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SYSTEMS ANALYSIS OF OPERATIONAL CONTROL OF WATER SUPPLY AND USE SYSTEM IN DROUGHT-TIME

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1. INTRODUCTION

In spite of increased recognition that water resources are limited and they should be developed and used more effectively than they used to be, there seems to be many problems which have scarecely received proper attention of researchers and practitioners. One typical example is the problem of water droughts. The purpose of this paper is to present the development of a systematic methodology for determining the optimal policy for water supply and demand control problems. "Optimal" in this context means a rule that leads to the operational policy most acceptable to the water users.

2. IDENTIFICATION OF THE PROBLEM

Since water supply cut-off is a degradation of water supply service, utmost efforts should be made to avoid it. But when it is found unavoidable by any means after a long continuation of water droughts, next best attempt should be made to alleviate as much as possible the extent of undesirable effects of water supply cut-off upon the water users.

A variety of types of approaches would be available for analyzing the problem, but there is little information so far on it.

3. SCOPE OF THE RESEARCH

In this paper we shall take the position that in coping with water droughts problems, operational control policies for alleviating unacceptable damages of droughts to the water users are as much important as long-range strategies for water resources development. In light of these considerations, our attention will be exclusively placed on the explorement of systems approaches for finding the optimal rule for drought-time water supply and demand controls. "Optimal" in this context means a rule that leads to the most "acceptable" operation policy to the water users.

The paper contains three major studies. Part 1 presents a pilot approach to the analysis of drought-time water demand and use characteristics with specific reference to the behavioral characteristics of the water users. An attempt is made to explore a methodology for measuring "acceptability" in terms of statistical variates. On the basis of this analysis follows presentation of a mathematical model in Part 2. Applicability and potentialty for further developments are also explained with a case study on Kyoto City. Part 3 deals with a mechanism of the dynamic interactions between the water supply and water demand sectors, during the periods of water droughts, thereby presenting a system dynamics approach to the problem with a case study on Toyonaka City, Osaka.

The papar closes with our assessment of the usefulness of the fundamental methodology proposed here for analyzing the drought-time operational control problems.

4. PART 1—STATISTICAL ANALYSIS OF BEHAVIORAL CHARACTERISTICS OF WATER USERS IN DROUGHT-TIME

In this part of the paper a statistical approach is presented for analyzing the behavioral characteristics of the water users in times of droughts. We shall construct a statistical model which will be applied to the analysis of the behavioral characteristics of the water users in Tenri City, Nara. We owe most of our basic data to the survey conducted there by the Kinki Branch of the Ministry of Construction with a view to gathering basic data relevant to the evaluation of water supply and demand control operations[1]. In this survey attempt was made to

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carry out a questionnaire which was so designed that it could present a basic framework in which statistical analyses would be made by use of the Multidimensional Quantification Analysis. In spite of much reliance on these data, or the same statistical technique used in that survey, the study that follows is unique and original in that it evolves an analytical approach from another angle, thus leading to different outputs and results. Another purpose of the study is to present a basis on which different systems approach will follow in Parts 2 and 3.

(1) Questionnaire

The questionnaire was so designed that it could collect basic information related to the behavioral characteristics of the water users in Tenri City, who had experienced right ahead of the time of the survey, one of the longest and severest droughts they ever had. The contents of the questionnaire are mainly related to the following problems: (See Tables 1, 2 and 3.)

(i) to identify the recognized service level of cut-off operations,

(ii) to comprehend the recognized service level of the provisional measures taken by the authority such as itinerant water-supply wagons, "save-water" campaigns, etc.,

(iii) to gauge the impact of fall in water supply on their everyday life,

(iv) to investigate the manner the water users adjusted themselves to the water-supply cut-offs.

Let us now turn to examine the results of questionnaire. Want of space allows us to touch merely upon the points.

(2) Preliminary Discussion

a. On Cut-off Operation

We here define the cut-off ratio as the ratio of the cut-off amount to the normal water supply. "Normal" in this context means the phase when water droughts do not occur and there is no need to cut-off water supply.

Table 1 shows that there is a certain discrepancy between the actual cut-off ratio and the recognized one. This gap tends to become smaller with rise in the actual cut-off ratio. Interestingly, there seems to be a smaller gap found between the changing rate of the actual cut-off ratio from one day to another and that of the recognized one. This means that the water users more easily recognize the present degree of cut-off, namely the cut-off ratio by recalling that of the preceding day by comparison. This provides a good theoretical basis for the following discussions where it will be assumed that the recognized condition of cut-off operations corresponds to its actual condition insofar as the qualitative characteristics of operational conditions are concerned.

Table 1 Comparison between Actual Operation Pattern and Recognized Pattern.

(a)

Stage		Initial	Middle	Final
Actual cut-off ratio (Condition of operation)		0.4~0.5 (Severe)	0.3~0.4 (Medium)	0.1~0.2 (Moderate)
Recognized condition of operation*	Severe	76.6 (%)	10.9 (%)	0 (%)
	Medium	24.4	47.2	35.3
	Moderate	0	45.3	64.7
Correct answers*		76.6	47.2	64.7

* percent of a specific choice among three alternatives

(b)

Period		Initial-Middle	Middle-Final
Change of actual operation		Improved	Improved
Recognized change of operation	Improved	89.1 (%)	82.5 (%)
	Unchanged	10.9	17.5
	Aggravated	0	0
Correct answers*		89.1	82.5

* percent of a specific choice among three alternatives

b. On Acceptable Cut-off Ratio

Fig. 1 which displays the upper limit to the acceptable cut-off ratio for different duration times of cut-off operations, shows that:

(i) The average of the upper limit ratio decreases as the duration time extends.

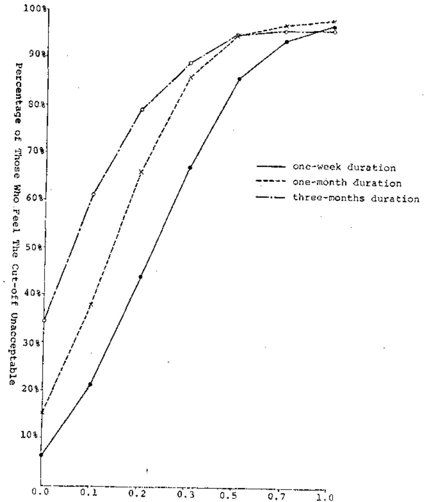

Fig. 1 Accumulated Numbers of Those who Feel Unacceptable to a Given Operation.

(ii) The distribution patterns of the upper limit ratio for the duration time of one week, and for that of one month are mutually similar, though the pattern for the duration time of three months completely differs from the above two.

