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CONSIDERATIONS OF WAVE CHARACTERISTICS IN SOIL ASSUMED AS A VISCOELASTIC MATERIAL

By Koichi AKAI and Masayuki HORI***

1. INTRODUCTION

As described in the previous papers[1),2)] a series of one-dimensional stress wave propagation tests has been performed with the following two main purposes;

(i) to know how vibrational displacements caused by earthquakes, traffics, construction machines or explosions propagate into ground and how they exert influence to structures and mankinds, and

(ii) to make clear the dynamic behaviors of soil.

Throughout these researches it has been found that the shock tube technique is suitable as an apparatus of dynamic loading system for applying sharp pulsative pressure on soil specimens.

In general, soil is a mixture which consists of three phases of solid, fluid and void. There are many unknown questions in dynamic characteristics of such a mixture. The authors have investigated experimentally how the wave characteristics change with moisture content by using sandy loam, and carried out the analytical considerations from a viewpoint of viscoelasticity for soil media.[3),4),5)] It has been found through these studies that the wave characteristics, such as attenuation of stress amplitude, wave velocity and wave patterns during stress propagation, can be approximately expressed by assuming soils as a three-parameter viscoelastic model when we select suitable rheological constants.

Biot explicitly induced the stress-strain relationships and the theory of three-dimensional consolidation of saturated porous elastic media standing on many assumptions.[6),7),8)] Further, he modified his theory to the problem of wave propagation with infinitesimally small amplitudes and found that there existed one shear wave, two compressional waves of the first and the second kind in such a medium.[9),10)] Although Biot's logical inference is unique, there are still left extremely difficult questions that the elastic constants expressed in his study and the amount of mass exchange between the solid and the fluid should be estimated. However, the idea that the dissipative force is proportional to the relative displacement between the solid and the fluid gives us distinct images for the characteristics of energy absorption of soil in dynamic state. While, the Biot's theory is similar to one of viscoelasticity and it is found that the characteristics of the compressional wave of the first kind are equivalent to those in spring-Voigt model[11)] and the second kind to Maxwell model.[12)]

There are many differences between the wave characteristics in soil and in metal. Consequently, the special treatment for the problems of wave propagation in soil should be made clear. In this paper are described the concepts of problems of wave propagation in soil and the experimental results in Section 2. In Section 3, it is assumed that the soil can be expressed as the spring-Voigt model and it is performed to estimate the wave characteristics quantitatively in porous media. In Sections 4 and 5 experimental results and conclusions are presented, respectively.

2. PROBLEMS OF WAVE PROPAGATION IN SOIL AND THE PREVIOUS EXPERIMENTAL STUDIES

2.1 Concept of Problems of Wave Propagation in Soil

The physical behaviors of soil are governed by the internal factors such as its physical properties and by the external factors such as applying forces. It is important to choose the main fac-

* Dr. Eng., Professor of Civil Engineering, Kyoto University.
** M.S.C.E., Doctorial course student, Kyoto University.

tors of them in order to simplify the problem. Hardin and Black[13] considered the shear modulus of soil G depending on the following factors;

$$G=f(\bar{\sigma}_0, e, H, S_r, \tau_0, c, A, f, s, \theta, T) \quad \cdots(1)$$

where, $\bar{\sigma}_0$ denotes the effective octahedral normal stress (average effective confining pressure), e the void ratio, H the ambient stress history and vibration history, S_r the degree of saturation, τ_0 the octahedral shear stress, c the grain characteristics, grain shape, grain size and grading mineralogy, A the amplitude of strain, f the frequency of vibration, s the secondary effect which is a function of time and magnitude of load increment, θ the soil structure and T the temperature. They also reported that, especially in the case of sand, G is independent of the factors except $\bar{\sigma}_0$ and e in the strain level less than 10^{-4}. By using the sand left by the No. 120 sieve, Hardin and Richart[14] reported that the grain size, arrangement and shape do not almost influence upon G, that the degree of saturation influences a little only at low pressure level and that the frequency less than 2 500 cps does not influence at all. The strain level used in their experiments is less than 10^{-4} where the behaviors of soil seem to be elastic. The viscoelastic or the plastic behavior is considered to be remarkable in strain level more than 10^{-4}.

Fig. 1 shows schematically an approach to the wave propagation phenomenon, the factors concerning it and the treatment for analytical study. Grain size, degree of saturation and void ratio are taken as the factors concerning the wave characteristics. The foundation of these representations is (i) to express the main physical properties of soil by these three factors and (ii) to consider the authors' wave propagation tests as mentioned later. The relationship between void ratio and confining pressure is obtained by soil testing.

When an external displacement or a force is applied to soil, the corresponding stress or strain occurs in it. In this case, Newton's second law holds between the displacement and the force, while there exists the kinetic equation between the displacement and the strain. The stress-strain relationship is characterized under all external actions such as strain rate, stress rate and strain level.

