PROC. OF JSCE,
No. 238, JUNE, 1975

A BASIC STUDY ON THE CONTROLLABILITY OF THE REGIONAL INCOME DISPARITY ARISING FROM THE SECOND OPTIMAL POLICY

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1. INTRODUCTION

The allocation of public facilities among regions is closely related to the promotion of social capital formation in each region. This is becoming one of the most important problems for civil planners constructing public facilities.

However, only a relatively small amount of research has been made on the regional allocation of public investments. In the literature, the regional development models recently were developed by Rahman-Sakashita.[1),2)]

One of the most important problems in the regional allocation of public investment is the regional income disparities. But, the detailed research of the regional income disparities has not been made.

In the author's paper 3), we have shown the detailed simulations of the regional income disparities concentrating on the minimum proportion of public investment and the regional rates of saving. In addition, we considered the second optimal policy in which the concept of distributive justice is introduced into the regional development model.

In this paper, we shall formulate a more generalized model arising from the second optimal policy and the local autonomy rate, and consider one theorem, four corollaries and the detailed simulations concentrating on the controllability of the minimum proportion of investment.

2. MATHEMATICAL FORMATION OF MODELS

This chapter presents the mathematical formulation of the regional development model arising from the second optimal policy, and a theorem and four corollaries on the controllability of the minimum proportion of investment.

First, we shall consider the mathematical formulation of regional development model arising from the second optimal policy which holds the following conditions.

(1) The allocation of regional investment is aimed at maximizing the total outputs when the outputs of each region should not bring about any wide disparity at the end of the planning period.

(2) The supply of funds for investment will be limited to the sum of savings in each region.

(3) The productivity of investment, saving ratio and local autonomy rate are given through central government.

(4) The investment for the dissolution of the maximum income disparity is given by the mutual consents of all regions.

The analysis is an explicit planning model for a closed economy and it is assumed that the planned saving equals the planned investment through the central government.

We define the notations as follows:

P_j^i : the productivity of investment of region j at i time.

S_j^i : the saving ratio of region j at i time.

U_j^i: the proportion of investment shared to the region j at i time.

$$\left(\sum_j^M U_j^i=1,\ i=1,\cdots,N\right) \qquad (1)$$

r : the local autonomy rate.

M : the number of regions.

N : the planning period time.

X_j^i: the regional income of region j at i time.

$$(X_j^i-X_j^{i-1}\geq 0,\ i=1,\cdots,N,\ j=1,\cdots,M) \qquad (2)$$

$C_j=X_j^0$: the regional income of region j at initial time.

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D_j^i: the minimum proportion of investment region j at i time.

$$(0 \leq D_j^i \leq 1/M) \quad \cdots\cdots (3)$$

Z^i : the national income at i time.

$$Z^i = \sum_j^M X_j^i, \quad (i=1, \cdots, N) \quad \cdots\cdots (4)$$

$$\text{Min } X_j^1: \left\{(1+r \cdot P_j^1 \cdot S_j^0) \cdot X_j^0 + P_j^1 \cdot (1-r)\left(\sum_{j=1}^M S_j^0 \cdot X_j^0\right) \cdot D_j^1\right\} \quad \cdots\cdots (5)$$

$$\text{Max } X_j^1: \left\{(1+r \cdot P_j^1 \cdot S_j^0) \cdot X_j^0 + P_j^1 \cdot (1-r)\left(\sum_{j=1}^M S_j^0 \cdot X_j^0\right)\left(1-\sum_{k \neq j} D_k\right)\right\} \quad \cdots\cdots (6)$$

Min X_j^1 (Max X_j^1) represents the minimum (maximum) value of the regional income of region j, at initial time plus the regional income of region j based on the local government investment and based on the central government investment.

The performance equations from condition (2) are as follows:

$$\sum_{j=1}^M (X_j^i - X_j^{i-1})/P_j^i = \sum_{j=1}^M S_j^{i-1} \cdot X_j^{i-1} \quad (i=1, \cdots, N) \cdots\cdots (7)$$

Where

$$X_j^i - X_j^{i-1} = P_j^i \cdot U_j^i \cdot (1-r)\left(\sum_{j=1}^M S_j^{i-1} \cdot X_j^{i-1}\right) + P_j^i \cdot r \cdot S_j^{i-1} \cdot X_j^{i-1} \quad \cdots\cdots (8)$$

The left-hand side represents total investment and the right-hand side represents total saving in the whole country at i time.

The boundary conditions are as follows:

$$C_j = X_j^0 \quad \cdots\cdots (9)$$

$$D_j^i \leq U_j^i \leq 1 - \sum_{k \neq j} D_k \quad \cdots\cdots (10)$$

The performance equations from condition (1) are as follows:

$$X_1^N = \cdots = X_M^N \quad \cdots\cdots (11)$$

$$J = Z^N \to \text{Max} \quad \left(Z^N = \sum_{j=1}^M X_j^N\right) \quad \cdots\cdots (12)$$

D_l : the limit point of controllability.

$[0, D_l]$: the feasible region of controllability.

With respect to the detail conception of computation and algorithm for the model, the reader may refer to the author's papers.[3),4),5)]

Next, we shall consider the controllability of the minimum proportion of investment. That is, whether or not the model is to be controlled depends on the increase of the minimum proportion of investment. And it is described in the theorem which follows.

Theorem

Assume that $S_j^i = S_j$, $D_j^i = D_j$ and $P_j^i = P_j$ $(i=1, \cdots, N,\ j=1, \cdots, M)$. The controllability of the minimum proportion of investment (D_j) is not realized if at least one of the following 2M cases such that $X_j^1 \geq \text{Min } X_j^1$ and $X_j^1 \leq \text{Max } X_j^1$ does not hold.