(iii) This can otherwise be interpreted in two different manners. ① The extent to which the water users manage to adjust themselves to cut-off operations, decreases as the duration time expands. ② The water users become less willing to spare water as the duration time expands.

The reader is required to recall the above findings in the model-building discussions in Parts 2 and 3.

(3) Drought-time Behavioral Patterning by Multidimentional Quantification Method

Next, we shall patternize the behavioral characteristics of the water users from a mathematico-statistical point of view. An approach is presented by the Multidimensional Quantification Method developed by Hayashi[2),3)]. That is to say that the primary concern of the analysis is to find out those dominant factors which determine the drought-time behavioral pattern of the water users.

a. Recognition Pattern Concerning Severity of Cut-off Operations

In order to find out the dominant factors determing the recognition pattern concerning the altereness of cut-off operations, we set as the external variable the item of "recognized severity of cut-off operations". Thereby the multicolinearities between these factors are examined by use of the "Contingency Matrix" due to Cramer, and those factors are excluded from the subsequent analysis which have proven to be closely interrelated. Table 2 shows the calculation results for this case. The theory underlying the quantification method tells us that the calculated range of each item (factor) stands for the degree of relevance to the patternization of recognized severity. In light of this theory we examine the calculated range of each item as listed in Table 2. It shows that the item of the biggest range, i.e., the most influential factor is "quantity and pressure of water from the tap", which is followed by "recognized cut-off ratio", and then by "recognized duration time of cut-off operations".

b. Recognition Pattern Concerning Willingness to Save Water

In likewise as the above, we shall patternize the recognition pattern concerning willingness to save water, thereby analyzing what items (factors) are most influential to it. In applying the quantification method to the problem, the item "willingness of the water users to save water" is set as an external variable. The calculation results are listed in Table 3. It must be noted here that among those influential items, "recognized level of campaign activities" proved to be the most influential to the "willingness of the water users to spare water". This seems to support our conclusion obtained in the preliminary discussion.

(4) Summary and Discussion

The findings of our study are summarized:

(i) The recognized condition of cut-off operations corresponds to its actual condition insofar as its qualitative characteristics are concerned.

(ii) The extent to which the water users practiced water-saving has much reliance on the recognized level of "save water" campaigns.

(iii) The water users became less willing to spare water as the duration time expands.

(iv) The recognition pattern concerning the severeness of cut-off operations is determined mainly by those factors such as ① "quantity and pressure of water from the tap", and ② "recognized cut-off ratio".

From this we may conclude that the mathematico-statistical analysis approach presented here provides a basic framework in which the studies to follow in Parts 2 and 3 are performed. It also presents basic information in policy-making for acceptable operational controls in times of droughts.

In spite of such fruits obtained from our study, more study is needed to overcome the difficulties mainly involved in the technical problems of the design of questionnaire contents. That is to say that more sophisticated manner of questioning needs to be examined so as to make the answerer understand exactly what the answer refers to, and also to identify more exactly what the answerer intends to mean.

5. PART 2—DYNAMIC PROGRAMMING APPROACH FOR DETERMINING OPTIMAL POLICY FOR DROUGHT-TIME OPERATIONAL CONTROLS

In this part of the paper our focus is shifted onto the problem of determining operational controls. In coping with the problem as specified below, an attempt is made to build up a mathematical model which will be applied to the case study on Kyoto.

(1) Description of the Problem

(i) The system under study is a tandem system which consists of two different types of "reservoirs".

Table 2 Recognition Pattern Concerning Severity of Cut-off Operations.

External criterion	Corresponding to the item of questionnaire	η	
Recognized degree of water supply cut-off	You felt the water supply cut-offs were	0.797	
	1 very severe		
	2 rather severe		
	3 medium		
	4 rather moderate		
	5 very moderate		
Candidate factors	Corresponding to the item of questionnaire	Score	Range
Quantity and pressure of water from the tap	You felt the water from the tap came out		1.020
	1 sufficient	−0.312	
	2 insufficient with low pressure	−0.371	
	3 sometimes sufficient, otherwise no water	−0.116	
	4 sometimes insufficient with low pressure, otherwise no water	0.641	
	5 no water all the time	0.168	
	6 otherwise	0.485	
Recognized percentage of water supply cut-off	You felt you were forced to reduce your usual water demand approximately at		0.295
	1 100%	0.109	
	2 90%	0.118	
	3 80%	0.148	
	4 70%	−0.009	
	5 60%	0.005	
	6 50%	0.156	
	7 40%	0.070	
	8 30%	−0.031	
	9 20%	−0.053	
	10 10%	−0.013	
	11 0%	−0.139	
Recognized duration time of cut-off operations	In your memory the cut-off operations lasted about		0.116
	1 below 1 week	−0.072	
	2 below 2 weeks	0.044	
	3 below 3 weeks	0.026	
	4 below 1 month	0.003	
	5 below 1.5 months	0.016	
	6 over 1.5 months	−0.020	
Recognized changing pattern of cut-off operations	In your memory the cut-offs seemed to be operated		0.070
	1 initially severest, then gradually more moderate	−0.031	
	2 severest in the middle of the period	0.039	
	3 initially most moderate, then gradually severer	0.036	
	4 always moderate	−0.007	
	5 otherwise	0.006	
Recognized level of supplementary water supply service by wagons	In your memory you had supplementaly water supply services by wagons		0.085
	1 never	−0.005	
	2 quite rarely	−0.020	
	3 once every two days	0.058	
	4 once a day	0.065	
	5 more than twice a day	−0.013	
Recognized level of save-water campaigns	You felt save-water campaigns worked		0.062
	1 very effective	0.040	
	2 rather effective	0.031	
	3 not effective, nor ineffective	−0.022	
	4 rather ineffective	0.014	
	5 very ineffective to lead yourself to voluntary curtailment	0.012	

Table 3 Recognition Pattern Concerning Willingness to Save Water.

