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FINITE ELEMENT ANALYSIS OF INTERFACE PROBLEM IN GROUNDWATER FLOW

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ABSTRACT

This paper describes the application of finite element method to the analysis of the interface problem in coastal groundwater flow.

Firstly are introduced the principle of finite element method in the analysis of groundwater flow and its application to the two-dimensional interface problem in steady groundwater flow, especially on the shape of interface and the position of seepage-out point.

Secondly are shown three examples of the finite element analysis. One of them is an interface problem in coastal groundwater flow without drain, then the numerical solution is compared with the theoretical one for the purpose of confirming the accuracy of the numerical method. The second one is the problem with a sink or a source, which is well-known as an upconing phenomenon. The last one is the intrusion of salt water into a coastal unconfined aquifer.

1. INTRODUCTION

One of the well-known interface problems appears concerning the exploitation of petroleum which is naturally stored on groundwater, and the other is known as the intrusion of salt water into a coastal aquifer.

The principles of the above phenomena are quite the same, which belong to the problem of the equilibrium between two liquids which have different densities and different viscosities. The free surface and the seepage surface of unconfined groundwater can be dealt with similarly because in that flow one liquid is groundwater and the other is air, namely it being the special case of the interface problem.

Hitherto the interface problems having been studied mainly with the complex variable theory, i.e. the method by use of the hodograph and conformal mapping, it is sometimes difficult to apply these theoretical formulae to the practical problems which have usually complicated boundary conditions.

On the other hand, the numerical methods, as finite element method in this paper, overcome the above demerit and are more powerful practically.

In this paper are described on the interface between salt- and fresh water in coastal groundwater, the free surface and seepage surface of unconfined groundwater. The practical problem of the intrusion of salt water into fresh groundwater is more complicated than the other interface problems because it includes the problem of diffusion of salt in the groundwater, namely should be considered the transition zone where the concentration of salt is variable strictly. But the description in this paper assumes, as usually done, that both the zones are separated abruptly at the interface without the transition zone. The above assumption does not make the generality of the problem missed.

2. CHARACTERISTICS OF INTERFACE

The interface between salt- and fresh water in coastal groundwater is considered (see Fig. 1).

The total head φ of groundwater is expressed as following,

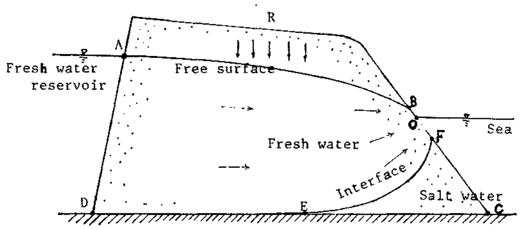

Fig. 1 Domain and boundaries of interface flow.

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$$\varphi = z + \frac{P}{\gamma} \quad \cdots\cdots(1)$$

where z is the height above the reference datum, P is the water pressure and γ is the specific weight.

Provided that the subscriptions f and s mean fresh- and salt groundwater respectively, one obtains the following equation (2),

$$\left.\begin{aligned} \varphi_f &= z_f + \frac{P_f}{\gamma_f} \ : \ \text{in fresh groundwater} \\ \varphi_s &= z_s + \frac{P_s}{\gamma_s} \ : \ \text{in salt water} \end{aligned}\right\} \quad \cdots\cdots(2)$$

On the interface (EF in Fig. 1) both the pressures of fresh- and salt groundwater have to be equal, then the equation (3) is formed,

$$P_f = P_s \quad \cdots\cdots(3)$$

Substituting Eq. (2) into Eq. (3) gives,

$$\left.\begin{aligned} \varphi_f &= \frac{\gamma_s}{\gamma_f}\varphi_s + z\left(\frac{\gamma_f - \gamma_s}{\gamma_f}\right) \\ &\text{or} \\ z &= \frac{\gamma_s\varphi_s - \gamma_f\varphi_f}{\gamma_s - \gamma_f} \quad (\gamma_s \neq \gamma_f) \end{aligned}\right\} \quad \cdots\cdots(4)$$

where z is the height of the interface above the reference datum.

