PROC. OF JSCE,
No. 244, DEC. 1975

DYNAMIC RESPONSE OF HORIZONTALLY CURVED GIRDER BRIDGES UNDER RANDOM TRAFFIC FLOWS*

By Hiroshi NAKAI** *and Hisao* KOTOGUCHI***

1. INTRODUCTION

It is a well-known phenomenon that the stresses induced to a highway bridge under the moving vehicles are greater than those in the case where the vehicles are applied statically. These increments of stresses due to the dynamical effects of vehicles are called as the impact, that must be considered in designing the highway bridges.

For a horizantally curved girder bridge, however, the dynamical behaviours are essentially distinguished from that of the ordinary straight girder bridges, because of the fact that the bending vibrations always couple with the torsional ones. Therefore, the impact of curved bridges shall be taken more or less different values from that of straight bridges.

The theoretical studies concerning these problems for highway bridges have already been reported by many researchers, where the following two typical methods are generally adopted. One is the deterministic method based upon the fundamental equation of motions for the bridge with the peculiar excitements of vehicles, while the other is the statistical method by taking into the considerations of random forces due to the vehicles.

By the former method, the dynamic response due to several vehicles, as is observed in the actual traffic flows, can exactly be simulated only when the excitement of vehicles is a definite condition. However, the various irregularities such as the variations of weight and spacing of vehicles as well as the roughness of roadway surfaces of the bridges will actually produce the random vibrations. It is so difficult to analyze these phenomena by the former method that the latter one will be powerful for the evaluation of the practical information with respect to the impact of highway bridges.

From such a point of view, this paper deals with the random vibration of the horizontally curved girder bridges. Although the accuracy of these analyses, of course, depends upon the correct data for the statistical properties of the traffic flows, there are almost no adequate records to be utilized for these purposes.

Accordingly, the field observations of the traffic flows were carried out on the typical national highways. With these experimental studies, a probability curve for the weight of vehicles and a power spectrum density of random forces acting upon the highway bridges were made clear numerically.

By applying these data, the coefficient of impact can approximately be given as the ratio between dynamic deflection and static deflection of which values will easily be found by means of the statistics. Finally, a few numerical calculations of the coefficient of impact for the horizontally curved girder bridges are illustrated. Furthermore, the coefficient of impact obtained from this method are evaluated and compared with the results which have been proposed by the authors on the basis of the deterministic simulation method.

2. OBSERVATION OF RANDOM TRAFFIC FLOW

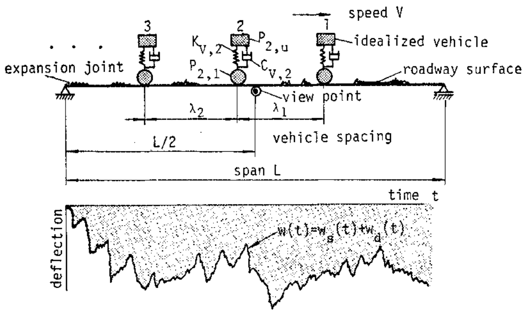
The dynamic response of a highway bridge subjected to the traffic flow can be treated by the mechanical model as shown in Fig. 1. When a vehicle, which is idealized as the simple vibration system as is seen in Fig. 1 (a), drives upon the bridge, the vehicle undergoes the oscillations due to the shock at expantion joint of the bridge

---

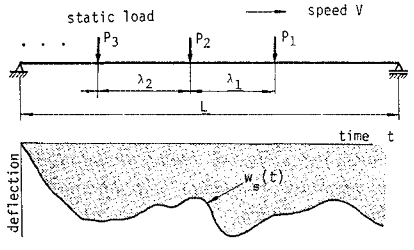
* The abstract of this paper has been reported at 26th. Annual Conference of JSCE, Oct., 1971.

** Dr. Eng., Professor, Department of Civil Engineering, Osaka City Univ.

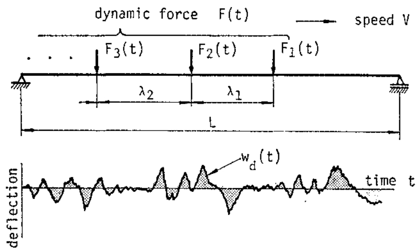
*** M. Eng., Research Associate, Department of Civil Engineering, Osaka City Univ.



(a) mechanical model



(b) static deflection



(c) dynamic deflection

Fig. 1 Random vibration due to moving vehicles.

and the roughness of roadway surfaces. For many vehicles, these oscillations are combined together and transmitted to the bridge as the random forces. Thus the vibrations of bridge result in random ones.

For example, Fig. 1 (a) illustrates a deflection curve $w(t)$ at the mid-span of bridge as the function of the time t. This deflection history $w(t)$ may be represented by the sum of static deflection $w_s(t)$ and dynamic deflection $w_d(t)$ as follows;

$$w(t)=w_s(t)+w_d(t) \quad \cdots\cdots(1)$$

where the static deflection $w_s(t)$ can be easily be found by considering the statical loading condition as shown in Fig. 1 (b), if the statistical distributions for the weight $P_1, P_2, \cdots$ and the spacing $\lambda_1, \lambda_2, \cdots$ of vehicles are given. On the other hand, the dynamic deflection $w_d(t)$ can be estimated by the random vibration theory, provided that the random properties of dynamical forces $F_1(t), F_2(t), \cdots$ due to the vehicles are also known.

To find these fundamental data of irregularities, the field experimental studies were carried out as below.

(1) Probability Curve of Vehicle Weight

Though there are many methods to measure the weight of vehicle in the actual traffic flows as devised by the references 1)~2), the most simple method will be performed by investigating the stresses or deflections of a bridge with comparatively short span as a load cell transducer. A set of these records can easily be converted into the original data for the static loads $P_1, P_2, \cdots$ of Fig. 1 (b), when the relationships between weight of vehicles and stresses or deflections of the bridge are calibrated by the various test vehicles previously. So, the statistical properties of $P_1, P_2, \cdots$ can be deduced by the theory of statistics.

Devoting our attentions to this concept the observations of traffic flow were conducted on a RC bridge (span 15.0 m, width 9.0 m) in Nishi-meihan National Highway in summer 1971. As this highway is an important route for connecting the industrial district in Osaka with that in Nagoya and the heavy trailers often pass through this route, the practical information is obtained as follows.

