PROC. OF JSCE, No. 212, APRIL, 1973

ON THE FATIGUE FRACTURE AND ITS STATISTICAL ASPECTS OF BITUMINOUS MIXTURE

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SYNOPSIS

It is generally known that the fatigue life of materials to be effected by repeated stress under fixed conditions fluctuates widely.

The phenomenon of its variation can not be interpreted only by the experimental error or nonhomogeneity of the materials; it is considered that the variation and its statistical properties are the essential attribute against the character of fracture of materials.

From this point, assuming that even the phenomenon of the fluctuation of fatigue life in the field of bituminous mixture could be satisfactorily explained, the author tried to explain it by using the probabilty theory.

The report is the experimental study of investigation of the statistical nature in fatigue life and process of fracture aginst the mixture of Dense-Graded Asphalt Concrete and Gap-Graded Asphalt Concrete under controlled load, temperature and support condition.

In experiment, the author used a stress-controlled bending machine in which an elastic leaf spring is installed at right angles to the center of the lower part of the specimen, the number of specimen is fourty per one kind mixture.

The results are as mentioned below:

- The distribution of fatigue life corresponds to "Weibull Distribution" led by the weakest link theory.
- It is possible that the phenomenon of fluctuation or wide scatter of fatigue life is explained by stockastic process.
- The deflection and modulus of deformation when a perceptible crack occures approximate to normal distribution. The coefficient variation is small as compared with its initial condition.
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4) Difference of the kind of mixture appears in the rate of crack propagation process, and the rate of Gap Graded Asphalt Concrete is greater than that of Dense-Graded Asphalt Concrete under the constant test conditions.

1. PREFACE

The problem related with asphalt pavement to be subjected to repeated stress is one of the important problems to be elucidated from the standpoint of rationalizing design and improving practical properties of pavement.

This kind of problem was first studied in 1942 when O. J. Porter investigated the allowable deflection amount¹⁾. In 1955, the WASHO road test was conducted, and since the researches on the deflection of pavement or bituminous mixture, especially the phenomenon of fatigue of material under repeated stress have been greatly expanded in both quality and quantity. Among them, especially great developments were achieved by the AASHO road test conducted in 1962 and the international conferences on the structural design of asphalt pavement which were held in 1962 and 1967 at the University of Michigan.

Paying attention to the researches on the fatigue phenomenon of bituminous mixture, it is observed that the number of repetition causing fatigue fracture greatly fluctuates even under the same condition and a constant stress amplitude. This fluctuation ranges from 50 to 100% in terms of coefficient of variance, and therefore it can not be considered that such fluctuation is caused merely due to errors in experiments or nonhomogeneity of materials.

The wide fluctuation of fatigue life is also observed in metal and other materials, which has been empirically known for many years. However, it is only recently that such fluctuation is an essential nature of fatigue fracture. Yokobori, Weibull, Freudenthal et al. proposed statistical methods independently during the period from

1950 to 19532). It is true that processes up to fatigue fracture, starting from crack formation and ending at fracture, consist of several mechanisms which are essentially different from each other. Furthermore, if the distribution of latent defects in materials constituting the source of stress concentration is taken into consideration, it is easily understood that development of fatigue fracture is a statistical phenomenon. Accordingly, it is considered that the statistical handling of fatigue life is quite significant from the basic standpoint of pursuing fatigue mechanism. Moreover from the informations of the reliability, as shown Fig. 7, it will be able to incorporate the theories utilizing stochastic variations into the pavement design process.

The author tried to investigate the fatigue fracture of bituminous mixture from the stochastic standpoint, assuming that such phenomenon can be satisfactorily explained from the statistical standpoint if the fluctuation of fatigue life is under a certain condition. The report deals with the results of our study on the statistical nature of fatigue life and various characteristics in processes up to fractue in two kinds of mixtures having different composition under a constant temperature and support condition.

