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# ESTIMATION OF TRAVELING TIME BETWEEN RAMPS AND DISCHARGE CONTROL ON EXPRESSWAY

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## SYNOPSIS

In this paper traffic congestion caused especially by traffic accident is analyzed theoretically and traffic control method for congestion on the expressway is considered.

The process of spread and vanishment of congestion is first analyzed by using Kinematic Wave Theory. A theoretical method of estimation of traveling time between ramps is also considered. The results show that the congestion does not vanish easily even after the accident is cleared away, and that the traveling time between ramps increases with spread of congestion and soon it becomes that drivers can get to their destination earlier by way of a surface road than by the expressway in case of heavy blockade of lanes or heavy traffic flow.

Therefore traffic control procedure should be implemented to prevent the spread of congestion and to shorten the traveling time. A method of recommending discharge from the upper off-ramp is introduced as a control procedure. The beginning time of the recommendation is determined by comparing traveling time on the expressway with that on the alternative surface road.

There is a possibility that the congestion continues spreading and reaches the upper off-ramp since not all drivers obey the recommendation. In such a case, a method of enforcing discharge from the off-ramp is introduced as a control procedure. Simple criteria for changing recommendation of discharge to enforcement and for removing the enforcement of discharge are shown.

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## 1. INTRODUCTION

Traffic accident on highways decreases the traffic capacity of highways, and causes congestion and more rear-end collisions. Such a situation is more serious on expressways due to the limited access and the higher running speed. Therefore the traffic control for preventing the spread of congestion should be implemented for expressways predominantly.

In case of rapid spread of congestion, the recommendation is done for drivers to discharge from the expressway through the sign of "congestion"[1]. However, since the degree of congestion is not informed to them, most of them may wait in a queue without discharging, expecting the sooner extinction of congestion, and as a result spreads the congestion contrary to their expectation. Such a situation is not desirable for both drivers and administrators of highway. And so the reliability of recommendation should be high to have an effect on drivers' choice of route.

Research Committee for Traffic Control on MEISHIN Expressway previously proposed a criterion of the discharge control as follows[2].

1) Until the congestion reaches the nearest off-ramp, any control policy is not carried out.
2) As soon as the congestion reaches the off-ramp, a policy of enforcement of discharge is carried out.

According to this principle, cars in a queue suffer much loss time and it takes a long time to recover the normal state after the source of congestion is taken off if the distance to the nearest upper off-ramp is long.

As a principle of discharge control, the recommendation of discharge for drivers will be better than enforcement of discharge, because drivers are able to select their route on their own responsibility.

This paper is devoted to the determination of the timing of recommendation of discharge. The beginning time of recommendation is determined by comparing the travel time on the expressway with that on an alternative surface road.

Sometimes, some drivers may stay on expressway disregarding the recommendation sign because they don't know any alternative routes. But it is not a problem in the control procedure shown in this paper.

## 2. BEGINNING TIME OF RECOMMENDATION OF DISCHARGE

Efficiency of discharge control through recommendation sign depends upon the usefulness of given information for the choice of route. If the discharge is recommended with high reliability, most of drivers will obey the recommendation. This is desirable for both drivers and administrators of highway, because the former will be able to reduce their traveling time and the latter will be able to prevent the extension of queue.

When should we recommend drivers to discharge?

When many lanes on one side of an expressway are blocked by an accident, congestion spread backward at high speed and it takes a long time for congestion to be extinguished after the removal of the obstacle. In such a case, the congestion should not be neglected for a long time. And when it takes a long time to clear the accident on the expressway, the congestion should not be also neglected, even if the traffic flow is not so heavy. Because some drivers are compeled to wait for a long time.

All drivers that obey the recommendation of discharge must expect that they will be able to reach their destination in a shorter traveling time by an alternative surface route than by the expressway. Therefore, the discharge should be recommended at a time when we can guarantee the shorter traveling time on the alternative route.

In Fig. 1 the two points, A and B, show the adjacent interchanges. A is the off-ramp of the upper interchange and B is the on-ramp of the down interchange.

