

AN ANALYTICAL STUDY OF TWO-LAYERED, RAPIDLY VARIED FLOW

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1. INTRODUCTION

Parallel streams of two-layered fluids exhibit various flow characteristics and are frequently observed in an aquatic environment. Understanding of this kind of phenomenon finds increasing applications in present-day world of industrialization and endangered eco-system.

Two-layered parallel streams may be or may not be accompanied by strong mixing. Specifically, the flow will resemble any of jet, hydraulic jump or immiscible two-layered fluids. Classification of these flow regimes and their possible occurrence are discussed by, among others, Koh¹⁾ and Stefan²⁾.

Two-layered rapidly varied flow encompasses jet and hydraulic jump. Characteristic to this class of flow is that the flow is internally supercritical and very often accompanied by strong mixing. On account of this mixing, the flow loses its momentum and often is followed by a two-layered flow system which is internally subcritical and accompanied by a negligible amount of mixing. Transition from supercritical to subcritical regime is tractable by the impulse-momentum principle.

Such an analysis was first presented by Schijf and Schönfeld³⁾ who solved a case when one fluid layer is in motion and there is no mixing between the layers. More general treatment to the case when two layers are in co-current motion is given by Yih and Guha⁴⁾. Shi-ighai⁵⁾, Hamada⁶⁾, and Iwasaki⁷⁾ studied the solution of the momentum equations under the premise that the free water surface remains level. Among them, Iwasaki gave energy relationship of internal hydraulic jump. On the basis of this Hayakawa⁸⁾ refined analysis given by Yih and Guha.

All the works mentioned above relate to internal

hydraulic jump of immiscible fluids. Mixing and turbulence associated with internal hydraulic jump has been studied only recently. Thus Iwasaki and Uehara⁹⁾, and Iwasaki and Abe¹⁰⁾ reported their extensive study on turbulent structure related to internal hydraulic jump. Wood¹¹⁾ has reported his experiments and some analysis on mixing internal hydraulic jump. Wilkinson and Wood¹²⁾ reported observation of various flow regimes with wide range of mixing when the lower layer was quiescent and the upper layer was covered by the solid wall. Similar discussion was presented by Stefan²⁾ regarding with the case the upper layer possesses free surface and an analysis of internal hydraulic jump with mixing considered and a lower layer quiescent was presented by Stefan and Hayakawa¹³⁾ who solved the impulse-momentum equation for the entire fluid surrounding jump phenomenon assuming hydrostatic pressure distribution in the lower layer across the control volume. Hayakawa and Stefan¹⁴⁾ slightly improved their earlier analysis deriving the impulse-momentum equations for both layers.

The present work tries to modify and expand the work of Hayakawa and Stefan. It is first asserted that the rapidly varied flow phenomenon generally consists of jet-type flow followed by internal hydraulic jump (Koh¹⁾ and Stefan²⁾). Therefore, the impulse-momentum equation is applicable to this system and gives the conjugate depth relationship with the rate of mixing as a parameter or vice versa. This is also a repetition of an assertion of Stefan and Hayakawa¹³⁾ that use of the impulse-momentum equation alone, i.e. without detailed knowledge of the flow inside of the control volume, does not yield a complete solution to the problem with interfacial mixing taken into consideration.

2. DERIVATION OF GOVERNING EQUATIONS

The problem treated in this work is shown dia-

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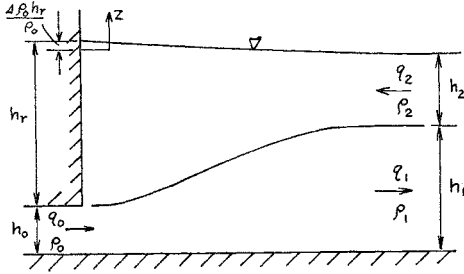


Fig. 1 (a) Definition Sketch of Subsurface Flow.

