cdots\cdots\cdots\cdots\cdots (2\text{-}2) \\ \sum_{j=1}^{n} h_{ij}x_j \leqq \underline{g}_i & \cdots\cdots\cdots\cdots\cdots\cdots (2\text{-}3) \end{array}\right.$$

for i's such that $\bar{g}_i > \underline{g}_i$

$$\left\{\begin{array}{ll} \sum_{j=1}^{n} h_{ij}x_j - \varepsilon_i + \eta_i = \bar{g}_i & \cdots\cdots\cdots\cdots\cdots\cdots (2\text{-}4) \\ \sum_{j=1}^{n} h_{ij}x_j \geqq \underline{g}_i & \cdots\cdots\cdots\cdots\cdots\cdots (2\text{-}5) \end{array}\right.$$

for i's such that $\bar{g}_i < \underline{g}_i$,

where $\bar{g}_i$ and $\underline{g}_i$ are the satisfied- and permitted-levels of a goal i, respectively, and ε_i and η_i, the deviational variables as illustrated in Fig. 1. As is clear from this figure, the following relations are also required to hold among ε_i's.

$$\varepsilon_1/\lambda_1 = \varepsilon_2/\lambda_2 = \cdots = \varepsilon_m/\lambda_m , \quad \cdots\cdots\cdots\cdots\cdots\cdots (2\text{-}6)$$

where $\lambda_i = |\bar{g}_i - \underline{g}_i|$ $(i=1, \cdots, m)$.

(3) objective function

Since all the constraints are formulated as such, the problem is stated as the minimization of the unattained degree of each goal that is representable by any one of the deviational variables, ε_i's $(i=1, \cdots, m)$. Therefore the objective function is expressed:

Minimize ε_{i^*} (i^* being any one of the ε_i's).
$\cdots\cdots\cdots\cdots\cdots\cdots$(2-7)

3. MODELING THE PROBLEM

(1) Identification of the Problem

(i) The area-wide, multiple river basins are considered where we assume that by taking account of local differences in water usages, hydrology, legislative and economic boundaries, etc., the area of each river basin is a priori divided into a couple of subareas which we call "downstream (demand) zone", "midstream zone", "upstream zone" and the like.

(ii) In the valleys up the rivers and above the upstream zones are constructed dams to develop freshwater.

(iii) Each zone on a given river is considered a geographical unit for water utilization in which a dual-modal water utilization system—a combined system for utilizing both fresh water and renovated water—is assumed to be implemented.

(iv) There are two types of water users in each zone, i.e., industrial water and domestic water users.

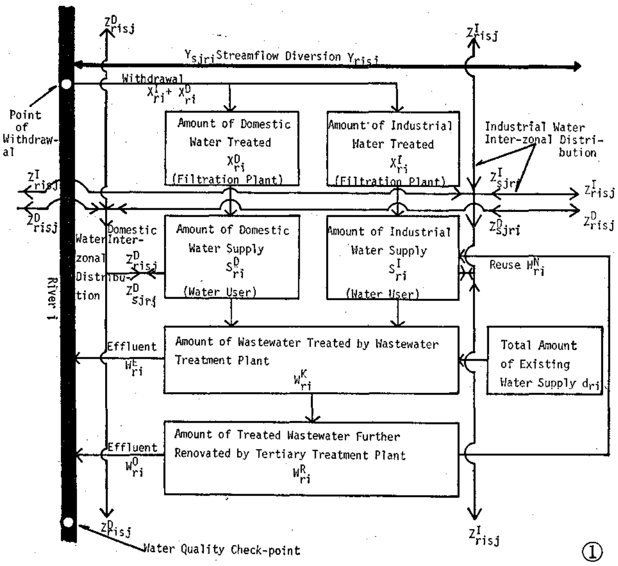


Fig. 2 Diagram for Modeling Water Use System of Each Zone.

(v) As shown in Fig. 2, in each zone water is collected from the nearest stream running through the area and then undergoes purification at two different utilization plants (one for industrial water supply and the other for domestic water supply), and is provided for both kinds of water users. The used waters with waste loads are carried by the sewers to a wastewater treatment plant and then undergo primary and secondary treatments. The effluents from the plant are partially returned back through an outlet to the same stream (receiving water body) at a point further down from the point of with drawal, the remainder of which being treated by the tertiary treatment plant and then returned back to the receiving water body at the point near the wastewater-discharge point, and partially provided again for industrial water use.

(vi) Between a given water body and its adjacent ones is constructed a diversion channel through which river flows are diverted from one stream to another, if necessary.

(vii) It is also admitted that between any two of all the zones, whether located on the same stream or different streams, can be constructed a distribution aqueduct through which some portions of purified industrial or domestic water are conveyed from one zone to the other.

(viii) At a given point on the stream in each zone which is assumed to be located farthest downstream in each zone, water quality of the stream is required to meet the set standard. We stand on the premise that water quality is checked with a single criterion, i.e., BOD ppm.

(ix) The water utilization system of each zone is assumed to consist of those facilities for withdrawal (1), filtration (2), wastewater treatment plant (1), tertiary treatment plant (1), discharge (2) and distribution of renovated waters (1). The numbers in the parentheses represent the number of facilities in question. In addition, there is another type of facilities to be considered, that is, the dams to be constructed on the farthest upstream of each river.

(x) The scales of the concerned facilities are to be determined so as to meet those incremental demands which are predicted a priori over a given time horizon.

(xi) The construction costs are rendered to the amortization cost per annum.

(xii) The set goals are the following three:

① minimizing total costs associated with the construction and operation of the concerned facilities (cost-goal),

② maximizing attainment of water supply to meet the projected water demands of each zone (supply-goal),

③ maximizing attainment of water quality conservation at the check-point on the stream in each zone (quality-goal).