In steady state it can be considered that only the fresh groundwater flows and the salt one is at rest, i.e. φ_s is constant in the domain of salt water. Then Eq. (4) is rewritten as the following form (5) with constant numbers C_1, C_2,

$$\left.\begin{aligned} \varphi_f &= C_1 z + C_2 \\ C_1 &= \frac{\gamma_f - \gamma_s}{\gamma_f}, \quad C_2 = \frac{\gamma_s}{\gamma_f}\varphi_s \end{aligned}\right\} \quad \cdots\cdots(5)$$

Eq. (5) means that the total head of fresh groundwater at the interface is expressed as a linear function of z, i.e. of the elevation head.

The free surface of unconfined groundwater can be considered now by reference of Eq. (5). The free surface corresponds to the interface, then γ_s must be interpreted to be the specific weight of air because the free surface is the interface between groundwater and air instead of the salt water in Eq. (5).

Then putting $\gamma_s = 0$ in Eq. (5) gives

$$\varphi_f = z \quad \cdots\cdots(6)$$

where the specific weight of air is so small that it can be neglected. Eq. (6) is the well-known equation on the free surface of unconfined groundwater.

From the above consideration it is concluded that the groundwater flow with free surface is a special case of the interface flow.

3. FINITE ELEMENT METHOD IN GROUNDWATER FLOW PROBLEM

(1) Finite Element Method

Finite element method is a kind of matrix method which has been used mainly in structural analysis, according to the variational principle. This numerical method becomes to be used not only in the original field of structural analysis but for the problems of soil mechanics and seepage[1], so on, because it has a lot of usefulness in its application.

One of the most beneficial advantages is that one can choose arbitrary shapes of triangles or of quadrangles in the procedure of dividing the domain into a large number of elements, then it is possible to make the boundary condition more satisfactory than the other numerical method, namely the finite difference method. The above advantage of the method makes it more powerful in the analysis of practical problems which have usually complicated boundary conditions.

(2) Analytical Method According to Variational Principle

The two-dimensional domain D of groundwater flow and the boundary S are shown in Fig. 2, where the boundary consists of S_1 and S_2.

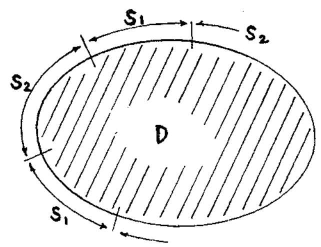

Fig. 2 Domain and boundaries.

(1) On the boundary S_1, the total head f is prescribed,

$$\varphi = f \ : \ \text{on } S_1 \quad \cdots\cdots(7)$$

(2) On S_2, the specific discharge v is prescribed,

$$k\frac{\partial\varphi}{\partial n} = v \ : \ \text{on } S_2 \quad \cdots\cdots(8)$$

where $\partial/\partial n$ is the derivative perpendicular to the boundary, the outward direction being taken as positive.

In the interior of the domain D, the well-known quasi-harmonic equation (9) is formed,

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial \varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial \varphi}{\partial y}\right)+q=0 \quad \cdots\cdots\cdots(9)$$

where q is the amount of water which is stored in unit area per unit time.

Consider the functions U, V, W defined as following,

$$V=\frac{1}{2}\iint_D\left\{k_x\left(\frac{\partial \varphi}{\partial x}\right)^2+k_y\left(\frac{\partial \varphi}{\partial y}\right)^2-2q\varphi\right\}dx\,dy \quad \cdots\cdots(10)$$

$$W=\int_{S_2} v\varphi\, ds \quad \cdots\cdots(11)$$

$$U=V+W \quad \cdots\cdots(12)$$

With the aid of the variational principle, the problem to solve Eq. (9) is reduced to the one of making U the minimum in the condition of Eq. (7). Hence,

$$\delta U=0 \quad \cdots\cdots(13)$$

The approximate solution of Eq. (9) is constructed as the following form with the n-parameters $(\varphi_1, \varphi_2, \cdots, \varphi_n)$ which are as yet unspecified.

$$\varphi=\varphi(x, y, \varphi_1, \varphi_2, \cdots, \varphi_n) \quad \cdots\cdots(14)$$

After substitution of Eq. (14) into Eq. (10), elaboration with Eqs. (11), (12), (13) leads to the following expression,

$$\sum_{i=1}^{n}\frac{\partial U}{\partial \varphi_i}\delta\varphi_i=0 \quad \cdots\cdots(15)$$

for all the combinations of $\delta\varphi_i$. Hence,

$$\frac{\partial U}{\partial \varphi_i}=0 \qquad (i=1, 2, \cdots, n) \quad \cdots\cdots(16)$$

From the n-equations of (16), the parameters $(\varphi_1, \varphi_2, \cdots, \varphi_n)$ being determined, substituting them into Eq. (14) gives the approximate solution of the problem.