Figure 2 shows the probability curve plotted as the histogram from the total numbers of observations $M=124$ without the light weight vehicles less than 2 tons. As the abscissa of this graph is taken by the logarithmic scale, the probability curve seems not normal but logarithmic normal distributions. Then, the mean value $\bar{P}$ and the standard deviation σ_P in the logarithmic scale are given by

$$\left.\begin{aligned} &\log \bar{P}=1.176 \\ &\log \sigma_P=0.152 \qquad (P:\ \text{in ton}) \end{aligned}\right\} \cdots\cdots(2)$$

By converting into the ordinally scale as shown in Fig. 2, mean value $\bar{P}$ is deduced as follows:

$$\bar{P}=10^{\log \bar{P}} \cong 15.0 \text{ tons}. \quad \cdots\cdots(3\text{-}1)$$

Maximum load P_{max} and minimum one P_{min} are respectively defined by taking three times of standard deviation $\pm 3 \log \sigma_P$ (probability fall within 99.7%) from mean value $\log \bar{P}$ as is seen in Fig. 2, thus

$$\left.\begin{aligned} P_{max}&=10^{(\log \bar{P}+3\log \sigma_P)} \cong 42.8 \text{ tons} \\ P_{min}&=10^{(\log \bar{P}-3\log \sigma_P)} \cong 5.2 \text{ tons} \end{aligned}\right\} \cdots\cdots(3\text{-}2)$$

The mean value $\bar{P}=15.0$ tons is approximately equivalent to the uniformly distributed load $p=$

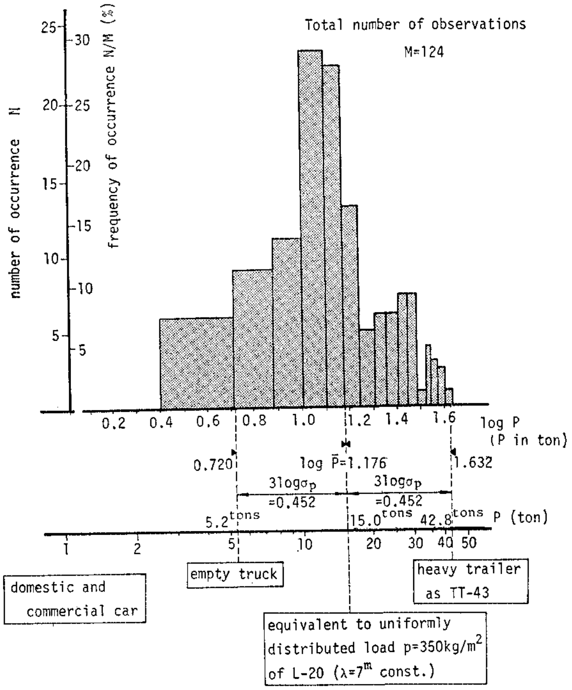


Fig. 2 Probability curve for vehicle weight.

350 kg/m^2 of L-loading provided by the Japanese specification for highway bridges when the vehicles are spaced at the constant interval $\lambda=7.0$ m. The maximum load P_{max} may correspond to the heavy trailer as TT-43, the minimum load P_{min} being to the empty trucks.

A set of these random values $P_1, P_2, \cdots$ in accordance with Eqs. (2)~(3) can easily be simulated by the computer program as shown in appendix.

(2) Power Spectral Density of Random Force

The dynamic forces $F_1(t), F_2(t), \cdots$ due to individual vehicle can be put as a resultant force $F(t)$ as shown in Fig. 1 (c) by combining the various dynamic factors of vehicles. The magnitude of this force $F(t)$ will be given by the following equilibrium condition of the dynamic actions for the bridge as idealized in Fig. 3.

$$F(t)=M_B\ddot{w}_d+C_B\dot{w}_d+K_Bw_d \quad \cdots\cdots\cdots(4)$$

where

$$\left.\begin{aligned}&\text{effective mass of bridge:} && M_B=\frac{1}{2}ql\\ &\text{stiffness of bridge} && : K_B=\frac{48EI}{l^3}\\ &\text{damping of bridge} && : C_B=2f_Bd_BM_B\end{aligned}\right\} \quad \cdots\cdots(5)$$

(l; span, q; dead load per unit length, EI; flexual rigidity, f_B; natural frequency of bridge$=\sqrt{K_B/M_B}/2\pi$, d_B; logarithmic damping) for a straight and simply supported girder bridge at mid-span.

Therefore, the dynamic force $F(t)$ can be obtained experimentally by the observations of acceleration $\ddot{w}_d$, velocity $\dot{w}_d$ and deflection w_d, being found on the device represented by the block diagram as shown in Fig. 4.

For these purpose, the tests were, now, conducted on a composite girder bridge having com-

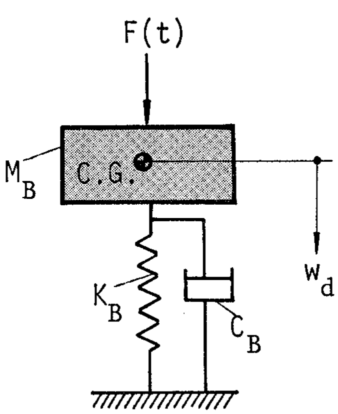


Fig. 3 Simplified vibration system for bridge.

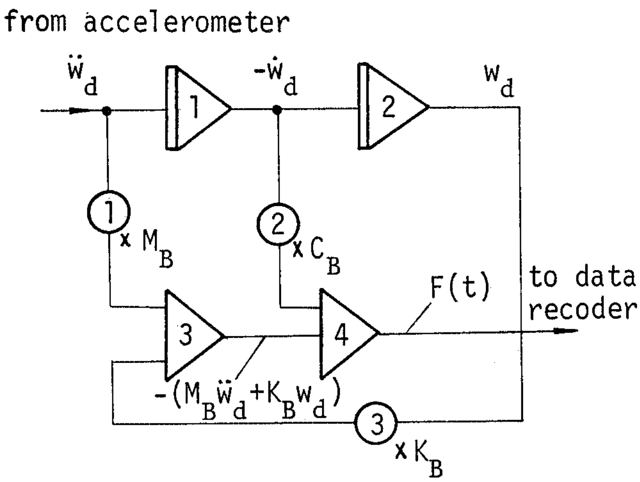
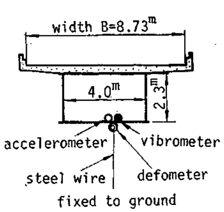
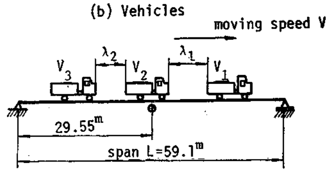


Fig. 4 Block diagram to compute $F(t)$.