(2) Model Formulation

a. Notation

(1) Variables

U : amount of water developed by dam
X : amount of water treated in the filtration plant
S : amount of suppliable water

Z : amount of inter-zonal purified water distribution

Y : amount of inter-basin streamflow diversion

W : amount of wastewater

Q : streamflow discharge

B : streamflow quality (measured in terms of BOD)

V : total cost

ξ, η : deviational variables

(2) Constants

c : upper bound on the capacity of dam

e : minimun streamflow requirement to be reserved to maintain its natural functions

f : amount of streamflow already being collected by existing water users

q : inflow from the tributary streams

d : total amounts of existing water supply for industrial and domestic uses

h : average water quality (measured in terms of BOD)

$\bar{g}$: satisfied-level of a given goal

$\underline{g}$: permitted-level of a given goal

$\alpha(U)$: cost function with respect to the implementation scale of a dam

$\beta(Y)$: cost function with respect to the implementation scale of a streamflow diversion channel

$\gamma(Z)$: cost function with respect to the implementation scale of an inter-zonal distribution aqueduct

$\delta(X)$: cost function with respect to the implementation scale of a filtration plant or its related facilities

$\mu(X)$: cost function with respect to the implementation scale of a (secondary or tertiary) wastewater treatment plant or its related facilities

(3) Indices

The following subscripts are used to specify the types of variables.

t : dam t (t being numbered from 1 to t_r, corresponding to the farthest downstream and the farthest upstream dams, respectively.)

r : river r ($r=1, \cdots, r_0$)

s : those rivers adjacent to river r

$i(j)$: zone $i(j)$ ($i=1, \cdots, m_s$)

The following superscripts are used to specify the types of variables.

I : industrial use

D : domestic use

K : treated in the secondary treatment plant

E : its effluent into the receiving water body

R : treated in the tertiary treatment plant

O : its effluent into the receiving water body

N : its return flow for industrial use (reclaimed water)

v : cost-goal

b : quality-goal

u : farthest upstream of the stretch of the river flowing through a given zone

l : farthest downstream of the same

p : tributary stream

Additionally, the following notations are used.

Γ_r : the set of those rivers, s, adjacent to river r

$i^*(j^*)$: the zone(s) nearest above zone $i(j)$

b. Physical and Technical Constraints

Otherwise stated, the following conditions hold for r, $s=1, \cdots, m_r$; $j=1, \cdots, m_s$.

In addition to the nonnegativity conditions involved in all variables, we set the following physical and technical constraints.

$$U_{rt} \leqq C_{rt} \quad (t=1, \cdots, t_r) \quad \cdots\cdots(3\text{-}1)$$

$$X_{ri}^{I}+X_{ri}^{D} \leqq Q_{ri}^{u}-(e_{ri}+f_{ri})+\sum_{s \in \Gamma_r}(Z_{sj,ri}-Z_{ri,sj}) \quad \cdots\cdots(3\text{-}2)$$

$$S_{ri}^{I} \leqq X_{ri}^{I}+\sum_{s \in \Gamma_r}(Y_{sj,ri}^{I}-Y_{ri,sj}^{I})+W_{ri}^{N} \quad \cdots\cdots(3\text{-}3)$$

$$S_{ri}^{D} \leqq X_{ri}^{D}+\sum_{s \in \Gamma_r}(Y_{sj,ri}^{D}-Y_{ri,sj}^{D}) \quad \cdots\cdots(3\text{-}4)$$

$$W_{ri}^{K} \leqq S_{ri}^{I}+S_{ri}^{D}+d_{ri} \quad \cdots\cdots(3\text{-}5)$$

$$W_{ri}^{R} \leqq W_{ri}^{K}-W_{ri}^{E} \quad \cdots\cdots(3\text{-}6)$$

$$W_{ri}^{N} \leqq W_{ri}^{R}-W_{ri}^{O} \quad \cdots\cdots(3\text{-}7)$$

$$Q_{ri}^{u}=Q_{ri^*}^{u}-(X_{ri^*}^{I}+X_{ri^*}^{D}+f_{ri^*})+W_{ri^*}^{E}+W_{ri^*}^{O}+Z_{sj,ri}-Z_{ri,sj} \quad (\text{for } i=1, \cdots, m_r-1) \quad \cdots\cdots(3\text{-}8)$$

$$Q_{ri}^{u}=\sum_{t=1}^{t_i} U_{rt} \quad (\text{for } i=m_r \text{ corresponding to the farthest upstream zone}) \quad \cdots\cdots(3\text{-}9)$$

$$Q_{ri}^{u}=Q_{ri^*}^{l} \quad (\text{for } i=1, \cdots, m_r-1) \quad \cdots\cdots(3\text{-}10)$$

Equation (3-1) states that each dam can be scaled at largest to its full technical limit. Equation (3-2) is the statement of the quantity-balance relation to be held between the streamflow discharges (the right-hand terms) and the amount of water withdrawn for water supply (the left-hand terms).

In Equation (3-3) it is prescribed that an available amount of water supply for industrial use is identical to the amount of water treated in the filtration plant plus the amount of purified water distributed from the adjacent zones to the zone of question minus the amount of purified water distributed from there to the adjacent zones plus the amount of reused wastewater for industrial use.

Equation (3-4) is the same kind of expression with particular reference to the amount of water supply for domestic use; the mere difference between Equations (3-3) and (3-4) lying in that the latter drops the last term concerning the amount of reused wastewater by allowing for the assumption that the reclaimed wastewater is used exclusively for industrial use.