External criterion	Corresponding to the item of questionnaire	η	
Willingness of the water users to save water	You reduced your water demands	0.795	
	1 rather willingly		
	2 unconsciously		
	3 rather unwillingly		
Candidate factors	Corresponding to the item of questionnaire	Score	Range
Existence of wells	Do you have wells in your house?		0.014
	1 yes	−0.008	
	2 no	0.006	
Quantity and pressure of water from the tap	You felt the water from the tap came out		0.139
	1 sufficient	0.083	
	2 insufficient with low pressure	−0.056	
	3 sometimes sufficient, otherwise no water	−0.045	
	4 sometimes insufficient with low pressure, otherwise no water	0.070	
	5 no water all the time	−0.009	
	6 otherwise	−0.052	
Recognized percentage of water supply cut-off	You felt you were forced to reduce your ordinary water demand approximately at		0.250
	1 100%	−0.002	
	2 90%	0.106	
	3 80%	0.055	
	4 70%	0.090	
	5 60%	0.096	
	6 50%	−0.139	
	7 40%	−0.091	
	8 30%	−0.069	
	9 20%	−0.087	
	10 10%	−0.144	
	11 0%	−0.101	
Recognized duration time of cut-off operations	In your memory the cut-off operations lasted about		0.128
	1 below 1 week	−0.058	
	2 below 2 weeks	−0.077	
	3 below 3 weeks	−0.050	
	4 below 1 month	0.051	
	5 below 1.5 months	0.009	
	6 over 1.5 months	0.022	
Recognized changing pattern of cut-off operations	In your memory the cut-offs seem to be operated		0.246
	1 initially severest, then graduallyless severe	0.073	
	2 severest in the middle of the period	−0.038	
	3 initially most moderate, then gradually severer	0.099	
	4 always moderate	−0.052	
	5 otherwise	0.194	
Frustration coming from prolonged cut-off operations	You get blues		0.255
	1 extremely	0.008	
	2 rather	0.122	
	3 a little	−0.112	
	4 little	−0.133	
	5 never	−0.051	
	as the cut-offs were prolonged		
Recognized level of save-water campaigns	You felt save-water campaigns worked		0.578
	1 very effective	0.440	
	2 rather effective	0.446	
	3 not effective, nor ineffective	−0.123	
	4 rather ineffective	−0.023	
	5 very ineffective to lead yourself to voluntary curtailment	−0.077	

One is a reservoir which serves as the water source of the second subsystem which is conceptualized as another reservoir, synonymously used to refer collectively to the purification facilities. For simplicity, let the former reservoir be called "the water-source reservoir", and the latter "the purification reservoir".

(ii) The magnitude of the droughts treated is prescribed as follows. Making allowance for the total amounts of water which are expected to be impounded over the given drought horizon, and comparing with it the total of the estimated water demands that would generate over the same period, we have already known that the latter exceeds the former, and our main concern is with the temporal allocation of the total amounts of available water to be withdrawn from the water-source reservoir for each stage of the period.

(iii) It is also postulated that this withdrawal from the water-source reservoir is identical to the collection of water into the purification reservoir. This implies that the question for the temporal withdrawal from the former reservoir is thus reduced to the determination of the temporal collection for the latter reservoir.

(iv) The amount of withdrawal from the latter reservoir at each stage of the period is assumed to be identical to the amount of supply to the water users.

(v) The demand at each stage of the period is treated as a stochastic variate obeying a normal distribution function.

(vi) The above assumptions have led to our observation that the system explicitly to be treated in this study is the purification reservoir and the water-source reservoir is treated only in implicit terms. Henceforth the term "reservoir" is used to refer to the purification reservoir, if otherwise specified.

(vii) Unless the water demand exceeds the storage in a given stage of the period, the release is so operated that it equates the demand at that time. Otherwise the release is so operated that it is identical to the storage at that time.

(viii) With this problem defined as such, it is considered a class of the optimal inventory of the (purification) reservoir whose inputs are the amount of collection and whose outputs the amount of supply to the water users with such water demand at each stage of the period as characterized by a normal distribution function.

(ix) The frame of reference with which to check the optimality of the policy is taken to be the minimization of the magnitude of unacceptableness perceived by the water users with respect to the policy.

(2) Notation

x_c : reservoir volume
x_{i1} : volume of remaining water in storage immediately after release in stage $i-1$ (or immediately before the initiation of stage i)
y_i : volume of water in storage immediately after replenishment (by collection) in the initiation of stage i
z_i : (available) amount of collected water in the initiation of stage i
ξ_i : (amount of) water supply which is equal to the amount of released water in stage i
q_i : (amount of) water demand in stage i
$\phi(q_i)$: probability distribution function for q_i

(3) Model Formulation

The volume of water in storage immediately after replenishment in the initiation of stage is formulated:

$$y_i=x_{i-1}+z_i \qquad (i=1,\cdots,n) \quad \cdots\cdots\cdots\cdots(2\cdot1)$$

The amount of water supply ξ_i which is equal to the amount of released water in stage i is determined by the following rule:

$$\xi_i=q_i \qquad \text{for } 0\leqq q_i<y_i \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots(2\cdot2)$$

$$\xi_i=y_i \qquad \text{for } q_i\geqq y_i\geqq 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots(2\cdot3)$$

The volume of water in storage immediately after release in stage $i/(x_i)$ can be written as follows:

$$x_i=y_i-q_i \qquad \text{for } 0\leqq q_i<y_i \quad \cdots\cdots\cdots\cdots\cdots(2\cdot4)$$

$$x_i=0 \qquad \text{for } q_i\geqq y_i\geqq 0 \quad \cdots\cdots\cdots\cdots\cdots(2\cdot5)$$

Additionally, there are constraints on the amount of replenished water y_i:

$$0\leqq y_i\leqq x_c \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(2\cdot6)$$

Then we set up the objective function which stands for the expected value of the degree of unacceptableness recognized by the water users in relation to the operational policies to be carried out with a given probability:

$$\text{Minimize } z=\sum_{i=1}^{n}\int_{q^i=\underline{q_i}}^{q_i=\overline{q_i}} p_i(r)\phi(q_i)dq_i \cdots\cdots\cdots(2\cdot7)$$

where

$\overline{q_i}$: upper bound on the water demand with respect to stage i
$\underline{q_i}$: lower bound on the same above

$$r_i=\frac{q_i-y_i}{q_i} \qquad \text{for } q_i>y_i \quad \cdots\cdots\cdots\cdots(2\cdot8)$$

$$=0 \qquad \text{for } q_i\leqq y_i \quad \cdots\cdots\cdots\cdots(2\cdot9)$$

r_i : cut-off ratio

$$p_i(r_i)=0 \quad \text{for } r_i=0 \cdots\cdots\cdots\cdots(2\cdot10)$$

$$0\leqq p_i(r_i)\leqq 1 \quad \text{for } r_i>0 \cdots\cdots\cdots\cdots(2\cdot11)$$

$p_i(r_i)$: penalty function with respect to r_i representing the degree of unacceptableness recognized by the water users

(4) Solution Technique

The mathematical model as formulated in the above has a multistage structure. One of the effective solution techniques which have already been developed and applied to different models of this type of mathematical structure, is dynamic programming developed by Bellman[4),5)]. We shall present a solution technique by use of dynamic programming.