Proof

First, we shall translate the model into the following equations. The equalities (8) and inequalities (10) can be replaced in terms of inequalities of X_j^i variables instead of U_j^i variables.

$$X_j^i - (1+P_j \cdot r \cdot S_j^{i-1}) \cdot X_j^{i-1} - D_j \cdot P_j \cdot (1-r)\left(\sum_{j=1}^M S_j^{i-1} \cdot X_j^{i-1}\right) \geq 0 \quad \cdots\cdots (13)$$

$$X_j^i - (1+P_j \cdot r \cdot S_j^{i-1}) \cdot X_j^{i-1} - \left(1-\sum_{k \neq j} D_k\right) \cdot P_j \cdot (1-r)\left(\sum_{j=1}^M S_j^{i-1} \cdot X_j^{i-1}\right) \leq 0 \quad (i=1, \cdots, N) \cdots\cdots (14)$$

The equations (1) can be replaced in the following equations.

$$\sum_{j=1}^M \{X_j^i - (1+P_j \cdot r \cdot S_j^{i-1}) \cdot X_j^{i-1}\} = \sum_{j=1}^M P_j \cdot (1-r)\left(\sum_{j=1}^M S_j^{i-1} \cdot X_j^{i-1}\right) \quad (i=1, \cdots, N) \cdots\cdots (15)$$

The boundary conditions are as follows:

$$C_j = X_j^0 \quad \cdots\cdots (16)$$

$$0 \leq D_j \leq 1/M \quad \cdots\cdots (17)$$

$$X_1^N = \cdots = X_M^N \quad \cdots\cdots (18)$$

It is clear from the system of equations that the equations (15) and (18) are the strong restrictions and the set of all feasible solutions of the inequalities (13) and (14) is the convex polyhedron. And the objective function is obtained as the sum of X_j^N at the planning period time N. Furthermore, the optimal solutions of X_j^i are realized in the order of the decreasing sequence such that $N, N-1, \cdots, 1$. Then, whether or not the model is to be controlled depends on the conditions in which X_j^i satisfy the restrictions of the system of equations from (13) to (18) with the characters mentioned above. And the restrictions on X_j^1 are the following 2M cases such that $X_j^1 \geq \text{Min } X_j^1$ and $X_j^1 \leq \text{Max } X_j^1$. Q.E.D.

Next, we shall attempt to examine in more detail the structure of the controllability of D_j with most of the emphasis of the two-region case. The following corollaries may be developed from the theorem mentioned above.

Corollary 1

Assume $P_1>P_2$, $S_1=S_2$, $C_1=C_2$, when the disparity of productivity of investment between P_1 and P_2 increases, the limit point of controllability (D_r) decreases and also the feasible region of controllability of D_r decreases.

Proof

Assume $P_1>P_1'>P_2$ $(P_1-P_2>P_1'-P_2)$, without loss of generality, the following equation is satisfied by the theorem.

$$\left\{D_r\Big/(1+r\cdot P_1\cdot S_1)\cdot C_1+P_1\cdot(1-r)\left(\sum_{j=1}^{2}S_j\cdot C_j\right)\times D_r=X_1^1 \text{ or } (1+r\cdot P_2\cdot S_2)\cdot C_2+P_2\times(1-r)\left(\sum_{j=1}^{2}S_j\cdot C_j\right)(1-D_r)=X_2^1\right\}$$

$$<\left\{D_r\Big/(1+r\cdot P_1'\cdot S_1)\cdot C_1+P_1'\cdot(1-r)\left(\sum_{j=1}^{2}S_j\cdot C_j\right)\times D_r=\acute{X}_1^1 \text{ or } (1+r\cdot P_2\cdot S_2)\cdot C_2+P_2\cdot(1-r)\times\left(\sum_{j=1}^{2}S_j\cdot C_j\right)(1-D_r)=\acute{X}_2^1\right\}$$

Where

X_j^1: the optimal solutions with P_1 and P_2.

$\acute{X}_j^1$: the optimal solutions with P_1' and P_2. $(j=1, 2)$

This equation represents that the limit point of controllability (D_r) with P_1 and P_2 is smaller than the one (D_r) with P_1' and P_2.

Q.E.D.

Corollary 2

Assume $P_1>P_2$, $S_1=S_2$, $C_1>C_2$, when the disparity of the regional income at initial time between C_1 and C_2 increases, the limit point of controllability (D_r) decreases and also the feasible region of controllability of D_r decreases.

Proof

Assume $C_1>C_1'>C_2$ $(C_1-C_2>C_1'-C_2)$, without loss of generality, the following equation is satisfied by the theorem.

$$\left\{D_r\Big/(1+r\cdot P_1\cdot S_1)\cdot C_1+P_1\cdot(1-r)\cdot\left(\sum_{j=1}^{2}S_j\cdot C_j\right)\times D_r=X_1^1 \text{ or } (1+r\cdot P_2\cdot S_2)\cdot C_2+P_2\cdot(1-r)\times\left(\sum_{j=1}^{2}S_j\cdot C_j\right)(1-D_r)=X_2^1\right\}$$

$$<\{D_r/(1+r\cdot P_1\cdot S_1)\cdot C_1'+P_1\cdot(1-r)\times(S_1\cdot C_1'+S_2\cdot C_2)\cdot D_r=\acute{X}_1^1 \text{ or } (1+r\cdot P_2\cdot S_2)\times C_2+P_2\cdot(1-r)(S_1\cdot C_1'+S_2\cdot C_2)(1-D_r)=\acute{X}_2^1\}$$

Where

X_j^1: the optimal solutions with C_1 and C_2.

$\acute{X}_j^1$: the optimal solutions with C_1' and C_2. $(j=1, 2)$

This equation represents that the limit point of controllability (D_r) with C_1 and C_2 is smaller than the one (D_r) with C_1' and C_2.

Q.E.D.

Corollary 3

Assume $P_1>P_2$, $S_1=S_2$, $C_1>C_2$, when the planning period time N decreases, the limit point of controllability (D_r) decreases and also the feasible region of controllability of D_r decreases.

Proof

It seems to be clear that this corollary 3 can be proved by the theorem and corollaries 1 and 2.

Q.E.D.

Corollary 4

Assume $P_1>P_2$, $S_1=S_2$, $C_1=C_2$, when the local autonomy rate r increases, the limit point of controllability (D_r) decreases and also the feasible region of controllability of D_r decreases.