The characterized wave propagation phenomena can be described by three equations; the equation of motion, kinetic equation and constitutive equation. If the wave with an infinitesimal amplitude is considered, the first two relationships always hold. So that, the stress-strain relationship is most important for treating the problem of wave propagation in soils and studying the wave characteristics. However, as long as there are many unknown factors in it, it is impossible to perfectly express the entire characteristics of waves in soil. Fortunately, since the soil strain level occurred by earthquakes, traffics or explosions is at most about 10^{-3},[15),16)] it can be allowed to assume soil as a certain simple model. For example, Ishihara[17] classified the change in the nature of soil with strain level into three classes as follows;

$<10^{-4}$	elastic
$10^{-4}\sim10^{-2}$	elasto-plastic
$>10^{-2}$	failure

While, the authors' classification is that the behaviors of soil are elastic in the strain range less than 10^{-4}, viscoelastic in the range of $10^{-4}\sim10^{-3}$ and plastic or viscoplastic more than 10^{-3} strain, refering to the above suggestion by Ishihara and experimental facts obtained in the one-dimensional stress wave propagation test by Shimogami.[18] In this test the peak stress attenuation does not occur in the strain range less than 10^{-4}, whereas we recognize the attenuation in the strain level of 10^{-3} where no plastic strains are observed on the stress-strain curves expressing a hysteresis

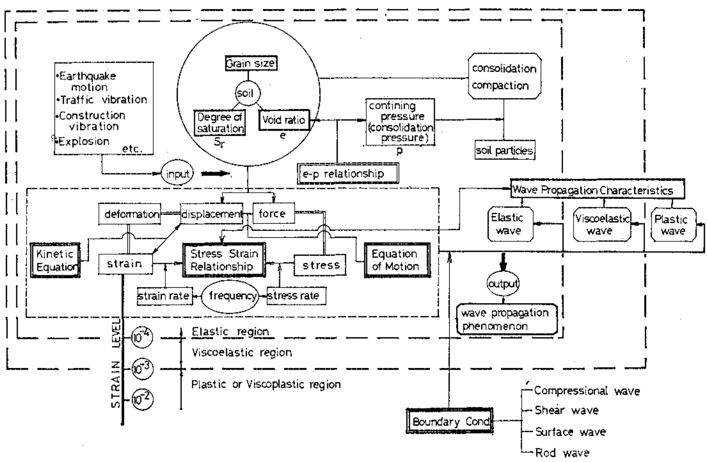

Fig. 1 Concepts of Treatment for the Problems of Wave Propagation in Soil,

loop. Consequently, the waves in the strain level with the above classification will indicate the characteristics of elastic, viscoelastic and plastic waves, respectively.

Since the strain level is limited in the range of $10^{-4}\sim10^{-3}$ in the wave propagation test by means of shock tube, the wave characteristics are considered to be viscoelastic. In the experiments, the authors make efforts to find out the relationships between the wave characteristics and the soil properties.

2.2 Shock Tube Study on Stress Wave Propagation in Confined Soils

(1) General description

In this section, the relationships between the characteristics of one-dimensional compressional wave and the soil properties such as compaction, void ratio and degree of saturation are described.[19] The experimental apparatus and procedure have been presented in the previous paper.[2] "The soil sample is sandy loam with the distribution of particle size of 70% in sand, 29% in silt and 1% in clay fraction." The optimum moisture content is 12% and the maximum dry density is 1.89 g/cm³. The tests have been performed for three kinds of water content, (a) dry, (b) 5~7% and (c) 11~13%, and the range of bulk density of 1.7~2.1 g/cm³. The degree of saturation is in the wide range of 20~80%. The pressure forms given by the shock tube are nearly pulsative, and the prominent frequency is about 100 cps.

(2) Peak stress attenuation

The attenuation of the peak stress amplitude normalized by the surface stress is expressed by the following equation;

$$\Sigma'=\exp(-\alpha x) \qquad \cdots\cdots(2)$$

where, Σ' denotes the dimensionless stress, x the depth (m) and α the attenuation constant (m^{-1}). Fig. 2 shows the relationship between the attenuation constant α and the void ratio e. It is found from this figure that there exists linear relationship on the logarithmic paper, and that the void ratio is very important factor for the energy absorption of soils. The amount of the attenuation changes with water content. At the same void ratio, the attenuation in the soil sample with water content of 11~13% is smaller than that of 5~7%. Further, the attenuation constant in the dry sample is $\alpha=0.16\,m^{-1}$ at the void ratio of 0.68. The attenuation is smaller in order of the sample in dry, 11~13% and 5~7% at the same void ratio. Although these tendency seems to be strange, it can be understood when arranged by the dry density. The attenuation constants, calculated from Eq. (2) using the experimental results for the clayey soils obtained by the other investigators from the same experiments as the authors, are also plotted in this figure. The soil properties, types of wave and experimental conditions used by each investigator are presented in Table 1. There is same tendency between α and e for the clayey soils as sandy loam. However, it is found that the amount of the attenuation in clayey soils is smaller than sandy loam. This fact supports the general concept concerning the mechanism of energy absorption that a viscous damping due to interaction between clay particles and water is predominant for clayey soils, while a frictional damping for sandy soils. It is also considered to be one of the reasons that the apparent void

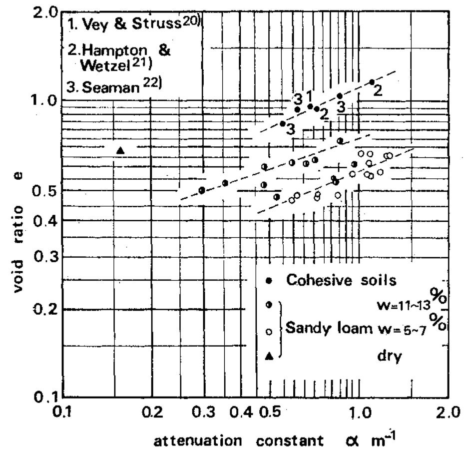

Fig. 2 Relationship between the Attenuation Constant and the Void Ratio.