2. OUTLINES OF EXPERIMENTS

Factors exerting influence on fatigue are comparatively complex and diversified. Among them, the following four factors are considered as foundamental; composition of bituminous mixture resiliency of base, magnitude of load and temperature. This report deals with the results of investigation on the composition of mixtures and magnitude of loads.

(1) Materials and Bituminous Mixtures Used in Experiments

a) Aggregate and Asphalt

Crushed limestone was used as coarse aggregate, and the sand obtained by crushing the stone of the same kind was used as fine aggregate. The specific gravity of each aggregate is shown in Table 1. The physical properties of asphalt are shown in Table 2.

b) Mixtures used in experiments Two kinds of mixtures were used: dense-graded

Table 1 Specific Gravity of Aggregate

Size	13-10	10-5	5-2.5	2.5-0.6	0.6- 0.0074	Filler
S. gravity	2.741	2.775	2.775	2.723	2.903	2.710

Table 2 Physical Properties of Asphalt

Penetration		25°C, 100 g, 5 sec	74
Softing poin	nt	R & B	48
Specific gra	vity	25/25°C g/cm ³	1.025
P.I.			-0.62
S.E.S. 150°	С	sec	130
155°	С	sec	102
160°	С	sec	85
			l

asphalt concrete (hereinafter to be refered to as "dense mixture") and gap-graded asphalt concrete (hereinafter to be referred to as "gap mixture") which are considered to be the commonest kinds of mixtures. The grading curves for testing are as shown in Fig. 1.

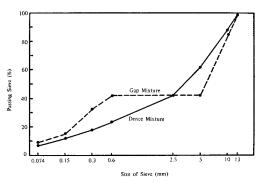


Fig. 1 Grading Curve for Testing.

The amount of asphalt used was 6.0% according to the Marshall test in the case of dense mixture and 7.0% in the case of gap mixture empirically. This mixture is different from the so-called hot rolled asphalt in B.S. 594.

(2) Test Specimen

The test specimen were 4cm in height, 4cm in width and 30cm in length. The test specimen were prepared by using a roller compactor, while controlling their weight so that the density was kept constant. The frequency distribution of density of test specimen is as shown Figs. 2 and 3. Out of them, 160 test specimen whose density is within the standard deviation 1σ were selected as test specimen. The present report deals with the results of experiments conducted by selecting 80 specimen out of the abovementioned specimen at random.

(3) Testing Machine and Experimental Method

The repeated bending test machine used in experiments consists of a lever arm having a weight, a cam connected to an infinitely variable speed motor and a loading unit. The increase or de-

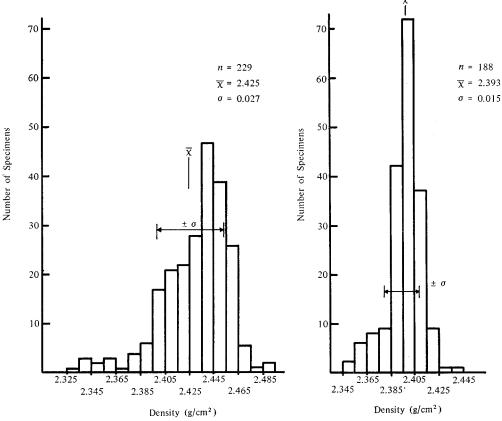


Fig. 2 Frequency of Density of Dense Mixture.

Fig. 3 Frequency of Density of Gap Mixture.

crease in load is accomplished by means of the weight. The specimen support units consists of a specimen supporting table and an elastic leaf spring which supports a test specimen at its center from beneath in the direction at right angles to the specimen. The supporting method of both test specimen and leaf spring is of simple support.

The load and displacement were measured according to a visigraph by using a calibrated load cell and a linear variable differential transformer type displacement meter (L.V.D.T.) as a pick-up.