(a) Test bridge



(b) Vehicles



dead load intensity q	100.6 kg/cm
moment of ineatia I_y	$5.529\times10^7 cm^4$
natural frequency f_B	1.51Hz
logarithmic damping d_B	0.07

item \ vehicle	V_1	V_2	V_3
weight (t)	9.31	12.95	7.68
natural frequency (Hz)	4.2	3.7	5.8
speed(km/h)	40.0	40.0	40.0
spacing λ(m)	12.0	12.0	

(C) Deflection history w(t)

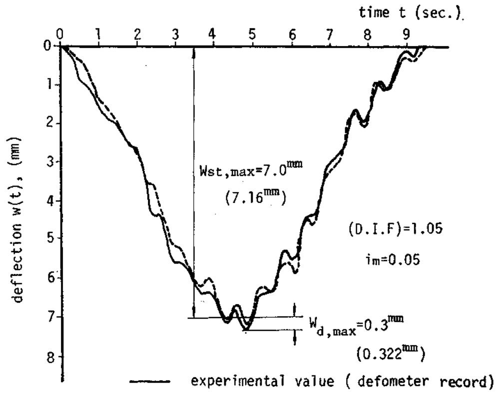
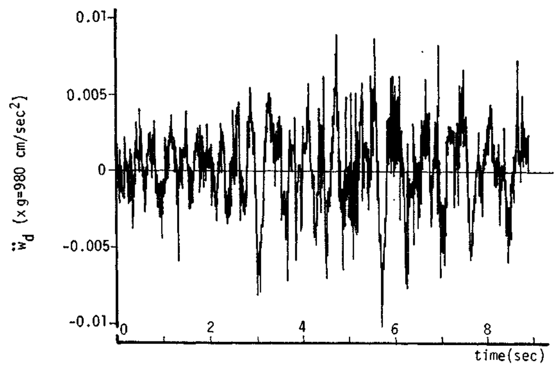
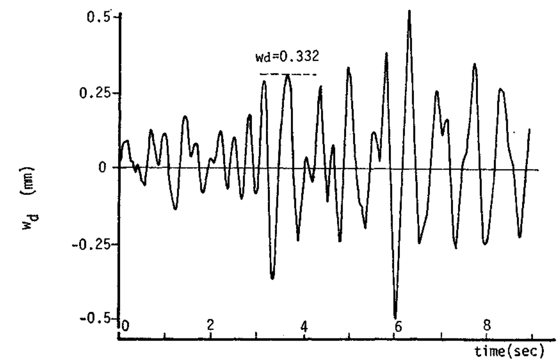


Fig. 5 Experiment under three moving vehicles.



(a) Acceleration

accelerometer record (frequency range 0.5~80c/s for ±2g)



(b) Deflection

vibrometer record (frequency range 0~50 c/s for ±6mm)

Fig. 6 Typical records of bridge vibration under moving vehicles.

paratively long span to make the vehicles drive upon the bridge as much as possible. The cross-sectional quantities of this bridge, and test vehicles are indicated in Fig. 5 (a), (b).

Figure 6 shows the typical records of acceleration and deflection of the test bridge. From these records, the dynamical force $F(t)$ is obtained by the above method as plotted in Fig. 7. Observing this figure, the randomness is so predominant that the dynamical force must be treated statistically.

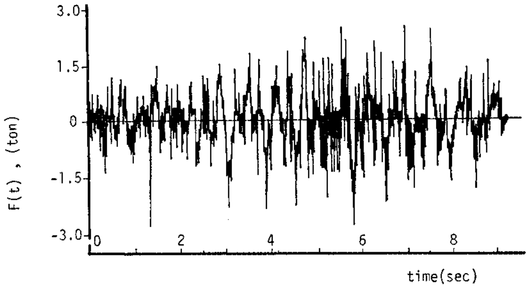


Fig. 7 Random force $F(t)$ (computed by Eq. (4)).

First, the autocorrelation function $R_F(\tau)$ for $F(t)$ under given time lag τ is obtained by

$$R_F(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} F(t)\cdot F(t+\tau)dt \quad \cdots\cdots(6)$$

Next, the power spectrum density $S_F(\omega)$, $\omega=2\pi f$, is calculated by performing Fourier's transformation of $R_F(\tau)$ as

$$S_F(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} R_F(\tau)e^{-i\omega\tau}d\tau \quad \cdots\cdots\cdots\cdots\cdots(7)$$

Since the data are converted into digital ones, $R_F(\tau)$ and $S_F(f)$[11] are calculated by the digital computer based on the block diagram as illustrated in Fig. 8.

According to these procedures, the power spectrum density $S_F(f)$ can be drawn as shown in Fig. 9, in which the spectrum is made smooth curve by the most suitable window.

It is obvious from this figure that the strong power arises near the natural frequencies of vehicles (see table in Fig. 5 (b)). While, the small peak near $f=10$ Hz may be caused by the roughness of roadway surfaces. Moreover, it seems that the power spectrum density $S_F(f)$ under test traffic loads can be put as the constant value within the frequency range $f=1\sim7$ Hz because the corresponding spectral density under the random traffic loads with different natural frequencies will considerably be reduced as indicated by the reference[6]. On the other hand, $S_F(f)$ is decreased gradually for $f>7$ Hz as shown by the dotted line in Fig. 9.

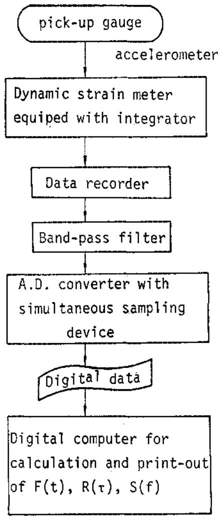


Fig. 8 Data processing procedure.

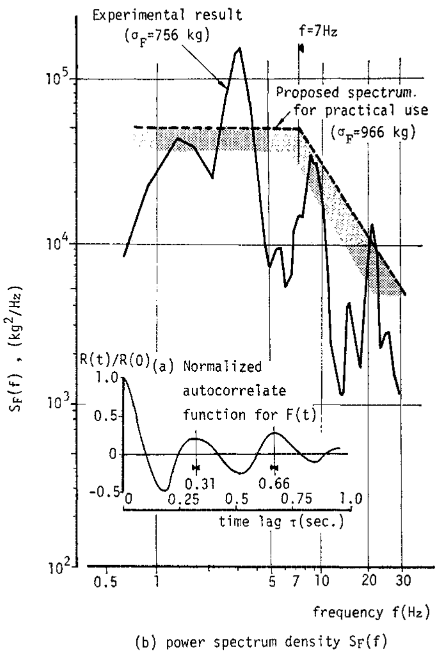


Fig. 9 Statistical property of random force $F(t)$.

