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ALLOWABLE STRESS FOR TWO-HINGED STEEL ARCH

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SYNOPSIS

An allowable stress formula for the conventional structural design is presented in this paper on the basis of the ultimate strength anaysis considered the finite deflection, the spread yeilded zones and the cooling residual stresses of arches as follows:

$$\sigma_n^{1st}/\sigma_a^{1st}+k\sigma_m^{1st}/\sigma_a^{1st}\leq(\text{F.S.})$$

$$\sigma_a^{1st}=\sigma_y/(\text{F.S.})\{0.51+4n-10n^2-0.1(\sigma_y/2\,400\,^{\text{kg/cm}^2})-0.5\times10^{-5}\lambda^2(\sigma_y/2\,400\,^{\text{kg/cm}^2})$$

$$k=0.55+(2\,400\,^{\text{kg/cm}^2}/\sigma_y)/4$$

For the 2nd order elastic analysis, an allowable stress formula is also proposed in a simple form:

$$\sigma_n^{2nd}+\sigma_m^{2nd}\leq0.9\sigma_y/(\text{F.S.})$$

1. INTRODUCTION

(1) General

Concerning the load carrying capacity of arches, a theoretical study, in which finite deformation, residual stresses and spread yielded zones were considered, was made by the author[1]. From his numerically calculated results the following behavior of arches loaded to the ultimate state in their plane could be concluded as a general tendency.

1) If an arch keeps perfectly elasticity, it makes no great difference in the ultimate load intensity for symmetrical loading pattern and unsymmetrical one. But if yielding occurs, the unsymmetrical loading plays a leading role in the load carrying capacity problem and decreases remarkably the strength of the arch.

2) The collapse load of an arch is considerably less than the elastic limit load determined according to the 1st order elastic analysis, especially for a nearly uniformly distributed load over the whole span.

3) The load carrying capacity of an arch depends chiefly on slenderness ratio of the arch rib, rise-span ratio and yielding point of material.

4) The collapse of an arch occurs rather at a low level of strain even in the plastic range, so that strain hardening does not affect its load carrying capacity.

Fig. 1 shows an arch and loading referred. In this paper, a design formula according to the conventional allowable stress method will be presented on the basis on the above mentioned ultimate strength analysis. Besides it, a comparison of the ultimate load vs. the elastic limit load determined according to the 2nd order elastic analysis is discussed and a critical stress level for design purpose is proposed to estimate the ultimate strength of arches.

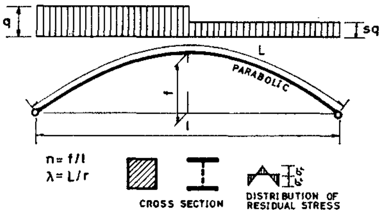

Fig. 1 Loading and configuration of arch.

(2) Criteria for Cross Section

Two types of cross section, sandwich cross sections with the residual stresses and rectangular cross sections were adopted for calculation to discuss the behavior of arches at the ultimate state. To generalize the results obtained, the three characteristic factors of a cross section A, W and I must be adjusted each other to get an equivalent sandwich cross section or rectangular cross section with an arbitrary cross section. A, W and I are cross sectional area, section modulus, and inertia moment of cross section respectively. But only two factors in three are possible to coincide with each other. In Fig. 2, compared

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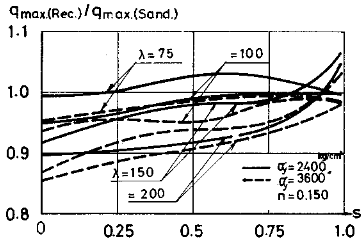

Fig. 2 Comparison curves of ultimated strength of arches with sandwich and rectangular cross sections with coincident A and I.

curves for the ultimate loads of arches with sandwich sections and rectangular ones are presented for the case of coincident A and I. In the figure, λ indicates the slenderness ratio of arch rib concerning the whole length of the arch. This case is in the best agreement of the ultimate loads among the three cases, so that, the use of an equivalent sandwich cross section, coinciding with sectional values in A and I respectively, leads to a good estimation of the ultimate strength of the arch. Since the sandwith cross section may be regarded to have more analogous characteristics with a conventional cross section of steel structures than rectangular ones.