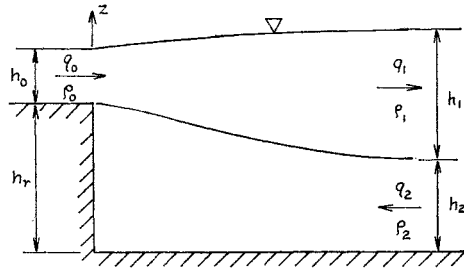


Fig. 1 (b) Definition Sketch of Surface Flow.

grammatically in Fig. 1(a) and Fig. 1(b). In Fig. 1(a) heavier fluid is discharged along the bottom surface of the reservoir (subsurface flow), whereas in Fig. 1(b) lighter fluid is discharged onto the surface of the reservoir (surface flow). The problem is two-dimensional and any interaction with atmosphere such as heat transport is not considered. Fluid is treated essentially as incompressible and miscible. To derive the impulse momentum equation for each layer, it is assumed that the interface is fairly well definable. Furthermore, following three important premises are elected: (I) Fluid being entrained from one layer to another carries a negligible amount of momentum in the longitudinal direction, (II) The pressure distribution at the upstream end of the reservoir is approximated by the hydrostatic distribution, and (III) The average pressure along the interface of the two fluids is the arithmetic average value of pressures at the upstream and downstream ends of the control volume. The premise (III) follows Yih and Guha⁴⁾ whose result is reproduced in the present work when the mixing is assumed to be negligible. Other two premises (I) and (II) exactly hold when mixing is of negligible amount. It should be regarded, therefore, that the present analysis is a good approximation when entrainment is small.

With this preparation the impulse-momentum equations for Fig. 1(a) are written as follows:

$$\begin{aligned} \beta_1 \frac{\rho_1 q_1^2}{h_1} - \beta_0 \frac{\rho_0 q_0^2}{h_0} &= \frac{1}{2} \rho_0 g h_0^2 + \rho_2 g h_r h_0 - \frac{1}{2} \rho_1 g h_1^2 \\ &\quad - \rho_2 g h_1 h_2 + \frac{(h_1 - h_0)}{2} (\rho_2 g h_r + \rho_2 g h_2) \end{aligned}$$

for lower layer(1)

$$\begin{aligned} \beta_2 \frac{\rho_2 q_2^2}{h_2} &= \frac{1}{2} \rho_2 g h_r^2 - \frac{1}{2} \rho_2 g h_2^2 \\ &\quad - \frac{(h_1 - h_0)}{2} (\rho_2 g h_r + \rho_2 g h_2) \end{aligned}$$

for upper layer(2)

where β is the momentum correction factor, ρ is fluid density, q is volumetric flow rate per unit width, h is fluid depth and g is the gravitational acceleration. In equations (1) and (2) suffixes 0 and 1 relate to quantities of the lower layer at the upstream and downstream end respectively and 2 and r those of the upper layer at the downstream and the upstream end respectively. Fluid densities are averaged in each layer.

Similarly the impulse-momentum equations for Fig. 1(b) are written as follows;

$$\begin{aligned} \beta_1 \frac{\rho_1 q_1^2}{h_1} - \beta_0 \frac{\rho_0 q_0^2}{h_0} &= \frac{1}{2} \rho_0 g h_0^2 - \frac{1}{2} \rho_1 g h_1^2 \\ &\quad + \frac{(h_r - h_2)}{2} (\rho_0 g h_0 + \rho_1 g h_1) \end{aligned}$$

for upper layer(3)

$$\begin{aligned} \beta_2 \frac{\rho_2 q_2^2}{h_2} &= \rho_0 g h_0 h_r + \frac{1}{2} \rho_2 g h_r^2 - \rho_1 g h_1 h_2 \\ &\quad - \frac{1}{2} \rho_2 g h_2^2 - \frac{(h_r - h_2)}{2} (\rho_0 g h_0 + \rho_1 g h_1) \end{aligned}$$

for lower layer(4)

where suffixes 0 and 1 relate to quantities of the upper layer at the upstream and downstream end respectively and 2 and r those of the lower layer at the downstream and upstream end respectively.