Table 1 Soils and Their Properties Used by Various Investigators

investigators	soil	water content %	degree of saturation %	void ratio e	kind of wave
E. Vey L. V. Strauss[20]	EPK-clay	34.1	93.0	0.97	rod wave
D Hampton R. A. Wezel[21]	EPK-clay	29.7	73.9	1.07	rod wave
	EPK-clay	33.2	93.1	0.95	rod wave
L. Seaman[22]	Kaolinite	34.7	88.4	1.02	compressional wave
	Kaolinite	31.7	87.8	0.94	compressional wave
	Kaolinite	18.8	58.9	0.83	compressional wave

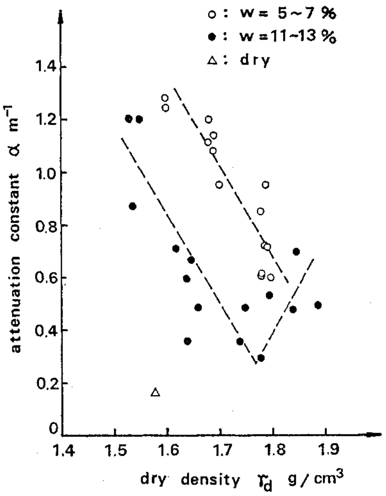

Fig. 3 Relationship between the Attenuation Constant and Dry Density of Sandy Loam.

ratio decreases due to absorbed water. Fig. 3 shows the relationship between the dry density γ_d and the attenuation constant α. Due to increase in dry density γ_d, the soil strengthens its structure with water content of 5~7%, while in 11~13% it is strongest at the dry density of about 1.75 g/cm^3. These tendencies represent well the characteristics of compaction for sandy loam.

(3) Rise time

The change in the wave pattern during stress wave propagation is an important factor to express the wave characteristics. The dimensionless rise time is defined by the following equation in order to estimate quantitatively;

$$T'=\frac{t_x}{t_0} \qquad \cdots\cdots(3)$$

where, t_0 denotes the rise time in surface pressure form (m sec) and t_x the rise time at a certain depth (m sec). In general, the spike pulsative wave form obtained by the shock tube becomes gently round and its periods grow longer during the wave propagation. The dimensionless rise time T' means that the larger of T' is, the larger in the change of wave form becomes and that as T' approaches unity, the wave form does not change. Fig. 4 shows the relationship between T' in each depth (40, 80, 120 and 160 cm) and the dry density γ_d. The relationship between α and γ_d is also drawn in this figure for reference. It can be understood that the tendency for the change in T' is similar to the

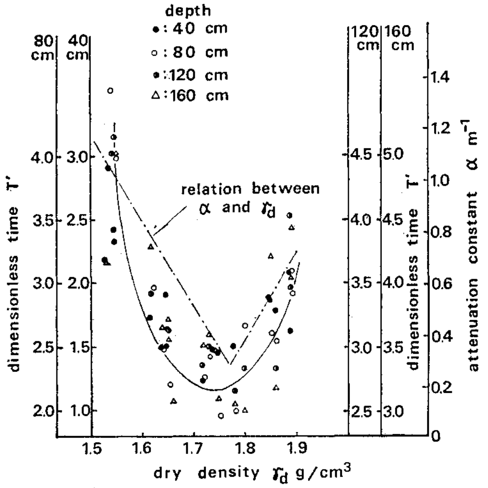

Fig. 4 Relationship between the Dimensionless Rise Time and Dry Density.

stress attenuation with γ_d. It is concluded that for the soil with stronger structure, both the attenuation and the change in the wave form are small during wave propagation through soils.

(4) Wave velocity

It is already reported in the previous paper[2] that the elastic wave velocity at the wave front increased with decrease in the void ratio and that the linear relationship between the velocity and the confining pressure exists on a logarithmic paper by using the samples with water content of 5~7%. Although there are many questions for the elastic wave velocity in unsaturated

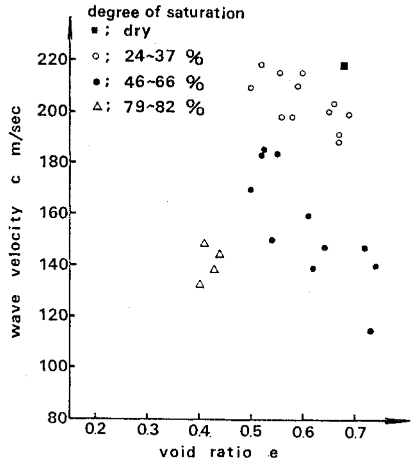

Fig. 5 Relationship between the Wave Velocity and the Void Ratio with the Parameter of Degree of Saturation.

soils to be solved, the distinct experimental conclusions are found out from Fig. 5 that when the wave velocities are plotted for the corresponding void ratio with parameters of the degree of saturation, they decrease with increase in the void ratio for the same degree of saturation and with increase in the degree of saturation for the same void ratio.