(3) Expression with Finite Element

In finite element method the domain D is divided into a large number (n) of sub-domains D^e which are called finite elements or simply elements. Each element has the contribution U^e to U in Eq. (12) as well as V^e, W^e to V, W respectively.

The shape of element is usually chosen as a triangle or a quadrangle and in this paper triangle elements are used.

The head in the interior of an element is approximated by a linear function of (x, y) as following,

$$\varphi=\lambda_1+\lambda_2 x+\lambda_3 y \quad \cdots\cdots(17)$$

where λ_1, λ_2, λ_3 are parameters.

In the triangular element $\Delta(k, l, m)$ as shown

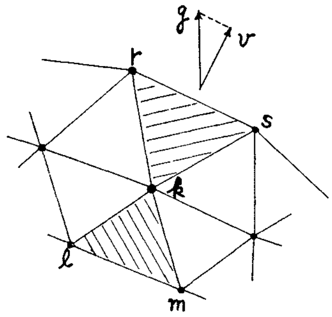

Fig. 3 Finite elements.

in Fig. 3, the subscriptions (k, l, m) are used for the nodes which are the corner points of the triangle. Then one obtains

$$\left.\begin{aligned}\lambda_1&=(c_k\varphi_k+c_l\varphi_l+c_m\varphi_m)/2\Delta\\ \lambda_2&=(a_k\varphi_k+a_l\varphi_l+a_m\varphi_m)/2\Delta\\ \lambda_3&=(b_k\varphi_k+b_l\varphi_l+b_m\varphi_m)/2\Delta\end{aligned}\right\} \quad \cdots\cdots(18)$$

where

$$\left\{\begin{aligned}a_k&=y_l-y_m\\ b_k&=x_m-x_l\\ c_k&=x_l y_m-x_m y_l\end{aligned}\right. \qquad \Delta=\frac{1}{2}|a_k x_k+a_l x_l+a_m x_m|$$

Substituting Eq. (18) into Eq. (10) gives

$$V^e=\frac{1}{2}(k_x\lambda_2^2+k_y\lambda_3^2)\iint_{D^e}dx\,dy$$

$$+q^e\iint_{D^e}(\lambda_1+\lambda_2 x+\lambda_3 y)\,dx\,dy \quad \cdots\cdots(19)$$

for the element, where q^e is the value of q in that element, which is assumed to be constant.

Moreover substituting Eq. (18) into Eq. (19) and elaborating give

$$\left.\begin{aligned}V^e&=\frac{1}{2}\sum_i\sum_j P_{ij}^e\varphi_i\varphi_j+\frac{q^e\Delta}{3}\sum_i\varphi_i\\ P_{ij}^e&=\frac{1}{4\Delta}\{k_x a_i a_j+k_y b_i b_j\} \qquad (i, j=k, l, m)\end{aligned}\right\} \quad \cdots\cdots(20)$$

where $\sum$ is the summation according to (k, l, m), then P^e is two-dimensional (3×3) array with the coefficient P_{ij}^e.

Substituting Eq. (20) into the following equation (21) and elaborating give Eq. (22),

$$V=\sum_{e=1}^{N}V^e \quad \cdots\cdots(21)$$

$$V=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}P_{ij}\varphi_i\varphi_j-\sum_{i=1}^{n}Q_i\varphi_i \quad \cdots\cdots(22)$$

where Q_i is the discharge at i-node, which is calculated by multiplying q by the corresponding area.

W in Eq. (11) is considered.