Equations (3-5) to (3-7) are the formulated conditions to be held between the amount of wastewater treated in the secondary treatment plant, the amount of its effluent, the amount of wastewater treated in the tertiary treatment plant, the amount of its effluent, and the amount of reclaimed wastewater reused for industrial use. Equations (3-8) to (3-10) are the formulated conditions to be held between streamflow discharges at different points of a river.

c. Goal Constraints

The cost-goal is formulated as:

$$V-\varepsilon^{v}+\eta^{v}=\bar{g}^{v} \tag{3-11}$$

$$V\leqq \underline{g}^{v} \tag{3-12}$$

$$\begin{aligned}V= &\sum_{r}\sum_{l}\alpha_{rl}(U_{rl})+\sum_{r}\sum_{i}\sum_{s\in\Gamma_r}\{\beta_{ri,sj}(Z_{ri,sj})\\ &+\beta_{sj,ri}(Z_{sj,ri})+\gamma^{I}_{ri,sj}(Y^{I}_{ri,sj})+\gamma^{I}_{sj,ri}(Y^{I}_{sj,ri})\\ &+\gamma^{D}_{ri,sj}(Y^{D}_{ri,sj})+\gamma^{D}_{sj,ri}(Y^{D}_{sj,ri})\}\\ &+\sum_{r}\sum_{i}\{\delta^{I}_{ri}(X^{I}_{ri})+\delta^{D}_{ri}(X^{D}_{ri})+\mu^{K}_{ri}(W^{K}_{ri})\\ &+\mu^{E}_{ri}(W^{E}_{ri})+\mu^{R}_{ri}(W^{R}_{ri})+\mu^{O}_{ri}(W^{O}_{ri})+\mu^{N}_{ri}(W^{N}_{ri})\}\end{aligned} \tag{3-13}$$

The supply-goal for each zone is written as follows.

$$S^{I}_{ri}+\varepsilon^{I}_{ri}-\eta^{I}_{ri}=\bar{g}^{I}_{ri} \tag{3-14}$$

$$S^{I}_{ri}\geqq \underline{g}^{I}_{ri} \tag{3-15}$$

$$S^{D}_{ri}+\varepsilon^{D}_{ri}-\eta^{D}_{ri}=\bar{g}^{D}_{ri} \tag{3-16}$$

$$S^{D}_{ri}\geqq \underline{g}^{D}_{ri} \tag{3-17}$$

The quality-goal for each zone is expressed as

$$B^{l}_{rl}-\varepsilon^{b}_{ri}+\eta^{b}_{ri}=\bar{g}^{b}_{ri} \tag{3-18}$$

$$B^{l}_{ri}\leqq \underline{g}^{b}_{ri} \tag{3-19}$$

$$\begin{aligned}B^{l}_{ri}=\frac{1}{Q^{l}_{ri}}\Big\{&B^{u}_{ri*}(Q^{u}_{ri*}-X^{I}_{ri}-X^{D}_{ri}-\sum_{s\in\Gamma_r}Y_{ri,sj}-f_{ri})\\ &+h^{E}_{ri}W^{E}_{ri}+h^{O}_{ri}W^{O}_{ri}+h^{p}_{ri}q_{ri}+B_{sj,ri}Y_{sj,ri}\Big\}\end{aligned} \tag{3-20}$$

$$B^{u}_{ri}=B^{l}_{ri*} \tag{3-21}$$

$$B_{sj,ri}=B^{l}_{sj*} \tag{3-22}$$

where h^{p}_{ri} represents the quality of the tributary streams joining with the main stream r at the point further upstream from the collection point for zone i.

Those terms contained in the bracket on the right hand of Equation (3-20) are the aggregate waste loads running down to the farthest downstream point of that stretch of the river running through the zone. Therefore Equation (3-20) means that the streamflow quality measured in BOD terms at that point is identical to those aggregate waste loads divided by the corresponding streamflow discharge.

Equation (3-21) states that the measurement of the streamflow quality at the farthest upstream point of a given zone is assumed to be equal to that at the farthest downstream point of the zone nearest above it.

Additionally Equation (3-22) prescribes that the measurement of the streamflow quality for the diversion channel leading from the adjacent river to the river in question is assumed to be identical to the quality measured at the diversion point of the adjacent river.

In addition to the above constraints there hold a relation between the deviational variables ε^{v}, ε^{I}_{ri}, ε^{D}_{ri} and ε^{b}_{ri} $(i=1,\cdots,m_r)$. That is,

$$\varepsilon^{v}/\lambda^{v}=\varepsilon^{I}_{ri}/\lambda^{I}_{ri}=\varepsilon^{D}_{ri}/\lambda^{D}_{ri}=\varepsilon^{b}_{ri}/\lambda^{b}_{ri} \tag{3-23}$$

where $\lambda^{v}=\underline{g}^{v}-\bar{g}^{v}$, $\lambda^{I}_{ri}=\bar{g}^{I}_{ri}-\underline{g}^{I}_{ri}$, $\lambda^{D}_{ri}=\bar{g}^{D}_{ri}-\underline{g}^{D}_{ri}$, $\lambda^{b}_{ri}=\underline{g}^{b}_{ri}-\bar{g}^{b}_{ri}$

d. Objective Function

The objective function is formulated as

$$\text{Minimize } \varepsilon^{v}, \tag{3-24}$$

where ε^{v} is substitutable by any other deviational variables, ε^{I}_{ri}, ε^{D}_{ri} and ε^{b}_{ri} $(i=1,\cdots,m_r)$.

e. Formulated Model

At this point let us sum up our formulated model.