Though much more experiments for the various traffic conditions should be carried out, let the power spectrum density of exciting force $F(t)$

Table 1 Comparison of dynamic deflection $w_{d,\max}$

Method		Sourse	Maximum dynamic deflection $w_{d,\max}$	Remarks
Experimental		Observed directly by records of defometer	0.3 mm	See Fig. 5 (c)
		Measured directly by records of vibrometer	0.332 mm	See Fig. 6 (b)
Theoretical	Deterministic	Simulated by using data of vehicles in Fig. 5 (b)	0.322 mm	Forcing terms of vehicles are assumed as $F_j=2$ tons[9] See Fig. 5 (c)
	Statistical	Estimated from spectrum of deflection in Fig. 10	0.335 mm	Max. deflection is deduced as twice value of standard deviation of response $S_{wd}(f)$, that probability fall within 95.45%
		Spectrum analysis by YAMADA and KOBORI[5]	0.356 mm	
		Estimated by approximate formula of spectrum of Eq. (8)	0.415 mm	

be assumed as the following expression for the sake of simplicity.

$$S_F(f)=5.0\times10^4\ \text{kg/Hz} \qquad (1\leq f\leq 7\ \text{Hz})$$
$$=\frac{1.09\times10^8}{f^{1.59}}\ \text{kg/Hz} \qquad (f>7\ \text{Hz}) \qquad \cdots\cdots(8)$$

To ensure the approximation of formula (8),

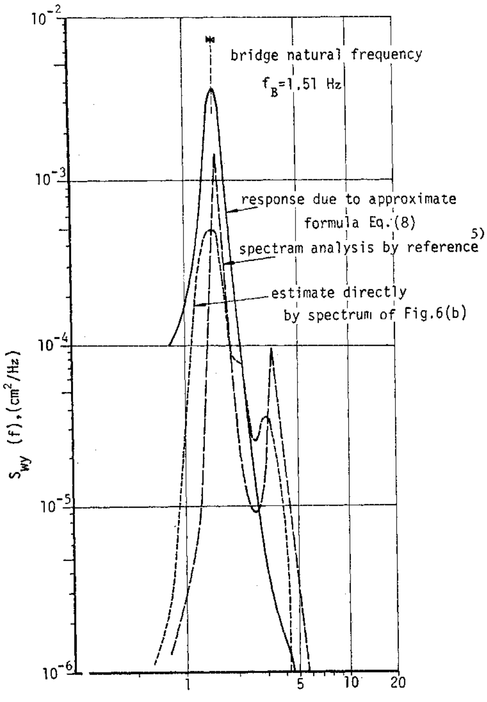


Fig. 10 Power spectrum density of response $S_{wy}(f)$.

the estimations of corresponding dynamic response, i.e. the maximum dynamic deflection $w_{d,\max}$ should be checked by the various experimental data and theoretical calculations. These values are summarized together as shown in Table 1, where the maximum dynamic deflection obtained by the statistical theory are deduced as twice values of the standard deviation calculated by spectrum $S_{wd}(f)$ that makes the probability fall within 95.45%.

With the comparisons of these results, the maximum dynamic deflections $w_{d,\max}$ have the tendency to coincide with another results, but the value estimated by an approximate formula Eq. (8) is somewhat greater than the other. It is caused by the power spectrum density of random force $S_F(f)$ takes as much as secure value to be applied for the practical purposes. Figure 10 shows the power spectrum density of response obtained by the various methods.

3. RESPONSE OF HORIZONTALLY CURVED GIRDER BRIDGE DUE TO RANDOM FORCE

The dynamic of a horizontally curved girder bridge may be estimated by dividing into static and dynamic components similar to the case of straight girder bridges as previously mentioned.

(1) Static Response

The static deflection $w_s(t)$ of a curved girder bridge due to moving loads $P_1, P_2, \cdots$ with the constant speed V may approximately be found in the reference[8] on the basis of the energy method.

For a curved girder bridge, however, the deformation can not be decided only by the ver-

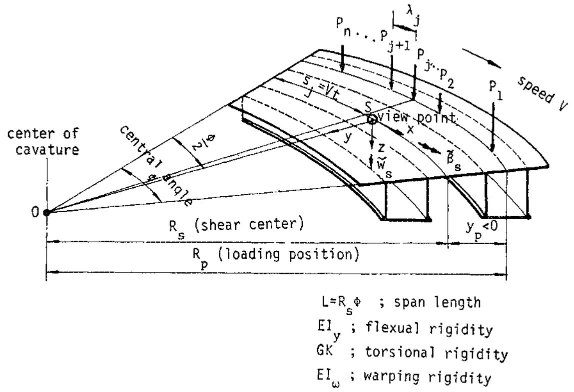


Fig. 11 Statical loading condition.

tical deflection $\bar{w}_s(t)$, but also by the angle of rotation $\bar{\beta}_s(t)$.

These values can be estimated by referring the sympols in Fig. 11 as follows;

$$\left.\begin{aligned}\bar{w}_s(t)&=\sum_{j=1}^{n}\left[\frac{R_p}{R_s}\cdot\frac{P_j}{K_{B,ww}}\cdot\frac{1-y_pk_\beta}{1-k_wk_\beta}\cdot U_j(s_j)\cdot\sin\left(\frac{R_p}{R_s}\cdot\frac{\pi s_j}{L}\right)\right]\\ \bar{\beta}_s(t)&=\sum_{j=1}^{n}\left[\frac{R_p}{R_s}\cdot\frac{P_j}{K_{B,\beta\beta}}\cdot\frac{y_p-k_w}{1-k_wk_\beta}\cdot U_j(s_j)\cdot\sin\left(\frac{R_p}{R_s}\cdot\frac{\pi s_j}{L}\right)\right]\end{aligned}\right\}\quad\cdots(9)$$

where $K_{B,ww}$, $K_{B,w\beta}$ and $K_{B,\beta\beta}$ designate the stiffness with respect to the mid-span of simply supported curved girder bridge, being determined by

$$\left.\begin{aligned}K_{B,ww}&=\frac{EI_y}{2L^3}\{\pi^2(\pi^2+\Phi^2\gamma)\}\\ K_{B,w\beta}&=\frac{EI_y}{2L^2}\{\pi^2\Phi(\gamma+1)\}\\ K_{B,\beta\beta}&=\frac{EI_y}{2L}(\pi^2\gamma+\Phi^2)\end{aligned}\right\}\quad\cdots(10)$$

and

$$\left.\begin{aligned}k_w&=L\Phi\cdot\frac{\gamma+1}{\pi^2+\gamma\Phi^2}\\ k_\beta&=\frac{\pi^2\Phi}{L}\cdot\frac{\gamma+1}{\gamma\pi+\Phi^2}\end{aligned}\right\}\quad\cdots(11)$$

in which

$$\gamma=\frac{GK+EI_\omega(\pi/L)^2}{EI_y}\quad\cdots(12)$$

means the ratio between effective torsional and flexual rigidities.