The specific discharge v along the segment between $r-s$ in Fig. 3 is prescribed, namely the segment is the boundary S_2 in Eq. (8). The vertical compornent of v is denoted by g, which enables rainfall and evaporation to be introduced.

If x-axis of (x, y) plane is chosen to horizontal direction, the following expression is derived,

$$v\,ds = g\,dx \qquad (23)$$

Hence from Eq. (11) one obtains

$$W = \int_r^s v\varphi\,ds = \int_{x_r}^{x_s} g\varphi\,dx \qquad (24)$$

where x_r, x_s $(x_s > x_r)$ are the x-coordinates of r, s respectively.

Assuming that g varies linearly along the boundary $r-s$ in the same way as φ, one obtains[2)]

$$W^e = \int_{x_r}^{x_s} \left[\left(\frac{x_s - x}{x_s - x_r}\right)\varphi_r + \left(\frac{x - x_r}{x_s - x_r}\right)\varphi_s\right] \times \left[\left(\frac{x_s - x}{x_s - x_r}\right)g_r + \left(\frac{x - x_r}{x_s - x_r}\right)g_s\right] dx$$
$$= \frac{1}{6}(x_s - x_r)[(2g_r + g_s)\varphi_r + (2g_s + g_r)\varphi_s] \qquad (25)$$

The summation of W^e in Eq. (25) for all the elements can be expressed as

$$W = \sum_{i=1}^{n} Q_i^* \varphi_i \qquad (26)$$

where the one-dimensional array Q_i^* is composed of the following terms,

$$\left.\begin{aligned}
Q_1^* &= \frac{1}{6}(x_2 - x_1)(2g_1 + g_2) \\
Q_2^* &= \frac{1}{6}(x_2 - x_1)(2g_2 + g_1) + \frac{1}{6}(x_3 - x_2)(2g_2 + g_3) \\
&\vdots \\
Q_{p-1}^* &= \frac{1}{6}(x_{p-1} - x_{p-2})(2g_{p-1} + g_{p-2}) \\
&\quad + \frac{1}{6}(x_p - x_{p-1})(2g_{p-1} + g_p) \\
Q_p^* &= \frac{1}{6}(x_p - x_{p-1})(2g_p + g_{p-1})
\end{aligned}\right\} \qquad (27)$$

namely Q_i^* is the concentrated discharge at i-node along the S_2-boundary, which is introduced instead of g_i.

Substituting Eqs. (22), (26) into Eqs. (12), (16) gives

$$\sum_{j=1}^{n} P_{ij}\varphi_j + Q_i + Q_i^* = 0 \qquad (i = 1, 2, \cdots, n) \qquad (28)$$

4. TECHNIQUE OF FINITE ELEMENT ANALYSIS

(1) The Way of Dividing to Elements

The way of making the network for finite element analysis is explained with reference to Fig. 4, where DE is the interface between salt- and fresh water, AC is the free surface on which the precipitation R exists, AD is an impermeable boundary, CO, OE are the seepage surfaces on which C, E are the seepage-out points, whereas OE faces to sea water.

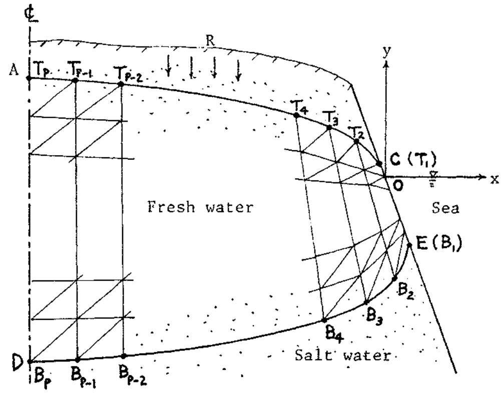

Fig. 4 Coastal groundwater flow.

The upper boundary AC (the position is assumed at first) is divided into $(p-1)$ segments by p-nodes denoted as $T_1, T_2, \cdots, T_p$, and the lower boundary DE is similarly done as $B_1, B_2, \cdots, B_p$.

Then the corresponding nodes T_i, B_i are connected by a straight line as $(T_1 - B_1)$, $(T_2 - B_2)$, $\cdots$, $(T_p - T_p)$ and on each line the same number of nodes are located. The combination between the nodes on the adjoining lines enables to make the triangular or rectangular elements, as shown in Fig. 4, where the division is done according to the program of a digital computer.