$$\text{Minimize } \varepsilon^{v} \tag{3-24}$$

subject to

$$U_{rt}\leqq C_{rt}\quad(t=1,\cdots,t_r) \tag{3-1}$$

$$X^{I}_{ri}+X^{D}_{ri}\leqq Q^{u}_{ri}-(e_{ri}+f_{ri})+\sum_{s\in\Gamma_r}(Z_{sj,ri}-Z_{ri,sj}) \tag{3-2}$$

$$S^{I}_{ri}\leqq X^{I}_{ri}+\sum_{s\in\Gamma_r}(Y^{I}_{sj,ri}-Y^{I}_{ri,sj})+W^{N}_{ri} \tag{3-3}$$

$$S^{D}_{ri}\leqq X^{D}_{ri}+\sum_{s\in\Gamma_r}(Y^{D}_{sj,ri}-Y^{D}_{ri,sj}) \tag{3-4}$$

$$W^{K}_{ri}\leqq S^{I}_{ri}+S^{D}_{ri}+d_{ri} \tag{3-5}$$

$$W^{R}_{ri}\leqq W^{K}_{ri}-W^{E}_{ri} \tag{3-6}$$

$$W^{N}_{ri}\leqq W^{R}_{ri}-W^{O}_{ri} \tag{3-7}$$

$$\begin{aligned}Q^{u}_{ri}=Q^{u}_{ri*}-(X^{I}_{ri*}+X^{D}_{ri*}+f_{ri*})+W^{E}_{ri*}+W^{O}_{ri*}\\ +Z_{sj,ri}-Z_{ri,sj}\\ (\text{for } i=1,\cdots,m_r-1)\end{aligned} \tag{3-8}$$

$$Q^{u}_{ri}=\sum_{t=1}^{t_r}U_{rt}\quad(\text{for } i=m_r) \tag{3-9}$$

$$Q^{u}_{ri}=Q^{l}_{ri*}\qquad(\text{for } i=1,\cdots,m_r-1) \tag{3-10}$$

$$V-\varepsilon^{v}+\eta^{v}=\bar{g}^{v} \tag{3-11}$$

$$V \leqq \underline{g}^{v} \quad \cdots\cdots (3\text{-}12)$$

$$\begin{aligned} V = & \sum_{r} \sum_{l} \alpha_{rl}(U_{rl}) + \sum_{r} \sum_{i} \sum_{s \in \Gamma_r} \{\beta_{ri,sj}(Z_{ri,sj}) \\ & + \beta_{sj,ri}(Z_{sj,ri}) + \gamma^{I}_{ri,sj}(Y^{I}_{ri,sj}) + \gamma^{I}_{sj,ri}(Y^{I}_{sj,ri}) \\ & + \gamma^{D}_{ri,sj}(Y^{D}_{ri,sj}) + \gamma^{D}_{sj,ri}(Y^{D}_{sj,ri})\} \\ & + \sum_{r} \sum_{i} \{\delta^{I}_{ri}(X^{I}_{ri}) + \delta^{D}_{ri}(X^{D}_{ri}) + \mu^{K}_{ri}(W^{K}_{ri}) \\ & + \mu^{E}_{ri}(W^{E}_{ri}) + \mu^{R}_{ri}(W^{R}_{ri}) + \mu^{O}_{ri}(W^{O}_{ri}) + \mu^{N}_{ri}(W^{N}_{ri})\} \end{aligned} \quad \cdots\cdots (3\text{-}13)$$

$$S^{I}_{ri} + \varepsilon^{I}_{ri} - \eta^{I}_{ri} = \bar{g}^{I}_{ri} \quad \cdots\cdots (3\text{-}14)$$

$$S^{I}_{ri} \geqq \underline{g}^{I}_{ri} \quad \cdots\cdots (3\text{-}15)$$

$$S^{D}_{ri} + \varepsilon^{D}_{ri} - \eta^{D}_{ri} = \bar{g}^{D}_{ri} \quad \cdots\cdots (3\text{-}16)$$

$$S^{D}_{ri} \geqq \underline{g}^{D}_{ri} \quad \cdots\cdots (3\text{-}17)$$

$$B^{l}_{ri} - \varepsilon^{b}_{ri} + \eta^{b}_{ri} = \bar{g}^{b}_{ri} \quad \cdots\cdots (3\text{-}18)$$

$$B^{l}_{ri} \leqq \underline{g}^{b}_{ri} \quad \cdots\cdots (3\text{-}19)$$

$$\begin{aligned} B^{l}_{ri} = \frac{1}{Q^{l}_{ri}} \Big\{ & B^{u}_{ri*}(Q^{u}_{ri*} - X^{I}_{ri} - X^{D}_{ri} - \sum_{s \in \Gamma_r} Y_{ri,sj} - f_{ri}) \\ & + h^{E}_{ri} W^{E}_{ri} + h^{O}_{ri} W^{O}_{ri} + h^{p}_{ri} q_{ri} + B_{sj,ri} Y_{sj,ri} \Big\} \end{aligned} \quad \cdots\cdots (3\text{-}20)$$

$$B^{u}_{ri} = B^{l}_{ri*} \quad \cdots\cdots (3\text{-}21)$$

$$B_{sj,ri} = B^{l}_{sj*} \quad \cdots\cdots (3\text{-}22)$$

$$\varepsilon^{v}/\lambda^{v} = \varepsilon^{I}_{ri}/\lambda^{I}_{ri} = \varepsilon^{D}_{ri}/\lambda^{D}_{ri} = \varepsilon^{b}_{ri}/\lambda^{b}_{ri} \quad \cdots\cdots (3\text{-}23)$$

(3) Nonlinear Goal Programming Based on Cutting Plane Method

Before delving into the details of our solution algorithm, the following two points need to be given prior consideration.