In this paper, all the calculated results and proposals are presented in reference to the case for sandwich cross sections with the residual stresses ($\sigma_r = \sigma_y/3$). Because the cross sectional shape affects so much the ultimate strength of arches that an almost infinitive amount of calculation is required to estimate it. But, of course, these may be impossible for us and unfit for engineering purpose. A unified cross section would be conveniently accepted as a basis of estimation and variations of the ultimate strength of arches due to the difference of the cross sectional shape can be included in the factor of safety.

Column-curves for a sandwich cross section with $\sigma_r = 0$, $\sigma_r/3$ and $2\sigma_r/3$ and other typical ones are presented in Fig. 3. The basic column curve at the Japanese Specification for Highway Bridge based on the buckling about a weak axis of H beam is added also in the figure. The effect of the residual stresses on the ultimate load q_{max} of an arch is shown by the ratio to the 1st order elastic limit load in Fig. 4 for the case of $\lambda = 100$, $n = 0.150$ and $\sigma_y = 2\,400\ \mathrm{kg/cm^2}$, $3\,600\ \mathrm{kg/cm^2}$, where s is a coefficient concerning the load distribution pattern as shown already in Fig. 1 and n is a rise-span ratio.

The ratios of the ultimate load q_{max} to the 1st

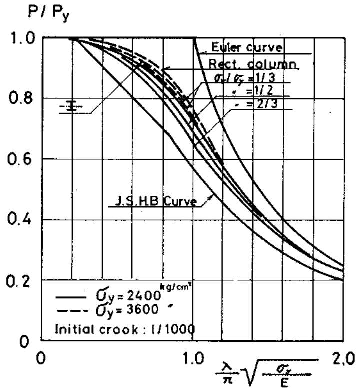

Fig. 3 Column curves with sandwich cross section.

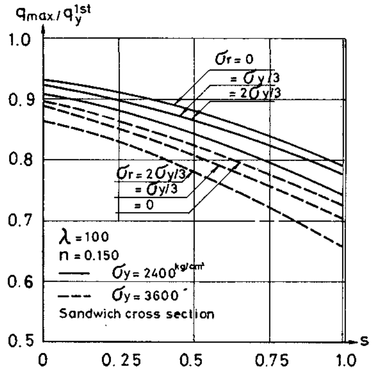

Fig. 4 Effect of residual stresses on strength of an arch.

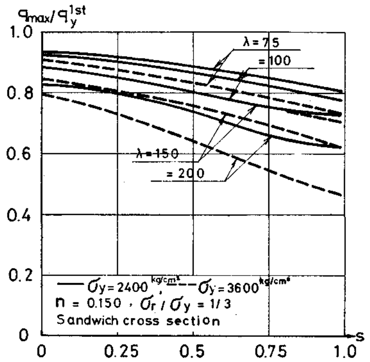

Fig. 5 Comparison of ultimate load vs. 1st order elastic limit load.

order elastic limit load q_y^{1st} are shown in Fig. 5 for $n = 0.150$, $\sigma_y = 2\,400\ \mathrm{kg/cm^2}$, $3\,600\ \mathrm{kg/cm^2}$.

(3) Applicable Arch and Loading

A numerical study was carried out using the following parameters, steel and loading:

$n = 0.125,\ 0.150,\ 0.200$

$\lambda=75,\ 100,\ 150,\ 200$

$s=0,\ 0.25,\ 0.5,\ 0.75,\ 0.99$

$\sigma_y=2\,400\ \mathrm{kg/cm^2},\ 3\,600\ \mathrm{kg/cm^2}$.

The proposed formula may be valid only in the above mentioned range.

The strength studied herein is concerning the inplane collapse of two-hinged steel arches, but it might be applicable rather safely to other types of arch, for instance, fixed arches or arches with stiffening girders. Only unsymmetrically distributed loads are applied herein, but concentrated loads or symmetrically distributed loads are not dangerous for arches calculated by the allowable stress based on the unsymmetrical load pattern.