(2) Determination of Interface

The procedure of the determination of interface in the finite element analysis is as following (see Fig. 4).

(1) Assume the shape and the position of interface DE.

(2) Make the nodes $B_1, B_2, \cdots, B_p$ on DE, of which y-coordinates are $Z_1(B_1)$, $Z(B_2)$, $\cdots$, $Z(B_p)$ respectively.

(3) Calculate φ in the interior of the domain $ACED$ by use of Eq. (28), where the bound-

ary DE has to be dealt with as an impermeable boundary, because the specific discharge through DE is zero.
(the boundaries AC, CE are described in **4.(3)**, **4.(4)** respectively)

(4) By substituting the above solutions $\varphi(B_2)$, $\varphi(B_3)$, $\cdots$, $\varphi(B_p)$ into Eq. (29), one obtains the Z-values which are denoted as $Z'(B_2)$, $Z'(B_3)$, $\cdots$, $Z'(B_p)$, whereas the seepage-out point B_1 is found out by the way described in **3.(4)**.

$$Z'=\frac{\gamma_s\varphi_s-\gamma_f\varphi_f}{\gamma_s-\gamma_f} \quad \cdots\cdots\cdots\cdots(29)$$

(5) Calculate the mean value of the initially assumed (Z) and the calculated one (Z') as following

$$[Z(B_2)+Z'(B_2)]/2,\ [Z(B_3)+Z'(B_3)]/2,\ \cdots,\ [Z(B_p)+Z'(B_p)]/2 \quad \cdots\cdots\cdots(30)$$

which are the y-coordinates of B_i $(i=2, \cdots, p)$ in the next step of the calculation.

(6) The node B_i on the interface in the next step are shifted along the straight line (T_i-B_i) and are located at the point which has the revised y-coordinate gotten by the above procedure (5).

The procedure (1)-(6) is repeated until the solution reaches to the convergent value.

(3) Determination of Free Surface

As described in **2.**, the free surface of groundwater is considered to be a special type of interface. Then instead of Eq. (29), one can use Eq. (6). Hence the following equation (31) is derived for the procedure of determination of free surface, instead of Eq. (30),

$$[Z(T_2)+\varphi(T_2)]/2,\ [Z(T_3)+\varphi(T_3)]/2,\ \cdots,\ [Z(T_p)+\varphi(T_p)]/2 \quad \cdots\cdots\cdots(31)$$

The seepage-out point T_1 is described in **4.(4)**.

(4) Determination of Seepage-out Point

A) *Seepage-out point of free surface*

It is convenient to be considered two types of seepage-out points. One of them is that the gradient θ of the slope on which the seepage-out point appears is smaller than 90° ($\theta \leqq 90°$), as shown in Fig. 5(a), whereas the other is that ($\theta > 90°$), as Fig. 5(b).

The conditions to be satisfied in the former type (a): the seepage-out point is on the free surface, moreover the tangent to the free surface at the seepage-out point is equal to the gradient of the downstream slope AB.

Then the following approximate way is proposed in the analysis. After the mean value θ_m of the gradients of (T_3-T_2) and of the slope AB being calculated, the straight line of which the gradient is θ_m and goes through the node T_2 is drawn, as in Fig. 5(a). The intersection C of the above straight line with the downstream slope AB is determined to be the seepage-out point. When the seepage-out point determined by the above way becomes under the lebel of the downstream reservoir, i.e. sea, the point is relocated at the sea level because the seepage-out point practically should not be under the sea lebel.

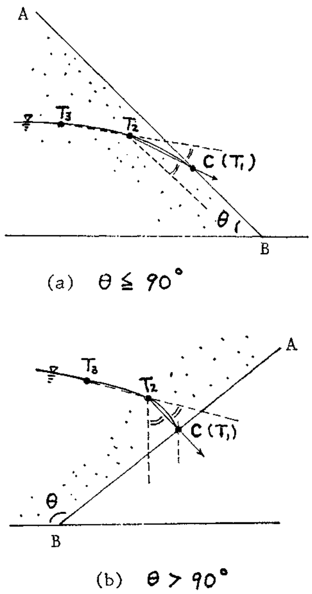

Fig. 5 Seepage-out point of free surface.

