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EXTENDED MONTE CARLO METHOD IN STRUCTURAL RELIABILITY

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INTRODUCTION

In the reliability theory of structural design, probability of structural failure P_f may be very small with order 10^{-3} to 10^{-6} in reality. This probability of failure P_f is defined as

$$P_f=P(S>R)=10^{-3} \text{ to } 10^{-6} \text{ order} \qquad (1)$$

where $P(\ )$ reads "probability that". R is the overall resistance of structure and S is the load applied upon structure.

In the case of normal distributions of both S and R, the theoretical solution of P_f may be evaluated as[1)]

$$P_f=1-\Phi\left\{\frac{\mu_R-\mu_S}{\sqrt{\sigma_R^2+\sigma_S^2}}\right\} \qquad (2)$$

where μ_R and μ_S are respectively the means of R and S. σ_R and σ_S are the standard deviations of R and S respectively. $\Phi(\ )$ is the probability distribution function of normalized Gaussian variables.

In general, R is given as a function of many structural elements which may be random variables. Therefore, the distribution of R is not necessarily normal nor a well known pattern of a probability model. The similar argument may ascertain that S may not be described with typical distribution laws.

For example, consider the reliability of an axially loaded reinforced concrete member. The ultimate strength of the column is given by

$$R=kf_c'A_c+f_{sy}A_s \qquad \text{(ACI code)}$$

If kf_c' (concrete strength), f_{sy} (yield strength of steel) and A_c (concrete area) are considered to be random variables, the ultimate strength (or we may call it the resistance R) also become random variables. In this case, even the probability distribution functions of kf_c', f_{sy} and A_c are given by well known distributions such as normal distribution functions, it is very difficult to obtain the theoretical distribution of R and consequently the evaluation of eq. (1) is very hard to be attained.

Another example is given by the analysis of a tainter gate problem (4). The resistance R of the gate is given by the following complicated function of random variables;

$$R=\frac{\pi^2 EI}{l^2}\left\{\frac{(\alpha_1^s+\alpha_2^s+\alpha_3^s+(\alpha_1^d+\alpha_2^d+\alpha_3^d)}{(\alpha_1^s+\alpha_1^d)^{1.5}+\sqrt{\frac{I_1}{I_2}}(\alpha_2^s+\alpha_2^d)^{1.5}+\sqrt{\frac{I_1}{I_3}}(\alpha^s{}_3+\alpha^d{}_3)^{1.5}}\right\}^2$$

where α_i^s, α_i^d $(i=1, 2, 3)$ are random variables. Again the evaluation of the probability distribution of R is not feasible.

In a case where the distribution pattern of overall resistance R is numerically given as an experimental study, these values are stored in computer for the use of the extended Monte Carlo method described in the next section.

In any one of the above cases, it is very difficult if not impossible to obtain the theoretical solution of P_f as in eq(2). Monte Carlo approach then becomes a very powerful means to evaluate the numerical value of P_f. However, it is noted that for the order 10^{-3} to 10^{-6} of P_f, at least more than 10^3 to 10^6 repeated trials are required and the computer time becomes a criteria to employ a Monte Carlo method.

In this paper, an extended Monte Carlo method is developed to evaluate P_f of order 10^{-3} to 10^{-6} with far less than 10^3 to 10^6 repeated trials. A simple conditional probability law is the basic concept in this approach.

EXTENDED MONTE CARLO METHOD

Equation (1) can be described as a Venn diagram shown in Fig. 1. The hatched area is the event of structural failure. A Monte Carlo method based upon eq. (1) is for the ith trial, first to generate random variables a_i and b_i independ-

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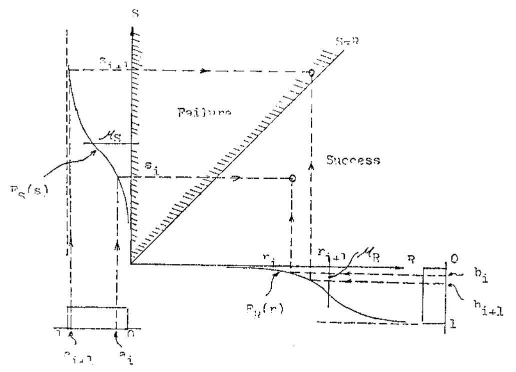