Two problems will be shown to be expressed in a single set of equations. Addition of equations (1) and (2) and the same procedure for equations (3) and (4) are combined as follows:

$$\begin{aligned} \beta_1 \frac{\rho_1 q_1^2}{h_1} - \beta_0 \frac{\rho_0 q_0^2}{h_0} + \beta_2 \frac{\rho_2 q_2^2}{h_2} &= \frac{1}{2} \rho_0 g h_0^2 - \frac{1}{2} \rho_1 g h_1^2 + \frac{1}{2} \rho_2 g h_r^2 \\ &\quad - \frac{1}{2} \rho_2 g h_2^2 + \left(\frac{\rho_2}{\rho_0} \right) g h_r h_0 - \left(\frac{\rho_2}{\rho_1} \right) g h_1 h_2 \end{aligned}$$

.....(5)

where the upper and lower values relate to the subsurface and surface flow respectively. Similarly,

Eqs. (2) and (4) are written in a single form as follows:

$$2\beta_2 \frac{\rho_2 q_2^2}{g h_2 (h_2 + h_r)} = \left(\frac{\rho_2}{\rho_0}\right) h_0 - \left(\frac{\rho_2}{\rho_1}\right) h_1 + \rho_2 (h_r - h_2) \dots\dots\dots (6)$$

Equations of continuity and incompressibility are the same for two problems and are written as follows:

$$\rho_0 q_0 + \rho_2 q_2 = \rho_1 q_1 \dots\dots\dots (7)$$

$$q_0 + q_2 = q_1 \dots\dots\dots (8)$$

Density excesses at the upstream end $\Delta\rho_0$ and at the downstream end $\Delta\rho_1$ are defined as follows:

$$\Delta\rho_0 = \pm(\rho_0 - \rho_2) \dots\dots\dots (9)$$

$$\Delta\rho_1 = \pm(\rho_1 - \rho_2) \dots\dots\dots (10)$$

where the upper and lower signs relate to subsurface and surface flow respectively. Continuity equation (7) is rewritten using Eqs. (8), (9) and (10) as follows:

$$\Delta\rho_0 q_0 = \Delta\rho_1 q_1 \dots\dots\dots (11)$$

Eliminating q_2 and $\Delta\rho_1$ in Eqs. (5) and (6) using Eqs. (8) and (11) and then combining and expressing them in terms of dimensionless quantities lead to the following:

$$2F_0^2 = H_2 \left[\left(1 - \left(\frac{0}{r}\right)\right) \left(Q - \left(\frac{0}{r}\right)\right) - \left(H_a - \left(\frac{0}{r}\right)\right)^2 + 2\varphi \left\{ H_2 \left(Q - \left(\frac{0}{r}\right)\right) + \left(Q + \left(\frac{r}{0}\right)\right) \left(H_a - \left(\frac{0}{r}\right)\right) \right\} - r \left(Q + \left(\frac{r}{0}\right)\right) \varphi^2 \right] \left(Q - \left(\frac{0}{r}\right)\right)^{-1} \left[\beta_2 (Q-1)^2 + \frac{\beta_1 H_2 \left(Q - \left(\frac{0}{r}\right)\right) \left(Q + \left(\frac{r}{-r}\right)\right)}{H_a - r\varphi - \left(\frac{0}{r}\right)} - \beta_0 \left(1 + \left(\frac{r}{-r}\right)\right) H_2 \right]^{-1} \dots\dots\dots (12)$$

Dimensionless quantities in Eq. (12) are defined as follows:

$$Q = \frac{q_1}{q_0}, \quad H_1 = \frac{h_1}{h_0}, \quad H_2 = \frac{h_2}{h_0}, \quad H_r = \frac{h_r}{h_0}, \\ r = \frac{\Delta\rho_0}{\rho_2}, \quad F_0 = \frac{q_0}{\sqrt{r g h_0^3}}, \quad H_a = 1 + H_r - H_2, \\ \varphi = 2\beta_2 \frac{F_0^2 (Q-1)^2}{H_2 (H_2 + H_r)} \dots\dots\dots (13)$$

In Eq. (12), the densimetric Froude number F_0 and the depth ratio H_2 are the primary independent variables. The flow ratio Q and the depth ratio H_a are the primary dependent vari-

ables. Eq. (12) cannot give a complete answer, say q_1 and h_1 to a given problem, say q_0, h_0, h_r and h_2 . With given, dimensionless parameters, F_0, H_r and H_2 , Eq. (12) gives the flow ratio Q as a function of the depth ratio H_a or vice versa. The complete answer to the physical problem cannot be given by the use of the impulse-momentum equations alone.