It has been reported that the above mentioned wave characteristics obtained from the experiments concerned with the sandy loam can be approximately expressed by assuming the spring-Voigt model.[3),5)] However, the problems how to obtain the viscoelastic constants and how to relate them to the soil properties are still left. The concept of Biot's wave propagation theory is an answer for such questions and it will give us a key to solve the problem by comparing with the theory of viscoelasticity. This will be discussed in the next section in detail.

3. MECHANICAL MODEL, ANALYSIS AND DISCUSSION

3.1 Problems of Wave Propagation in Saturated Porous Media

When waves with infinitesimally small amplitude propagate in a saturated porous elastic medium, Biot supposed that there occurred mass exchange between the solid and the fluid phase and that the equivalent mass should be added to both phases due to coupling in each other. Consequently, the negative apparent mass has been introduced in his theory. This plays a role as equivalent parts for both phases in kinetic energy and a dissipation function. Under these assumptions, he derived the following equation of motion;[9)]

for the solid phase:

$$\sigma_{ij,j}=\frac{\partial^2}{\partial t^2}(\rho_{11}u_i+\rho_{12}U_i)+b\frac{\partial}{\partial t}(u_i-U_i) \qquad \cdots(4)$$

for the fluid phase:

$$-p_{,i}=\frac{\partial^2}{\partial t^2}(\rho_{12}u_i+\rho_{22}U_i)-b\frac{\partial}{\partial t}(u_i-U_i) \qquad \cdots(5)$$

where, σ_{ij} denotes the tensor components of the stress acting on the solid part, $-p$ the stress acting on the fluid part, u_i the displacement components of the solid, and U_i the displacement components of the fluid. The following relations exist among ρ_{11}, ρ_{12} and ρ_{22};

$$\rho_{11}=\rho_1+\rho_a,\quad \rho_{12}=-\rho_a,\quad \rho_{22}=\rho_2+\rho_a \qquad \cdots(6)$$

where, ρ_1 and ρ_2 denote the mass of solid and fluid per unit volume of aggregate, respectively, ρ_a the additional mass, and ρ_{12} the additional apparent mass. ρ_1 and ρ_2 are given by the following equations with the terms of mass densities ρ_s and ρ_f for the solid and the fluid, respectively, and the porosity n:

$$\rho_1=(1-n)\rho_s,\qquad \rho_2=n\rho_f \qquad \cdots(7)$$

Further, b is given by;

$$b=\frac{\eta n^2}{k} \qquad \cdots(8)$$

where, η denotes the fluid viscosity and k the Darcy's coefficient of permeability.

Biot found that there existed one shear wave and two dilatational waves in a saturated porous elastic medium by starting from Eqs. (4) and (5). Two dilatational waves propagate in the elastic structure and the fluid, respectively. The former is called the wave of the first kind and the latter the second kind. The wave of the first kind is a common wave in solids, while the wave of the second kind has a nature of wave in higher frequency range but loses the nature in the lower frequency range.

Ishihara supposed the interacting force proportional to the time rate of relative displacement between the solid and the fluid acting equivalently to the two phases, in place of the mass exchange in Biot's theory. He introduced the following equations of motion;[11)]

for the solid phase:

$$\rho_s\frac{\partial^2 u_i}{\partial t^2}=\sigma_{ij,j}+b\frac{\partial}{\partial t}(u_i-U_i) \qquad \cdots(9)$$

for the fluid phase:

$$\rho_w\frac{\partial^2 U_i}{\partial t^2}=-p_{,i}-b\frac{\partial}{\partial t}(u_i-U_i) \qquad \cdots(10)$$

where, ρ_s and ρ_w are bulk densities of the solid and the fluid, respectively. In his case b is given by,

$$b=\frac{n^2\gamma_w g}{k} \qquad \cdots(11)$$

where, γ_w denotes the specific mass density of the fluid and g the acceleration of gravity. Further, he expressed the elastic constants in the stress-strain relationships by the terms of four kinds of compressibility (bulk and skeleton compressibilities of the solid skeleton and the solid particles, respectively, pore and fluid compressibilities) which have been proposed by Nagumo.[23)] According to his approach,[24)] it is possible to estimate the elastic constants quantitatively, because the compressibilities of soil can be easily obtained from experiments, and in fact, they have been measured by many investigators.

The equation of wave propagation for the compressional wave of the first kind is derived as;

$$\left[b(\rho_s+\rho_w)\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial t^2}-V_1^2\nabla^2\right)-\rho_s\rho_w\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2}{\partial t^2}-V_n^2\nabla^2\right)\right]e=0 \quad \cdots\cdots\cdots(12)$$

where, e denotes the volumetric strain and V_1 and V_n the phase velocities at the zero and the infinite frequency, respectively.[11] It is found that Eq. (12) is equivalent to the following wave equation for the rod wave in a bar whose properties are represented by the three-parameter viscoelastic model as shown in Fig. 6;

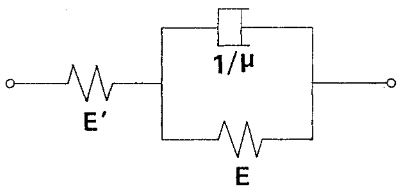

Fig. 6 Three-parameter Linear Viscoelastic Model (Spring-Voigt Model).

$$\left[\rho(E'+E)\left\{\frac{\partial^2}{\partial t^2}-\frac{EE'}{\rho(E'+E)}\frac{\partial^2}{\partial x^2}\right\}-\frac{\rho}{\mu}\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial t^2}-\frac{E'}{\rho}\frac{\partial^2}{\partial x^2}\right)\right]u=0 \quad \cdots\cdots\cdots(13)$$

where, ρ denotes the density and u the displacement along the propagation.