Since dynamic programming requires to define the recurrence functions, let us rewrite the objective function to introduce them as follows.

$$\begin{aligned} z=f_n(x_0)=\underset{x_0\leq y_0\leq x_c}{\mathrm{Min}}\Bigg\{&\int_{q_1=\underline{q_1}}^{q_1=\overline{q_1}} P_1(r_1)\phi(q_1)dq_1 \\ &+\int_{q_2=y_2}^{q_2=\overline{q_2}} f_{n-1}(0)\phi(q_2)dq_2 \\ &+\int_{q_2=\underline{q_2}}^{q_2=y_2} f_{n-1}(y_2-q_2)\phi(q_2)dq_2\Bigg\} \end{aligned}$$

Here the first term on the right-hand side of the above equation refers to the evaluation of the penalty function with respect to the first stage, the second and the third terms the integrated evaluation of those penalty functions with respect to the subsequent stages. The second term is equivalent to the statement that if in the first stage the water demand exceeds the upper limit to the suppliable amount of water, then it follows that the subsequent stages initiate with the condition that the volume of water on storage, x_1, is equal to zero, with probability of $\phi(q_2)$. It also states that if the water demand happens to be lower than the upper limit to the suppliable amount of water, then it follows that the initial condition of the subsequent stages is so determined that the volume of water in storage, x_2, equates the volume in storage immediately after replenishment, y_2, less the amount of released water, q_2, with probability of $\phi(q_2)$.

Similar discussion applies to the formulation of the general recurrence relation between a given stage and its subsequent stage.

$$\begin{aligned} f_i(x_{n-i})=\underset{x_{n-i}\leq y_{n-i+1}\leq x_c}{\mathrm{Min}}\Bigg\{&\int_{q_{n-i+1}=\underline{q_{n-i+1}}}^{q_{n-i+1}=\overline{q_{n-i+1}}} \\ &\times P_{n-i+1}(r_{n-i+1})\phi(q_{n-i+1})dq_{n-i+1} \\ &+\int_{q_{n-i+2}=\underline{y_{n-i+2}}}^{q_{n-i+2}=\overline{q_{n-i+1}}} f_{i-1}(0)\phi(q_{n-i+2})dq_{n-i+2} \\ &+\int_{q_{n-i+2}=\underline{q_{n-i+2}}}^{q_{n-i+2}=y_{n-i+2}} f_{i-1}(y_{n-i+2}-q_{n-i+2})\phi(q_{n-i+2})dq_{n-i+2}\Bigg\} \end{aligned}$$

$$(i=1, 2, \cdots, n)$$

where

$$f_1(x_{n-1})=\underset{x_{n-1}\leq y_n\leq x_c}{\mathrm{Min}}\left\{\int_{q_n=\underline{q_n}}^{q_n=\overline{q_n}} P_n(r_n)\phi(q_n)dq_n\right\}$$

The recurrence relation thus obtained yields a theoretical basis for obtaining the sequence, $f_i(x)$, inductively once $f_1(x)$ is known. We see that $f_1(x)$ determines $f_2(x)$, that $f_2(x)$ leads to an evalution of $f_3(x)$, and so on. This procedure called the technique of dyamic programming assures a quick and accurate sollution to the multistage (stochastic) model.

(5) Case Study on Kyoto

a. Model Data

(1) Probability Distribution of Water Demand (See Fig. 2.)

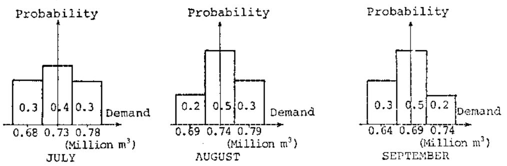

Fig. 2 Probability Distribution of Water Demand (Kyoto City).

(2) Penalty Function

We employ the function obtained in Part 1 representing the number of those who would not accept the operation insofar as the cut-off ratio remains fixed at a given percentage or more than that for a limited duration time. (See Fig. 1.)

(3) Calculation Cases

Before performing calculation, we preplanned the following analytical policies. (See Table 4.)

(i) Three different kinds of cases were established with respect to the duration time of the water-droughts.

(ii) Two different cases were established for two distinct "degrees of availability in collection" which are defined as the ratios of the total amounts of collected water over the period to the total amounts of the potential water demands over the period.

(iii) For a given degree of availability in collection,

Table 4 (a) Calculation Cases.

		Pattern	$Z_1 : Z_2$	Stage 1		Stage 2		Total	Case number
Degree of availability in collection	0.2	1	1 : 2	July	Million m^3 0.393	August	Million m^3 0.787	Million m^3 1.18	I-1
		2	1 : 1		0.590		0.590	1.18	
		3	2 : 1		0.787		0.393	1.18	
	0.2	1	1 : 2	August	0.381	September	0.763	1.14	I-2
		2	1 : 1		0.572		0.572	1.14	
		3	2 : 1		0.762		0.381	1.14	
	0.4	1	1 : 2	July	0.295	August	0.590	0.89	I-3
		2	1 : 1		0.443		0.443	0.89	
		3	2 : 1		0.590		0.295	0.89	
	0.4	1	1 : 2	August	0.286	September	0.572	0.86	I-4
		2	1 : 1		0.429		0.429	0.86	
		3	2 : 1		0.572		0.286	0.86	

Table 4 (b) Calculation Cases.

		Pattern	$Z_1 : Z_2 : Z_3$	Stage 1	Stage 2	Stage 3	Total	Case number
				July	August	September		
Degree of availability in collection	0.2	1	1 : 2 : 4	Million m^3 0.247	Million m^3 0.494	Million m^3 0.987	Million m^3 1.73	II-1
		2	1 : 2 : 2	0.346	0.691	0.691	1.73	
		3	1 : 2 : 1	0.432	0.864	0.432	1.73	
		4	1 : 1 : 2	0.432	0.432	0.864	1.73	
		5	1 : 1 : 1	0.576	0.576	0.576	1.73	
		6	2 : 2 : 1	0.691	0.691	0.346	1.73	
		7	2 : 1 : 2	0.691	0.346	0.691	1.73	
		8	2 : 1 : 1	0.864	0.432	0.432	1.72	
		9	4 : 2 : 1	0.987	0.494	0.247	1.73	
	0.4	1	1 : 2 : 4	0.185	0.370	0.741	1.30	II-2
		2	1 : 2 : 2	0.258	0.519	0.519	1.30	
		3	1 : 2 : 1	0.324	0.648	0.324	1.30	
		4	1 : 1 : 2	0.324	0.324	0.648	1.30	
		5	1 : 1 : 1	0.432	0.432	0.432	1.30	
		6	2 : 2 : 1	0.519	0.519	0.258	1.30	
		7	2 : 1 : 2	0.519	0.258	0.519	1.30	
		8	2 : 1 : 1	0.648	0.324	0.324	1.30	
		9	4 : 2 : 1	0.741	0.370	0.185	1.30	

the assignment of available water to be collected is patternized according to the extent the water is assigned to each stage. This assignment pattern is termed "operation pattern", or simply "pattern" which represents how much water is collected in each stage.

b. Calculation Results

Out of space consideration we merely refer to the results of Cases II-1 and II-2.