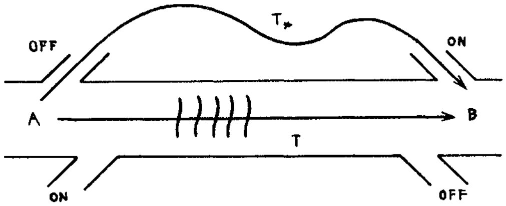

Fig. 1 $T$ and $T_*$

It is assumed that an accident is brought on at a point on the section AB.

Let $T$ and $T_*$ be the traveling time from A to B on the expressway and on an alternative surface road, respectively. The recommendation of discharge may be done if we can guarantee $T_* < T$.

**Estimation of $T$**

The traveling time $T$ on the expressway can be estimated easily by the Kinematic Wave Theory, if the time of removal of the accident is given. But it is too difficult to estimate the time when the accident is removed, because it depends on the content of the accident.

In this paper, therefore, the traveling time of a car is estimated on the assumption that the accident is just removed at the time when the car arrives at A.

**Estimation of $T_*$**

The traveling time of a car which has discharged to the alternative surface road from A by the recommendation of discharge must be also estimated. The estimation is more difficult because there are many intersections on the alternative surface road.

If the traveling time is not so much effected by the increase of traffic flow due to recommendation of discharge, the traveling time on the alternative road can be estimated by the data of travel time obtained by a floating car study or a simulation. If it depends very much on the discharged flow, it should be modified.

## 3. ESTIMATION OF TRAVELING TIME BETWEEN RAMPS ON THE EXPRESSWAY

### (1) Assumptions of Traffic Behaviour

The traffic flow is treated as a compressible fluid in this paper.

As is known in Kinematic Wave Theory, flow $q$, speed $v$ and traffic density $k$ have the following relations each other,

$$v = v_f(1 - k/k_j) = v_f(1 - p), \quad \cdots\cdots (1)$$

$$q = kv = v_f k_j p(1 - p), \quad \cdots\cdots (2)$$

where $p = k/k_j$ and $v_f$ and $k_j$ are free running speed and jam density, respectively.

It is also known that the propagation speed of disturbance of density $k$ is a differential coefficient $(dq/dk)$ and the shock wave propagates with a speed of tangent of a straight line connecting two points on the flow-density curve[3].

### (2) Traffic Behaviour Before the Obstacle is Cleared Away

When all the vehicles are traveling with a uniform speed $v_0$, as shown in Fig. 2, an accident is assumed to occur at the point $x_0$.

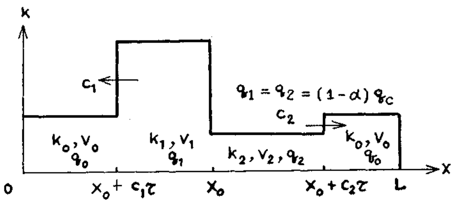


**Fig. 2** Traffic Density at Time $\tau$

The $x$-axis is taken downstream and the subscript 0 represents the condition before the accident occurs. The subscripts 1 and 2 represent that of upstream and downstream, respectively, after the accident occurred. The origin of $x$-axis is taken on the position of the upper off-ramp and the down on-ramp is at a distance of $L$ from the origin.

Fig. 2 shows traffic density at the time $\tau$ elapsed from the occurrence of accident.

Now we assume that the degree of blockade in the bottleneck is represented by $\alpha$, and that the capacity of expressway is reduced to $q_1$ from the normal value of $q_c$. Then we have

$$q_1=(1-\alpha)q_c\,, \qquad 0\leqq\alpha\leqq1 \quad \cdots\cdots\cdots\cdots\cdots\cdots(3)$$

and

$$k_1/k_j=(1+\sqrt{\alpha})/2 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(4)$$

from Eq. (2).

Therefore, the speed of shock wave propagating upstream is described as

$$c_1=\frac{q_1-q_0}{k_1-k_0}=-\frac{v_f}{2}(\sqrt{\alpha}-1+2p_0)\,, \quad \cdots\cdots(5)$$

where,

$$p_0=k_0/k_j\,. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(6)$$

### (3) Traffic Behaviour After the Obstacle Was Cleared Away

P. I. Richards studied on starting waves and the process of vanishment of the congestion on the assumption that all lanes are blocked[4]. We introduce the degree of blockade of lanes and discuss traffic behavior in this section referring to his results.

It is assumed that the obstacle is cleared away at the time $\tau$. The traffic situation at the time $\theta$ after the clearance of the obstacle is considered. Then the time-distance co-ordinates $(\theta, \xi)$ is used. The origins of time and distance are taken at the cleared time and the position of obstacle, respectively.