Proof

Assume $r_1>r_2$, without loss of generality, the following equations are satisfied by the theorem.

$$\left\{D_r\Big/(1+r_1\cdot P_1\cdot S_1)\cdot C_1+P_1\cdot(1-r)\cdot\left(\sum_{j=1}^{2}S_j\cdot C_j\right)\times D_r=X_1^1 \text{ or } (1+r_1\cdot P_2\cdot S_2)\cdot C_2+P_2\cdot(1-r_1)\times\left(\sum_{j=1}^{2}S_j\cdot C_j\right)\cdot(1-D_r)=X_2^1\right\}$$

$$<\left\{D_r\Big/(1+r_2\cdot P_1\cdot S_1)\cdot C_1+P_1\cdot(1-r_2)\times\left(\sum_{j=1}^{2}S_j\cdot C_j\right)\cdot D_r=\acute{X}_1^1 \text{ or } (1+r_2\cdot P_2\cdot S_2)\times C_2+P_2\cdot(1-r_2)\left(\sum_{j=1}^{2}S_j\cdot C_j\right)(1-D_r)=\acute{X}_2^1\right\}$$

Where

X_j^1: the optimal solutions with r_1.

$\acute{X}_j^1$: the optimal solutions with r_2. $(j=1, 2)$

This equation represents that the limit point of controllability (D_r) with r_1 is smaller than the one (D_r) with r_2.

Q.E.D.

3. SIMULATIONS OF THE MODELS

In this chapter, we shall consider several typical simulations of the models of two-region case to clear the meanings of the corollaries mentioned above. In these models, the productivity of investment and saving ratio are assumed to be a constant over time.

(1) Model 1

In this model, we shall consider two simulations concentrating on the controllability of D_r. And two simulations are shown as follows: One is a simulation in which the productivities of investment are $P_1=1.400$ and $P_2=1.300$, and the other is a simulation in which the productivities of investment are $P_1=1.400$ and $P_2=1.200$. And the planning period times N are assumed as $N=8$, $N=5$ and $N=3$.

a) The first simulation

The data used in the computation is shown as follows:

$P_1=1.400$, $P_2=1.300$, $S_1=S_2=0.200$, $X_1^0=X_2^0=10$ (Billion dallars), $r=0.0$.

Where, the minimum proportion of investment are changed in the order of magnitude from 0.00 to 0.500. The results of the simulation at $N=8$, $N=5$ and $N=3$ are shown in Fig. 1 to Fig. 6.

It is clear from Fig. 1 that the decreasing rate of national income is small, but the maximum income disparity shows a rapidly decreasing rate as the minimum proportion of investment increases. And the value of the limit point of

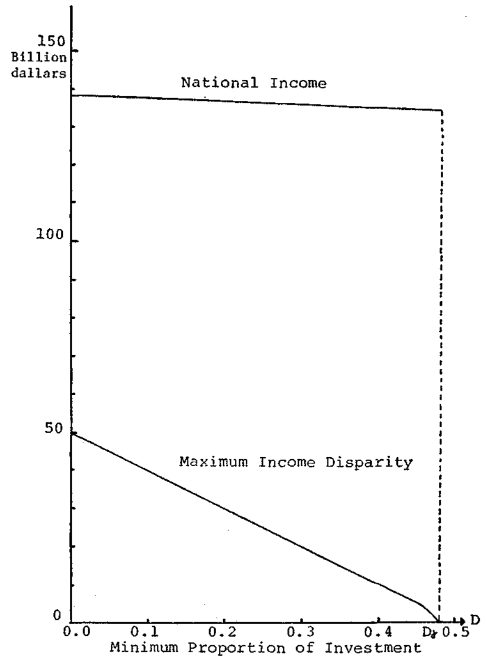

Fig. 1 The National Income and Maximum Income Disparity at $N=8$ by $P_1=1.400$, $P_2=1.300$ and $X_1^0=X_2^0=10$.

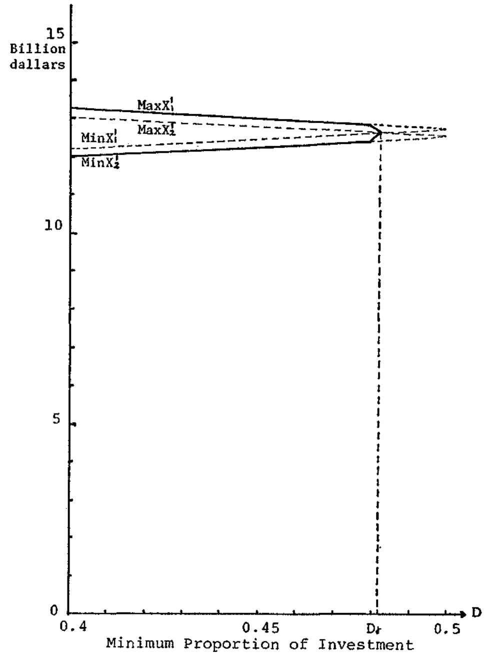

Fig. 2 The Feasible Region of Controllability of D_r at $N=8$ by $P_1=1.400$, $P_2=1.300$ and $X_1^0=X_2^0=10$.

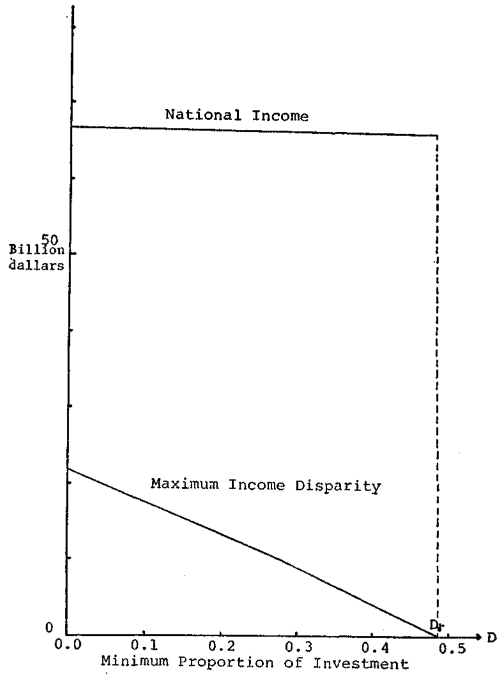

Fig. 3 The National Income and Maximum Income Disparity at $N=5$ by $P_1=1.400$, $P_2=1.300$ and $X_1^0=X_2^0=10$.