In similar, the equation for the wave of the second kind is;

$$\left[\omega_c\frac{\partial}{\partial t}+\frac{\partial^2}{\partial t^2}-V_d^2\nabla^2\right]e=0 \quad \cdots\cdots\cdots\cdots\cdots\cdots(14)$$

where, ω_c is the characteristic frequency given by the following equation;

$$\omega_c=\frac{b(\rho_s+\rho_w)V_1^2}{\rho_s\rho_w V_n^2} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(15)$$

and V_d is the phase velocity at the infinite frequency. It is understood from Eq. (14) that as the contribution of the second-order time derivative is large, it has a nature of hyperbolic partial differential equation, while it shifts to parabolic partial differential equation in the lower frequency range, since the influence of inertia has little role there. Therefore, the physical phenomenon changes from the wave propagation to the quasi-steady state of consolidation. Eq. (14) is equivalent to the following wave equation for a rod wave in the bar whose properties are represented by Maxwell model as shown in Fig. 7;

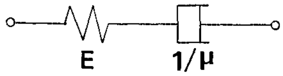

Fig. 7 Maxwell Model

$$\left[E\mu\frac{\partial}{\partial t}+\frac{\partial^2}{\partial t^2}-\frac{E}{\rho}\frac{\partial^2}{\partial x^2}\right]u=0 \quad \cdots\cdots\cdots\cdots\cdots(16)$$

Comparing Eq. (16) with Eq. (14), the terms of E/ρ and $E\mu$ correspond to V_d^2 and ω_c, respectively. Fig. 8 shows the wave characteristics for the rod wave by Maxwell model. In this figure, V denotes the phase velocity, V_0 equals to $\sqrt{E/\rho}$, δ the logarithmic decrement, p the circular frequency and τ equals to $1/E\mu$. The behavior of Voigt model is also shown in the figure for reference.

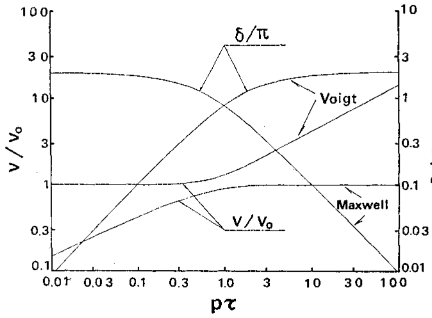

Fig. 8 Wave Characteristics of Rod Wave by Maxwell and Voigt Models.

3.2 Mechanical Model and Analysis of Soil Behavior Assumed as Viscoelastic Material

In **3.1**, we understand that the wave characteristics in a saturated porous elastic medium are similar to those in viscoelastic media. Also, we are interested in the wave of the first kind whose characteristics are equivalent to those of the spring-Voigt model. Let us here analyse the wave characteristics.

It is assumed that the medium is homogeneous, isotropic and viscoelastic, its volume change is

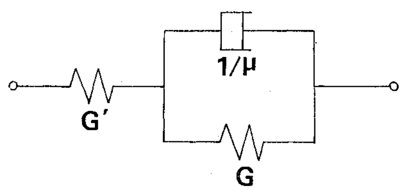

Fig. 9 Mechanical Model

elastic and the relationship between the deviatoric stress and the deviatoric strain is expressed by the spring-Voigt model as shown in Fig. 9. The constitutive equations are;

for the volume change:

$$\sigma=3Ke \qquad \cdots\cdots(17)$$

for the deviatoric stress and strain:

$$2G\cdot G'\gamma_{ij}+2\frac{G'}{\mu}\dot{\gamma}_{ij}=(G+G')\tau_{ij}+\frac{1}{\mu}\dot{\tau}_{ij} \qquad \cdots\cdots(18)$$

where, K denotes the bulk modulus and τ_{ij} and γ_{ij} the components of the deviatoric stress and strain, respectively. They are related with the stress tensor components σ_{ij} and the strain tensor component e_{ij}, respectively, by the following equations;

$$\tau_{ij}=\sigma_{ij}-\sigma\delta_{ij}, \qquad \sigma=\frac{1}{3}\sigma_{kk} \qquad \cdots\cdots(19)$$

$$\gamma_{ij}=e_{ij}-e\delta_{ij}, \qquad e=\frac{1}{3}e_{kk} \qquad \cdots\cdots(20)$$

where, e_{ij} is given by the following equation by using the displacement components u_i;

$$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \qquad \cdots\cdots(21)$$

From Eqs. (17) and (18),

$$\sigma_{ij}=\left\{K+\frac{\frac{2}{3}G'\left(G+\frac{1}{\mu}\frac{\partial}{\partial t}\right)}{G+G'+\frac{1}{\mu}\frac{\partial}{\partial t}}\right\}e_{kk}\delta_{ij}+\frac{2G'\left(G+\frac{1}{\mu}\frac{\partial}{\partial t}\right)}{G+G'+\frac{1}{\mu}\frac{\partial}{\partial t}}e_{ij} \qquad \cdots\cdots(22)$$