Change in water surface level is given as follows:

$$W \equiv \frac{H_1 + H_2 - 1 - H_r}{r} = \left(\frac{H_1}{Q} - 1\right) \left(\frac{0}{1}\right) - \varphi \dots\dots (14)$$

Eq. (14) indicates that the change in water surface level is of the order of r . It is to be noted that Eq. (14) with $Q=1$ for surface flow was first given by Shi-ighai⁵⁾. Eqs. (12) and (14) for surface flow were first obtained by Hayakawa and Stefan¹⁴⁾.

Energy loss of this flow system may be given by calculation of energy flux as follows:

$$\Delta E = \frac{q_0 E_0 + q_2 E_2 - q_1 E_1}{q_0 E_0} \dots\dots\dots (15)$$

where E_0, E_1 and E_2 are total pressures of each layer. The reference datum planes to calculate the total pressures are selected so that the resultant equations exhibit the maximum symmetry. They are at the free surface of the upstream discharge for the surface flow (Fig. 1 (b)) and at the depth $\Delta\rho_0 h_r / \rho_0$ of the upstream end for the subsurface flow (Fig. 1 (a)). With this choice Eq. (15) is calculated as follows:

$$\Delta E = 1 + \frac{\alpha_2}{\alpha_0} \frac{(Q-1)^3}{\left(1 + \left(\frac{r}{-r}\right)\right) H_2^2} - \frac{\alpha_1}{\alpha_0} \frac{\left(Q + \left(\frac{r}{-r}\right)\right) Q^2}{\left(1 + \left(\frac{r}{-r}\right)\right) H_1^2} - \frac{2W}{\alpha_0 F_0^2} \frac{1 - \frac{r}{Q} \left(\frac{0}{1}\right)}{1 - \left(\frac{0}{r}\right)} - \frac{2}{\alpha_0 F_0^2} \frac{1}{1 + \left(\frac{r}{-r}\right)} \left(\frac{H_r - H_2}{\left(1 - \frac{1}{Q}\right) H_a} \right) \dots\dots\dots (16)$$

Hereafter the parameter r is considered to be very small. This is the case for engineering application of this problem to temperature- or salinity-stratified flow. Therefore, the terms involving r in Eqs. (12) and (16) are neglected in the ensuing analysis. Glance on these equations indicates that Eq. (12) is reduced to a single

dimensionless equation when $r=0$ whereas Eqs. (14) and (16) give slightly different forms for subsurface and surface flow. Change in water surface level given in Eq. (14) is of the order of r or higher. Therefore, substitution of quantities which satisfy Eq. (12) into Eq. (14) yields this value in the lowest order of r .

3. THEORETICAL ANALYSIS FOR $r=0$

Eq. (12) for $r=0$ can be further rearranged as follows to enable investigation of its analytical behavior:

$$\phi(Q) \equiv -2F_0^2 \frac{(H_a + H_2)(H_a - 2H_2 + 1)}{H_a(H_a + 2H_2 - 1)} Q^3$$