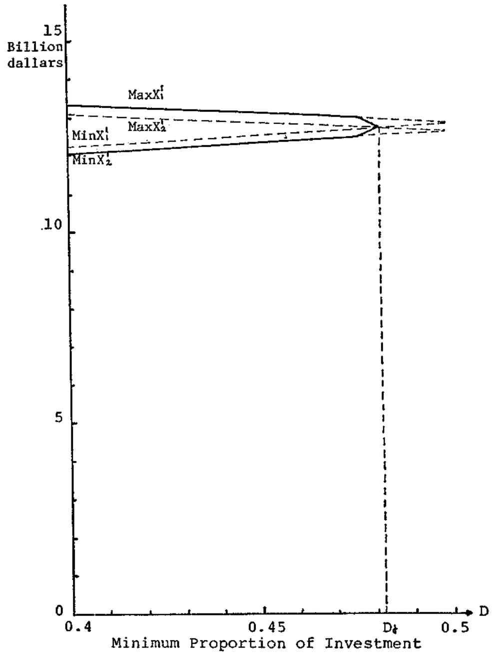

Fig. 4 The Feasible Region of Controllability of D_r at $N=5$ by $P_1=1.400$, $P_2=1.300$ and $X_1^0=X_2^0=10$.

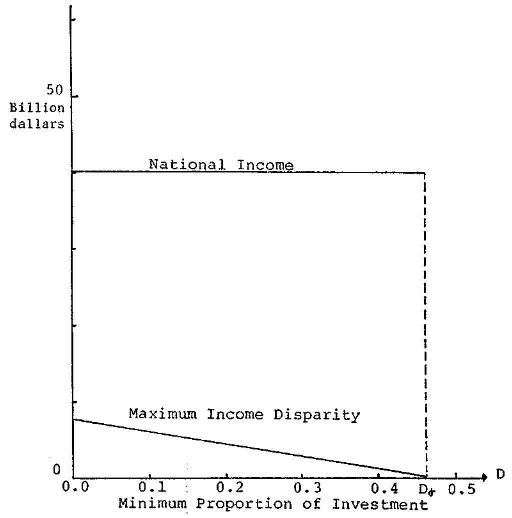

Fig. 5 The National Income and Maximum Income Disparity at $N=3$ by $P_1=1.400$, $P_2=1.300$ and $X_1^0=X_2^0=10$.

controllability (D_r) is 0.481 and the feasible region of controllability of D_r is from 0.000 to 0.481. Then, it is impossible to control the model when the minimum proportion of investment is more

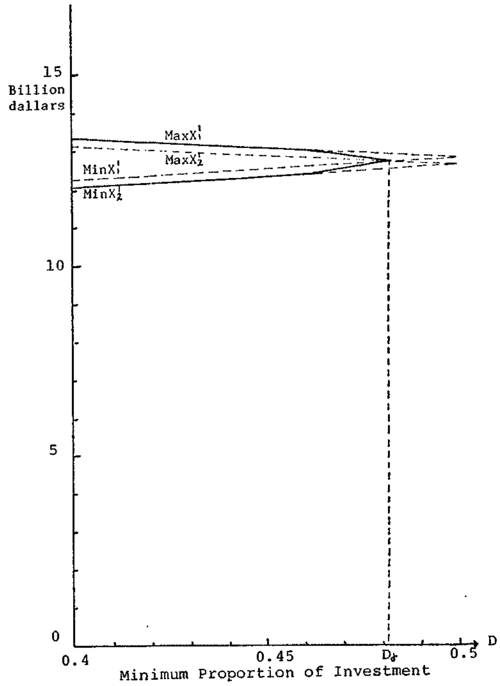

Fig. 6 The Feasible Region of Controllability of D_r at $N=3$ by $P_1=1.400$, $P_2=1.300$ and $X_1^0=X_2^0=10$.

than D_r.

Next, to clarify the uncontrollable cause, the detailed process of the simulation is shown in Fig. 2. In the graph, the dotted lines represent the values of $\text{Max}\,X_j^1$ and $\text{Min}\,X_j^1$ and the solid lines represent the optimal solution of the model based on the Decomposition Method according to the increase of the minimum proportion of investment D.

From the facts presented in the graphs and corollary 1, the uncontrollable cause is based on the conditions in which the restrictions such that $X_1^1 \geq \text{Min}\,X_1^1$ and $X_2^1 \leq \text{Max}\,X_2^1$ does not hold.

The results of the simulations at $N=5$ and $N=3$ have a similar interpretation mentioned above. And the value of D_r is indicated as same value at $N=8$.

b) The second simulation

The data used in the computation is shown as follows:

$P_1=1.400$, $P_2=1.200$, $S_1=S_2=0.200$, $X_1^0=X_2^0=10$ (Billion dallars), $r=0.0$.

The results of the simulations at $N=8$, $N=5$ and $N=3$ are shown in Fig. 7 to Fig. 12. It is clear from Fig. 7 that the decreasing rate of

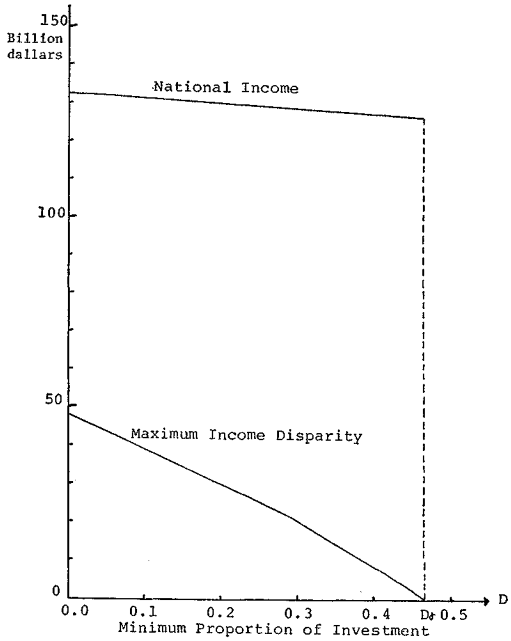

Fig. 7 The National Income and Maximum Income Disparity at $N=8$ by $P_1=1.400$, $P_2=1.200$ and $X_1^0=X_2^0=10$.