Fig. 1 $P_f=P(S>R)$

ently from a uniform probability distribution with the range of zero to one as shown in Fig. 1. Then sample realizations s_i and r_i may be obtained from the relations $s_i=F_S^{-1}(a_i)$ and $r_i=F_R^{-1}(b_i)$ respectively. $F_S(s)$ and $F_R(r)$ are the probability distribution functions of S and R respectively. If $s_i<r_i$, the structure is safe and the next trial is to be proceeded. Whenever $s_i>r_i$, keep the tally of the structural failure. After sufficient trials, the probability of failure P_f in eq. (1) can be approximated by the following estimator

$$\hat{P}_f=n/N \quad \cdots\cdots(3)$$

where n is the number of failure and N is the total number of independent trials. As can be seen in the above description, the method is very simple except for the many repetitions of trials which are not realistic because of the computer time.

To avoid this technical difficulty, this paper developes an extension of the Monte Carlo method in the following paragraphs.

Consider now the probability of an event of $(S>R\cap S>S_0\cap R<R_0)$ where S_0 and R_0 are deterministic such that $S_0<R_0$.

Following a conditional probability law, we have

$$P(S>R\cap S>S_0\cap R<R_0)$$
$$=P(S>R)P(S>S_0\cap R<R_0/S>R) \quad \cdots\cdots(4)$$

or

$$P(S>R\cap S>S_0\cap R<R_0)$$
$$=P(S>S_0\cap R<R_0)P(S>R/S>S_0\cap R<R_0) \quad \cdots\cdots(5)$$

Consequently from eqs. (4) and (5)

$$P_f=P(S>R)$$
$$=\frac{P(S>S_0\cap R<R_0)P(S>R/S>S_0\cap R<R_0)}{P(S>S_0\cap R<R_0/S>R)} \quad \cdots\cdots(6)$$

For the improvement (shortening) of computer time, we will consider the use of the right hand side of eq. (6) to evaluatet he probability of failure P_f.

Let

$$P_1=P(S>S_0\cap R<R_0) \quad \cdots\cdots(7)$$

$$P_{1/f}=P(S>S_0\cap R<R_0/S>R) \quad \cdots\cdots(8)$$

$$P_{f/1}=P(S>R/S>S_0\cap R<R_0) \quad \cdots\cdots(9)$$

Then eq. (6) can be expressed as

$$P_f=P_1P_{f/1}/P_{1/f} \quad \cdots\cdots(10)$$

P_1 is the probability that $S>S_0$ and $R<R_0$ simaltaneously. If the events $(S>S_0)$ and $(R<R_0)$ are independent, then

$$P_1=P_{11}P_{12}=P(S>S_0)P(R<R_0) \quad \cdots\cdots(11)$$

In the Monte Carlo evaluation of P_1, it is obvious that the number of trials can be greatly reduced if appropriate values S_0 and R_0 are assigned.

$P_{1/f}$ is the probability that sample realization S is greater than S_0 and at the same time R is less than R_0 under the condition of $S>R$ When the failure occurs, it may be highly expected that the load is great $(S\gg S_0)$ and the resistance R is small $(R\ll R_0)$. Therefore, in the Monte Carlo simulation, the number of trials can be also reduced for the stable estimation of $P_{1/f}$.

On the contrary to $P_{1/f}$, $P_{f/1}$ is the probability that the structure fails under the condition of of $S>S_0$ and $R<R_0$. The number of trials is also expected to be small.

Consequently, once P_1, $P_{f/1}$, and $P_{1/f}$ are evaluated with reasonable number of trials, the probability of failure P_f can be determined from eq. (10). In other words, with much less than 10^3 to 10^6 trials, the probability of failure of the order 10^{-3} to 10^{-6} can be estimated.

It is noted that if S_0 is chosen to be very small and R_0 very large, eq. (10) becomes trivial, since in this case

$$P_1=P(S>0\cap R<\infty)=1.0\ ,$$
$$P_{1/f}=P(S>0\cap R<\infty/S>R)=1.0$$

and

$$P_{f/1}=P(S>R/S>0\cap R<\infty)=P(S>R)\ .$$

Thus, both sides of eq. (10) is of identical form; $P(S>R)=P(S>R)$.