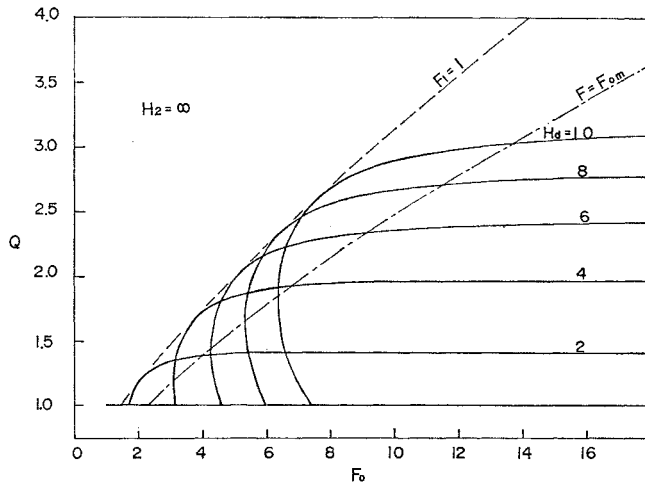


Fig. 2 (a) Theoretical Results for Q as a function of F_0 , $H_2 \rightarrow \infty$.

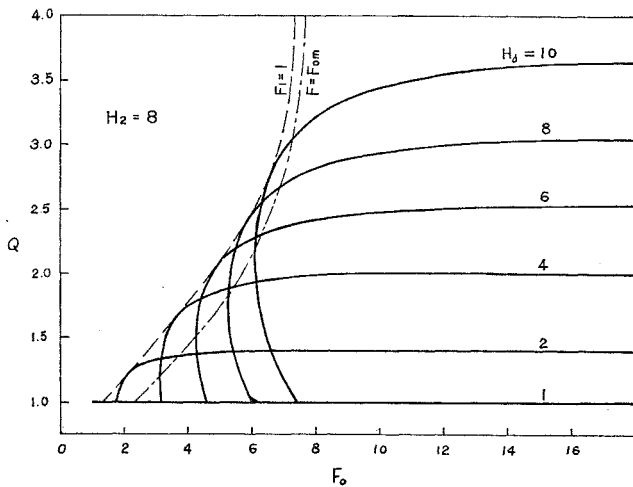


Fig. 2 (b) Theoretical Results for Q as a function of F_0 , $H_2 = 8$.

$$+4F_0^2 \frac{H_a + 1}{H_a + 2H_2 - 1} Q^2 - 2F_0^2 \left(\frac{H_a + 1}{H_a + 2H_2 - 1} + H_2 + \frac{H_2}{2F_0^2} \right) Q + H_2 H_a^2 = 0 \dots\dots\dots(17)$$

If $Q=1$ —i.e. with no entrainment—Eq. (17) is solved with respect to H_a as follows:

$$H_a = 1 \dots\dots\dots(18a)$$

or

$$H_a = \frac{1}{2}(-1 + \sqrt{1 + 8F_0^2}) \dots\dots\dots(18b)$$

Equation (18b) is identical to the solution for the non-mixing internal hydraulic jump given by Yih and Guha⁴⁾. When $Q > 1$, Eq. (17) is a cubic equation with respect to Q and can be easily verified that if $H_a < 2H_2 + 1$, Eq. (17) has two roots larger than one for Q when $F_{0m} < F_0 < F_{\infty}$, where F_{∞} is given by the inverse relation of Eq. (18b) as a function of H_a and F_{0m} is a number greater than one, and one root when $F_0 > F_{\infty}$. This point is illustrated in Figs. 2 (a) and 2 (b) which give numerical results of $Q-F_0$ relationship for $H_2 = \infty$ and $H_2 = 8$ respectively. For a given H_a the value of Q approaches a constant as F_0 increases infinitely. The curves of Eq. (17) are bounded by envelopes represented by broken lines labelled as $F_1 = 1$ in Figs. 2 (a) and 2 (b). F_1 is the downstream densimetric Froude number, defined as

$$F_1 = F_0 \left(\frac{Q^3}{H_a^3} \right)^{1/2} \left(1 + 2 \frac{(Q-1)^2}{Q^2} \times \frac{(H_2 - 1)H_a^2}{H_2(H_a + 2H_2 - 1)} \right) \dots\dots(19)$$