The reason why the elastic leaf spring was used is to study the fatigue of bituminous mixture to be subjected to repeated deflection but not to give the same effect to be obtained from the base in which

Table 3 Test Condition

Time of loading	0.2 sec
Load	80 and 96 kg
Intensity	5 and 6kg/cm ²
Spring constant	100 kg/mm
Temperature	10°C

the actual pavement is taken into consideration.

Table 3 summarizes the conditions of the experiments conducted by us. Fig. 4 shows an example of the relationship between the load, deflection and time which was recorded by the visigraph.

During the repeated test, the deflection was

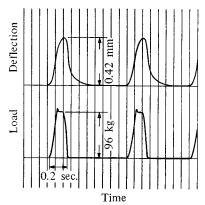


Fig. 4 Relationships of Load and Deflection vs. Time.

measured at an arbitrary number of repetition and attention was paid to the discovery of cracks. After crack formation was observed, the test was continued until the test specimen was broken. White water paint was applied to test specimen at their center so as to facilitate observation of cracks.

3. RESULTS OF EXPERIMENS AND DISCUSSION

(1) Statistical Properties of Fatigue Life

a) Adaptability to Weibull Distribution

As shown in Figs. 5 and 6, the frequency distribution of fatigue life fluctuates greatly. It will be understood that this distribution is comparatively complex.

This fluctuation phenomenon can be explained as follows by the socalled Weibull distribution which is developed from the theory of extreme probability (weakest link theory) introduced by Weibull for the first time in 1939 3).

In this kind of fatigue test, N is generally larger than 1 and can be regarded as a continuous variable. Therefore, assuming that the random variable is N and that the cumulative distribution

function F(N) is expressed as follows:

$$F(N)=1-[\exp -\phi(N)]$$
(1)

then, the probability density function f(N) will be as follows:

$$f(N) = \phi'(N) \exp[-\phi(N)] \cdots (2)$$

On the other hand, let the failure rate of single-element connected portion under the external force applied by N times be P. Thus, the reliability rate of n-numbered connected elements (1-Pn) will be equal to the probability that n-numbered connecting elements are not broken simultaneously, and therefore the following equations will be obtained according to the multiplicative theory of probability:

Assuming that P = F(N),

$$Pn=1-\{1-F(N)\}^n$$

=1-exp[-n\phi(N)]\dots\dots\dots(4)

Let's now assume that the value of N is always positive and its lower limit value is Nc, and then $\phi(N)$ in the equation (4) will satisfy the following conditions:

$$\phi(N) \ge 0 \qquad N \ge Nc$$

$$\phi(N) = 0 \qquad 0 < N < Nc$$

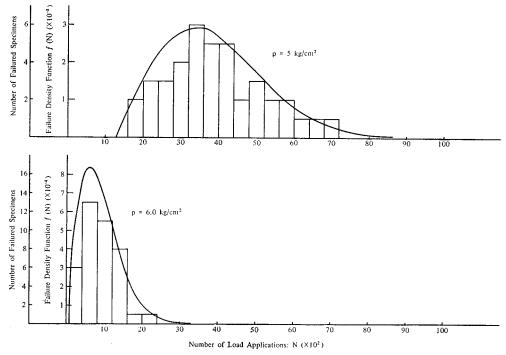
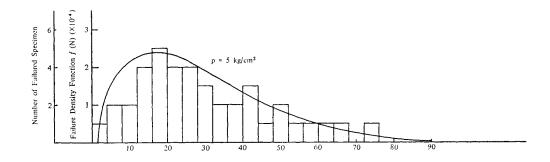


Fig. 5 Failure Density Functions of Dense Mixture vs. Number of Load Applications, for Different Bending Stress.



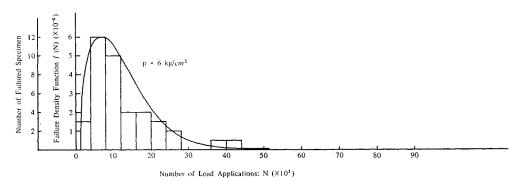


Fig. 6 Failure Density Functions of Gap Mixture vs. Number of Load Applications, for Different Bending Stress.