As soon as the obstacle is cleared away at the time $\theta=0$, the wave of starting cars will propagate upstream with a speed of $(dq/dk)$ through the queue. The locus of the wave of density $k$ on the $\theta-\xi$ plane is expressed by the equation $\xi^1=\xi^1(\theta)$. Then we have

$$d\xi^1/d\theta=dq/dk\,. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(7)$$

Therefore, the locus of the starting wave is described by

$$\xi^1(\theta)=v_f(1-2k/k_j)\theta \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(8)$$

or

$$\xi^1(\theta)=(2v-v_f)\theta\,, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(9)$$

using Eqs. (2) and (1) under the initial condition, $\xi^1=0$ at $\theta=0$.

The locus of the starting wave becomes a linear function of $k$ and $v$ as a result of linear speed-density relationship.

The position of shock front of queue at time $\theta$ will be expressed in

$$\xi^2(\theta)=c_1(\tau+\theta)\,, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

because the tail of queue has already been $c_1\tau$ at $\theta=0$.

Hence, the position where the starting wave will catch up with the shock front of queue is given by

$$\theta_1=\frac{\sqrt{\alpha}-1+2p_0}{\sqrt{\alpha}+1-2p_0}\tau\,, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(11)$$

$$\xi_1=-v_f\sqrt{\alpha}\,\theta_1\,, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(12)$$

as a crossing point $(\theta_1, \xi_1)$ of Eq. (10) and Eq. (8) in which $k$ is replaced by $k_1$. The point is shown in Fig. 3.

After the shock front was caught up with the starting wave, it will propagate upstream with a lower speed and attain the most upstream point of $(\theta_2, \xi_2)$. And then it will turn to move downstream and return the position of $x_0$ when the congestion vanishes.

Now the locus of shock front on the $\theta-\xi$ plane after $\theta=\theta_1$ is considered. The propagating speed of the shock front is $d\xi^3/d\theta=(q_0-q)/(k_0-k)$. Eliminating $k$ from this equation and Eq. (8), we have a first linear differential equation:

$$d\xi^3/d\theta-\xi/(2\theta)=v_f(1-2p_0)/2\,.$$

The locus of shock front can be obtained by solving above equation under the initial condi-

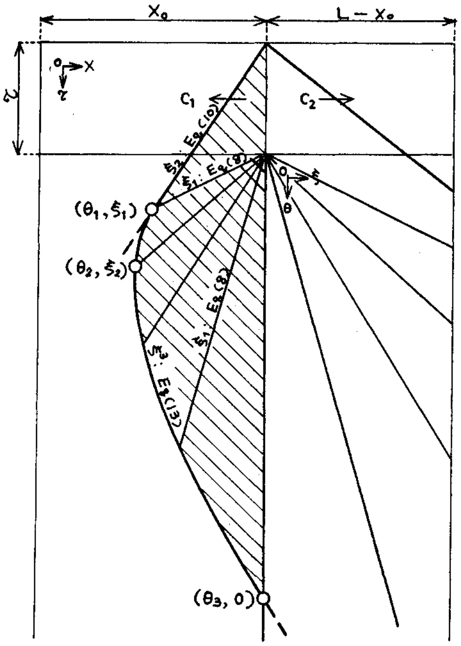


**Fig. 3** Propagation of the Shock Wave and the Starting Waves

tion of $\xi=\xi_1$ at $\theta=\theta_1$, and is described by

$$\xi^3(\theta)=v_f(1-2p_0)\theta - v_f(\sqrt{\alpha}+1-2p_0)\sqrt{\theta_1}\sqrt{\theta}. \quad \cdots\cdots(13)$$

Therefore, the most upstream point of shock front is represented by

$$\theta_2=\frac{(\sqrt{\alpha}+1-2p_0)^2}{4(1-2p_0)^2}\theta_1 = \frac{(\sqrt{\alpha}-1+2p_0)(\sqrt{\alpha}+1-2p_0)}{4(1-2p_0)^2}\tau, \quad \cdots\cdots(14)$$

$$\xi_2=-v_f(1-2p_0)\theta_2 = -\frac{(\sqrt{\alpha}-1+2p_0)(\sqrt{\alpha}+1-2p_0)}{4(1-2p_0)}v_f\tau, \quad \cdots\cdots(15)$$

as the solution of $d\xi^3/d\theta=0$.