Each curve for $H_a = \text{constant}$ in Figs. 2 (a) and 2 (b) has two segments relative to the envelope $F_1 = 1$, one below the point tangent to the envelope and the other above. Only the segment below refers to an internally sub-critical flow downstream from the mixing zone, which is necessary to produce an internal hydraulic jump. Thus only the lower segments of the curves $H_a = \text{constant}$ in Figs. 2 (a) and 2 (b) are physically meaningful as solutions of the internal jump. In Figs. 3 (a) and 3 (b), which give $H_a - F_0$ relationship, this implies that in each curve for $Q = \text{constant}$ the seg-

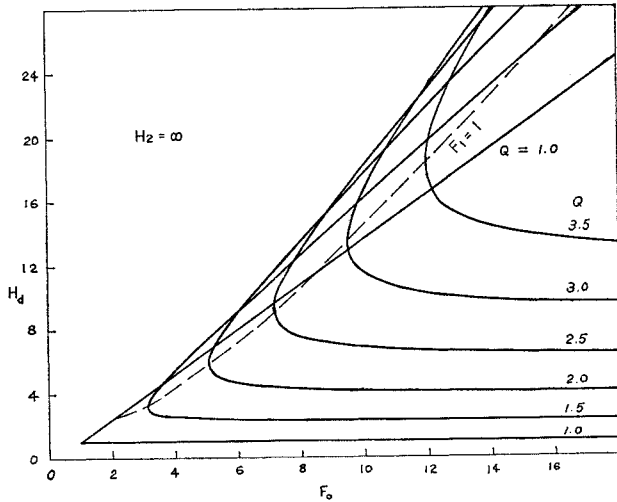


Fig. 3 (a) Theoretical Results for H_d as a function of F_0 , $H_2 \rightarrow \infty$.

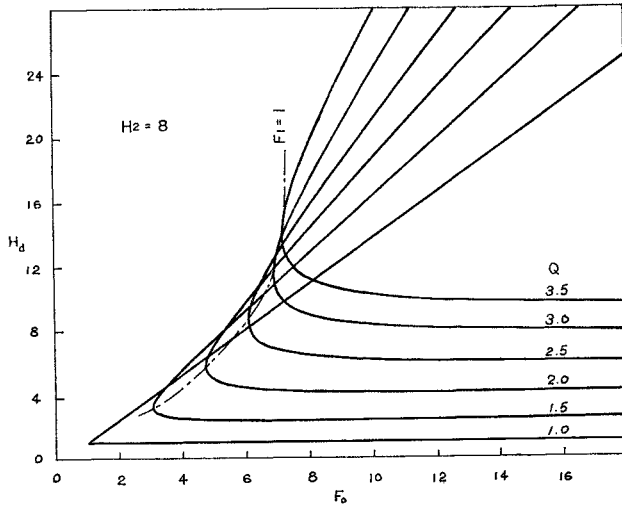


Fig. 3 (b) Theoretical Results for H_d as a function of F_0 , $H_2=8$.

ment above the point of intersection with $F_1=1$ is physically meaningful.

Figs. 3(a) and 3(b) further illustrate that a point (F_0, H_d) above the line $F_1=1$ is generally associated with two different Q values which gives the subcritical downstream condition. The experimental data suggest that the smaller value of Q occurs. This point will be discussed further in the next section.

4. COMPARISON WITH EXPERIMENTAL DATA

Only Stefan and Hayakawa^{13,14)} reported the data of the entrainment rate of this type of flow. Figures 4 and 5 show the depth ratio H_d and the

flow ratio Q , respectively, as functions of the densimetric Froude number F_0 . Since the total depth of the experimental tank was limited, the relative thickness of the denser (colder) layer H_2 varied within the range $3.5 < H_2 < 13.5$. Fig. 4 indicates that due to the entrainment the downstream depth H_d is larger than the depth for a non-mixing internal hydraulic jump. Furthermore, a majority of the data points fall below the curve of the maximum observed flow ratio $Q=1.6$ with the average value of the cold layer $H_2=8$. Fig. 4 includes a number of data which relate to submerged internal hydraulic jumps. (For description of submerged internal hydraulic jumps see Stefan and Hayakawa¹³⁾.) The submerged internal hydraulic jump involves an additional variable, namely the depth of submergence, and therefore is not tractable in the present analysis. Fig. 4 indicates that the submerged internal hydraulic jump is usually observed in the range $F_0 < 4$ and is associated with greater downstream depths than are predicted by the theory.