Moreover, from the monotonous increasing property of F(N) and the Eq. (1), the following simple function will be obtained:

$$\phi(N) = (N - N_c)^m / N_0, \quad (m > 0, N_0 > 0)$$
.....(5)

Consequently, the Eqs. (1) and (2) will be rewritten as follows:

$$F(N) = 1 - \exp \left[-(N - N_c)^m / N_0 \right] \quad \dots \quad (6)$$

$$f(N) = m(N - N_c)^{m-1} / N_0$$

$$\cdot \exp \left[-(N - N_c)^m / N_0 \right] \quad \dots \quad (7)$$

From the above equation, we will obtain:

$$R(N)=1-F(N)=\exp[-(N-N_c)^m/N_0]$$
(8)

$$\lambda(N) = f(N)/R(N) = m(N - Nc)^{m-1}/N_0 \cdots (9)$$

where:

F(N): Cumulative failure rate until number of repetition N

f(N): Failure density function

R(N): Reliability until number of repetition N

 $\lambda(N)$: Instataneous hazard rate (probability that failure is caused per cycle at an arbitrary number of repetition N)

m: Shape parameter

 N_c : Location parameter N_0 : Scale parameter

Shown in Figs. 5 and 6 are the curves obtained by estimating these parameters and determining f(N) for two kinds of mixtures. It will be seen from these curves that the phenomenon of fluctuation of fatigue life well conforms to the Weibull distribution.

Then, R(N) is obtained from the frequency distribution curve f(N) obtained from the results of experiments. In case of a comparatively small number of test pieces, it is also, obtained from the following equation which was independently proposed by Gumbel and Weibull:

$$R = 1 - \frac{\nu}{n+1}$$
(10)

where, R is the expected value at the ν order in case when the total number n of test specimen is arranged in the order of smaller life, that is, N_1 , N_2 , $N_3 \cdots N_{\nu} \cdots N_n$.

Figure 7 show the relation between the reliability (probability that fracture will not occur) and the number of repetition according to the above-described method. The curves shown in these figures are obtained from the Eq. (8). It

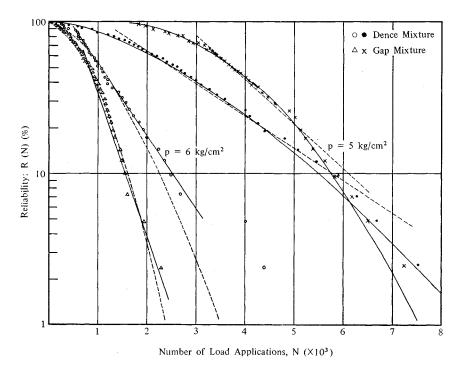


Fig. 7 Reliability vs. Number of Load Applications, for Different Mixes and Different Bending Stress.

will be seen that these curves reflect the results of experiments faithfully.

The parameters N_0 and N_0 are merely related with the scale of the abscissa of the probability density function. It can be said that the fundamental parameter is m. In other words, this relation conforms to the exponential distribution in case when m equals 1, but it approximates to the normal distributions as m is increased. As shown in Table 4, these parameters show different values depending upon the composition of mixture and the stress level.

As far as the instantaneous hazard rate $\lambda(N)$ is concerned, the relation between the hazard rate and the number of repetition shows a definite difference depending on the kind of mixture, as shown in Fig. 8. In case of dense mixture,

the instantaneous hazard rate, that is, crack formation speed is gradually saturated. On the other hand, in case of gap mixture, the crack formation speed is high and the failure rate is also high.

b) Adaptability to stochastic process theory

The problem related with starting of failure of material is generally explained also as the stochastic process. This explanation was confirmed by Yokobori as to various breakages of metal and the yielding phenomenon of steel. Thus, the phenomenon of fluctuation of fatigue life was explained as a stochastic process in 1954.