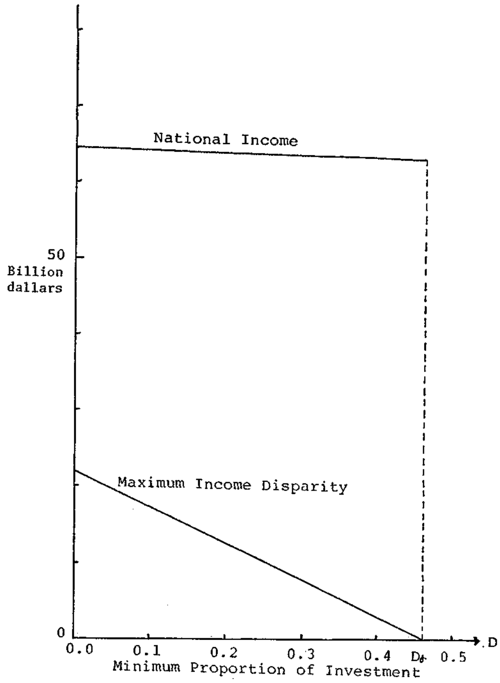

Fig. 9 The National Income and Maximum Income Disparity at $N=5$ by $P_1=1.400$, $P_2=1.200$ and $X_1^0=X_2^0=10$.

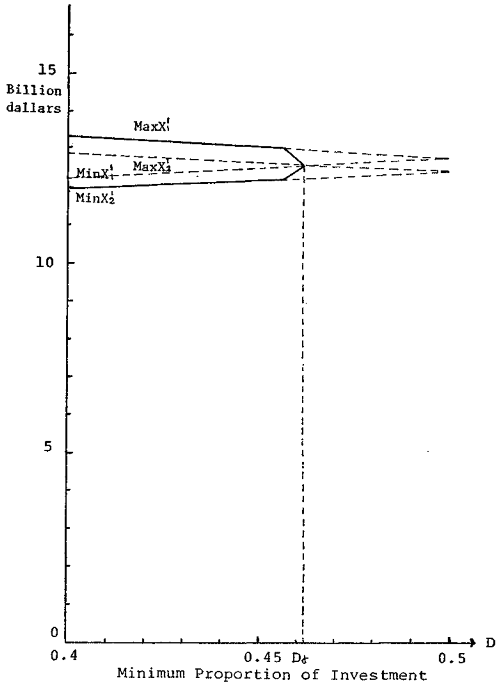

Fig. 8 The Feasible Region of Controllability of D_r at $N=8$ by $P_1=1.400$, $P_2=1.200$ and $X_1^0=X_2^0=10$.

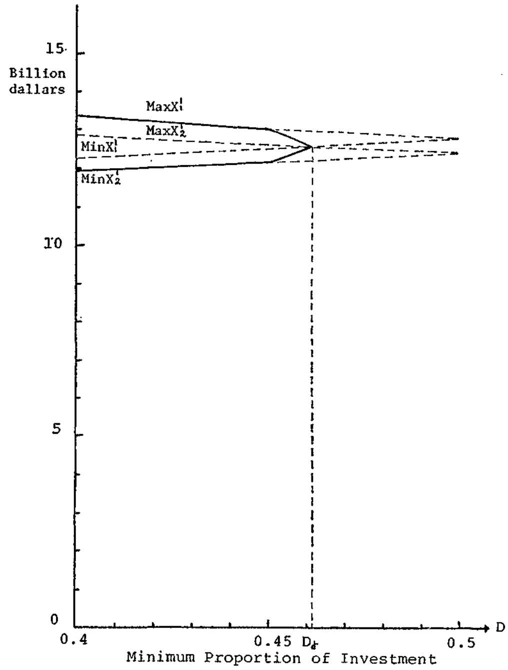

Fig. 10 The Feasible Region of Controllability of D_r at $N=5$ by $P_1=1.400$, $P_2=1.200$ and $X_1^0=X_2^0=10$.

national income is small, but the maximum income disparity shows a rapidly decreasing rate as the minimum proportion of investment in-

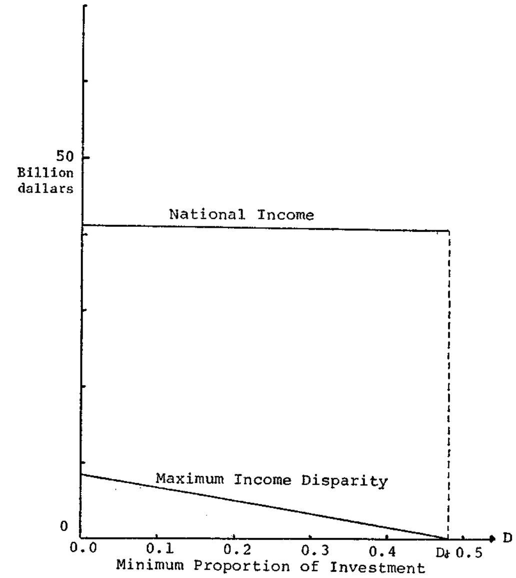

Fig. 11 The National Income and Maximum Income Disparity at $N=3$ by $P_1=1.400$, $P_2=1.200$ and $X_1^0=X_2^0=10$.

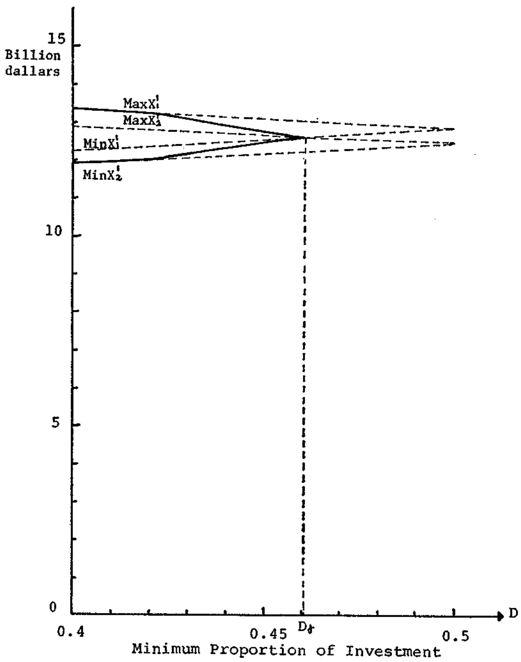

Fig. 12 The Feasible Region of Controllability of D_T at $N=3$ by $P_1=1.400$, $P_2=1.200$ and $X_1^0=X_2^0=10$.

creases. And the value of the limit point of controllability (D_T) is 0.462. The uncontrollable cause and the results of the simulations at $N=5$ and $N=3$ have a similar interpretation of the simulation a) mentioned above.