Fig. 5 shows a theoretical prediction of the flow ratio, Eq. (17), with $H_2=8$, which fits the experimental data remarkably well. In Fig. 5 the theoretical curves, except for the smallest H_d value, belong to the portion lower than F_{0m} line in Fig. 2. Therefore, for a specific set of values for F_0 and H_d (see Fig. 3), the smaller of the two

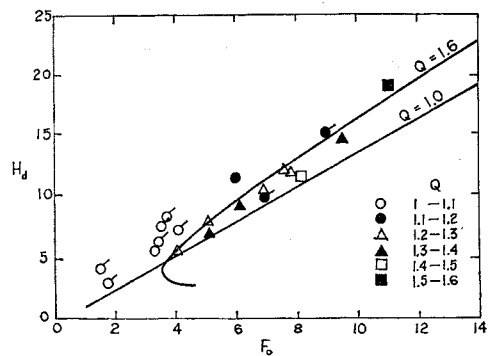


Fig. 4 Comparison with Experimental Data. H_d versus F_0 . Solid lines represent Eq. (17) with $H_2=8$. Flagged symbols relate to submerged hydraulic jump.

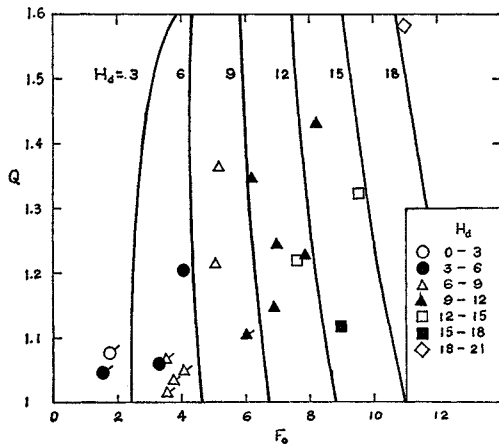


Fig. 5 Comparison with Experimental Data. Q versus F_0 . Solid lines represent Eq. (17) with $H_2=8$. Flagged symbols relate to submerged hydraulic jump.

values of Q provided by the theory fits the experimental data. This implies that in Fig. 2 only those segments of the curves which sprout from the $Q=1$ line and terminate at $F_0=F_{0m}$ are physically meaningful.

The flow ratio Q of the experiments never exceeded 1.6, a value which is considerably less than either non-buoyant jets or three-dimensional buoyant surface jets would have produced under similar outfall conditions. The former can be obtained as a limiting value of Q at $F_0=\infty$ in Fig. 5, and the latter is reported as high as 10 or more by Harlemen and Stolzenbach¹⁵⁾. Fig. 5 further indicates that for a given value of H_a , an increase in F_0 results in a sharp decrease in Q except for cases of small H_a values. For a given value of F_0 , an increase in H_a results in an increase in Q .

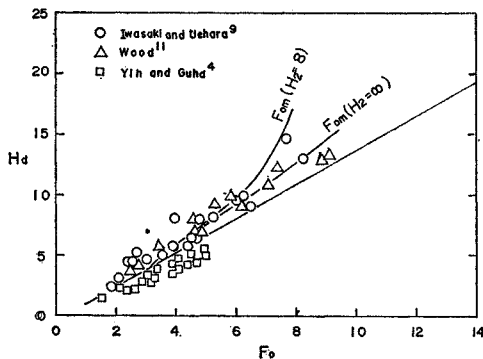


Fig. 6 Comparison with Experimental Data of Other Sources.