Substituting Eq. (22) into the equation of motion

$$\rho\ddot{u}_i=\sigma_{ij,j} \qquad \cdots\cdots(23)$$

we obtain the following equation;

$$\rho\frac{\partial^2 u_i}{\partial t^2}=\frac{G'\left(G+\frac{1}{\mu}\frac{\partial}{\partial t}\right)}{G+G'+\frac{1}{\mu}\frac{\partial}{\partial t}}\nabla^2 u_i+\left\{K+\frac{\frac{1}{3}G'\left(G+\frac{1}{\mu}\frac{\partial}{\partial t}\right)}{G+G'+\frac{1}{\mu}\frac{\partial}{\partial t}}\right\}\text{grad. div. }\boldsymbol{u} \qquad \cdots\cdots(24)$$

where, $\boldsymbol{u}$ denotes the displacement vector. Applying the divergence operator to Eq. (24), we obtain the following equation for the compressional wave;

$$\frac{\partial^2 e}{\partial t^2}=\left\{K+\frac{\frac{4}{3}G'\left(G+\frac{1}{\mu}\frac{\partial}{\partial t}\right)}{G+G'+\frac{1}{\mu}\frac{\partial}{\partial t}}\right\}\nabla^2 e \qquad \cdots\cdots(25)$$

On the other hand, using the curl operator to Eq. (24) becomes;

$$\rho\frac{\partial^2 \boldsymbol{\omega}}{\partial t^2}=\frac{G'\left(G+\frac{1}{\mu}\frac{\partial}{\partial t}\right)}{G+G'+\frac{1}{\mu}\frac{\partial}{\partial t}}\nabla^2\boldsymbol{\omega} \qquad \cdots\cdots(26)$$

where, $\boldsymbol{\omega}$ is the rotation of the displacement vector $\boldsymbol{u}$. By solving Eqs. (25) and (26), the phase velocities and the logarithmic decrement are obtained, by means of infinite harmonic waves as follows;

A. Compressional wave

$$\frac{V_C}{V_1}=\left(\frac{2}{\sqrt{A}+A'}\right)^{1/2}, \qquad \frac{\delta_C}{\pi}=2\left(\frac{\sqrt{A}-A'}{\sqrt{A}+A'}\right)^{1/2} \qquad \cdots\cdots(27)$$

where, A and A' are given by;

$$A=\frac{1+\theta_C{}^2\tau_C{}^2p^2}{1+\tau_C{}^2p^2}, \qquad A'=\frac{1+\theta_C\tau_C{}^2p^2}{1+\tau_C{}^2p^2} \qquad \cdots\cdots(28)$$

B. Rotational wave

$$\frac{V_H}{V_s}=\left(\frac{2}{\sqrt{B}+B'}\right)^{1/2}, \qquad \frac{\delta_H}{\pi}=2\left(\frac{\sqrt{B}-B'}{\sqrt{B}+B'}\right)^{1/2} \qquad \cdots\cdots(29)$$

where, B and B' are given by;

$$B=\frac{1+\theta_H{}^2\tau_H{}^2p^2}{1+\tau_H{}^2p^2}, \qquad B'=\frac{1+\theta_H\tau_H{}^2p^2}{1+\tau_H{}^2p^2} \qquad \cdots\cdots(30)$$

Throughout Eqs. (27)~(30), V_C and V_H denotes the phase velocities, δ_C and δ_H the logarithmic decrements related with the attenuation constant α by;

$$\delta_C=2\pi V_C\alpha/p, \qquad \delta_H=2\pi V_H\alpha/p \qquad \cdots\cdots(31)$$

V_1 and V_s are the phase velocities at the zero frequency expressed by the following equations, respectively;

$$V_1{}^2=\frac{K}{\rho}+\frac{\frac{4}{3}G'G}{\rho(G+G')}, \qquad V_s{}^2=\frac{G'G}{\rho(G'+G)} \qquad \cdots\cdots(32)$$

The phase velocities V_n and V_p at the infinite frequency are respectively given by;

$$V_n{}^2=\frac{K+\frac{4}{3}G'}{\rho}, \qquad V_p{}^2=\frac{G'}{\rho} \qquad \cdots\cdots(33)$$

θ_C and θ_H are the parameters defined by;

$$\theta_C=\left(\frac{V_1}{V_n}\right)^2, \quad \theta_H=\left(\frac{V_s}{V_p}\right)^2 \quad \cdots\cdots(34)$$

Finally, τ_C and τ_H are given by the following equations, respectively;

$$\tau_C=\frac{\frac{1}{\mu}\left(K+\frac{4}{3}G'\right)}{G\left(K+\frac{4}{3}G'\right)+KG'}=\frac{\frac{1}{\mu}}{G+G'}\cdot\frac{1}{\theta_C} \quad \cdots\cdots(35)$$

$$\tau_H=\frac{1}{\mu G} \quad \cdots\cdots(36)$$

3.3 Numerical Analysis and Discussion

Fig. 10 shows the calculted results from Eqs. (27) and (29) in the range of θ_C and θ_H of 0.1 to 0.9. The wave characteristics of both com-