$U_j(s_j)$ is the unit step function defined by the following form.

$$\left.\begin{aligned}U_j(s_j)&=1, &&\text{for } (0\leq s_j\leq L)\\ &=0, &&\text{for } (s_j<0,\ s_j>L)\end{aligned}\right\}\quad\cdots(13)$$

Consequently, the static deflection at the position with the eccentricity y from the shear center S can be decided by

$$w_s(t)=\bar{w}_s(t)+y\cdot\bar{\beta}_s(t)\quad\cdots(14)$$

(2) Dynamic Response

The dynamic response of a horizontally curved girder bridge can be estimated by the model as shown in Fig. 12 instead of Fig. 3. Owing to the reference 8), the simultaneous differential equations with respect to the dynamic defiection $\tilde{w}_d$ and angle of rotation $\tilde{\beta}_d$ at the mid-span of curved bridge are given by d'Alembert's principle as follows;

$$\left.\begin{aligned}&M_B\ddot{\tilde{w}}_d+C_{B,w}\dot{\tilde{w}}_d+K_{B,ww}\tilde{w}_d+Z_B\ddot{\tilde{\beta}}_d+K_{B,\beta w}\tilde{\beta}_d=\frac{R_p}{R_s}F(t)\\ &I_B\ddot{\tilde{\beta}}_d+C_{B,\beta}\dot{\tilde{\beta}}+K_{B,\beta\beta}\tilde{\beta}_d+Z_B\ddot{\tilde{w}}_d+K_{B,w\beta}\tilde{w}_d=\frac{R_p}{R_s}y_pF(t)\end{aligned}\right\}\quad\cdots(15)$$

$$(\dot{}=d/dt)$$

The notations M_B, I_B, and Z_B denote the mass terms, and are obtained approximately by

$$\left.\begin{aligned}M_B&=\frac{1}{2}mA_sL+\frac{L}{2g}\cdot\sum_{j=1}^{n}\frac{P_{j,l}}{\lambda_j}\\ I_B&=\frac{1}{2}mI_sL+\frac{L}{2g}y_p^2\cdot\sum_{j=1}^{n}\frac{P_{j,l}}{\lambda_j}\\ Z_B&=\frac{1}{2}mS_zL+\frac{L}{2g}y_p\cdot\sum_{j=1}^{n}\frac{P_{j,l}}{\lambda_j}\end{aligned}\right\}\quad\cdots(16)$$

where the cross-sectional quantities of curved bridge are defined by referring Figs. 11~12 as

weight and mass per unit volume;

$$\rho,\quad m=\frac{R_G}{R_s}\cdot\frac{\rho}{g}$$

cross-sectional area;

$$A_s,\quad A_z=A_s-\frac{S_z}{R_s}$$

polar moment of inertia with respect to G and S;

$$I_G,\quad I_s=I_G+A_s(y_G^2+z_G^2)$$

statical moment with respect to z-axis;

$$S_z=A_sy_G,\quad S_z'=S_z-\frac{I_s}{R_s}$$

acceleration of gravity;

$$g=980\ \text{cm/sec}^2\quad\cdots(17)$$

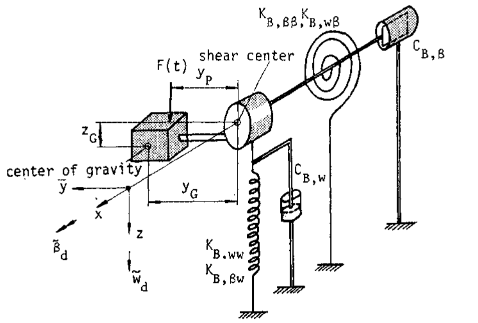


Fig. 12 Simplified vibration system for curved girder bridge.

The second terms $P_{j,l}/g$ of right hand-side of Eq. (16) are due to the effect of unsprung mass for the idealized vehicles as is seen in Fig. 1 (a).

Remaing damping terms $C_{B,w}$, $C_{B,\beta}$ may be represented in the following form as

$$\left.\begin{aligned} C_{B,w}&=2M_B d_{B,\mathrm{I}} f_{B,\mathrm{I}} \\ C_{B,\beta}&=2I_B d_{B,\mathrm{II}} f_{B,\mathrm{II}} \end{aligned}\right\} \quad \cdots\cdots(18)$$

Hence, $f_{B,\mathrm{I}}$ and $f_{B,\mathrm{II}}$ are the natural frequencies of curved girder gained by

$$f_{B,\mathrm{I\,or\,II}}=\sqrt{(b\mp\sqrt{b^2-4ac}/2a)}/2\pi \quad \cdots\cdots(19)$$

provided that

$$\left.\begin{aligned} a&=A_sI_G \\ b&=A_zI_s(p^2_{ww}+p^2_{\beta\beta})-S_zS'_z(p^2_{\beta w}+p^2_{w\beta}) \\ c&=A_zI_sp^2_{ww}-S_zS'_zp^2_{\beta w}p^2_{w\beta} \end{aligned}\right\} \quad \cdots\cdots(20)$$

and

$$\left.\begin{aligned} p^2_{ww}&=EI_y\{\pi^2(\pi^2-\Phi^2)\}/(mA_zL^4) \\ p^2_{\beta w}&=EI_y\{\Phi(\pi^2-\Phi^2)\}/(mS'_zL^3) \\ p^2_{w\beta}&=EI_y\{\pi^2\Phi(\gamma+1)\}/(mS_zL^3) \\ p^2_{\beta\beta}&=EI_y(\gamma\pi^2+\Phi^2)/(mI_sL^2) \end{aligned}\right\} \quad \cdots\cdots(21)$$

The symbols $d_{B,\mathrm{I}}$ and $d_{B,\mathrm{II}}$ are the logarithmic decrement factors corresponding to the free vibrations with the frequencies $f_{B,\mathrm{I}}$ and $f_{B,\mathrm{II}}$ respectively.

Let us now consider the dynamic force $F(t)$ of the right hand-side of Eq. (15) defined Dirac's delta function as follows;

$$F(t)=\delta(t) \quad \cdots\cdots(22)$$

and the initial condition at time $t=0$ are assumed as

$$\left.\begin{aligned} \tilde{w}_d(0)&=\dot{\tilde{w}}_d(0)=0 \\ \tilde{\beta}_d(0)&=\dot{\tilde{\beta}}_d(0)=0\,, \end{aligned}\right\} \quad \cdots\cdots(23)$$

to estimate the steady state solutions of Eq. (15).