The absolute value of $\xi_2$, $|\xi_2|$, shows the maximum length of congestion.

On the other hand, the time $\theta_3$, at which congestion will vanish, is given by

$$\theta_3=\frac{(\sqrt{\alpha}+1-2p_0)^2}{(1-2p_0)^2}\theta_1 = \frac{(\sqrt{\alpha}-1+2p_0)(\sqrt{\alpha}+1-2p_0)}{(1-2p_0)^2}\tau \quad \cdots\cdots(16)$$

because of $\xi^3(\theta_3)=0$.

From Eqs. (14) and (16), we have

$$\theta_3=4\theta_2. \quad \cdots\cdots(17)$$

It means that it takes four times of $\theta_2$ to disolve congestion even after the tail of congestion began to move downstream.

The duration of congestion, $\theta_3$, is proportional to duration of blockade, $\tau$, as shown in Eq. (16). This matter will emphasis us the importance of earlier removal of the obstacle.

### (4) Estimation of Traveling Time of a Noticed Car

A car will be noticed which reached the upper off-ramp of the accident section at time $\theta=0$.

The locus of the car on $\theta-\xi$ plane, $\xi^4(\theta)$, will be taken into consideration. Since the car travels with a constant speed $v_0$ before catching up with the tail of congestion, the position of the car at time $\theta$ is given by

$$\xi^4(\theta)=v_0\theta-x_0, \quad 0\leqq 0<\theta_0, \quad \cdots\cdots(18)$$

where $\theta_0$ is the time when it catched up with congestion.

There are several cases in the position of catching up with congestion according to the arrival time at off-ramp.

Now it is assumed that $\tau_1$ is the arrival time of the noticed car at upper off-ramp which catches up with congestion just on the point of obstacle, as shown in Fig. 4 (a).

Since we can put $\xi^4(\theta)=0$ at $\theta=\theta_3$ in Eq. (18) and substitute Eq. (16) for $\theta_3$,

$$\tau_1=\frac{1}{1-p_0}\cdot\frac{(1-2p_0)^2}{(\sqrt{\alpha}+1-2p_0)(\sqrt{\alpha}-1+2p_0)}\cdot\frac{x_0}{v_f}. \quad \cdots\cdots(19)$$

Next it is assumed that the noticed car reached the off-ramp at time $\tau_2$ catches up with congestion just on the position where the shock front is caught up with a starting wave propagating upstream, as shown ia Fig. 4 (b).

Then, since we have $\xi^4(\theta)=\xi_1$ at $\theta=\theta_1$ in Eq. (18), the value of $\tau_2$ is described as

$$\tau_2=\frac{1}{\sqrt{\alpha}+1-p_0}\cdot\frac{\sqrt{\alpha}+1-2p_0}{\sqrt{\alpha}-1+2p_0}\cdot\frac{x_0}{v_f} \quad \cdots\cdots(20)$$

by the substitution of both Eqs. (11) and (12) into Eq. (18).

In addition, if it is assumed that the tail of congestion propagates on the upper off-ramp at time $\tau_3$ as shown in Fig. 4 (c), we have

$$\tau_3=\frac{2}{\sqrt{\alpha}-1+2p_0}\cdot\frac{x_0}{v_f} \quad \cdots\cdots(21)$$