Thus, the reduction of trials can not be retained. If on the other hand, S_0 and R_0 are chosen such that $S_0>R_0$, the condition $S>S_0\cap R<R_0$ automatically means $S>R$. Therefore we have

$$P(S>R/S>S_0\cap R<R_0)=1.0$$

and

$$P(S>S_0 \cap R<R_0/S>R)$$
$$=P(S>S_0 \cap R<R_0)/P(S>R)$$

since

$$(S>S_0 \cap R<R_0) \subset (S>R).$$

Thus, eq. (10) reduces again that $P(S>R)=P(S>R)$ which is trivial. Therefore, the optimum S_0 and R_0 may be determined through the parametric analysis.

For the evaluation of P_1 in eq. (11), the sample realization s_i is to be filtered out through $F_S(s)$ into which random number a_i in the range of zero to one is supplied. Repeating the trials and keeping the tally whenever we have $s_i>S_0$, $P_{11}=P(S>S_0)$ can be obtained by an estimator;

$$\hat{P}_{11}=n_1/N_1 \quad \cdots\cdots(12)$$

where n_1 is the number of the event $(s_i>S_0)$ occuring in N_1 independent trials. Similarly $P_{12}=P(R<R_0)$ can be obtained by

$$\hat{P}_{12}=n_2/N_2 \quad \cdots\cdots(13)$$

where n_2 is the number of the event $(r_i<R_0)$ in N_2 trials.

In what follows, the evaluation of $P_{1/f}$ and $P_{f/1}$ is discussed.

Consider the Monte Carlo evaluation of P_1/f. The event $(S>S_0 \cap R<R_0/S>R)$ is illustrated in Fig. 2. First generate a_i from a uniform dis-

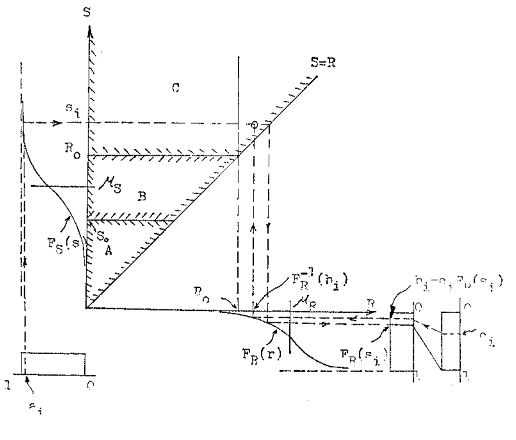

Fig. 2 $P_{1/f}=P(S>S_0 \cap R<R_0/S>R)$

tribution between zero to one. The sample realization s_i can be obtained from $s_i=F_S^{-1}(a_i)$. Since the condition $S>R$ is required in this case, b_i is to be generated from a uniform distribution between zero to $F_R(s_i)$. This means first to generate c_i from a uniform distribution from zero to one and then to obtain b_i such that $b_i=c_iF_R(s_i)$. The sample realization r_i is now filtered out from $r_i=F_R^{-1}(b_i)$ so that the condition $s_i>r_i$ is satisfied. For the sample set (s_i, r_i), examine if the event $(s_i>S_0 \cap r_i<R_0)$ occcurs. If it does, keep the tally and go to the next trial. After many repetition one may have the probability of event $(S>S_0 \cap R<R_0/S>R)$ by the following estimator.

$$P_{1/f}=n_3/N_3 \quad \cdots\cdots(14)$$

where n_3 is the number of the event occuring in N_3 trials.

It is noted that the range of distribution of b_i is variable since the upper bound $F_R(s_i)$ is dependent upon the sample realization s_i. It is also observed that if s_i is less than S_0, the event A indicated in Fig. 2 occurs against any realization of r_i. Thus the interesting event can not be possible. If $S_0<s_i<R_0$, the event B occurs against any r_i. Thus the interesting event always occurs. Therefore it is not necessary to generate r_i in these cases except for the recording the event in the tally.