By taking into the considerations of these equations (22)∼(23), the expanded deformations $W_d(i\omega)$ and $B_d(i\omega)$ by means of Fourier's transformations with frequency ω (rad./sec) and $i=\sqrt{-1}$ are rewritten a set of the following equations.

$$\left.\begin{aligned} &[(i\omega)^2M_B+i\omega C_{B,w}+K_{B,ww}]\cdot W_d(i\omega) \\ &\quad+[(i\omega)^2Z_B+K_{B,\beta w}]\cdot B_d(i\omega)=\frac{R_p}{R_s} \\ &[(i\omega)^2Z_B+K_{B,w\beta}]\cdot W_d(i\omega) \\ &\quad+[(i\omega)^2I_B+i\omega C_{B,\beta}+K_{B,\beta\beta}]\cdot B_d(i\omega)=\frac{R_p}{R_s}\,y_p \end{aligned}\right\} \quad \cdots\cdots(24)$$

The solutions of this simultaneous equations give

$$\left.\begin{aligned} W_d(i\omega)&=\frac{R_p}{R_s}\cdot\frac{h_{\beta\beta}-h_{w\beta}y_p}{h_{ww}h_{\beta\beta}-h^2_{w\beta}} \\ B_d(i\omega)&=\frac{R_p}{R_s}\cdot\frac{h_{ww}y_p-h_{w\beta}}{h_{ww}h_{\beta\beta}-h^2_{w\beta}} \end{aligned}\right\} \quad \cdots\cdots(25)$$

where

$$\left.\begin{aligned} h_{ww}&=(i\omega)^2M_B+i\omega C_{B,w}+K_{B,ww} \\ h_{w\beta}&=(i\omega)^2Z_B+K_{B,\beta w} \\ h_{\beta\beta}&=(i\omega)^2I_B+i\omega C_{B,\beta}+K_{B,\beta\beta} \end{aligned}\right\} \quad \cdots(26)$$

In the similar way to the case of static load, the response function $H_B(i\omega)$ at an arbitrary point with the eccentricity y from the shear center S can also be put as

$$H_B(i\omega)=W_d(i\omega)+y\cdot B_d(i\omega) \quad \cdots\cdots(27)$$

Then, the weighting function $W_B(\omega)$ of the curved girder bridge can easily be obtained by the product of $H_B(i\omega)$ with it's conjugate function $\overline{H_B(i\omega)}$ as follows;

$$W_B(\omega)=H_B(i\omega)\cdot\overline{H_B(i\omega)}=|H_B(i\omega)|^2 \quad \cdots(28\text{-}1)$$

or with $\omega=2\pi f$

$$W_B(f)=|H_B(i2\pi f)|^2 \quad \cdots\cdots(28\text{-}2)$$

Consequently, the power spectrum density $S_{wd}(f)$ for the dynamic deflection $w_d(t)$ can be estimated by the following expression.

$$\begin{aligned} S_{wd}(f)&=W_B(f)\cdot S_F(f) \\ &=|H_B(i2\pi f)|^2\cdot S_F(f) \end{aligned} \quad \cdots\cdots(29)$$

(3) Estimation of Impact

The dynamic increment factor (D.I.F.) for a horizontally curved girder bridge may derived on the definition analogous to that for a straight girder bridge as follows;

$$(\mathrm{D.I.F.})=\frac{w_d}{w_s} \quad \cdots\cdots(30)$$

From this equation, the coefficient of impact can be set as w_d/w_s. The rational impact, however, shall be determined for the condition where the vehicles are loaded throughout the bridge (see Fig. 5 (c)).

Since the static deflection w_s due to random traffic loads $P_1, P_2, \cdots, P_n$ shown in Fig. 1 (b) is not a deterministic value, the following statistical treatments must be needed. First, the various combinations of random loads $(P_1, P_2, \cdots, P_n)_1$, $(P_1, P_2, \cdots, P_n)_2, \cdots, (P_1, P_2, \cdots, P_n)_k$ are generated by the computer program in appendix. Then, the corresponding maximum static deflections $w_{s,1}, w_{s,2}, \cdots, w_{s,k}$ are calculated by Eq. (9). For the most probable value of the static deflection w_s to be used in Eq. (30) will be decided by

$$w_s = \boldsymbol{E}[w_s] = \frac{1}{k}\sum_{j=1}^{k} w_{s,j} \quad \cdots\cdots(31)$$

On the other hand, the dynamic deflection w_d will be deducted by

$$\sigma_{wd} = \sqrt{\int_0^{\infty} S_{wd}(f)df} \quad \cdots\cdots(32)$$

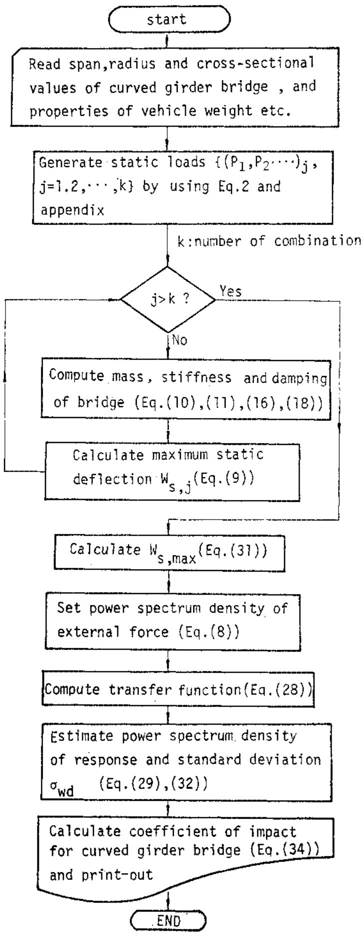


Fig. 13 Flow chart for computation of impact subjected to random force.

and by taking the twice values of this standard deviation σ_{wd} for the dynamic deflection w_d as illustrated in Table 1, thus

$$w_d = \boldsymbol{E}[w_d] = 2\sigma_{wd} \quad \cdots\cdots(33)$$

The coefficient of impact i_m will be, thereby, put as

$$i_m = \frac{2\sigma_{wd}}{w_s} \quad \cdots\cdots(34)$$

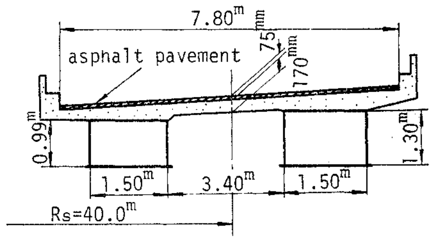
This expression will be resonable, because the determination of impact is not the problem for determining the absolute amplitude of vibration such as in the seismic load, but the relative one for clarifying the dynamic effect of live load as the ratio of dynamic deflection to static one.