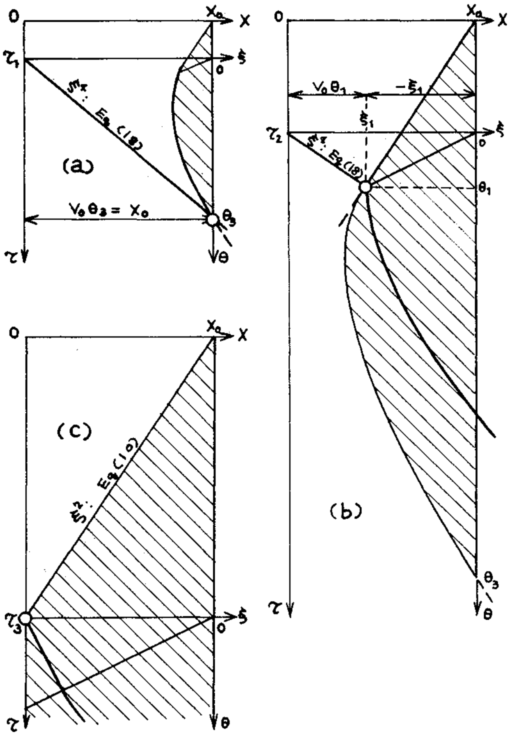


**Fig. 4** $\tau_1$, $\tau_2$ and $\tau_3$

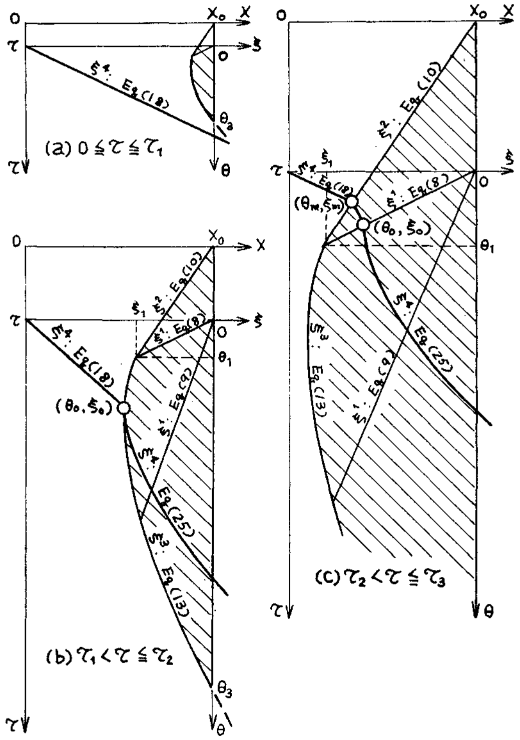


**Fig. 5** Loci of the Noticed Car

putting $\xi^2(\theta) = -x_0$ at $\theta=0$ in Eq. (10) and substituting Eq. (5) into Eq. (10).

Hence, the position that the noticed car catches up with congestion is classified to four categories corresponding with time of $\tau$.

1) $0 \leqq \tau \leqq \tau_1$

The car arrived at off-ramp in this interval can not catch up with the tail of congestion as shown in Fig. 5 (a). Therefore this case can be taken out of consideration, because the car will not suffer from the congestion.

2) $\tau_1 < \tau \leqq \tau_2$

The car reached the off-ramp at time $\tau$ will be compelled to follow through the congestion as shown in Fig. 5 (b).

3) $\tau_2 < \tau \leqq \tau_3$

The car reached the off-ramp at time $\tau$ will catch up with the tail of queue, and run at low constant speed first, and then increase its speed gradually through starting waves, as shown in Fig. 5 (c).

4) $\tau_3 < \tau$

The policy of enforcement discharge will be implemented when the congestion reaches the off-ramp. Therefore, this case will be also taken out of consideration.

**The Locus of a Noticed Car**

In the case of $\tau_1 < \tau \leqq \tau_2$, the position of a crossing point brought by Eqs. (13) and (18) is given by

$$\theta_0 = \frac{1}{p_0}\cdot\frac{x_0}{v_f} + \frac{(\sqrt{\alpha}+1-2p_0)(\sqrt{\alpha}-1+2p_0)}{2p_0^2}\tau - \frac{\sqrt{\alpha}+1-2p_0}{p_0}\sqrt{\frac{(\sqrt{\alpha}-1+2p_0)^2}{4p_0^2}\tau^2 + \frac{1}{p_0}\cdot\frac{\sqrt{\alpha}-1+2p_0}{\sqrt{\alpha}+1-2p_0}\cdot\frac{x_0}{v_f}\tau}\,, \quad \cdots\cdots(22)$$

$$\xi_0 = v_0\theta_0 - x_0\,, \quad \cdots\cdots(23)$$

as shown in Fig. 5 (b).