If the variable N is to be measured from N_c by assuming that the occurrence of failure after a certain number of repetition N_c is a problem of stochastic process, the following equations will

Table 4	Parameters	of	Weibull	Distribution
	1	_		

	Dense	mixture	Gap mixture		
Load intensity (p)	5 kg/cm ²	6 kg/cm ²	5 kg/cm ²	6 kg/cm ²	
Shape parameter (m)	1.50	1.48	2.00	1.64	
Scale parameter (Nc)	150	120	1 300	70	
Location parameter (N ₀)	1.71×10 ⁵	3.73×10 ⁴	8.59×10 ⁶	7.26×10 ⁴	

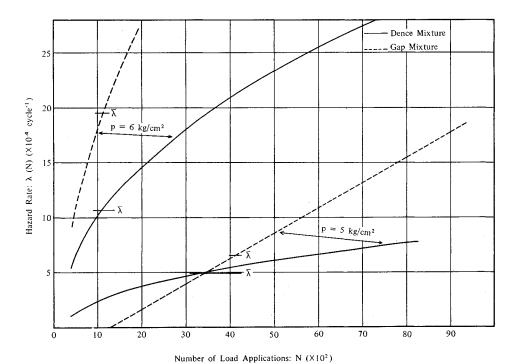


Fig. 8 Hazard Rate vs. Number of Load Applications, for Different Mixes and Different Bending Stress.

be obtained:

$$q(N) = \frac{Z_n}{Z_t}$$
 (11)
$$R(N) = \frac{\sum_{N}^{\infty} Z_n}{Z_t}$$
 (12)
$$\lambda(N) = \frac{q(N)}{R(N)} = \frac{Z_n}{\sum_{N}^{\infty} Z_n}$$
 (13)

wherein:

 $\lambda(N)$: Failure rate per cycle at an arbitrary number of repetition N

q(N): Failure rate between N and N+1 cycles after the number of repetition N

R(N): Reliability rate up to the number of repetition N

 Z_t : Total number of specimen tested

 Z_n : Number of test specimen in which failure occurred at the number of repetition N

In this case, assuming that N is the continuous variable and substituting q(N) with q(N)dN, the following equation is obtained.

$$R(N) = \int_{N}^{\infty} q(N)dN \quad \dots (14)$$

On the other hand, the probability that failure will not occur until the number of repetition N is reached but will occur at the next cycle will be

$$R \cdot \lambda dN$$

From the equation q(N) = -R'(N)

$$R \cdot \lambda dN = -dR$$
(15)

or

Accordingly, the following relation will be obtained:

$$\lambda(N) = d(\log R)/dN \quad \cdots \qquad (17)$$

This equation shows that $\lambda(N)$ is obtained by exchanging the sign of the gradient of the $\log R$ versus N diagram. Thus, the instataneous hazard rate or the speed of failure occurrence is obtained in the unit per cycle.

Assuming that the results of experiments show the linear relationship as shown by a straight line in Fig. 7 and that λ is constant, regardless of the value of N, the following equation is obtained from the Eq. (16):

$$R(N) = \exp(-\lambda N)$$
(18)

Accordingly, we will obtain

$$q(N) = \lambda \exp(-\lambda N)$$
(19)

The average life is expressed by

$$\bar{N} = \int_0^\infty N\lambda \cdot \exp(-\lambda N) dN = \frac{1}{\lambda} \cdots (20)$$

The variance is given by

$$\sigma^2 = \frac{1}{\lambda^2} \quad \cdots \qquad (21)$$

R(N) and q(N) shown here correspond to R(N) and f(N) described with respect to the Weibull distribution, respectively. $\lambda(N)$ in the Eq. (9) is expressed by

$$\lambda = \frac{1}{N_0} \dots (22)$$

when m=1 and is constant, regardless of the number of repetition N. As \bar{N} is apparent from the Eq. (20), (N_0) becomes equal to the average life. In this case, the dispersion is given by (N_0^2) .