(i) The original class of algorithms developed for the solution of the goal programming model based on L-type utility function resorts to the direct utilization of the simplex method or its variants which have been developed to treat a class of linear programming models.

(ii) Our model as formulated above involves some nonlinear goal constraints with respect to the cost-goal and the quality-goals.

(iii) Although a number of solution techniques developed for the nonlinear programming problems of different special structures seem to be on our hands, we shall stick to the use of the cutting plane method due to Kelley[6] and present its extension to our goal programming problem for the reasons that follow: ① It is necessary to retain the structural property of the original type of linear goal progrmming. ② It is often the case that our common way of evaluation for a highly complicated problem is likely to be performed at the level of limited locality and not at the level of totality. And if it is found to be inefficient in terms of a given criterion, then the process is repeated iteratively until the efficient condition holds. This seems to have the strong analogy with the solution algorithm which gives a first linear approximation to the nonlinear goal constraints by making use of the local solution in hand and which solves the linear goal programming as derived from the original nonlinear goal programming, thus modifying the problem based on the updated solution which has been obtained at the preceding step and finding the modified solution, until a precribed convergence condition holds. ③ This implies the validity for the explicit use of the cutting plane method which consists of the repeated process of approximating the nonlinear constraints by the linear constraints derived from the supporting hyperplanes obtained from the base point that is the solution to the approximated linear programming which has immediately preceded.

The above discussion has naturally lead to the idea that our nonlinear goal programming would be approchable by making use of the cutting plane method. Putting this in mind, we have developed our solution algorithm that can roughly be stated as follows.

(1) Find an initial base point which is not necessarily required to be a feasible point.

(2) Apply the cutting plane method to our problem and find a solution to it.

(3) Set this solution as the secondary base point and apply the cutting plane method to our modified problem and renovate the solution. (A new solution is obtained.)

(4) Compare the old and new solutions and check whether they are considered identical to each other (in approximate terms).

(5) If this is the case, step up to (6). Otherwise, set the new solution as a base point and the old solution is replaced by this new one. Return to step (3) and repeat the steps from (3) to (5), until the above condition holds. If it holds, the solution is considered a candidate for the required efficient solution.

(6) Return to step (1) by renovating the initial base point and matters proceed between (1) to (5) to obtain another candidate for the efficient solution. Repeat this process in a number of times until it is seen that the fundamental set of different candidates have already been covered.

(7) Compare these candidates and single out the alternative yielding the highest attainment of the goals. This alternative is regarded as the efficient solution to our goal programming problem in practical terms.

The creative idea imbedded into this algorithm is found in ① the induction of the cutting plane method into the core of the algorithm for one part, and ② its extension to a general type of nonlinear programming model where all the constraints are not necessarily convex by repeated uses of this method for another part.

A more clear-cut illustration of this algorithm is given by the flowchart in Fig. 3. For the details of the cutting plane method, see reference 6).

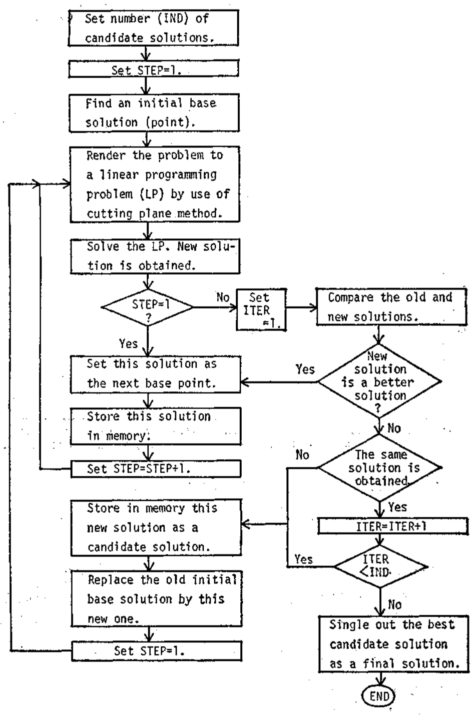


Fig. 3 Flowchart of Algorithm.

5. CASE STUDY ON SOUTHERN PART OF HYOGO PREFECTURE

In likewise as our earlier reports,[8),9),10),11),12),13)] the southern part of Hyogo Prefecture was selected as a case-study area, where the five major streams (the Chigusa, Ibogawa, Yumesaki, Ichikawa and Kakogawa Rivers) run in parallel with one another from the northern hilly countries down through the southern flat countries into the Seto Inland Sea.

(1) Model Data

a. Zoning

Taking account of the difference in water use and hydrological conditions as well as institutional and economical boundaries, the zoning of the study area was preplanned as shown in Fig. 4.

b. Projected Water Demands

Based on the data concerning the exisiting water demands as well as the water-related activities for the study area, which have been provided by the Ministry of Construction and the Prefecture of Hyogo, the water demands at the year of 1985 were projected (See Table 1).

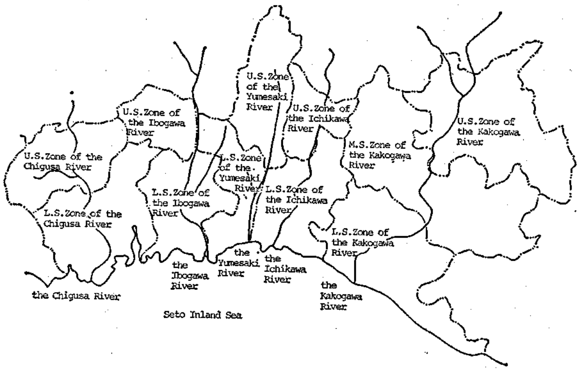


Fig. 4 Zoning of Study Area.