The speed of the car at any time $\theta$, $d\xi^4/d\theta$, must be equal to the $v$ in Eq. (9). Therefore, we have the equation of locus

$$\frac{d\xi}{d\theta} = \frac{1}{2}\left(v_f + \frac{\xi}{\theta}\right). \quad \cdots\cdots(24)$$

Its solution is given by

$$\xi^4(\theta) = v_f\theta - C\sqrt{\theta}\,, \quad \theta_0 \leqq \theta \leqq T\,, \quad \cdots\cdots(25)$$

under the initial condition of $\xi=\xi_0$ at $\theta=\theta_0$, where, $T$ is the arrival time of car at the on-ramp downstream and

$$C = (v_f\theta_0 - \xi_0)/\sqrt{\theta_0}\,. \quad \cdots\cdots(26)$$

In the case of $\tau_2<\tau\leqq\tau_3$, the position of a crossing point brought by Eqs. (18) and (10) is given by

$$\theta_m=\frac{2}{1+\sqrt{\alpha}}\cdot\frac{x_0}{v_f}-\frac{\sqrt{\alpha}-1+2p_0}{1+\sqrt{\alpha}}\tau \quad \cdots\cdots(27)$$

$$\xi_m=v_0\theta_m-x_0\,, \quad \cdots\cdots(28)$$

as shown in Fig. 5 (c).

After the noticed car catched up with the queue at the point of Eqs. (27) and (28), it moves with a constant speed $v_1$ before meeting the starting wave. So, its locus is given by

$$\xi^4(\theta)=v_1(\theta-\theta_m)+\xi_m\,, \qquad \theta_m\leqq\theta<\theta_0\,. \quad \cdots\cdots(29)$$

Therefore, the meeting position with the starting wave is determined as a solution of Eqs. (8) and (29), that is,

$$\theta_0=\frac{4p_0}{(1+\sqrt{\alpha})^2}\cdot\frac{x_0}{v_f}+\frac{(\sqrt{\alpha}+1-2p_0)(\sqrt{\alpha}-1+2p_0)}{(1+\sqrt{\alpha})^2}\tau \quad \cdots\cdots(30)$$

$$\xi_0=-v_f\sqrt{\alpha}\,\theta_0\,. \quad \cdots\cdots(31)$$

After the car passed over the point of Eqs. (30) and (31), it will trace on the locus of the same expression as written in Eq. (25).

**The Traveling Time Between Ramps**

When the car just passed over the off-ramp

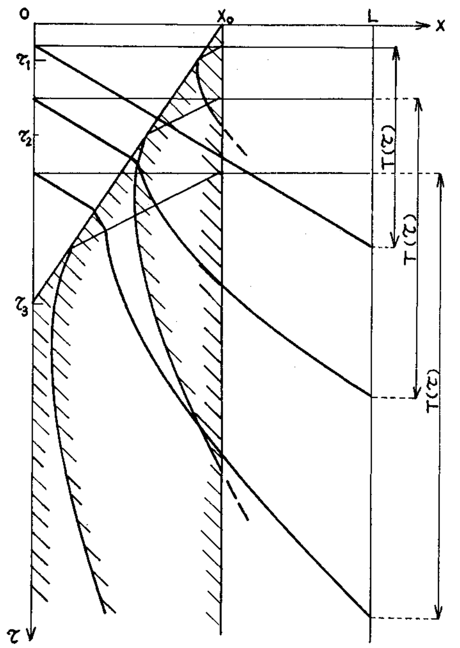


**Fig. 6** Summerized Loci of the Noticed Car

upstream at time $\tau$ (i.e. $\theta=0$), the arrival time at the on-ramp downstream will be

$$T(\tau)=\frac{2v_f(L-x_0)+C^2+\sqrt{C^4+4v_fC^2(L-x_0)}}{2v_f^2} \quad \cdots\cdots(32)$$

by putting $\xi^4=L-x_0$, and $\theta=T$ in Eq. (25). Where, $C^2$ is reduced by substituting Eqs. (23) and (31) into (26).

$$C^2=\begin{cases} v_f^2(p_0\theta_0+x_0/v_f)/\theta_0 & \text{for } \tau_1<\tau\leqq\tau_2\,, \\ v_f^2\theta_0(1+\sqrt{\alpha})^2 & \text{for } \tau_2<\tau\leqq\tau_3\,, \end{cases} \quad \cdots\cdots(33)$$

where, $\theta_0$ is given by Eq. (22) for $\tau_1<\tau\leqq\tau_2$ and Eq. (30) for $\tau_2<\tau\leqq\tau_3$.