Figure 13 shows the flow chart for computation of impact coefficient for horizontally curved girder bridges under random traffic loads.

4. NUMERICAL EXAMPLES

In order to investigate static and dynamic deflections of horizontally curved girder bridge under random traffic loads, a few numerical examples are carried out. Fig. 14 shows a cross-section of curved girder bridge used in these examples.

The static loads $P_1, P_2, \cdots, P_{1000}$ are simulated by using Eq. (2) and computer program shown in appendix, and divided into fifty traffic flows consisting of twenty vehicles $(P_1, P_2, \cdots, P_{20})_1$, $(P_1, P_2, \cdots, P_{20})_2, \cdots, (P_1, P_2, \cdots, P_{20})_{50}$. The inter-



span L	30.0^m
radius Rs	40.0^m
As	$1.271\times10^4 cm^2$
Sz	$5.353\times10^5 cm^3$
Iy	$1.383\times10^8 cm^4$
Is	$4.220\times10^9 cm^4$
K	$9.990\times10^6 cm^4$
Iω	$8.214\times10^{11} cm^6$

Fig. 14 Cross-section of typical bridge.

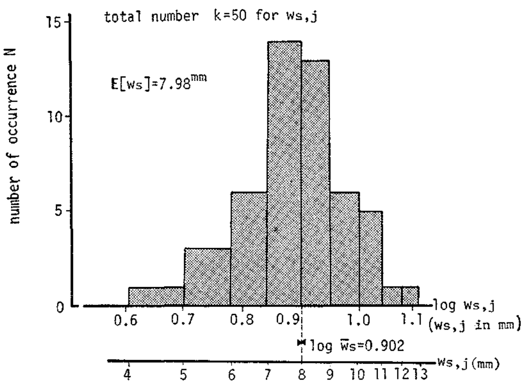


Fig. 15 Probability curve for static deflection.

vals of vehicles are assumed as constant $\lambda=10$ m.

For these set of vehicles, the probability curve of maximum static deflection $w_{s,1}, w_{s,2}, \cdots, w_{s,50}$ is shown in Fig. 15. As the abscissa of this graph is taken by the logarithmic scale, the probability curve of $w_{s,j}$ ($j=1\sim50$) is also logarithmic normal distribution.

Figures 16~17 shows the transfer function $W_B(f)$ and the power spectrum density $S_{wd}(f)$ of dynamic deflection for this bridge respectively.

Next, the relationships between impact coeffi-

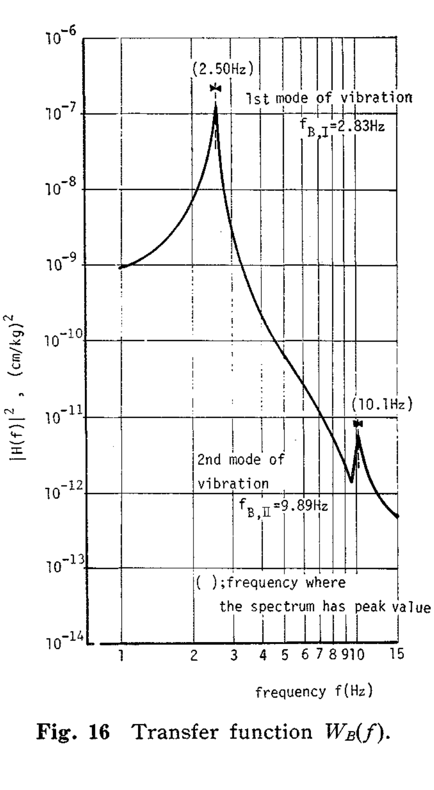


Fig. 16 Transfer function $W_B(f)$.

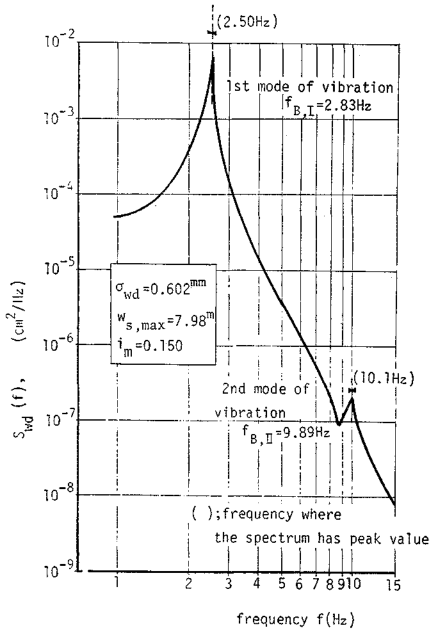


Fig. 17 Power spectrum density of deflection $S_{wd}(f)$.

cient and span length for various types of curved girder bridge[9)] calculated by this method are plotted as shown in Fig. 18, in which the symbol × denotes multiple curved I-girder bridges, the symbol ○ being curved box girder bridges.

In this figure, the values of impact provided by the specifications as well as the proposed formulae in references 6) and 9) are also plotted.

Especially, the following expression is a proposed formula for the horizontally curved girder bridges by the authors on the basis of deterministic simulation method;

$$i_m=0.5\left(\frac{R_s}{R_p}\right)\frac{10}{L}\leq 0.4 \quad \cdots\cdots\cdots\cdots(35)$$

where R_s: radius of curvature in shear center
R_p: radius of curvature of loading position
$L=R_s\Phi$: span length of horizontally curved girder bridge
Φ : central angle (see Fig. 11)

Comparing Eq. (35) with the values obtained by this random vibration method, Eq. (35) is somewhat under-estimated for comparatively short span $L\cong20$ m, whereas Eq. (35) is over-estimated for large span $L>35$ m. For short span, it seems that the random vibrations due to the roughness of road-way surfaces with high frequencies $f=7\sim10$ Hz more or less affect upon