In Fig. 3 is plotted the calculated values of the objective function against different initial storages. It shows that the most acceptable pattern for a given initial storage as plotted at the foot of the figure, varies with the value for initial storage. Those specified points denoted by * are the cross points for two different pattern curves at which their ranks of acceptableness are reversed. These remarks also apply to Fig. 4.

(1) Case II-1

From the results as depicted in Fig. 3, we understand:

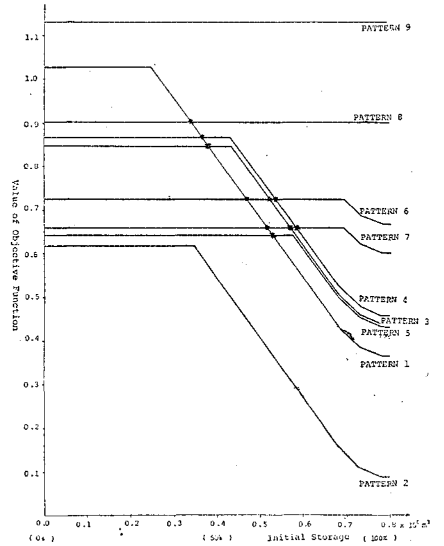

Fig. 3 Calculation Results of Case II-1.

(i) Irrespective of the initial storage, the most acceptable operational policy is such that relatively smaller amount of water is collected in the first stage, then increased amount of water in either of the subsequent two stages. (pattern 2)

(ii) In contrast to this, the least acceptable policy irrespective of the initial level of storage is such that relatively larger amount of water is collected in the first stage, then it decreases with the passage of time, and the smallest amount of water in the last stage. (pattern 9)

(iii) Pattern 1 belonging to the type of operation similar to pattern 4 proved to be the second best alternative if and only if the initial storage exceeds three-fourths of the potential water demand, while it proved to be the least acceptable one next to pattern 9 if and only if the storage falls short of half of the demand.

(2) Case II-2

Fig. 4 shows graphically the results of this case. From this we see:

(i) If the storage exceeds one-third of the potential water demand in the initial stage, the most acceptable policy is pattern 1. Pattern 2 which belongs to the same type of operations is the second best policy next to pattern 1. The difference between the two patterns of operations lies in that pattern 1 takes a more drastically-changing pattern than pattern 2. Herewith attention needs to be paid to the fact that if the storage goes down from one-third of the demand, both of the two patterns become no more acceptable policy at all but fall in a category of the most unacceptable policies;

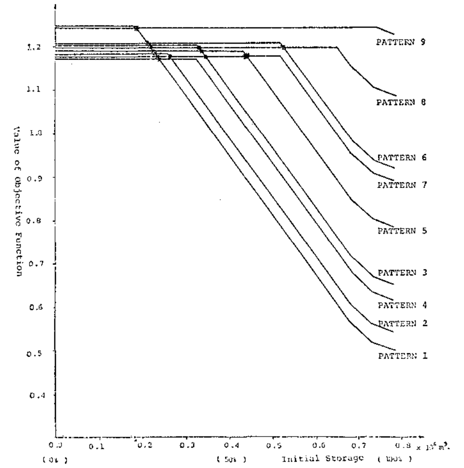

Fig. 4 Calculation Results of Case II-2.

(ii) If the storage falls short of one-third of the demand, the most acceptable policy proved to be Pattern 4. This pattern belongs to the type of operation similar to patterns 1 and 2, the difference being that the former assumes less drastically-changing pattern than the latter.

(6) Summary and Discussion

The results of this part of the paper have indicated that;

(i) The problem of determining optimal policy for drought-time operational controls can be formulated as multi-stage stochastic programming to which dynamic programming is most efficiently applicable.

(ii) The "optimal" policy in this context means that it is an operation which is regraded as the most acceptable one to the water users.

(iii) The measurement of "acceptableness" in the above sense of the word can be performed by making use of the functional relationship obtained in Part 1. But it must also be noted that there remains difficult problems left to be discussed from another viewpoint, i.e.,

① Whether or not it is reasonable to take such a qualitative factor as the objective function;

② Whether or not it is appropriate to measure such a qualitative factor by the manner presented here. In this connection, more study is needed to overcome the difficulties.

The treated system assumed that although the total amounts of water is limited to be collected throughout the period from the water-source reservoir which was not explicitly included in the system, the choice of the optimal operational policy concerning the allocation of the total collected water in each stage of the period is under control of the water sector. This assumption inevitably precludes the study of such conditions that the droughts are so severe that we cannot freely control collecting water any more.

Such being the case, our concern should be shifted from the control of collected water onto the controls of both water distribution and water demands. Let us now consider this type of problem in the next part of the paper.

6. PART 3—SYSTEMS DYNAMICS APPROACH TO THE ANALYSIS OF DROUGHT-TIME WATER SUPPLY-DEMAND CONTROL MECHANISM

Part 3 describes the problem of drought-time water supply-demand control mechanism and presents a systems dynamics approach[6)~9)] to its analysis. To begin, we define the terminology which will be frequently found in this part and which provides basic idea underlying our study.

(1) Terminology and Specification of the Problem

a. Water Droughts

To specify our definition of "water droughts", it will be called "abnormal water droughts". The term "abnormal" means that the droughts considered are so severe that controllability of collected water does not hold any more. But if otherwise specified, "abnormal water droughts" are simply referred to as "water droughts".

b. Water Supply-Demand Control System (Mechanism)

We conceptualize the "water supply control system" as consisting of three components; ① collection, ② storage, and ③ distribution. It must be noted here that we assume that collection is uncontrollable. As a result, impoundage of water by dams or reservoirs is given a priori. Then it follows that storage and distribution can be conceived as the major functions of the system. Here the storage is conceptualized as the total amount of water existing in water purification ponds and water transmission conduits as well as distributing reservoirs.

The time lag in transmission or distribution is neglected simply because our time scale for computation runs is selected as "day" as will be explained later.

(2) Water Demand Control System (Mechanism)

The water demand system is a conceptualized system which describes the mechanism of the drought-time behavioral structure of the water users. It is assumed that the system is composed of four processes as follows.

(i) Potential Water Demand Generation Process

The "potential water demand" is defined as a "normal" water demand which would totally come out unless "water droughts" occurred. Here the term "normal" is used to refer to the state where no drought is being encountered. The normal water demand is so estimated that it is set equal to the average of the actual amounts of water used in the period June-August, 1970-1975 (excluding 1973 when the water cut-off operations were actually conducted. Accordingly the "potential water demand generation process" describes the generation process of the "potential water demand".