Table 5 shows the comparison of the expected mean value and variance obtained from the primary and secondary moments at the origin of the Weibull distribution with those obtained from the stochastic process. The difference between these values is increased as the value of m is increased, resulting in an error of about 30% from the actually measured value. However, the phenomenon of fatigue life can be explained as a stochastic process, as suggested by ASTM⁵).

The modulus of resiliency of dense mixture and gap mixture at the early stage are $23\,900\,\mathrm{kg/cm^2}$ and $20\,700\,\mathrm{kg/cm^2}$ on the average, respectively. It is considered that the difference between these two figures appears as the magnitude of $\bar{\lambda}$ and as the trend of its gradual increase. Such a trend is also observed in metal and is explained by the stochastic process²⁾.

(2) Modulus of Resiliency of Mixture in Fatigue Process

The modulus of resiliency was obtained from deflection under the load intensity of $6 \,\mathrm{kg/cm^2}$ by assuming the neutral axis at the center of test specimen. No definite relationship was observed between the modulus of resiliency at the early stage of repeated loading (N=3), the deformation coefficient at the time of crack formation and the density of test specimen. Between the modulus of resiliency at the time of crack formation and the number of repetition up to that time, however, a trend of increasing the modulus of resiliency with the increase in the number of repetition was observed as shown in Figs. 9 and 10. This trend is also reported by Deacon⁴).

It is interesting to note the variation of initial deflection and deflection at the time of crack formation. It is needless to say that such variation is also observed in modulus of resiliency. As

	Dense mixture				Gap mixture			
	Weibull		Stochastic process		Weibull		Stochastic process	
Load intensity	5 kg/cm ²	6 kg/cm ²	5 kg/cm ²	6 kg/cm ²	5 kg/cm ²	6 kg/cm ²	5 kg/cm ²	6 kg/cm²
Hazard late (λ)	4.6×10-4	1.13×10-3	4.9×10-4	1.1×10-3	6.1×10-4	1.7×10-3	6.6×10-4	2.0×10-3
Mean life (\vec{N})	2 930	1 230	2 190	1 050	3 900	890	2 820	580
Variance (σ²)	3.6×10 ⁶	9.5×10 ⁵	2.1×10 ⁶	8.7×10 ⁵	1.8×10 ⁶	3.0×10 ⁵	2.3×10 ⁶	2.6×10 ⁵

Table 5 Specific Character of Fatigue Life

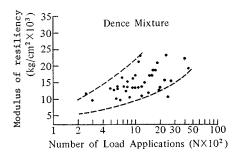
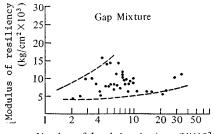


Fig. 9 Relationship of Number of Load Applications vs. Modulus of Resiliency (crack initiated).



Number of Load Apprications (NX102)

Fig. 10 Relationship of Number of Load Applications vs. Modulus of Resiliency (crack initiated).

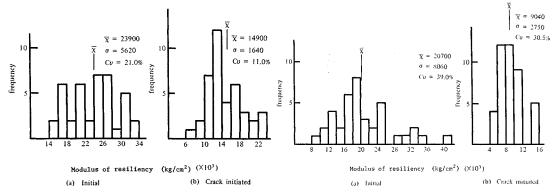


Fig. 11 Frequency of Modulus of Resiliency of Dense Mixture.

shown in Figs. 11 and 12, the variation forms a distribution which is very close to the normal distribution, and the variation of modulus of resiliency fluctuating widely at the initial stage is decreased by about 10% in the terms of variance coefficient at the time of crack formation. As is apparent from the above-mentioned values, it is considered that the modulus of resiliency of bituminous mixture is gradually decreased when the mixture is subjected to a constant repeated stress, and when the modulus of resiliency is decreased to a certain value, cracks are formed. After that the deflection is rapidly increased since crack formation, thus resulting in fracture at a certain limit.