On the other hand the necessary conditions of the later type (b) in Fig. 5: the tangent to the free surface at the seepage-out point is vertical. Then after the mean value θ_m of the gradients of (T_3-T_2) and of the vertical direction being calculated, the other treatment and interpretation are the same as the former.

B) *Seepage-out point of interface*

For the general interface problem, the approximate way how to determine the seepage-out point described in the above section A) is possible to be used similarly.

For the interface between salt- and fresh groundwater the procedure of determining the seepage-out point is shown in Fig. 6.

It is also possible to determine the seepage-out point by assuming that the shape of the interface is a parabola in the vicinity of the seepage-out point. For example one considers the case that the sea bottom is horizontal or there is a horizontal drainage as shown in Fig. 7. If the shape of the interface is parabola, the following equation (32) is formulated,

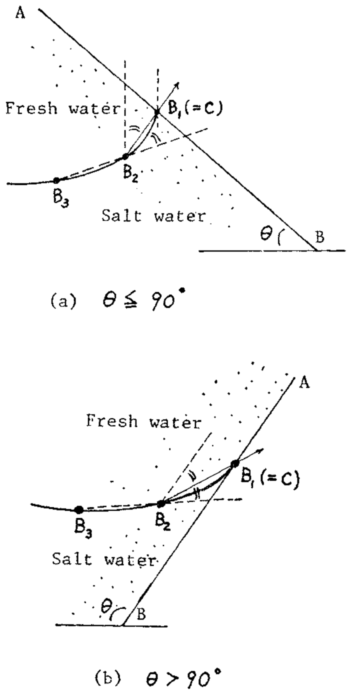

Fig. 6 Seepage-out point of interface between salt- and fresh water.

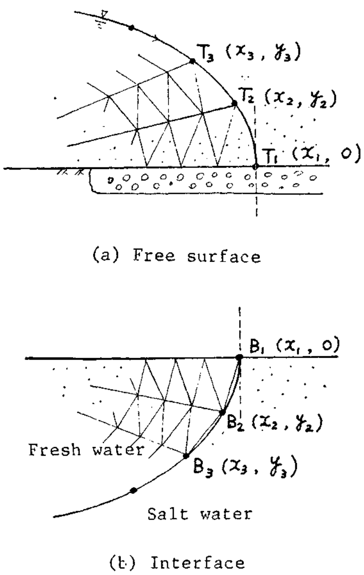

Fig. 7 Seepage-out to horizontal drainage.

$$x_1=\frac{x_2y_3-x_3y_2}{y_3-y_2} \quad \cdots\cdots\cdots\cdots(32)$$

where (x_i, y_i) is the coordinate of B_i or T_i ($i=1, 2, 3$).

5. EXAMPLES

(1) Comparison with Theoretical Solution

The interface problem in coastal groundwater as shown in Fig. 8 was solved theoretically by use of the method of conformal mapping and the hodograph[3]. In order to compare the numerical solution by finite element analysis with the theoretical one, the calculation by use of finite element method is executed.

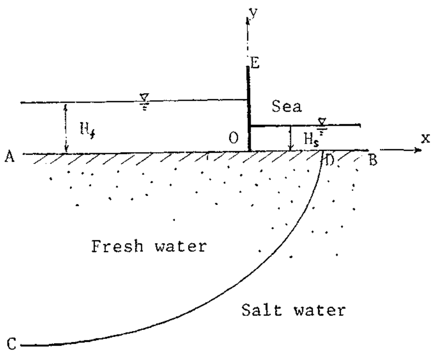

Fig. 8 Boundary condition of problem.

In Fig. 8, *AOB* is the ground surface where *AO* is the bottom of the fresh water reservoir and *OB* is the sea bottom, *EO* is the impermeable wall (the point *O* is the singular point) and *CD* is the interface between salt- and fresh water. The depth of the fresh water reservoir is H_f and the one of sea is H_s, and the origin of (x, y) coordinate is chosen at the point *O*.