Table 1 Projected Water Demands.

Demands / Zone	Projected Industrial Water Demand	Projected Domestic Water Demand
U.S. Zone of the Chigusa R.	(10^3 m^3/day) 3.99	(10^3 m^3/day) 1.05
L.S. Zone of the Chigusa R.	42.55	10.75
U.S. Zone of the Ibogawa R.	7.95	2.07
L.S. Zone of the Ibogawa R.	72.82	16.67
U.S. Zone of the Yumesaki R.	3.89	1.22
L.S. Zone of the Yumesaki R.	40.27	9.87
U.S. Zone of the Ichikawa R.	13.75	4.65
L.S. Zone of the Ichikawa R.	161.09	39.27
U.S. Zone of the Kakogawa R.	38.14	9.72
M.S. Zone of the Kakogawa R.	76.40	19.85
L.S. Zone of the Kakogawa R.	189.78	119.40

c. Estimated River Discharge

Based on the data concerning the estimated average river discharges (low-flow), they are set as listed in Table 2.

d. Estimated Cost Functions

The practical experience by a certain authorita-

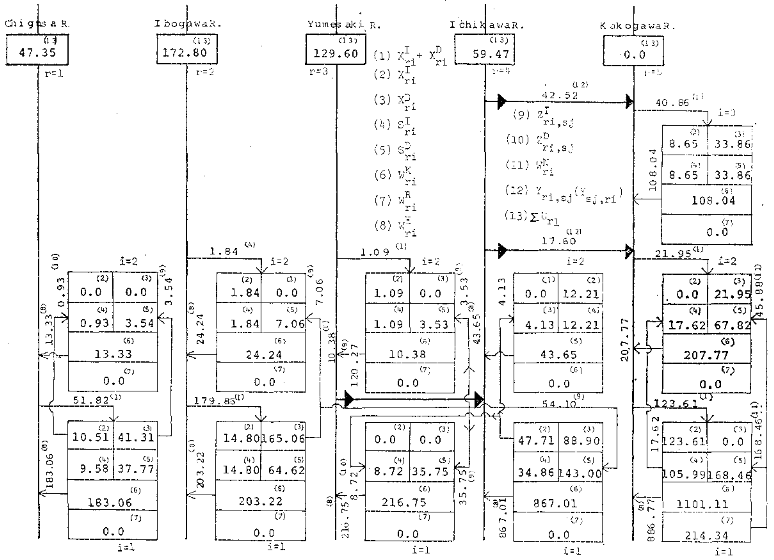


Fig. 8 Calculation Results for Standard Case (Case A-(1)).

(iii) In the basin of the Chigusa R. it seems better to establish an inter-basin (single basin) water utilization system, separated from the other basins.

(iv) In each basin some of those purified industrial waters are distributed to its upstream zone.

(v) So far as the reclamation system is concerned, it is taken limitedly in the downstream zones of the Kakogawa R., from which 21 percent of the renovated waters are further distributed to midstream zone to supplement the provisions of water for industrial use there.

b. In-depth Analysis of the Above Findings

By referring to the above findings let us carry out an in-depth analysis with an aim to probe into the underlying mechanism operating to yield the results. Once this is done, we shall be able to obtain some basic yardsticks to formulate our water resources management policies.

① Taking account of scales of economics related to the facilities of purification, wastewater treatment and tertiary treatment, and in view that inter-basin river-flow diversions are not as much economical as inter-zonal distributions chiefly owing to a "round-about" way of supply involved in the former as compared to the latter, one might intuitively expect that if economic efficiency alone is pursued to its full extent, inter-basin water transfers, if necessary, will take the form of inter-zonal purified water distributions rather than that of inter-basin streamflow diversions. The findings of **a**-(i) through **a**-(iii), however, show that the results derived from our model come out a little bit against our expectation, because among those transported from one basin to another totalling 316 thousand m^3/day (102 thousand m^3/day in net quantity), only 43 percent (36 percent) are taken up by the inter-zonal distribution system, the remaining 57 percent (64 percent) being conveyed by diversion channels.

From the findings **a**-(i) through **a**-(v), to the end of the cost-goal, our alternative derived from our model proved to be a second-best policy, because a more extended scale of the inter-zonal distribution system would be more economical. Likewise we might well say that in the light of the quality-goals our alternative is also a second-best policy in the sense that higher quality of freshwater could be more extensively developed in the upstream valleys.

Here recalling the fact that the best (optimal) policy judged from the cost-goal has been set identical to its satisfied-level which is derived from the solution of the linear programming problem with an objective function taken to be the total cost, one might easily understand that our goal programming model is thought of as an effective tool in producing a second-best policy other than the optimal one that is obtainable from the linear programming approach.

② Next let us assume a case where the supply-goals were excluded and only their permitted-levels were set as technical constraints. Then one might readily understand that the amount of water supplied for the downstream zone on each river would be reduced to their permitted-levels in order to augment the stream discharges, because it would lead to the increased attainment of both the cost- and quality-goals.

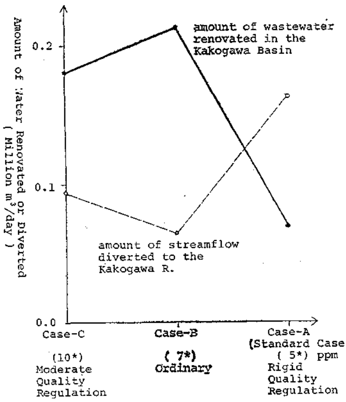
③ From what have been studied above it will be obvious that our alternative obtained in our standard case study can be conceived as an efficient solution, because a further improvement in the attainment of any one kind of goals would be achievable only at the expense of the others.