4. CONCLUSION

The simple bending fatigue test was conducted on two kinds of bituminous mixtures under the constant test conditions, and the following conclusion was obtained.

- (1) The distribution of bituminous mixtures under the constant stress, temperature and supporting conditions follows the Weibull distribution developed from the weakest link theory.
- (2) At the same time, the phenomenon of variation of fatigue life can be explained as a stochastic process.
- (3) The difference depending on the kind of mixture is significantly reflected in the crack formation speed (instantaneous hazard rate) and the crack forming speed in gap mixture is higher than that in dense mixture under the constant stress, temperature and supporting conditions.
- (4) The variation of life of gap mixture is smaller than that of dense mixture, and when

Fig. 12 Frequency of Modulus of Resiliency of Gap Mixture.

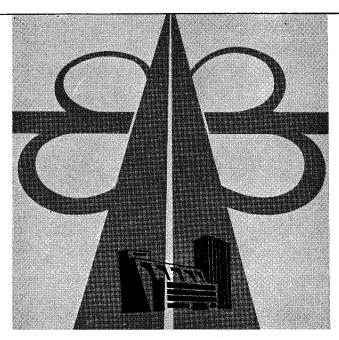
the load intensity is small, the median fatigue life (50% Reliability) of former mixture is longer than that of the latter.

- (5) The deflection or modulus of resiliency at which perceptible cracks are formed approximate to the normal distribution. Its variation at that time is smaller than that at the initial stage and is decreased by 10% in terms of variance coefficient.
- (6) The problems to be studied in the future may be the significance of shape parameter m and hazard rate λ varying in accordance with the kind of mixture and the magnitude of repeated stress, as well as the relation between m, λ and the kind of mixtures. When examining the fatigue of actual pavement, it is impossible to demonstrate fatigue or the whole process up to fatigue fracture with only on mechanism. It will be necessary to subdivide the whole mechanism into several stages and to investigate such problem by taking the pavement structure into consideration.

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(Received Aug. 28, 1972)





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性能の混和剤…それはポゾリスの代名詞です

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研究所で、一定の環境の下に行われるテストでは、良い性能を出す混和剤は他にもあります。然し現場で研究所に於て予知した通りの立派な性能を(地域、材料、コンクリートの性質の条件がいかなるものであっても)つくりだすことをポゾリスは実証しております。これらが、ポゾリスをして「性能の混和剤」の名をほしいままにしている理由であります。



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共和のデータコーダは、マイクロオーダの技 術、ひずみ測定器づくりの技術と精密な工作 技術を生かし、またどのような使用条件でも 最高の性能を発揮するように徹底した信頼性 試験を行なって設計製作されています。

共和のデータコーダは、高精密電子サーボ機 構によるSN比50dB以上、周波数特性、DC~ 10KHz、非直線性0.5%、小型軽量、テープ速 度は駆動中でもワンタッチ切換できる、プラグ インヘッド、サーボモニタ、位相、振巾平坦 切換のフィルタなど性能、操作性ともすぐれ 特に電算機でのデータ処理に抜群の性能を発 揮します。

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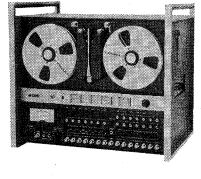
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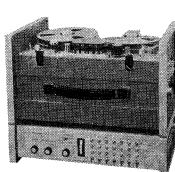
33チャンネル、5速度、連続6時間記録(4.75cm/

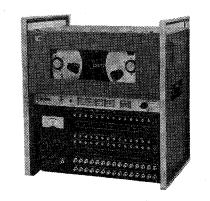
sec) 調整はサーボガンにより自動的に行なえる

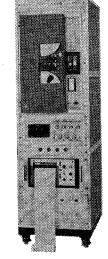
ので、準備時間は非常に短い。

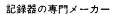












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