The major cause for the difference of D_T between two simulations a) and b) is based on the difference of productivity of investment of region II.

(2) Model 2

In this model, we shall consider a simulation in which the regional incomes at initial time indicate different values $C_1 \neq C_2$.

The data used in the computation is shown as follows:

$P_1=1.400$, $P_2=1.200$, $S_1=S_2=0.200$, $X_1^0=20$, $X_2^0=10$ (Billion dallars), $r=0.0$.

The results of the simulation at $N=8$, $N=5$ and $N=3$ are shown in Fig. 13 to Fig. 18. It is clear from Figs. 13, 15 and 17 that the decreas-

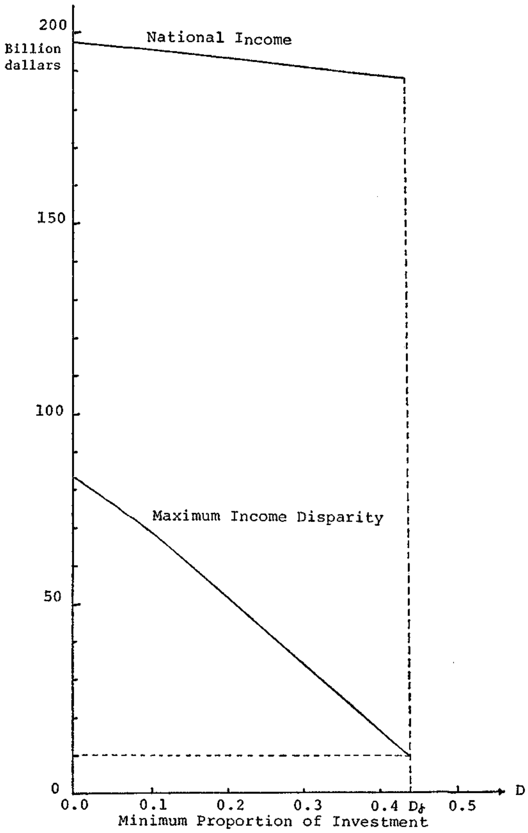

Fig. 13 The National Income and Maximum Income Disparity at $N=8$ by $P_1=1.400$, $P_2=1{\cdot}200$, $X_1^0=20$ and $X_2^0=10$.

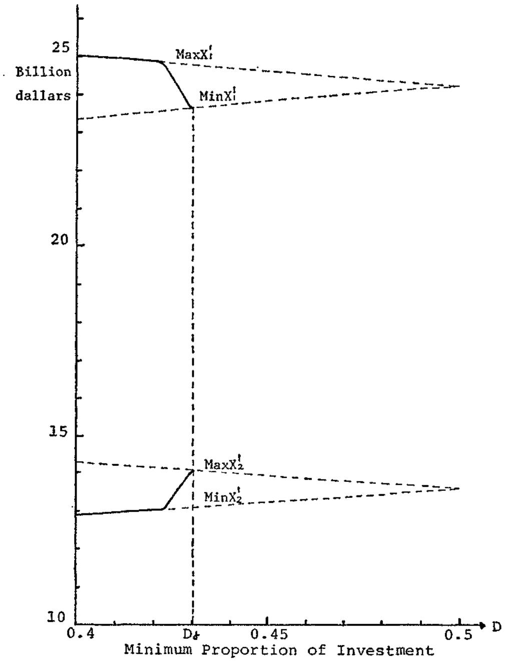

Fig. 14 The Feasible Region of Controllability of D_r at $N=8$ by $P_1=1.400$, $P_2=1.200$, $X_1^0=20$ and $X_2^0=10$.

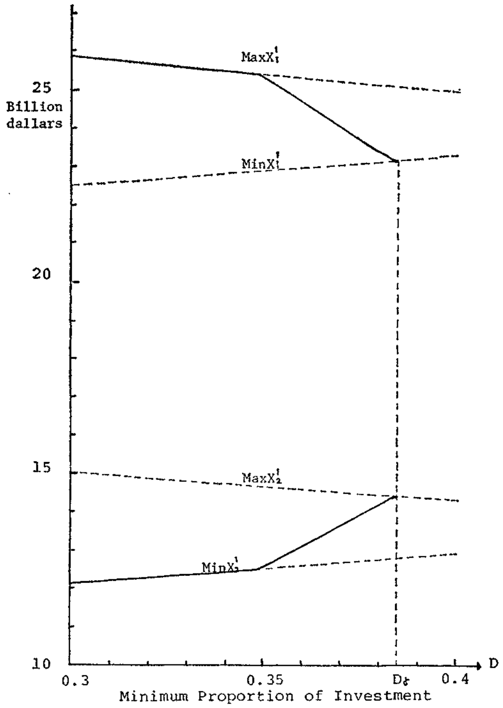

Fig. 16 The Feasible Region of Controllability of D_r at $N=5$ by $P_1=1.400$, $P_2=1.200$, $X_1^0=20$ and $X_2^0=10$.