This efficient solution can be characterized by its well-balanced attainments of all the goals, mainly because of the stipulated *L*-type utility curves. The attainment ratio for each goal which is defined as the ratio of the deviation between the attained- and permitted-levels to that between the satisfied- and permitted-levels, is calculated as 0.40 to 0.50 for the different goals.

④ Our modified goal programming approach which was developed by the authors by applying the cutting plane method proved to yield a very reasonable solution after a couple of iterations. The nonconvexities involved in our nonlinear goal constraints can be overcomed if a good approximation to the optimum (efficient solution) is a priori obtained by our model and it is used as our first base point.

(4) Comparative Study

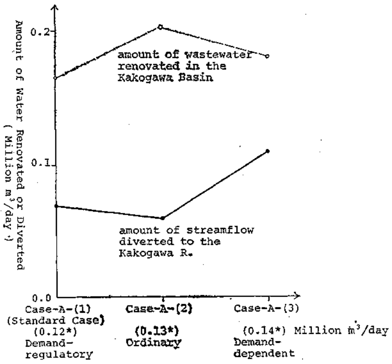
(i) Let us first compare those results of Cases C-1, B-1, and A-1, which are characterized by passive, ordinary and active actions, respectively, in light of the conservation of water quality. Furthermore our discussions will be limited to the Kakogawa R. which seems to provide us with the most suggestive information for the policy-formulations. From Fig. 9 it is obvious that the augmentation of stream discharges by promoting the inter-basin streamflow diversion systems seems to be relatively effective for the purpose of quality alleviations, but a higher standard for water quality is effectively attainable chiefly by promoting the reclamation recycling systems.



* represents the permitted quality level for the downstream of the Kakogawa R.

Fig. 9 Comparative Study of Cases A-(1), B-(1) and C-(1).

(ii) We shall next take Cases A-(1), A-(2) and A-(3) to analyze the results by comparison. Let us first observe that as illustrated in Fig. 10, cases A-(1), A-(2) and A-(3) might well be called "demand-regulatory alternative", "medium alternative" and "demand-dependent alternative", respectively. The comparison of the results shows that if one takes a demand-dependent alternative system, a balanced mix of the two modes of utilizations is most concerted, whereas if one seeks for an ordinary alternative, main sources of supplies are shared by dams and renovated waters play soley a subordinate part in providing water. And for a demand regulatory alternative system a little more increase in the amount of renovated water with a little more decrease in the amount of fresh water seems to be most propitious.



* represents the permitted demand level for the downstream of the Kakogawa

Fig. 10 Comparative Study of Cases A-(1), A-(2) and A-(3).

6. CONCLUSION

The central question to which this paper is addressed is a problem of coordinating the attainments of the different kinds of goals assigned to the planning of the water resources systems of inter-basin development and dual-modal utilization. At the outset of our study we placed into perspective those different goals involed in the planning of this field and gave primary consideration to the specification of those goals.

In these terms we concerned ourselves with a problem of coordinating multiple-goal attainment in planning the specified water resources system.

Let us first summarize our findings.

(i) In the basin of the Chigusa R. it seems better to establish an inter-basin system which is separated from the other basins.

(ii) So far as the Kakogawa R. is concerned and if the water quality is confined to be less than 3 ppm in the upstream, 5 to 6 ppm in the midstream and 7 to 10 ppm in the downstream, it appears that the inter-basin streamflow diversion system should be implemented from the Yumesaki R. via the Ichikawa R. to the downstream basin of the Kakogawa R. so as to meet the given standards of quality as well as to put renovated water supplies into practice.

But things turn out to be somewhat different if one wishes to achieve a higher quality management especially in the upstream and midstream. That is to say that a relatively small amount of streamflow is diverted from the upstream of the Ichikawa R. to the upstream and middlestream of the Kakogawa R. On the contrary the diversion is not implemented from the downstream of the Yumesaki R. via the Ichikawa to that of the Kakogawa R. Instead, the reclamation system needs to be implemented on a larger scale. This seems to be derived from the fact that the increased necessity for augmenting streamflow might be more efficiently met by extending the scale of the reclamation system than increasing the amounts of diverted water or developing more fresh water in its upstream valley.

(iii) So far as the Ichikawa R. is concerned, the implementation of the inter-basin river-flow diversion system as well as the inter-zonal industrial water distribution system seems to be consistently required irrespective of the difference in the water quality management.

(iv) Anyhow the inter-basin water transfer system is believed to be necessary insofar as the Ichikawa R. and Kakogawa R. are concerned. The diversion system should be introduced primarily for the purpose of improving streamflow quality rather than to the end of water supplies. On the other hand the implementation of the inter-zonal distribution system seems to be most adequate for securing increased amount of water supply but not propitious to the conservation of stream water quality.

(v) Our goal programming model is considered one of the most effective tool in producing a second-best policy which guarantees the maximal attainment of all the goals involved at the level of gross equity.

(vi) The variation in the permitted-levels of supply-goals has proven to lead to the comparatively different modes of water utilizations, thus indicating the strong likelihood that the above findings might not well be envisaged as general conclusions but should be considered a kind of a frame of reference in the choice of an efficient water utilization system for the area under study.

(vii) The problems which are still open to us are: ① How high and in what specific a manner should we set the levels of the goals to aim for? ② Is it an acceptable assumption to the planner of the practical field of water resources that all the goals should be well developed?