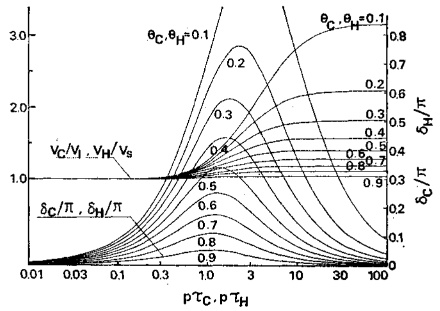

Fig. 10 Wave Characteristics of the Assumed Model.

pressional and rotational waves are identical in the expression. If τ_C is replaced by the reciprocal of the characteristic frequency given by Eq. (15) as

$$\frac{1}{\omega_C}=\frac{\rho_s\rho_w V_n^2}{b(\rho_s+\rho_w)V_1^2}=\frac{\rho_s\rho_w}{b(\rho_s+\rho_w)}\cdot\frac{1}{\theta_C} \quad \cdots\cdots(37)$$

in the compressional wave, while τ_H is replaced by the reciprocal of the characteristic frequency as

$$\frac{1}{\omega_H}=\frac{\rho_w}{b} \quad \cdots\cdots(38)$$

in the rotational wave, then the wave characteristics shown in Fig. 10 are similar to those in the saturated porous elastic medium calculated by Ishihara.[11] In the case of rod waves in a bar whose properties are represented by the spring-Voigt model as shown in Fig. 6, Fig. 10 can express the wave characteristics,[25] if τ_C or τ_H is replaced by the retardation time defined by the following equation;

$$\tau=\frac{1}{E\mu} \quad \cdots\cdots(39)$$

It is found that the physical behaviors or the wave characteristics change widely by the parameters θ_C or θ_H and the dimensionless frequency $p\tau$. $\tau=0$ or $\tau=\infty$ corresponds to the perfectly elastic body. By refering Fig. 9, $\tau=0$ corresponds to state where the dashpot in the model is invalid and the resulting rigidity for the entire body G'' is

$$G''=\frac{1}{\frac{1}{G}+\frac{1}{G'}} \quad \cdots\cdots(40)$$

while, $\tau=\infty$ corresponds to where the dashpot becomes rigid and the resulting rigidity for the entire body G'' is

$$G''=G' \quad \cdots\cdots(41)$$

For the above two extreme cases, the logarithmic decrements approach zero.

In the middle range between the two extreme cases where the material behaves vicoelastically, the physical behaviors are uniquely determined by τ_C or τ_H and θ_C or θ_H, respectively. θ_C or θ_H is determined only by the ratio of the spring constants as represented by Eqs. (32), (33) and (34), and depends on the entire rigidity of the material. The rigidity of the material increases as θ_C or θ_H becomes large, and the behaviors become elastic. While, τ_C or τ_H is concerned with the nature of viscosity by Eqs. (35) and (36). The materials have the characteristic frequency and exhibit the maximum viscous damping at $p\tau_C$ or $p\tau_H$ of about 1.0. The dependence upon the frequency for the logarithmic decrement is belltype whose peak appears at $p\tau_C$ or $p\tau_H=1.0$.

Comparing Eqs. (35) and (36) with Eqs. (37) and (38) with each other, it is understood that the viscoelastic constants, $1/\mu$, G and G', are related with the coefficient b given by Eq. (11) for the saturated porous elastic medium. In other words, the viscoelastic constants depend upon the coefficient of permeability and the porosity.

From the above considerations, we conclude that, if soil is assumed as the spring-Voigt model in the dynamic state, the free spring in the model expresses elastic behavior which is independent of the coupling between solid and fluid phases, while the Voigt part expresses viscoelastic coupling behaviors depending on the relative displacement between two phases.

4. CONSIDERATIONS OF EXPERIMENTAL RESULTS

4.1 Changes in Time Constants with Frequency

Since the wave characteristics in a saturated porous elastic medium are equivalent to those in a viscoelastic material as described in the previous section, τ_G or τ_H is considered to be the same as $1/\omega_G$ or $1/\omega_H$ in the other, respectively. The values of τ_G or τ_H are investigated by experiments. The manner of them is described below.

The forced vibration tests were performed by means of the apparatus for the vibrational triaxial compression test in the frequency range of 0.04 to 4 cps. The soil sample is Fukakusa clay consolidated under the pressure σ_G of 1.0 to 2.5 kg/cm^2. The sample was almost saturated. The amplitude of shear strain was in the range of 2.4×10^{-4} to 1.4×10^{-3}. Measuring the area surrounded by the hysteresis loop in the shear stress-strain curve, the time constant $\tau=1/\mu G$ can be calculated from the following equation;[26)]

$$\tau=\frac{\Delta W}{\pi p \tau_0 \gamma_0\left\{1-\frac{1}{2}\left(\frac{\Delta W}{\pi \tau_0 \gamma_0}\right)^2\right\}} \qquad \cdots\cdots\cdots\cdots(42)$$

where, ΔW denotes the area surrounded by the hysteresis loop, p the circular frequency, γ_0 the amplitude of shear strain, and τ_0 the amplitude of shear stress. Fig. 11 shows the relationship between τ calculated from Eq. (42) and the frequency. In this figure, there are also expressed the time constants τ calculated from $\tau=1/E\mu$ by using the viscoelastic constants which Hatano and Watanabe[27)] obtained by the forced vibration tests assuming the soil as Burgers model as shown in Fig. 12, also from the experimental results by Akai and Okano[28)] and the time constants τ_G calculated from results in the wave propagation tests with dry sandy loam by the following equation derived from Eq. (35);