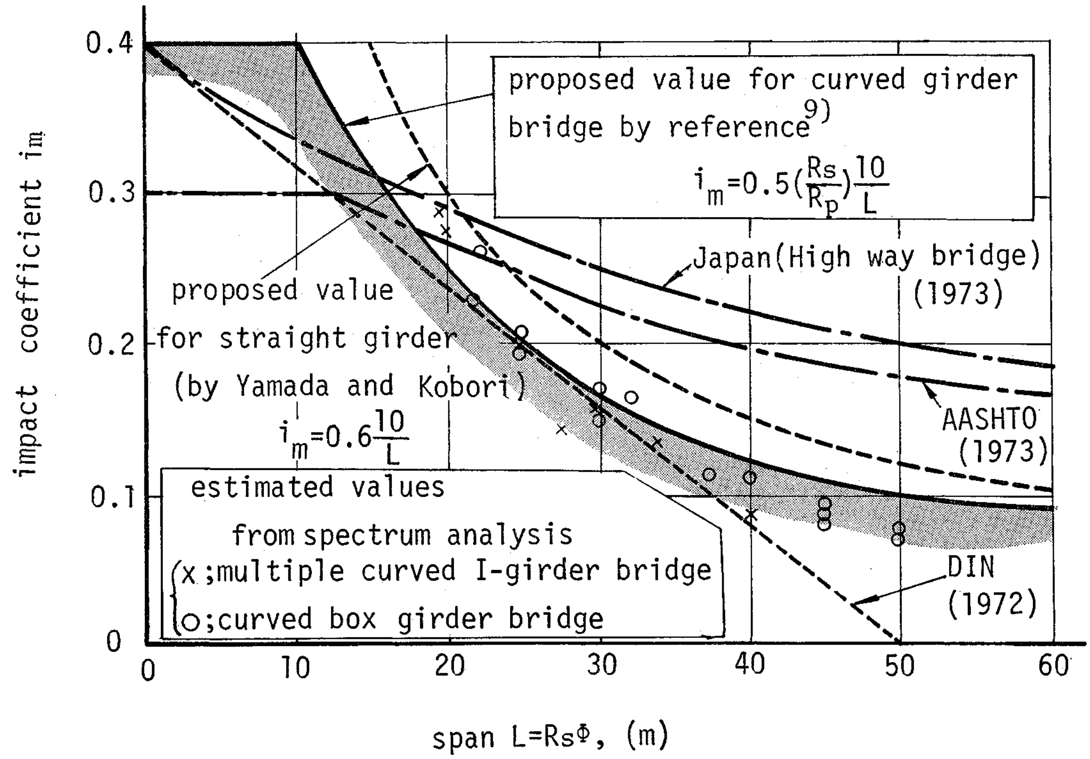


Fig. 18 Impact coefficient.

the dynamic deflections w_d of curved bridge, since the natural frequencies of bridge $f_{B,\mathrm{I}}$ and $f_{B,\mathrm{II}}$ in Eq. (19) take same order as this frequency range.

To investigate these phenomena, we should undertake much more field experimental studies under the various traffic conditions, for the results of Fig. 18 depend upon the correct data of statistical properties of traffic flows.

5. CONCLUSION

This paper deals with the random vibration of the horizontally curved girder bridges. The main conclusions are summarized as follows;

(1) The probablility curve for the weight of vehicles is estimated as shown in Fig. 2 from the field observations of traffic flows, and power spectrum density of exciting force is approximately determined by Eq. (8). These formulae are checked by the various methods on experimental and theoretical studies.

(2) The practical equations for determining dynamic response of curved girder bridges under random traffic loads are derived and a few numerical calculations are carried out.

(3) Comparisons of impact by this method with the reference 9) show that the random vibrations due to the roughness of roadway surfaces should be considered for the horizontally curved girder bridges with comparatively short span.

(4) As the results of these analyses depend entirely upon the correct data for statistical properties of traffic flows, much more experiments for various traffic conditions should be conducted.

ACKNOWLEDGEMENTS

In writing this paper the authors are grateful to the Japan Ministry of Education for the scientic research funds. Moreover, the authors wish to thank Professor Dr. S. Komatsu of Osaka University for his helpful advice. Our thanks are also due to Japan Highway Public Cooperation and Hanshin Highway Public Cooperation in experimental study.

REFERENCES

1) Novak, M. E., Jr. C. P. Heins and C. T. G. Looney: Induced Dynamic Strains in Bridge Structures due to Random Truck Loadings,

J-74

Maryland State Road Commission and U.S. Bureau of Public Road, Feb., 1968.

2) Cudney, G. R.: Stress Histories of Highway Bridges, Proc. of ASCE, Vol. 94, ST12, pp. 2725-2737, Dec., 1968.

3) Tan, C. P. and S. Shore: Response of Horizontally Curved Bridges to Moving Load, Proc. of ASCE, Vol. 94, ST9, pp. 2135-2151, 1968.

4) Fryba, L.: Vibration of Solids and Structures under Moving Loads, Nordhoff, pp. 410-427, 1971.

5) Yamada, Y. and T. Kobori: Dynamic Response of Highway Bridges due to Live Load by Spectral Analysis, Trans. of JSCE No. 148, pp. 40-50, Dec., 1967.

6) Yamada, Y. and T. Kobori: Studies on Highway Bridge Impact due to Random Moving Vehicles, Trans. of JSCE No. 119, pp. 1-9, July, 1965.

7) Komatsu, S. and H. Nakai: Study on Free Vibration of Curved Girder Bridge, Trans. of JSCE, No. 136, pp. 35-60, 1966.

8) Komatsu, S. and H. Nakai: Fundamental Study on Forced Vibration of Curved Girder Bridge, Proc. of JSCE, No. 174, pp. 41-55, Feb., 1970.

9) Komatsu, S., H. Nakai and H. Kotoguchi: Study on Dynamic Response and Impact of Horizontally Curved Girder Bridges under Moving Vehicles, Proc. of JSCE, Vol. 192, pp. 55-68, Aug., 1971.

10) Nakai, H., H. Kotoguchi and R. Ominami: Dynamic Response of Highway Bridges under Traffic Loads, Memoirs of the Faculty of Engineering, Osaka City Univ., Vol. 12, pp. 219-237, Dec., 1971.

11) Jenkins, G. and P. Watts: Spectral Analysis and its Application, Holden-Day, Aug., 1969.

12) Blackman, R. B. and J. W. Tukey: The measurements of power spectra, Dover, 1958.

Appendix—Computer program for generating the static loads $P_1, P_2, \cdots, P_{1000}$

```
   DIMENSION A(1000)
   READ(5,10) IX,AM,SD
10 FORMAT(I5,2E10.3)
   DO 20 I=1,1000
   CALL NORRNS(AM,SD,IX,RY)
   A(I)=10.0**RY
20 CONTINUE
   WRITE (6,30)
30 FORMAT (1H1,//,30X,17HTRAFFIC LOAD (KG),/)
   WRITE (6,40) (A(I),I=1,1000)
40 FORMAT (5(4X,E16.5))
   STOP
   END
```

Remarks

IX : initial value less than 32767

AM : mean value ($\log \bar{P}$)

SD : standard deviation ($\log \sigma_P$) ……………………Ref. Eq. (2)

A(I) : answer for static loads $P_1, P_2, \cdots, P_{1000}$

(*Received March 19, 1975*)