2. FORMULATION OF ALLOWABLE STRESS

(1) Allowable Stress for the 1st Order Elastic Analysis

Axial force N^{1st} and bending moment M^{1st} acting on an arch subjected to the ultimate load can be calculated applying the 1st order elastic analysis. Usually there is a small difference of position between two points the maximum value of M^{1st}/M_y or the maximum sum of M^{1st}/M_y and N^{1st}/N_y are produced, where N_y and M_y are axial yield force and yield moment respectively. In this paper the values at the point where the bending moment takes the maximum value, are used and denoted by M_m^{1st} and N_m^{1st}. Because the sandwich cross section has a tendency to be estimated the effect of the axial force too high and the difference of the values at the two point is not so large. For some slender arches, the maximum stress is calculated at their springings. But nevertheless the point where the bending moment becomes maximum, is referred to the analysis considering that axial forces at the springings affect little the deformation of the arch.

An interaction equation relating N^{1st} to M^{1st} obtained may be expressed by the following conventional design formula:

$$N^{1st}/N_y+kM^{1st}/M_y\leq\alpha^{1st} \quad \cdots\cdots\cdots\cdots\cdots(1\text{-a})$$

or

$$\sigma_n^{1st}/\sigma_a^{1st}+k\sigma_m^{1st}/\sigma_a^{1st}\leq(\mathrm{F.S.}) \quad \cdots\cdots\cdots\cdots\cdots(1\text{-b})$$

$$\sigma_a^{1st}=(\alpha^{1st}\sigma_y)/\mathrm{F.S.}$$

where σ_n^{1st} and σ_m^{1st} are working axial and bending stresses respectively, σ_a^{1st} is the allowable stress for the 1st order elastic design, F.S. is a factor of safety and α^{1st} and k are coefficients depending on λ, n and σ_y. The coefficients α^{1st} and k are determined for the maximum bending moment M_m^{1st} and the axial force N_m^{1st} at the same point as mentioned above.

Fixing λ, n and σ_y, the coefficients α^{1st} and k can be calculated for the various loading pattern parameter s by an iterative method. At the first step, supposing $M_m^{1st}=0$ for $s=0.99$ (a nearly uniformly distributed load over the whole span) and providing $N_m^{1st}/N_y=\alpha^{1st}$, α^{1st} and k are calculated for a certain s. Using these α^{1st} and k, we can get improved α^{1st} and k again for $s=0.99$. This calculation process is repeated to obtain the sufficiently converged values of α^{1st} and k.

For a certain slenderness ratio, the values of α^{1st} show very small variations with $s=0$, 0.25, 0.5 and 0.75 and scattering of the calculated values is less than 0.5% of the mean value of α^{1st}. Fig. 8 shows the interaction curves of these mean values vs. slenderness ratios taking $\sigma_y=2\,400\ \mathrm{kg/cm^2}$ and $3\,600\ \mathrm{kg/cm^2}$, and $n=0.125$, 0.150 and 0.200.

The value of k varies depending on s, i.e. depending on the ratio of N_m^{1st}/N_y and M_m^{1st}/M_y.

Fig. 6 and Fig. 7 show the interaction curves k vs. s, λ and σ_y.

The value of α^{1st} may be expressed also by a conventional parabolic curve equation as follows:

$$\alpha^{1st}=A-B\lambda^2 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

After an appropriate estimation of the curves, B may be written by the following simplified equation:

$$B=0.5\times10^{-5}\times(\sigma_y/2\,400^{\ \mathrm{kg/cm^2}}) \quad \cdots\cdots\cdots\cdots(3)$$

This equation is valid only for $\sigma_y=2\,400\ \mathrm{kg/cm^2}$ and $3\,600\ \mathrm{kg/cm^2}$, but it may apply approximately to an intermediate value of σ_y.

Substituting the assumed value of B into Eq.

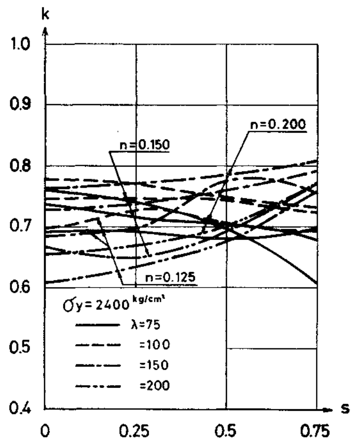

Fig. 6 Effect of n, λ and s on k ($\sigma_y=2\,400\ \mathrm{kg/cm^2}$).