$$\tau_G=\frac{\frac{1}{G\mu}}{1+\frac{G'}{G}}\cdot\frac{1}{\theta_G} \qquad \cdots\cdots\cdots\cdots(43)$$

where, θ_G equals to $(V_1/V_n)^2$ as expressed by Eq. (34). Using $V_1=127$ m/sec calculated from the density and the modulus of elastic deformation 270 kg/cm^2 obtained by the static confined compression test and the velocity of the compressional wave obtained by the wave propagation test $V_n=218$ m/sec, $\theta_G=0.34$ is calculated. $\tau_G=0.03$ is calculated from Eq. (43) with the value of $\theta_G=0.34$, $1/\mu G=0.03$ and $G'/G=2.0$. It is found from this figure that τ is not proper in the soil but changes with the frequency. On the logarithmic paper τ increases linearly with decrease in the frequency in the range less than 1 cps and tends to constant of about 3×10^{-2} sec in the range more than 1 cps.

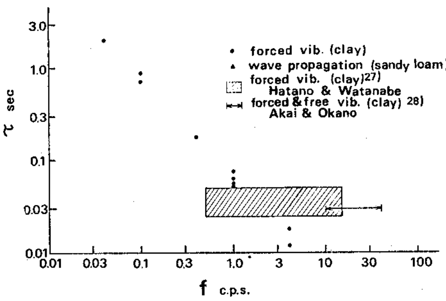

Fig. 11 Change in the Time Constant with Frequency.

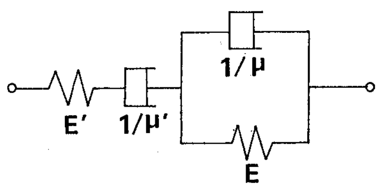

Fig. 12 Burgers Model

According to Ishihara,[24)] the order of the characteristic frequency is 10^3 (sec^{-1}) in the case of sand and $10^7\sim10^8$ (sec^{-1}) in clay. These values are extremely smaller than the values shown in Fig. 11. The coefficient b given by Eq. (11) can be derived by supposing the Poiseuille flow and Darcy's law for the water in the void. However, it is well known that the effective porosity through which the pore water can flow under certain hydraulic gradient is extremely smaller than the total porosity.[29)] By taking the above information into account, it is found that the value of b becomes much smaller, so that the calculated results of $1/\omega_G$ or $1/\omega_H$ becomes larger as known from Eq. (37) or (38), respectively, and should correspond to the experimental results as shown in Fig. 11 obtained from the viscoelastic approach. However, in order to recognize this matter, we need to carry out a great deal of experiment on the relationships between the wave characteristics and the physical properties of soils.

4.2 Estimation of Logarithmic Decrement

In Fig. 13, the logarithmic decrement δ/π is plotted versus $p\tau$ where p is the circular frequency given in the experiments and τ is expressed in Fig. 11. The logarithmic decrements can be obtained in the free vibration test directly, or in the forced vibration tests where the loss energy and the maximum stored energy per cycle should be taken into account, and calculat-

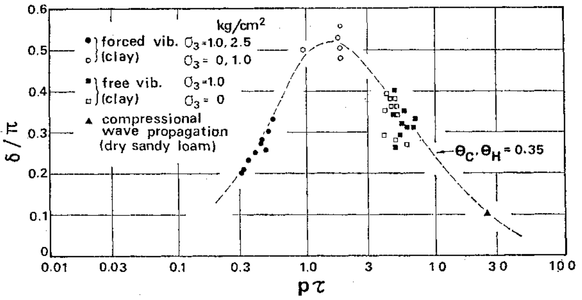

Fig. 13 Logarithmic Decrement Obtained by Experiments.

ed from Eq. (31) by using the attenuation constants given in the wave propagation tests. The dotted line in this figure shows the calculated results of δ/π in the case of θ_C or θ_H=0.35. Although the soil sample is different from each other, the dynamic characteristics of soil materials in wide range of frequency can be well recognized.

It is considered that θ_C or θ_H is different between in the normally consolidated clay and the over-consolidated clay, and that τ and the rheological constants change with the confining pressure. These are still left for further investigation.

5. CONCLUSIONS

Main conclusions obtained in this research can be summarized as follows;

(1) The physical behaviors of soil are elastic in the strain level less than 10^{-4} and viscoelastic in 10^{-4} to 10^{-3}.

(2) The wave characteristics in the saturated porous elastic medium investigated by Biot and Ishihara are equivalent to those in viscoelastic material, where the compressional wave of the first kind corresponds to the wave in the spring-Voigt model and the wave of the second kind to the wave in Maxwell model.

(3) It is useful to assume soil as the spring-Voigt model in the dynamic state. By using this model, we can express the behaviors of soil in wide frequency range.

(4) The free spring in the spring-Voigt model shows the elastic nature which is independent of the coupling between the solid and fluid phases, while the Voigt part expresses viscoelastic coupling behaviors depending on the relative displacement between two phases.

(5) The time constant or the characteristic frequency changes with circular frequency of vibration. The order is a few second in the quasi-static state with the frequency range of 10^{-2} cps and becomes a constant of 10^{-2} sec in the frequency range more than 1 cps.

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