Next, let us discuss of $P_{f/1}$. The event $(S>R/S>S_0 \cap R<R_0)$ is shown in Fig. 3. Since the

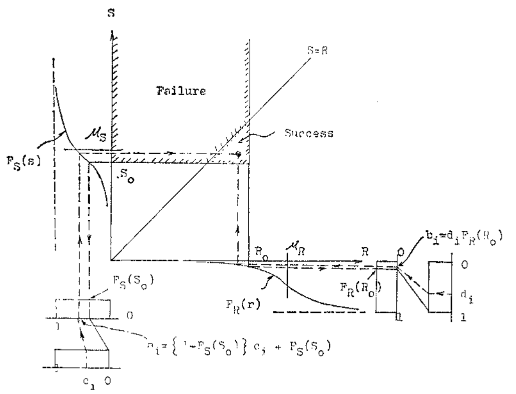

Fig. 3 $P_{f/1}=P(S>R/S>S_0 \cap R<R_0)$

conditions $S>S_0$ and $R<R_0$ are required, a_i is to be generated from a uniform distribution between $F_S(S_0)$ to unity, whereas b_i is generated from a uniform distribution between zero to $F_R(R_0)$. Next generate s_i and r_i such that $s_i=F_S^{-1}(a_i)$ and $r_i=F_R^{-1}(b_i)$. Keep the tally whenever the sample realization of s_i and r_i satisfies $s_i>r_i$. After sufficient trials, the conditional probability of failure $P_{f/1}$ can be estimated in the similar manner;

$$\hat{P}_{f/1}=n_4/N_4 \quad \cdots\cdots(15)$$

where n_4 is the number of the event $(S>R/S>S_0 \cap R<R_0)$ in N_4 trials. As described above, eq. (10) attains a reasonable Monte Carlo evalua-

tion of the probability of structural failure.

Suppose the external load S is considered to be deterministic as $S=a$ and only the the resistance R is probabilistic. Then, the extended Monte Carlo method can be reduced to the following way;

$$P(a>R\cap R<R_0)=P(a>R)P(R<R_0/a>R) \quad \cdots(16)$$

or

$$P(a>R\cap R<R_0)=P(R<R_0)P(a>R/R<R_0) \quad \cdots(17)$$

From eqs. (16) and (17),

$$P_f=P(a>R)=\frac{P(R<R_0)P(a>R/R<R_0)}{P(R<R_0/a>R)} \quad \cdots(18)$$

If R_0 is chosen such that $a<R_0$, then

$$R<R_0\supset a>R$$

Hence

$$P(R<R_0/a>R)=1.0$$

Therefore eq. (18) becomes

$$P_f=P(a>R)=P(R<R_0)P(a>R/R<R_0) \quad \cdots(19)$$

It is clear that use of the right hand side of eq. (19) reduces the number of trials in the Monte Carlo approach.

ERROR STATISTICS

For the evaluation of true probability of failure P_f, estimators $\hat{P}_{11}$, $\hat{P}_{12}$, $\hat{P}_{f/1}$ and $\hat{P}_{1/f}$ are employed in the extended Monte Carlo method. In other words, the true probability of failure from eqs. (10) and (11)

$$P_f=P_{11}P_{12}P_{f/1}/P_{1/f} \quad \cdots(20)$$

are estimated either by the direct approach

$$\hat{P}_f=n/N \quad \cdots(21)$$

or by the extended Monte Carlo approach

$$\hat{P}_f'=\hat{P}_{11}\hat{P}_{12}\hat{P}_{f/1}/\hat{P}_{1/f}$$
$$=(n_1/N_1)(n_2/N_2)(n_3/N_3)/(n_4/N_4) \quad \cdots(22)$$

Although it was discussed that use of eq. (22) can save considerable amount of computer time, the accuracy of $\hat{P}_f'$ of eq. (22) must be examined in the comparison with the accuracy of P_f of eq. (21).