(ii) Water Users' Behavioral Response Process

It is assumed that the water users who would require the total amount of the potential water demand are forced to spare part of it so as to adjust themselves to the water droughts. This process represents the following two processes: ① a process through which the water users are assumed to come to curtail their potential water demand to a certain extent in response to the "save-water" campaigns. This process will be referred to as the "economizing process"; ② a process through which the water users who get tired of cut-off operations are assumed to push up again their water demand closer to the potential water demand. This process will be referred to as the "anti-economizing process".

At this time the reader is required to recall that the setting of these two processes describing the water users' behavioral responses to cut-off operations is based on the results obtained in Part 1.

(iii) Actual Water Demand Generation Process

The "actual water demand" is defined as a demand which is synonymous to the amount of water actually used by the water users and therefore synonymous to that actually supplied by the water supply sector. In this context the "actual water demand generation process" describes the process through which the amount of actual water demand generates and the amount of actual water supply is determined to meet it.

In this connection we shall explain the definition of the "water supply-demand gap". This term is defined as the difference in amount between the potential water demand and the actual water demand.

It needs to be added that the term "water demand sector" is used to mean collectively the water users.

(3) Fundamental Structure of the Model

For limited space, the fundamental structure consiting of major feedback loops is depicted in Fig. 5.

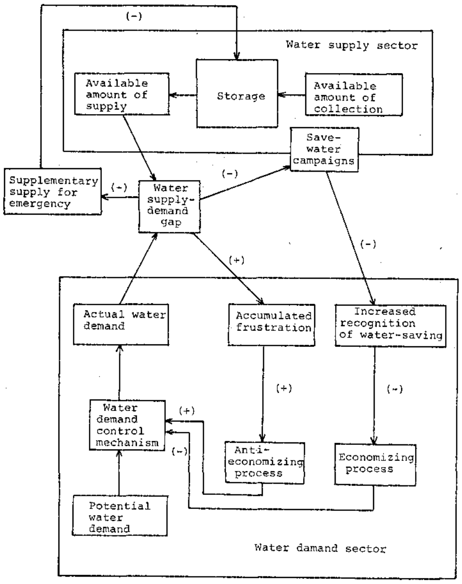

Fig. 5 Basic Structure of Model.

a. Feedback Loop I (see Fig. 6)

① Suppose the available amount of supply (SWL) falls short of the potential water demand (GWD), then the water supply-demand gap (RQCUT) exists. ② Then the save-water campaigns (PR) are conducted. ③ The emergent need for water-saving is increasingly recognized by the water use sector (PATB). ④ They become prepared to economize on water to a certain extent (SAVE). ⑤ They actually curtail their potential water demand (WWD). ⑥ Finally the supply-demand gap (RQCUT) decreases. Go back to ① and the process is iterated until the gap vanishes.

This repetitive cycle is called Feedback Loop I and plays an essential function in the mechanism under study.

b. Feedback Loop II (see Fig. 6)

① Suppose the water supply-demand gap (RQCUT) has persisted for a certain period and exists still now. ② Then frustration due to the continued suppression of their potential water demand (GWD) increases. This frustration is assumed to be derived from both the strength (ACUT) and the prolongation (DB) of cut-off operations. ③ The water users become more reluctant to economize on water and get prepared to push up their demand toward their potential one (PUSH 1+PUSH 2). ④ They actually push up their demand to ⑤ produce the actual water demand (WWD).

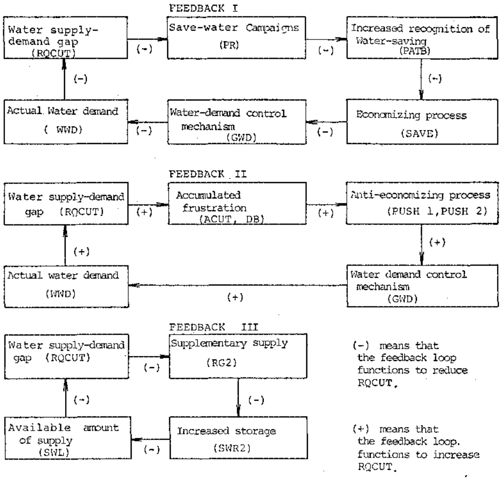

Fig. 6 Essential Feedback Loops.

This repetitive cycle is called Feedback Loop II.

c. Feedback Loop III (see Fig. 6)

① Suppose the water supply-demand gap (RQCUT) exists. ② Then the water supply sector calls for the supplementary supply for emergency (RG 2). ③ It gives an increase to storage (SWR 2). ④ The amount of actual water supply (SWL) increases. ⑤ Thus it tends to reduce the gap (RQCUT). The process is repeated until the gap vanishes. This repetitive cycle is called Feedback Loop III.

In formulizing the mechanism outlined as such, a systems dynamics approach is attempted for the reasons as follows. (i) The fundamental structure of the mechanism to be incorporated in model is characterized by several feedback loops which are mutually interrelated in a complicate manner. (ii) Accordingly the system will behave against our intuitive expectations. (iii) The performance of the mechanism varies with time. (iv) In this context the system can be conceived as a dynamic system. (v) The incorporated structure involved in rather qualitative characteristics of the water users can hardly be formulated otherwise than system dynamics.

(4) Study Area

Toyonaka City in Osaka was selected as the study area.

(5) Model Formulation

a. Water Supply Control Mechanism

The storage in the reservoir is expressed by the system equations in Table 5.

Table 5 Water Supply Control Mechanism.

SWR 1. K=SWR 1. J+(DT) (PG 1. JK-RS 1. JK)		L-1
SWR 2. K=SWR 2. J+(DT) (PG 2. JK-RS 2. JK)		L-2
PG. JK=PG 1. JK+PG 2. JK		A-1
SWR 1:	storage of water collected from the main water source (m^3)	
SWR 2:	that from the minor water source (m^3)	
PG 1 :	amount of collection from the main water source (m^3/day)	
PG 2 :	that from the minor source (m^3/day)	
PG :	total amount of collection (m^3/day)	
RS 1 :	amount of distribution from SWR 1 (m^3/day)	
PG 2. KL=TABLE (TABG, TIME. K, 0, 46, 1)		R-1
PG 1. KL=(FIT) (YTAC. K)		R-2
YTAC=TABLE (TAB 5. TIME)		
YTAC:	amount of supply by the Osaka Prefectural Water Supply Agency (m^3/day)	
FIT :	ratio of supply for Toyonaka City by O.P.W.S. to YTAC	

b. Anti-economizing Process

In formulating the mechanism of this process, let us assume on the basis of the results obtained in Part 1, that the process consists of two parallel processes, i.e., ① directly-anti-economizing process, and ② indirectly-anti-economizing process. The former describes a mechanism such that the accumulated frustration of the water demand sector due to the strength of cut-off operations (i.e., cut-off ratio) leads to an immediate push-up to its water demand. The latter is concerned with a mechanism such that the accumulated frustration of the water demand sector due to the prolongation of cut-off operations will cause, after a certain time lag, another push-up to its water demand, in combination with the former type of push-ups.