Fig. 6 shows plotting of experimental data on conjugate depth of different sources. These data all relate to the internal hydraulic jump of the subsurface flow. Enough information is not provided in these sources as to enable close comparison with the theoretical prediction developed in this paper. However, following remarks can be observed. First, all data except those of Yih and Guha fall above the curve representing the non-mixing internal hydraulic jump. This is in agreement with theoretical predictions of Fig. 3. Even the data of Yih and Guha may not contradict with Fig. 3 which suggest that some data with small F_0 may be found below $Q=1$ curve. Secondly these data are scattered around the curves giving maximum H_a values for a given F_0 value, i.e. F_{0m} curves or envelopes of curves in Fig. 3. In view of this discussion the meaning and use of Figs. 2 and 3 are summarized herein. At first it should be noted again that for given values of F_0 and H_2 these two figures give only the relationship between Q and H_a . An additional knowledge on the mechanism of mixing is necessary to make the problem physically determinate. In Fig. 2, as the preceding discussion indicates, only the lower segments of the curves $H_a=\text{constant}$ are physically meaningful. In Fig. 3 for a given set of values of F_0 and H_a , the latter to be found only above the line $F_1=1$, two values of Q will be found and the smaller of them will actually be realized. For a given set of values of F_0 and Q , the higher of two values of H_a obtained in Fig. 3 will be observed.

In practice the densimetric Froude number F_0 and H_2 are the only prescribed parameters. Fig. 3 then gives such a useful information as a range of H_a values and upper bound of Q value for a given F_0 .

5. CONCLUSIONS

Two-dimensional, rapidly varied flow of two layers has been analyzed with use of the impulse-momentum equations. It has been shown that the conjugate depth relationship for both surface and subsurface flow merges under the condition of small density difference. The solution to this problem has been compared with experimental data and it is found that the rate of entrainment induced by the rapidly varied-internal flow is very sensitive to changes in both the upstream and the downstream conditions and that buoyancy reduces the total entrainment to a substantially smaller amount than would be found either under neutrally buoyant condition or with three-dimen-

tional buoyant jet flow. The conjugate depth is found to be slightly increased by entrainment of flow. Comparison with experimental data shows that theory predicts the tendency remarkably well.

6. ACKNOWLEDGEMENTS

Major part of this work has been carried out while the author was associated with Construction Technique Institute, Foundation, Tokyo, Japan. The author is grateful to their support.

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(Received June 5, 1974)

コンクリートライブラリー一覧

No.	編著者	題 目	定価	〒
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10	委員会編	構造用軽量骨材シンポジウム	500	140
11	樋口芳朗	微細な空気を充てんするためのセメント注入における混和材料に関する研究	120	60
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21	委員会編	パウル・レオンハルト工法設計施工指針(案)	700	210
22	委員会編	レオバ工法設計施工指針(案)	700	140
23	委員会編	BBRV 工法設計施工指針(案)	900	210
24	委員会編	第2回構造用軽量骨材シンポジウム	1100	210
25	丸安・小林 阪本	高炉セメントコンクリートの研究	550	210
26	松本嘉司	鉄道橋としての鉄筋コンクリート斜角げたの設計に関する研究	200	80
27	岡村甫	高張力異形鉄筋の使用に関する基礎的研究	200	60
28	尾坂芳夫	コンクリートの品質管理に関する基礎研究	200	60
29	委員会編	フレシネー工法設計施工指針(案)		
30	委員会編	フープコーン工法設計施工指針(案)	1000	210
31	委員会編	OSPA 工法設計施工指針(案)	1100	210
32	委員会編	OBC 工法設計施工指針(案)	1100	210
33	委員会編	VSL 工法設計施工指針(案)	1000	210
34	委員会編	鉄筋コンクリート終局強度理論の参考	1600	210
35	委員会編	アルミナセメントコンクリートに関するシンポジウム	1300	210
36	委員会編	SEEE 工法設計施工指針(案)	1300	210

37	委員会編	コンクリート標準示方書（昭和49年度版）改訂資料	1500	210
38	委員会編	コンクリートの品質管理試験方法	1500	210
39	委員会編	膨張性セメント混和材を用いたコンクリートに関するシンポジウム	2000	210