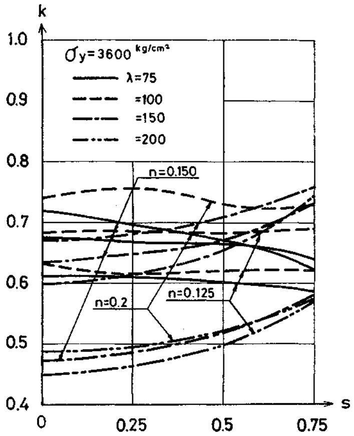

Fig. 7 Effect of n, λ and s on k (σ_y=3 600 kg/cm²).

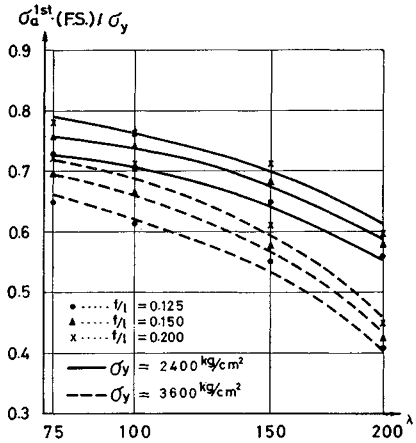

Fig. 9 Proposed curves for apparent unelastic buckling stress.

(2), A can be obtained by the least square method. But A depends on n and σ_y as shown in Fig. 5. Estimating appropriately the curves obtained by the least mean square method and simplifying coefficients so as not to exceed far beyond the calculated points, A is expressed by the following equation:

$$A=0.51+4n-10n^2-0.1(\sigma_y/2\,400\,^{\mathrm{kg/cm^2}}). \qquad \cdots\cdots(4)$$

Combining Eq. (3) and Eq. (4), the following formula is obtained for α^{1st}:

$$\alpha^{\mathrm{1st}}=0.51+4n-10n^2-0.1(\sigma_y/2\,400\,^{\mathrm{kg/cm^2}}) -0.5\times10^{-5}(\sigma_y/2\,400\,^{\mathrm{kg/cm^2}})\lambda^2. \cdots\cdots(5)$$

The curves calculated by the above formula are plotted in Fig. 9 with the aimed values presented in Fig. 8.

Even for an arch applied uniformly distributed loads, there is produced some bending moment due to shortening of the arch rib it by the axial force. But as is clear from Eq. (1), $\alpha^{\mathrm{1st}}\sigma_y$ provides the apparent inelastic buckling stress of arch.

It seems to be a little difficult to formulate the value of k, because k varies complicatedly according to the values of s, n, λ and σ_y. Using Eq. (5), k can be recalculated. Scattering of k obtained covers also the range of about ±0.05 for $0\leq s\leq0.75$. The maximum values of k in the range of $0\leq s\leq0.75$ are plotted in relation to λ in Fig. 10. These interaction curves vary also complicatedly with n and σ_y, so that it may be hard to express these curves in a simple equation.

Estimating boldly these curves not to exceed beyond the maximum points of them, we can take k=0.8 for σ_y=2 400 kg/cm² and 0.71 for σ_y =3 600 kg/cm² in safety sides.

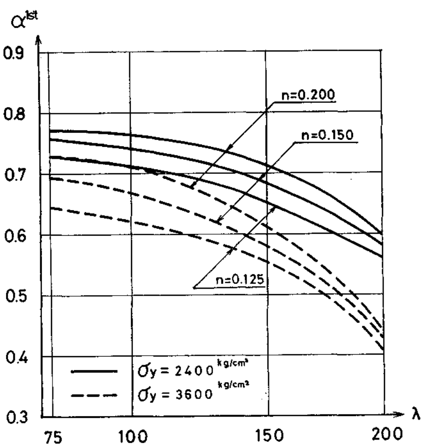

Fig. 8 Appearent inelastic buckling stress.