The estimators $\hat{P}_f$, $\hat{P}_{11}$, $\hat{P}_{12}, \ldots$ are all random variables since they are given as the fraction of number of interesting events occuring in the independent trials and in fact they are binomially distributed[2] as, for example;

$$P(\hat{P}_f=n/N\leqq b)=\sum_{n=0}^{Nb}\binom{N}{n}P_f^n(1-P_f)^{N-n} \quad \cdots(23)$$

with the mean

$$E(n/N)=P_f \quad \cdots(24)$$

and the variance

$$\sigma^2 n/N=P_f(1-P_f)/N \quad \cdots(25)$$

(1) Confidence Interval of P_f based on eq. (21)

Despite of the binomial distribution of $\hat{P}_f=n/N$, $\hat{P}_f$ can be assumed to be approximately normally distributed due to the central limit theorem, when N is large number although P_f is very small value.

Thus, we put

$$\hat{P}_f=N\left(P_f, \sqrt{\frac{P_f(1-P_f)}{N}}\right) \quad \cdots(26)$$

where $N(a, b)$ means normal distribution with mean a and standard deviation b. Therefore $(1-\alpha)$ % confidence interval can be obtained as follows.

$$P\left(\left|\frac{\hat{P}_f-P_f}{\sqrt{\frac{P_f(1-P_f)}{N}}}\right|\leqq K_{\alpha/2}\right)=1-\alpha$$

Solving this for P_f and after some approximation,

$$P\left(\hat{P}_f-K_{\alpha/2}\sqrt{\frac{\hat{P}_f}{N}}\leqq P_f\leqq \hat{P}_f+K_{\alpha/2}\sqrt{\frac{\hat{P}_f}{N}}\right)$$
$$=1-\alpha \quad \cdots(27)$$

Or the confidence interval is

$$|P_f-P_f|\leqq K_{\alpha/2}\sqrt{\frac{P_f}{N}} \quad \cdots(28)$$

For example, 95% confidence interval is given by

$$|P_f-\hat{P}_f|\leqq 1.96\sqrt{\frac{\hat{P}_f}{N}} \quad \cdots(29)$$

(2) Confidence Interval of P_f based on eq. (22)

Estimators $\hat{P}_{11}$, $\hat{P}_{12}, \ldots$ are similarly assumed to be normally distributed. For example,

$$\hat{P}_{11}=N\left(P_{11}, \sqrt{\frac{P_{11}(1-P_{11})}{N_1}}\right) \quad \cdots(30)$$

Expand into Taylor series[3] the multivariate function of $\hat{P}_f'$ given by eq. (22). Then we can approximate

$$E(\hat{P}_f')\cong\frac{P_{11}P_{12}P_{f/1}}{P_{1/f}}=P_f \quad \cdots(31)$$

Table 1 Comparison of 95% Confidence Interval

For Example $\hat{P}_f=10^{-4}$, $\hat{P}_{11}=\hat{P}_{12}=\hat{P}_{f/1}=\hat{P}_{1/f}=10^{-2}$

N	95% Confidence Interval of $\lvert P_f-\hat{P}_f\rvert$	Equivalent to N	95% Confidence Interval of $\lvert P_f-\hat{P}_f'\rvert$
10^4	$1.96*10^{-4}$	$N_i=10^2$	$3.96*10^{-4}$
10^5	$0.63*10^{-4}$	$N_i=10^3$	$1.22*10^{-4}$
10^6	$0.20*10^{-4}$	$N_i=10^4$	$0.39*10^{-4}$
10^7	$0.06*10^{-4}$	$N_i=10^5$	$0.12*10^{-4}$
10^8	$0.02*10^{-4}$	$N_i=10^6$	$0.04*10^{-4}$

$i=1,2,3,4$

$$\sigma^2_{\hat{P}_f'}\cong\left\{\frac{P_{12}P_{f/1}}{P_{1/f}}\right\}^2\sigma^2_{\hat{P}_{11}}+\left\{\frac{P_{11}P_{f/1}}{P_{1/f}}\right\}^2\sigma^2_{\hat{P}_{12}}+\left\{\frac{P_{11}P_{12}}{P_{1/f}}\right\}^2\sigma^2_{\hat{P}_{f/1}}+\left\{\frac{P_{11}P_{12}P_{f/1}}{P^2_{1/f}}\right\}^2\sigma^2_{\hat{P}_{1/f}} \quad \cdots\cdots(32)$$

$$\cong\left\{\frac{P_{11}P_{12}P_{f/1}}{P_{1/f}}\right\}^2\{1/(P_{11}N_1)+1/(P_{12}N_2)+1/(P_{f/1}N_3)+1/(P_{1/f}N_4)\} \quad \cdots\cdots(33)$$