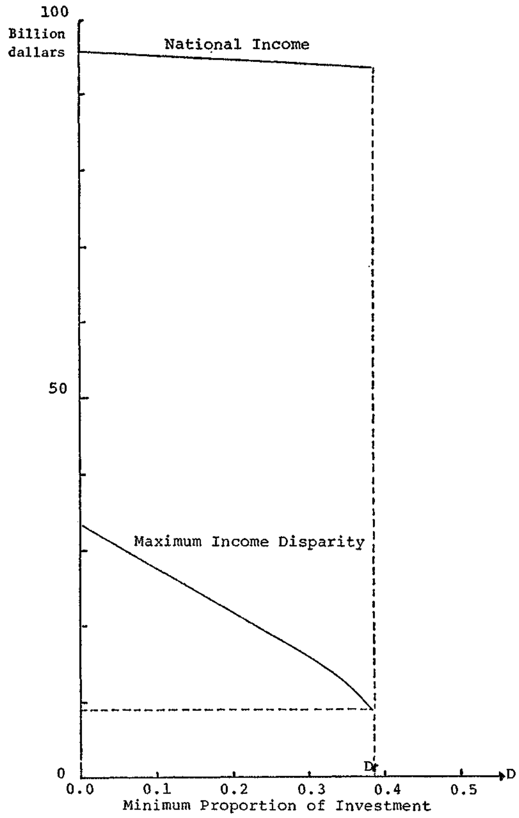

Fig. 15 The National Income and Maximum Income Disparity at $N=5$ by $P_1=1.400$, $P_2=1.200$, $X_1^0=20$ and $X_2^0=10$.

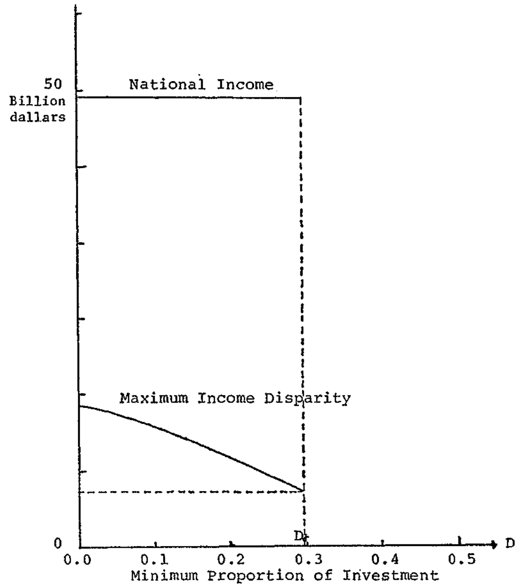

Fig. 17 The National Income and Maximum Income Disparity at $N=3$ by $P_1=1.400$, $P_2=1.200$, $X_1^0=20$ and $X_2^0=10$.

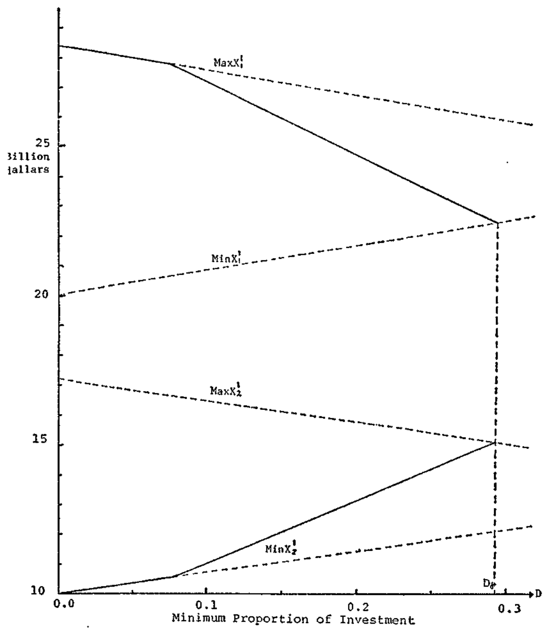

Fig. 18 The Feasible Region of Controllability of D_r at $N=3$ by $P_1=1.400$, $P_2=1.200$, $X_1{}^0=20$ and $X_2{}^0=10$.

ing rate of national income is small, but the maximum income disparity shows a rapidly decreasing rate as the minimum proportion of increases. Next, it is clear from Figs. 14, 16 and 18 that the values of the limit point of controllability (D_r) are indicated as 0.430 at $N=8$, 0.384 at $N=5$ and 0.293 at $N=3$. Then, we shall compare the differences between this simulation and the simulation b) in Model 1. The values of D_r are indicated as a larger decline than the value of D_r of the simulation b) in Model 1 as the planning period time N decreases. Thus, the major cause for these differences is based on the difference of the regional incomes at initial time.

It is clear from the facts presented above that the difference of the regional income at initial time plays an important role in the controllability of D_r.

(3) Model 3

In this model, we shall consider two simulations concentrating on the controllability of D_r in which the local autonomy rates are $r=0.2$ and $r=0.8$.

a) The first simulation

The data used in the computation is shown as follows:

$P_1=1.400$, $P_2=1.200$, $S_1=S_2=0.200$, $X_1{}^0=X_2{}^0=10$ (Billion dallars), $r=0.2$.

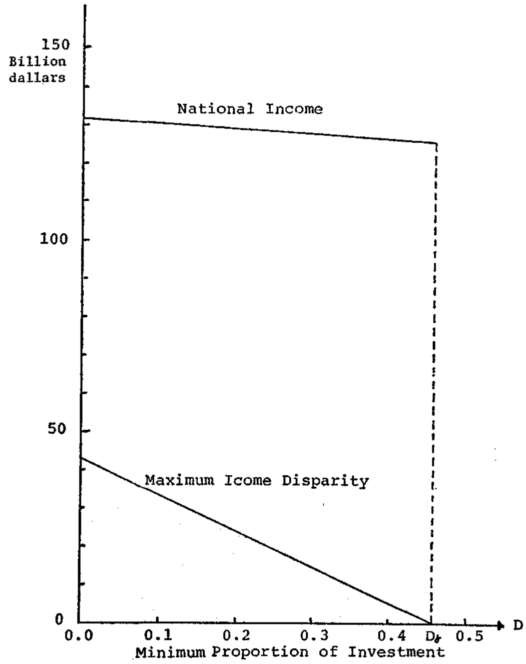

Fig. 19 The National Income and Maximum Income Disparity at $N=8$ by $P_1=1.400$, $P_2=1.200$, $X_1{}^0=X_2{}^0=10$ and $r=0.2$.