(viii) The problems which fall beyond the potential use of this approach and which need different angles other than this study are: ① capacity expansion problems involving the multi-stage dynamic programming treatment, ② stochastic treatments calling for a more qualified level of the input data as well as an increased number of time-variant variables incorporated into the model, ③ more precise treatment of technical problems including water-quantity mechanisms.

For all these questions to be further analyzed in future, it seems to be important to go as far as possible with models presented in this paper and it is believed that enough information would come from this kind of analysis.

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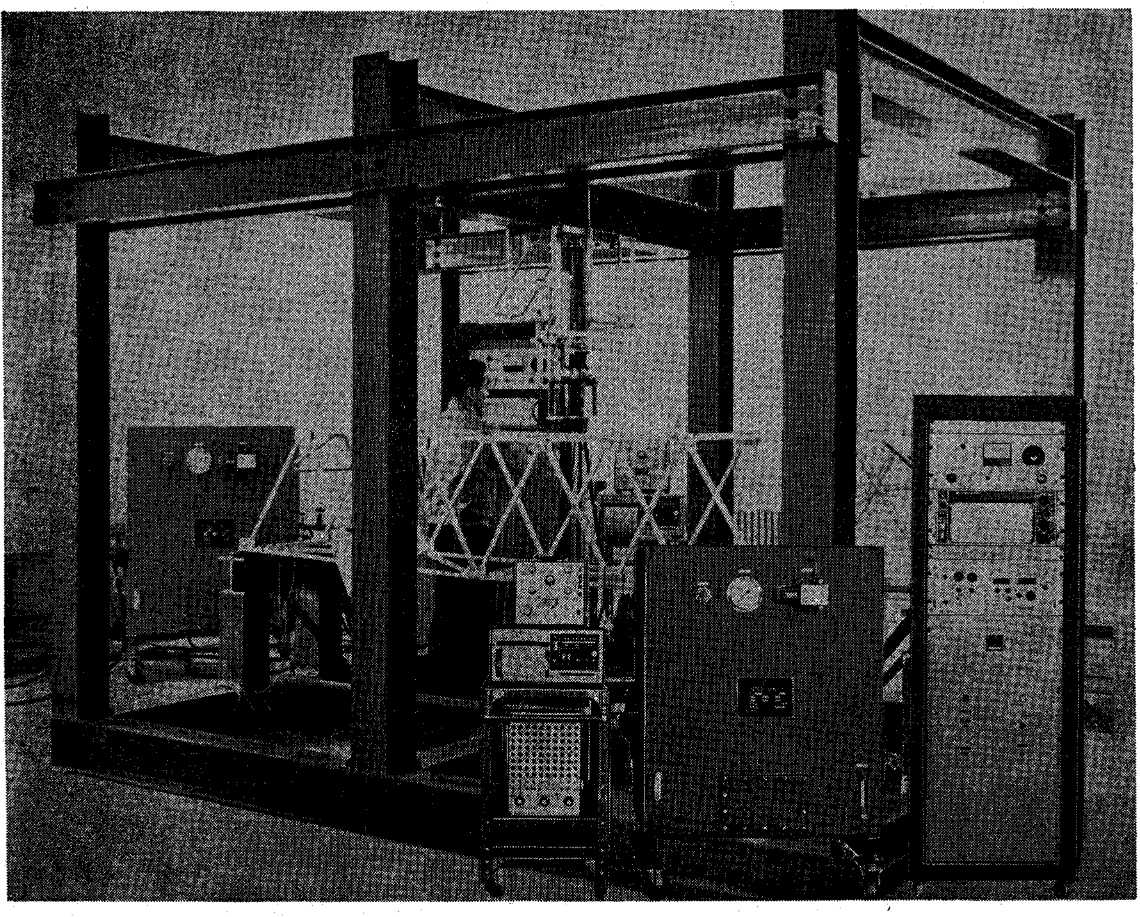

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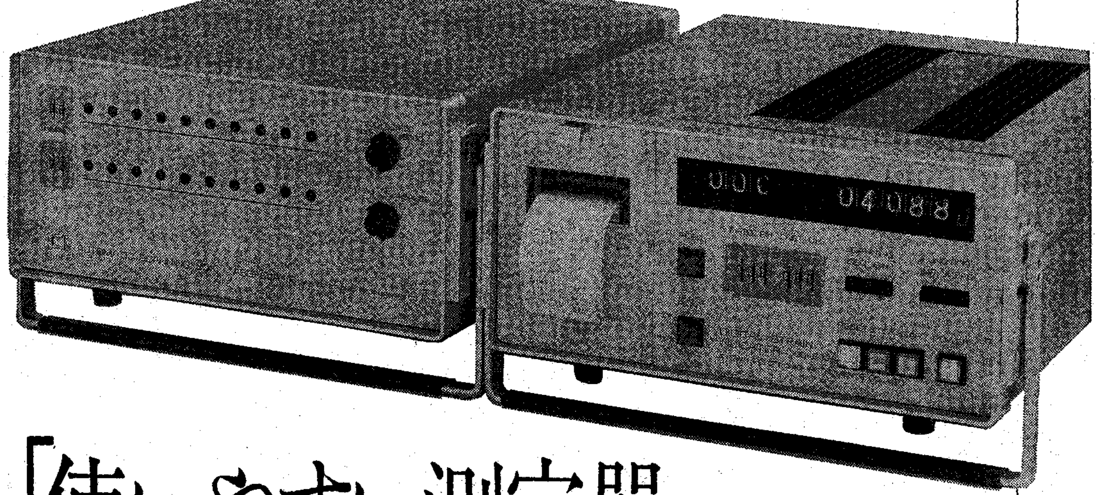

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