Thus,

$$\sigma^2_{\hat{P}_f'}\cong P_f^2\beta^2 \quad \cdots\cdots(34)$$

where

$$\beta^2=1/(P_{11}N_1)+1/(P_{12}N_2)+1/(P_{f/1}N_3)+1/(P_{1/f}N_4) \quad \cdots\cdots(35)$$

Since $\hat{P}_f'$ is approximated as a linear function of $\hat{P}_{11}$, $\hat{P}_{12}$ etc. by the Taylor series and since $\hat{P}_{11}$, $\hat{P}_{12}$ etc. can be nearly normally distributed, $\hat{P}_f'$ is also normally distributed. Thus,

$$\hat{P}_f'\cong N(P_f,\ P_f\beta) \quad \cdots\cdots(36)$$

The $(1-\alpha)$% confidence interval is now obtained as follows.

$$P\left(\left|\frac{\hat{P}_f'-P_f}{P_f\beta}\right|\leqq K_{\alpha/2}\right)=1-\alpha \quad \cdots\cdots(37)$$

Therefore,

$$|\hat{P}_f'-P_f|\leqq K_{\alpha/2}\beta P_f$$

For example, the 95% confidence interval is

$$|\hat{P}_f'-P_f|\leqq 1.96\beta P_f \quad \cdots\cdots(38)$$

It is noted that since P_f and β are unknown, the evaluation of eq. (38) is impossible in the strict sense. However, for the purpose of comparison of the accuracy of $\hat{P}_f$ and $\hat{P}_f'$, the right hand side of eq. (38) is replaced by

$$1.96\beta P_f\approx 1.96\hat{P}_f\times\sqrt{\frac{1}{\hat{P}_{11}N_1}+\frac{1}{\hat{P}_{12}N_2}+\frac{1}{\hat{P}_{f/1}N_3}+\frac{1}{\hat{P}_{1/f}N_4}} \quad \cdots\cdots(39)$$

Based on eqs. (29) and (39), the comparison are made in Table 1.

Table 1 shows the confidence intervals of estimation of the true probability of failure by both the direct approach and extended Monte Carlo approach. For example, the probability of failure P_f is supposed to be estimated as $\hat{P}_f=10^{-4}$ by the direct approach eq. (21) after $N=10^7$ trials. On the other hand, $N_1=N_2=N_3=N_4=10^5$ trials are supposed to be needed to estimate $P_{11}=P_{12}=P_{f/1}=P_{1/f}=10^{-2}$ respectively. Then from the table 1, it can be seen that the first approach gives 0.06×10^{-4} of the 95% confidence interval of $|P_f-\hat{P}_f|$, whereas the second approach gives 0.12×10^{-4} although this approach needs all together 4×10^5 trials at most. If the computer program is effectively coded to include some of the four trials in a single loop, the number of trials becomes much less than 4×10^5. Thus, it is clear that use of P_f' sacrifices the degree of prediction accuracy of P_f. However, it is noted that at least the order estimation of P_f can be attained only by P_f' unless otherwise the lengthy computer time is not permitted.

NUMERICAL EXAMPLE

For the demonstration purpose, both structural resistance R and load S are assumed to be normally distributed with the reason that prior to the Monte Carlo evaluation, the true probability of structural failure P_f can be obtained by eq. (2). Therefore, the accuracy of $\hat{P}_f$ can be examined. It is noted, however, that even for any distribution of R and S, the extended Monte Carlo method can be applied.

Assume $R=N(10.0,\ 1.0)$ and $S=N(6.0,\ 1.0)$. The first and second arguments are mean and standard deviation respectively. The dimensions are not specified herein because of the demonstration. From eq. (2), then the true P_f is calculated as $P_f=0.233*10^{-2}$.

For several conbinations of S_0 and R_0, P_f values were estimated by eq. (22). For a few sets of S_0 and R_0 values, the results are summarized in Table 2 together with the confidence interval.