Provided that the ground is homogeneous and isotropic, the position of the interface is given as following by theoretical formula (33),[3]

$$\left.\begin{aligned} x&=-\frac{\gamma_f \Delta H}{\pi(\gamma_s-\gamma_f)}\ln\left(\frac{\lambda+1}{4}\right)\\ y&=\frac{2\gamma_f \Delta H}{\pi(\gamma_s-\gamma_f)}\tan^{-1}(\lambda)\end{aligned}\right\} \quad \cdots\cdots\cdots(33)$$

where λ is a parameter, $\Delta H=(H_f-H_s)$.

Now $\Delta H=5$ m, $\gamma_s=1.03$, $\gamma_f=1.00$ being prescribed, the result gotten by the finite element analysis is shown in Fig. 9 and the theoretical solution by Eq. (33) is drawn by a dotted line.

One can understand that the both of the results agree satisfactorily, namely the numerical solution

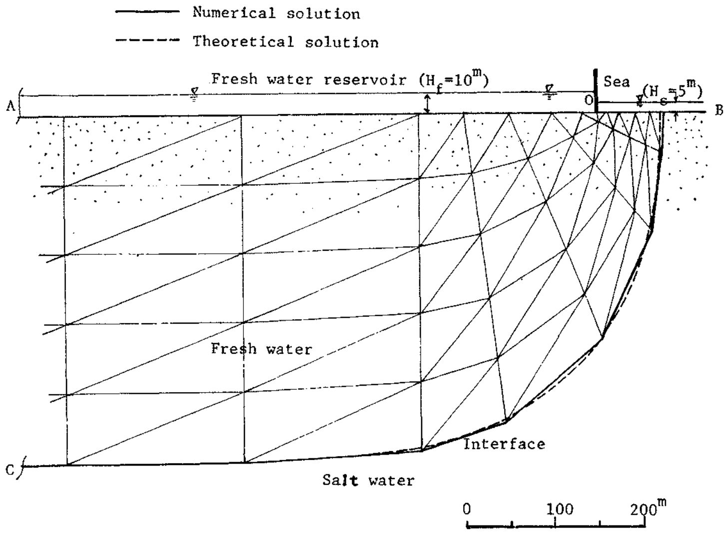

Fig. 9 Example 1, shape of interface without drain.

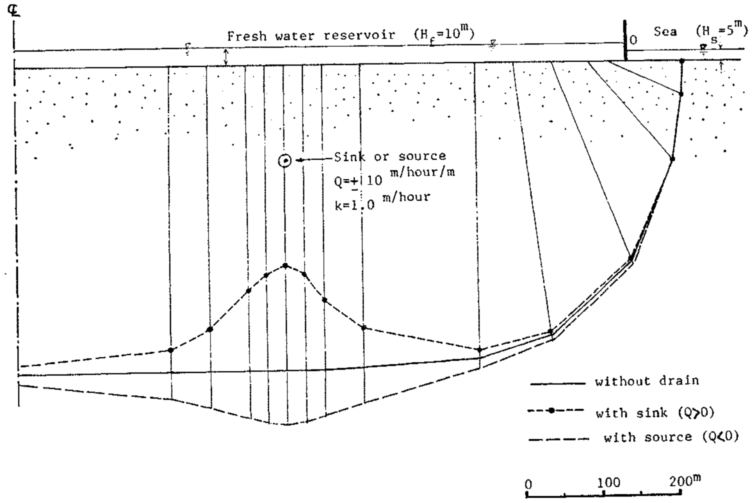

Fig. 10 Example 2, shapes of interfaces.

by finite element analysis is well-approximated.

(2) Upconing

When a drain is operating in the fresh groundwater zone, the phenomenon of raising the interface, which is called "upconing", appears in coastal groundwater.

Provided that the coefficient of permeability: $k=1.0$ m/hour, the discharge of a well: $Q_p=\pm 10$ m^3/hour/m, the coordinate of the well: $(x_p, y_p)=$ (-450 m, -50 m) so on the other boundary conditions are prescribed as equal to the ones in (1). The result gotten by the finite element analysis is shown in Fig. 10.