In this context the directly anti-economizing process is expressed as shown in Table 6.

The indirectly-anti-economizing process is expressed as shown in Table 7.

Table 6 Directly-Anti-Economizing Process.

ACUT. K=(WEEKLY. K) (RRQT. K)		A-25
PUSH 1. K=(CONST 1) (ACUT. K)		A-26
WEEKLY. K=TABHL (TAB 8, TIME, K, 0, 90, 10)		A-27
RDB. KL=CLIP (AUR. K, RDB 1. K, TIME. K, 7)		A-28
AUR. K=CLIP (RDB 3. K, RDB 2. K, TIME. K, 30)		A-29
RDB 1. K=TABHL (TAB 11, RRQT. K, 0, 0, 6, 0, 1)		A-30
RDB 2. K=TABHL (TAB 12, RRQT. K, 0, 0, 6, 0, 1)		A-31
RDB 3. K=TABHL (TAB 13, RRQT. K, 0, 0, 6, 0, 1)		A-32
ACUT :	accumulated frustration of the water demand sector due to the magnitude of cut-offs	
PUSH 1 :	direct push-up ratio (ratio of directly-pushed-up amount to the potential water demand)	
CONST 1 :	factor of transformation	
WEEKLY:	coefficicient of RRQT	
RDB :	dummy for a boxcar-train	
RDB :	dummy for table functions	
AUR :	dummy for a clip function	

Table 7 Indirectly-Anti-Economizing Process.

DB=BOXLIN (7, 1)		B-32
DB*1. K=DB*1. K+(DT) (RDB. JK-DB*1. J)		L-32
SUDB. K=SUM (7, DB. K)		A-34
SSDB. K=(SUDB. K) (INT*1. K)/WEEK		A-35
DB :	accumulated frustration of the water demand sector due to prolonged cut-off operations	
DB*1 :	dummy for a boxcar-train	
SUDB :	dummy representing total amounts of the levels stored in the boxcar-trains	
SSDB :	average of DB over the past one week	
WEEK:	7 days	
INT*1 :	dummy which takes on 1 for every 7th day, or 0 otherwise	

Table 8 Actual Water Demand Generation Process (1).

VAD. KL=(MEAN. K) (SSDB. K)		R-40
LAT. K=LAT. J+(DT) (VADB. JK-OUT. JK)		L-41
OUT. KL=LAT. K/DAB		R-42
PUSH 2. K=(CONST 2) (LAT. K)		A-40
VADB :	weighted average of SSDB	
LAT :	accumulated level of VADB	
OUT :	its decreasing rate	
DAB :	lag constant	
CONST 2:	factor of transformation	
RD. K=PUSH 1. K+PUSH 2. K-SAVE. K		A-43
WD. KL=(GWD. K) (1+RD. K)		R-45
WWD. K=WWD. J+(DT) (WD. JK-WWD. J)		L-46
WD :	dummy	
WWD :	reduced water demand	

c. Actual Water Demand Generation Process

The average of the accumulated frustration which is weighted by the average of cut-off ratio over every one week is transformed into the weighted average of accumulated frustrations. Then the weighted average which is accumulated with a certain time lag, gives rise to a push-up to the water demand. The formulation of this process is given in Table 8.

For another kind of important system equations related to this process, see Table 9.

Table 9 Other Mechanism.

SWL. K=MIN (WWD. K, SWR. K)	A-51
SWR. K=SWR 1. K+SWR 2. K	A-52
RS 1. KL=(1−COF. K) (SWL. K)	R-8
RS 2. KL=(COF) (SWL. K)	R-9
COF. K=SWR 2. K/SWR. K	A-53
SWL : amount of actual water supply (m³/day)	
SWR : total amounts of stored water (m³/day)	
WWD : reduced water demand (m³/day)	
COF : ratio of SWR 2 to SWR	
RQCUT. K=(1/GWD. K) (GWD. K−SWL. K)	A-53
RRQT. K=MAX (RQCUT. K, 0)	A-54
RQCUT : ratio of water deficiency	
GWD : potential water demand (m³/day)	
RRQT : cut-off ratio	

(6) Simulation Results

a. Model Validation

As explained above, the model's validity can be tested only by examining how well the model can produce past performance of the real world. As is obvious from Fig. 7, the dynamic patterns of the outputs show close analogy with those of the real world, taken away the minor differences in the absolute values for the former and those of the latter. Accordingly, we arrived at the observation that the model behaves very well in reproducing what have occurred in the past, allowing for the attainable accuracy in measuring our input data used as well as the hypothesized structure of the model that was constructed on the basis of rather qualitative inspection of the real-world phenomena.

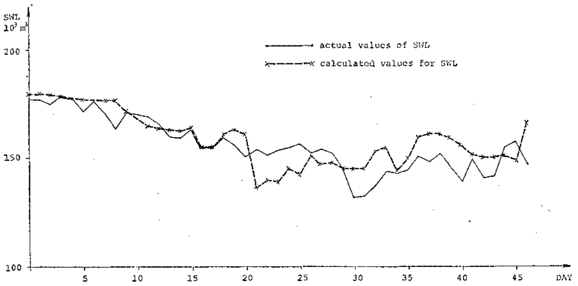

Fig. 7 Validity Test.

b. Standard Case

The outputs of this case are plotted in Figs. 8, 9, 10. At first blush the followings can be easily understood.

(i) The amount of actual water supply (SWL) takes such a changing pattern quite similar to that of the total amount of collection (RG). This can be explained by the uncontrollability of collection which tends to lead to the uncontrollability storage. Consequently the water supply-demand gap can be adjusted only by reducing water demands indirectly (by doing "save-water" campaigns) or directly (by cutting off part of water supply).

(ii) A sharp rise or drop in the changing pattern of PRQ 1 as plotted in Fig. 8 corresponds to the phase of an ill-balance in the amount of water be-

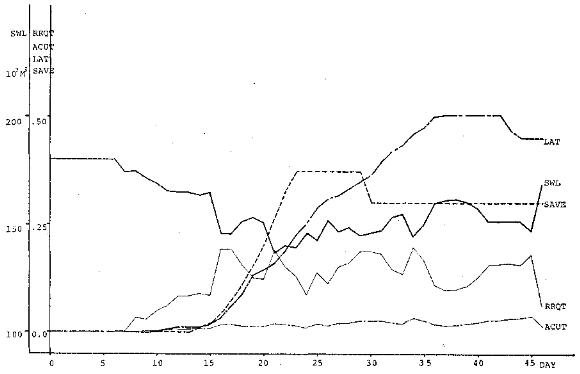

Fig. 8 Calculation Results (1) of Standard Case.