Table 2 Results of Numerical Example

S_0	R_0	N_i	$\hat{P}_{11}$	$\hat{P}_{12}$	$\hat{P}_{1/f}$	$\hat{P}_{f/1}$	$\hat{P}_f'$	95% Confidence Interval
6.0	8.0	1000	0.539	0.280*10-1	0.519	0.781*10-1	0.227*10-2	0.104*10-2
		2000	0.549	0.285*10-1	0.518	0.815*10-1	0.245*10-2	0.073*10-2
6.0	9.0	1000	0.539	0.186	0.531	0.290*10-1	0.548*10-2	0.095*10-2
		2000	0.549	0.187	0.532	0.293*10-1	0.562*10-2	0.067*10-2
6.0	10.0	1000	0.539	0.454	0.531	0.501*10-2	0.231*10-2	0.207*10-2
		2000	0.549	0.454	0.532	0.501*10-2	0.234*10-2	0.146*10-2
6.0	11.0	1000	0.539	0.785	0.531	0.200*10-2	0.159*10-2	0.325*10-2
		2000	0.549	0.783	0.532	0.250*10-2	0.202*10-2	0.206*10-2
6.0	12.0	1000	0.539	0.965	0.531	0.400*10-2	0.392*10-2	0.231*10-2
		2000	0.549	0.962	0.532	0.250*10-2	0.248*10-2	0.205*10-2
7.0	10.0	1000	0.212	0.454	0.224	0.140*10-1	0.602*10-2	0.131*10-2
		2000	0.217	0.454	0.227	0.175*10-1	0.758*10-2	0.084*10-2
7.0	11.0	1000	0.212	0.785	0.224	0.501*10-2	0.372*10-2	0.209*10-2
		2000	0.217	0.783	0.227	0.816*10-2	0.611*10-2	0.118*10-2

True Probability $P_f=0.233*10^{-2}$

For example, choose $S_0=6.0$ and $R_0=10.0$. If $N_1=N_2=N_3=N_4=1000$ trials are made, the estimator $\hat{P}_f'=0.231\times10^{-2}$ can be obtained with 0.207×10^{-2} of 95% confidence interval.

In the case of $N_1=N_2=N_3=N_4=2000$ trials, $P_f'=0.234\times10^{-2}$ with 0.146×10^{-2} of 95% confidence interval.

Since only a small computer was available at hand, optimum values of S_0 and R_0 were not discussed herein. However, it is obvious that if S_0 and R_0 are close to their mean values with the condition $S_0<R_0$, the reduction of computer time can be expected.

It is observed that the order estimation is at least successfully obtained within less repititions of trials than by the direct approach. Note that the confidence interval in Table 2 is purely of statistical nature and does not consider the error of computer calculation.

CONCLUSIONS

An extended Monte Carlo method is developed by applying a simple conditional probability theory for the evaluation of very low probability of structural failure. Numerical examples are given for the purpose of demonstration.

As the summary, the following conclusions are made;

(1) An extended Monte Carlo method makes it feasible to technically obtain a very low probability of structural failure within reasonable computer time.

(2) The accuracy of prediction of true probability of failure by this approach is discussed and an order estimation of P_f is assured through the discussion of error statistics.

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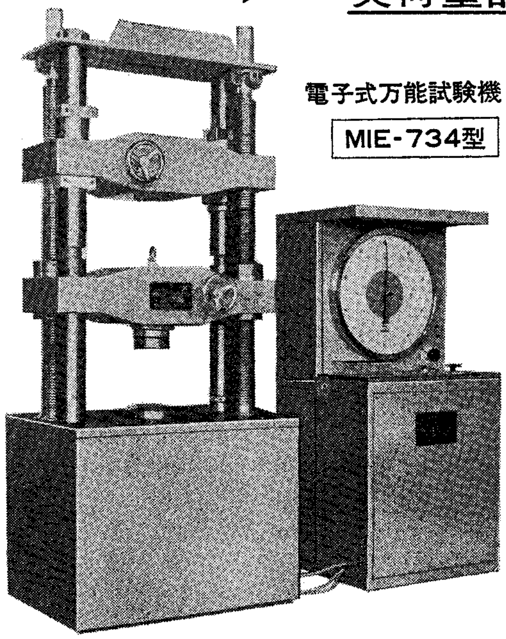
MARUI 創業50年 **電気・油圧サーボシステム・自記計測のマルイ** 1UP&UP