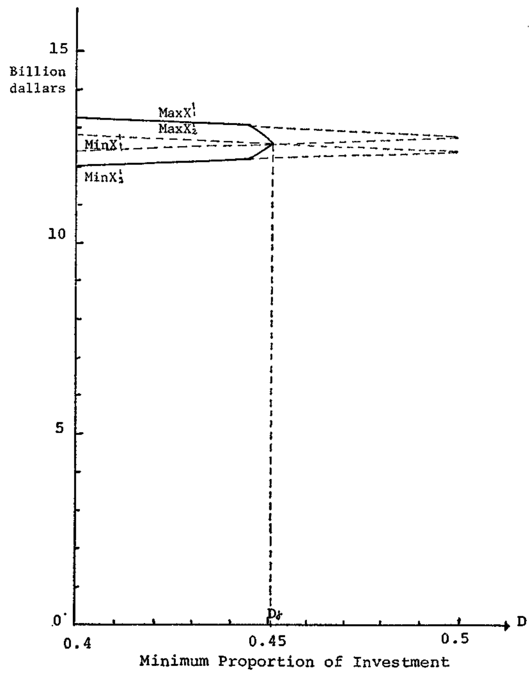

Fig. 20 The Feasible Region of Controllability of D_r at $N=8$ by $P_1=1.400$, $P_2=1.200$, $X_{1_0}=X_2{}^0=10$ and $r=0.2$.

The results of the simulation at $N=8$ are shown in Fig. 19 to Fig. 20. It is clear from Fig. 19 that the decreasing rate of national income is small, but the maximum income disparity shows a rapidly decreasing rate as the minimum proportion of investment increases. And from Fig. 20, the value of the limit point of controllability (D_r) is 0.451.

b) The second simulation

The data used in the computation is shown as follows:

$P_1=1.400$, $P_2=1.200$, $S_1=S_2=0.200$, $X_1^0=X_2^0=10$ (Billion dallars), $r=0.8$.

The results of the simulation at $N=8$ are shown in Fig. 21 to Fig. 22. It is clear from Fig. 21 that the decreasing rate of national income and the maximum income disparity show a slight decreasing rate as the minimum proportion of investment increases. And from Fig. 22, the value of the limit point of controllability (D_r) is 0.307.

Next, we shall compare the difference between two simulations a) and b). The value of D_r of the simulation b) is indicated as a larger decline than the value of D_r of the simulation a). Thus, the major cause for the difference of D_r is based on the difference of the local autonomy rates.

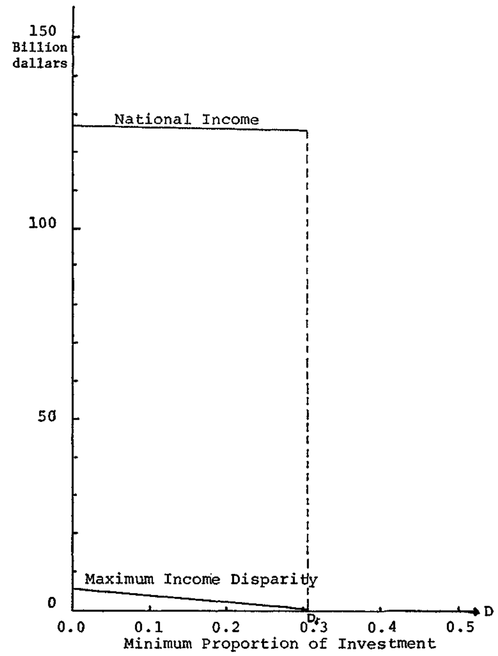

Fig. 21 The National Income and Maximum Income Disparity at $N=8$ by $P_1=1.400$, $P_2=1.200$, $X_1^0=X_2^0=10$ and $r=0.8$.

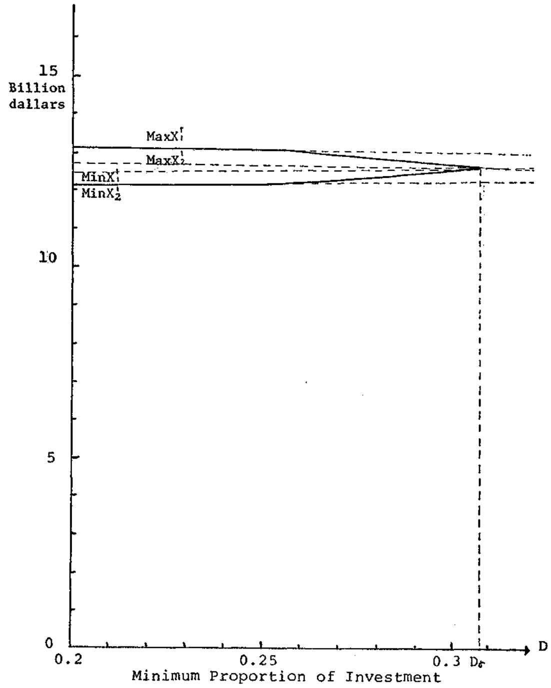

Fig. 22 The Feasible Region of Controllability of D_r at $N=8$ by $P_1=1.400$, $P_2=1.200$, $X_1^0=X_2^0=10$ and $r=0.8$.

4. CONCLUSION

In summary, we have investigated several typical simulations to clear the controllability of the minimum proportion of investment.

From the facts obtained in the theorem, four corollaries and the simulations mentioned above, the following three points may be concluded.

First, in Model 1, it seems to be clear that the limit point of controllability (D_r) is affected remarkably by the increase of disparity of the productivity of investment between P_1 and P_2.

Second, in Model 2, when the planning period time N decreases, the limit point of controllability (D_r) is not so much changed at $C_1=C_2$, but D_r shows a rapidly decreasing rate at $C_1>C_2$. It indicates that the difference of the regional income at initial time plays an important role in the controllability of the minimum proportion of investment.

Third, in Model 3, it seems to be clear that when the local autonomy rate r increases, the limit point of controllability D_r decreases and the maximum income disparity shows a small value,

ACKNOWLEDGEMENT

The author wishes to express his gratitude to Professor Ogawa, H., Igarashi, H. and Dr. K. Yamagata of Hokkaido University for suggesting the problem and for their encouragement throughout the work. And the numerical calculations in chapter 3 were made on Hokkaido University Computing Center.

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(*Received Sept. 13, 1974*)