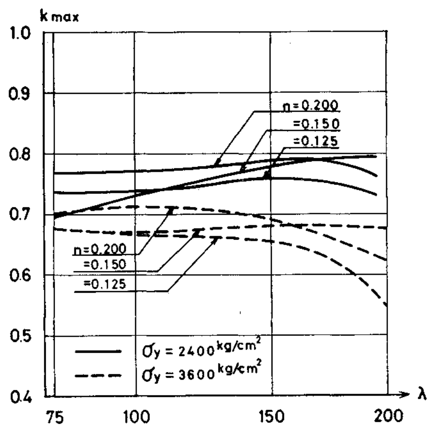

Fig. 10 Maximum k curves in Fig. 6 and Fig. 7.

Because of smaller W of on arbitrary cross section compared with that of the equivalent sandwich cross section with the same values of A and I, the value of k may be reduced by about 10%. But in case of braced rib arches, the above mentioned values are seem to be appropriate ones.

So we can propose the following value for k in safety:

$$k=0.55+0.25(2\,400^{\,\mathrm{kg/cm^2}}/\sigma_y) \quad \cdots\cdots\cdots\cdots(6)$$

If application of the above equation is confined to the conventional cross sections of solid rib steel arches, this value may be reduced by 10% also.

(2) Allowable Stress for the 2nd Order Elastic Analysis

In the preceding chapter, the allowable stresses were presented relating to the conventional structural theory. But of course, the so-called secant formula approach is applicable to estimation of the strength of arches.

In Fig. 11, the upper and lower bound curves for the ratio α^{2nd} of the ultimate loads to the elastic limit loads according to the 2nd order elastic analysis are presented. The difference of α^{2nd} for each case is so small that it is impossible to identify each curve distinctly in the figure. In any case, these ratios fall in the range

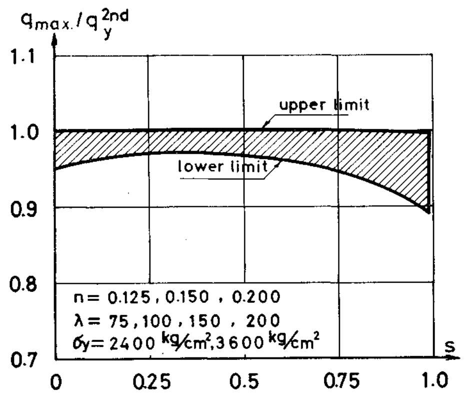

Fig. 11 Range of ratio of ultimate load and the second order elastic limit load.

between 1.0 and 0.9.

Considering not so large nonlinearity between the load and the bending moment or the axial force obtained by the 2nd order analysis, the following allowable stress is given:

$$\sigma_a^{2nd}=0.9\sigma_y/(\mathrm{F.S.})\,. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(7)$$

This value may be raised a little for a conventional cross section also and the values of q_{max}/q_y^{2nd} are presented in Fig. 12, for reference, concerning the case of the rectangular cross section.

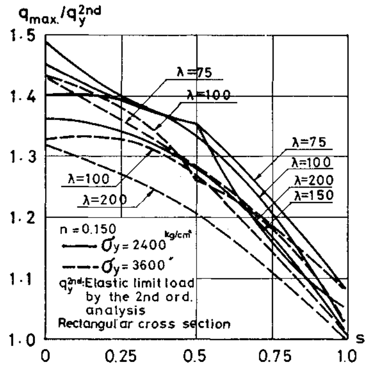

Fig. 12 Ratio of ultimate load and second order elastic limit load for rectangular cross sections.

3. CONCLUSION

Rearranging the results obtained by the ultimate strength analysis of two-hinged steel arches[1], we were able to formulate reasonable design equations as given by Eq. (1), (5) and (6) for the conventional structural design method.

If the secondary elastic analysis is applied, Eq. (7) makes a good estimation of the strength of two-hinged steel arches and other types of arches also.

REFERENCE

1) Kuranishi, S. and Le-Wu, Lu: "Load Carrying Capacity of Two-Hinged Steel Arch," Proc. of J.S.C.E., No. 204, Aug., 1972.

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