新しい万能材料試験機！

→ 電子式
実荷重計測式



電子式万能試験機
MIE-734型

計測機構と負荷機構の分離

直接計測して、従来の間接的計測の不可抗力的要素を省きました。

※負荷荷重の検出は特殊型ロードセル
※温度変化除く特殊電気回路
※特殊ロードセルは引張強度の½以下で使用
※荷重負荷は多連式ポンプにて行う
※計測指示は自動平衡装置利用

電子式？

特殊ロードセル方式のための完全な電子式機構

特殊ロードセルは、D・T・Fを利用精度0.1μ指針の動きはタコゼネレーターによる自動平衡方式。このように計測はすべて電子回路を駆使しています

実荷重計測

多くの利点を生みだします。

①正確な計測
②故障発生減少
③操作簡単
④感度上昇
⑤再現性いちじるしい
⑥負荷中レンジ切換えできる
⑦「0」調容易になった
⑧応答性早く05秒以内
⑨破断ショック影響受けない
⑩自記自動化が容易になった

油圧系統は負荷するだけ計測値は関係ありません

※詳細ご一報下さい。すぐ参上します。

——自記自動化のトップをめざす——

株式會社 **圓井製作所**

営業品目

土質試験機	非破壊試験機
アスファルト試験機	温調試験機
コンクリート試験機	水理試験装置
セメント試験機	材料試験機

——信頼を旨とす——

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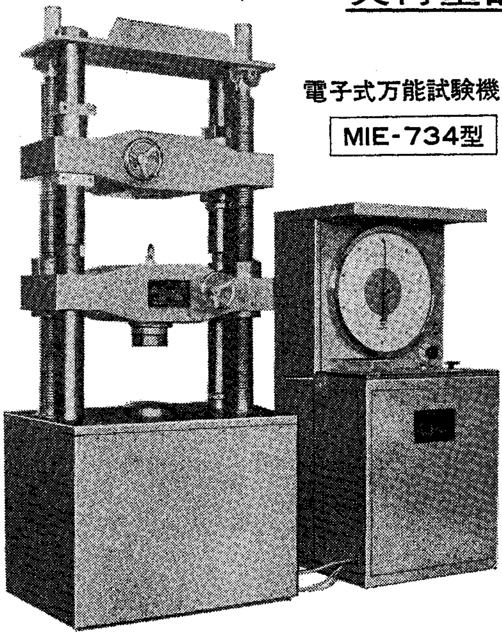

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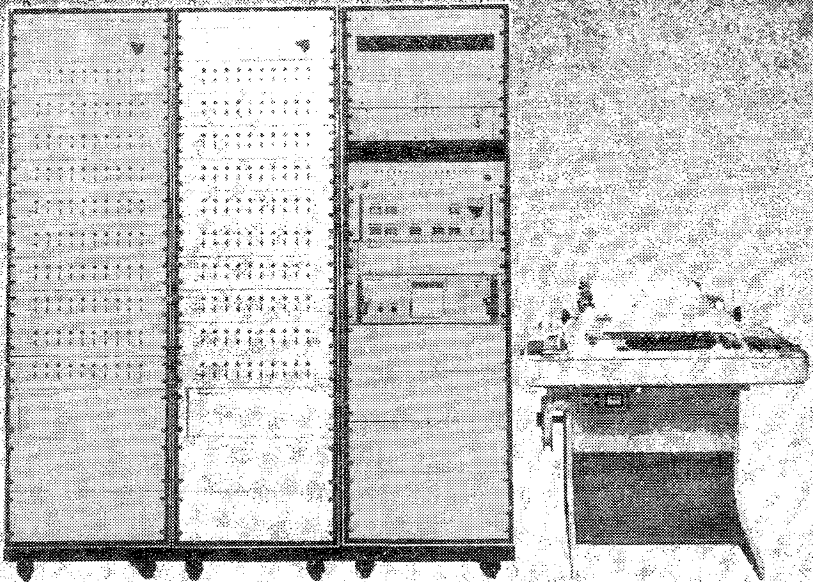

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