(3) Intrusion of Salt Water into Aquifer

Fig. 11 shows an example of the intrusion of salt water into an unconfined aquifer with bed rock at the depth of -200 m under the ground surface, which is gotten by the finite element analysis.

The way of finding out the point *P*, which is the end point of the interface, is succeeded by checking whether the position of the interface is above the bed rock.

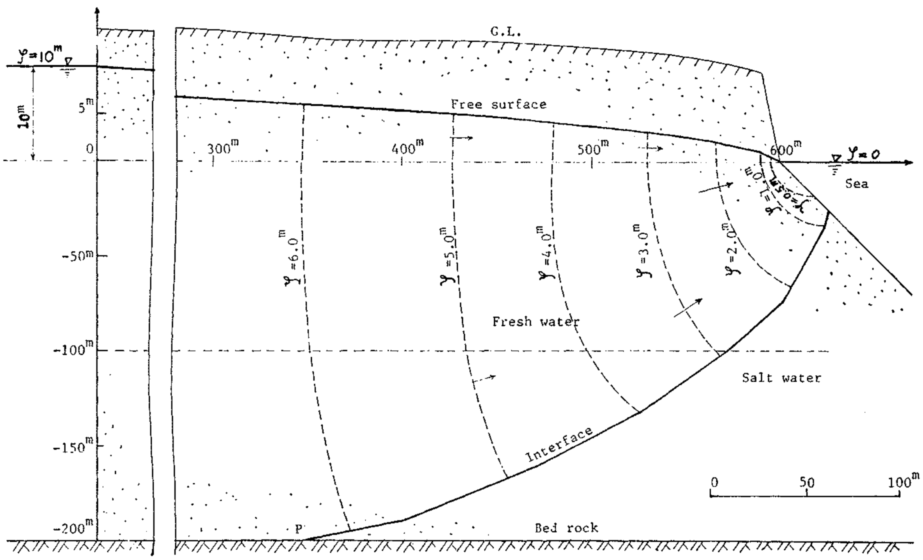

Fig. 11 Intrusion of salt water into a coastal unconfined aquifer.

6. CONCLUSION

This paper described the application of finite element method to the interface flow problem in coastal groundwater and some examples. Consequently it is confirmed that the finite element analysis is useful and powerful to solve these interface problems numerically.

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コンクリートライブラリー一覧

No.	編著者	題目	定価	〒
3	委員会編	異形鉄筋を用いた鉄筋コンクリート構造物の設計例	1000	140
10	委員会編	構造用軽量骨材シンポジウム	500	100
11	樋口芳朗	微細な空げきてん充のためのセメント注入における混和材料に関する研究	120	60
15	委員会編	ディビダーク工法設計施工指針（案）改版	900	110
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21	委員会編	バウル・レオンハルト工法設計施工指針（案）	700	110
22	委員会編	レオバ工法設計施工指針（案）	700	110
23	委員会編	BBRV 工法設計施工指針（案）	900	140
24	委員会編	第2回構造用軽量骨材シンポジウム	1100	140
25	丸安・小林 阪本	高炉セメントコンクリートの研究	550	110
26	松本嘉司	鉄道橋としての鉄筋コンクリート斜角げたの設計に関する研究	200	80
27	岡村甫	高張力異形鉄筋の使用に関する基礎的研究	200	60
28	尾坂芳夫	コンクリートの品質管理に関する基礎研究	200	60
29	委員会編	フレシネー工法設計施工指針（案）		
30	委員会編	フープコーン工法設計施工指針（案）	1000	110
31	委員会編	OSPA 工法設計施工指針（案）	1100	140
32	委員会編	OBC 工法設計施工指針（案）	1100	110
33	委員会編	VSL 工法設計施工指針（案）	1000	110
34	委員会編	鉄筋コンクリート終局強度理論の参考	1600	140
35	委員会編	アルミナセメントコンクリートに関するシンポジウム	1300	140
36	委員会編	SEEE 工法設計施工指針（案）	1300	140

37	委 員 会 編	コンクリート標準示方書（昭和49年度版）改訂資料	近　刊
38	委 員 会 編	コンクリートの品質管理試験方法	
39	委 員 会 編	膨張性セメント混和材を用いたコンクリートに関するシンポジウム	