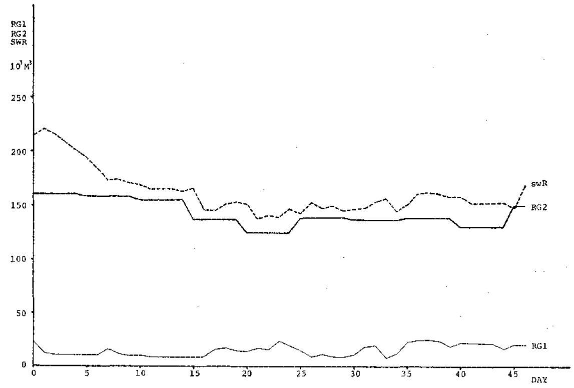

Fig. 9 Calculation Results (2) of Standard Case.

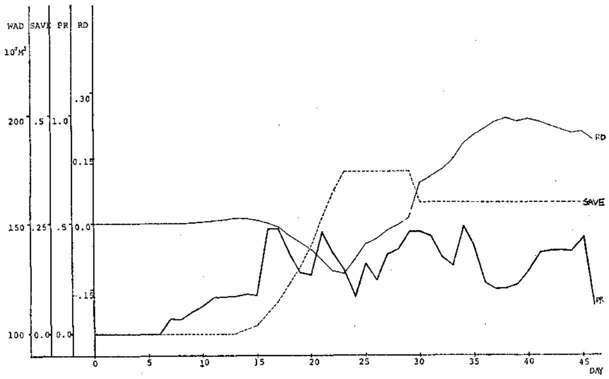

Fig. 10 Calculation Results (3) of Standard Case.

between supply and actual demand.

(iii) The cut-off ratio (RRQT) changes with time in a manner quite contrary to that the amount of actual supply (SWL) changes. This can well be accounted for by the fact that the smaller SWL becomes, the more (RRQT) increases, and vice versa.

(iv) This seems to suggest that the macroscopic changing trend (RRQT) is dependent on that of (SWL). A closer examination of Fig. 8 reveals, however, that the microscopic trend of (RRQT) is governed by that of the reduction ratio (SAVE). This is particularly the case for periods of 24th day to 29th and 34th to 40th in which (RRQT) sharply fluctuates and tends to decrease while (SWL) changes slightly between 20 and 25 ($\times 10^3 m^3$). In other words, the fluctuation indicates that the water demand is in control to such extent that (RRQT) is reduced owing to the increased reduction ratio.

(v) By comparing those changing patterns of RRQT, LAT and ACUT as plotted in Table 8, it will be clear that the pattern of ACUT bears a close resemblance to that of RRQT in a qualitative sense, difference being that the change in RRQT follows that of LAT with a time lag of about ten days and that LAT becomes higher whereas RRQT fluctuates between 0.1 and 0.2. This seems to be derived from the fact that LAT reflects the integral (accumulative) impacts of the fluctuation in RRQT. Furthermore the changing pattern of ACUT seems to reflect a minor change in RRQT.

(vi) Accordingly the combined changing pattern of the anti-economizing process which is obtained by superposing the above two patterns; i.e., LAT and ACUT, shows a pattern quite similar to that of RRQT with a time lag of about ten days.

(vii) From 25th day to 45th, RD which represents the changing rate of the actual water demand, seems to take such a changing pattern similar to the combined changing pattern. But, from 15th to 23rd, it takes a decreasing pattern, reflecting the effect of the foregoing save-water campaigns. (See Figs. 8 and 10)

c. Modified Cases

Mainly out of space consideration, we merely summarize the points.

A closer inspection of the results reveals that so far as RRQT is concerned, there is slight difference between the two cases. This verifies that the assumed structure is qualitatively valid. In this context the mechanism incorporated seems to be valid in the sense that it is at least consistent with the standard case. This can be explained by a heavy reliance of SWL on RG owing to the uncontrollability of the latter.

(7) Summary and Discussion

The above findings seem to suggest that the assumed structure underlying the model, though invisible and intangible directly, proved to be valid enough in the sense that it well explains the real world and the results obtained from it are consistent with the actual phenomena. On this basis, we can expect that the model presents an effective tool by which a deeper insight can be given into the mechanism functioning in the drought-time water supply-demand controls, and with which effective operational control policies can be examined when a situation occurs to which our assumptions adopted here roughly applies.

Lastly it should be added that the model is incomplete in that:

(1) The parameters of the model applied to Toyonaka City, are identified on the basis of data collected in the survey of the drought-time behavioral characteristics of the water user of Tenri City. But properly speaking, the parametric identification should be performed on the basis of those data which exactly reflect the behavioral characteristics of the water users in Toyonaka, although it was attempted in vain, mainly because of limited data availability.

(2) The validity of the model needs to be more rigorously examined, because one approach attempted in this study is considered to guarantee merely the satisfaction of some necessary conditions for the validity of the model.

7. CONCLUSION

The interest in this paper has been to throw light on a special type of water supply problems, i.e., drought-time water supply and demmand control problems, which have scarcely been treated so far by researchers and practitioners concerned.

The burden of our presentation is to elucidate the multifacets of these problems, the form of its presentation and analysis being varied, depending on both the magnitude of the "water droughts" and the interest of the investigator and the planner.

In light of these considerations, we have presented three kinds of analytical models with which to facilitate the explanation and interpretation of the unexplored problems, thus giving some empirical content to them. The basic idea that has consistently been placed at the bottom of this paper is that the most essential factor to be allowed for in evaluating the operational policies for the drought-time supply and demand controls is not the efficiency of operation (costs, available manpowers, maintenance availabilities, etc.), but the impact of cut-off operations on the water users. So we have started with the analysis of the behavioral characteristics of the water users, and have shown that some disadvantageous impacts can be gauged in terms of statistical variates. On this basis, we have developed two kinds of models to determine the most appropriate (optimal) operational policy.

Our experience with these models is encouraging in that this kind of problems are accessible by our model which can be operationally handled to produce different alternatives for different assumptions. It also shows (1) what are the most influential factors that would sharply alter the magnitude of the outputs, and (2) what of the outputs are the most sensitive to the assumptions from which they were derived.

We make no claim to being exempt from bias. However, our attempt has been to point out assumptions as explicitly as possible, so that the analysis and presentation of data in this paper are amenable to reinterpretation under different sets of conditions.

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