Fig. 6 shows the summarized loci of the noticed car. The traveling time, $T$, on expressway has been obtained here necessary to issue the recommendation of discharge.

## 4. SWITCHING OVER AND EXPIRATION OF DISCHARGE CONTROL

Some cars will make their way towards the bottle-neck without discharging, even if the recommendation of discharge has been issued for drivers. Therefore, the recommendation should be changed to the enforcement of discharge, when the tail of congestion reaches to the off-ramp upstream.

Then there is one problem in the expiration of enforcement of discharge. The problem will be considered in this paragraph.

The proposal for the expiration of control is that the first car which can advance on expressway after the expiration of enforcement of discharge is able to catch up with the last car having passed the upper off-ramp just before the beginning of enforcement of discharge on the position of vanishment of congestion at the disappearance time of congestion.

If the obstacle is cleared away in the queue length of $l$ at time $\theta=0$, the last car in the queue will pass over the position at the time of $\theta_4$ which is given by

$$k_1 l=q_c\theta_4 \quad \cdots\cdots(34)$$

where, $k_1$ and $q_c$ are the density of queue and the capacity of expressway, respectively.

Then, since

$$q_c=k_jv_f/4 \quad \cdots\cdots(35)$$

from Eq. (2) and $k_1$ is given by Eq. (4), the $\theta_4$ is reduced as follows.

$$\theta_4=\frac{4k_1}{k_j}\cdot\frac{l}{v_f}=2(1+\sqrt{\alpha})\frac{l}{v_f}\,. \quad \cdots\cdots(36)$$

On the other hand, the first car after expiration can travel to the position of past obstacle with a free speed $v_f$.

According to the criterion in this paper, hence, the expiration of enforced discharge is issued at the time of $\theta_*$,

$$\theta_* = \theta_4 - x_0/v_f = 2(1+\sqrt{\alpha})l/v_f - x_0/v_f \ . \qquad \cdots\cdots(37)$$

The expressway will be opened again at the time of $\theta_*$ after the obstacle was cleared away.

Although $T$ gets smaller than $T_*$ before $\theta_*$, it is considered that the enforced discharge should not be expired at the time.

## 5. EXAMPLES

As a simple example, it is assumed that an accident occured on the middle point between two ramps. For the following values and notations,

$L = 24$ km, $v_f = 80$ km/h,

$k_j = 120$ vehicles/km/lane,

$T_* = 40$ minutes,

Fig. 7 is obtained in which the travel time on expressway is shown under the variation of traffic volume $q_0$, degree of blockade $\alpha$ and position of accident $x_0$.

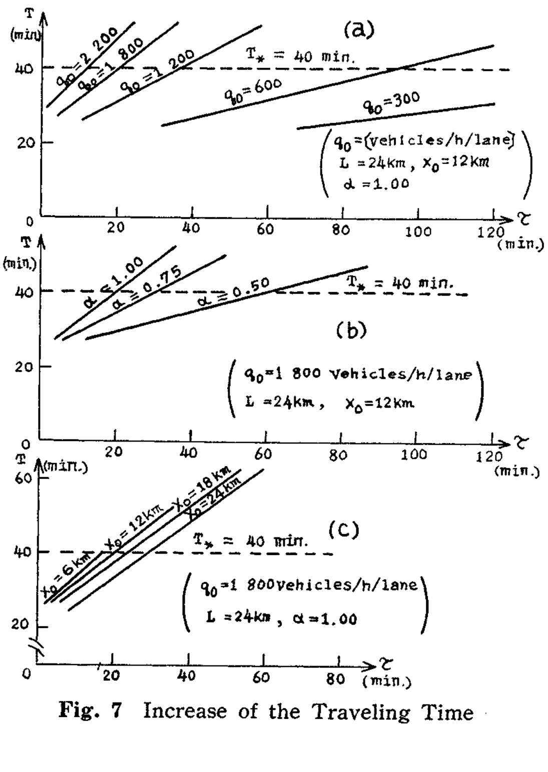


**Fig. 7** Increase of the Traveling Time

The recommendation time of discharge control is given by 93, 37, 20 and 12 minutes corresponding to the traffic volume on a lane per hour of $q_0 = 600$, 1200, 1800 and 2200 under the conditions of $\alpha = 1.0$ and $x_0 = 12$ km, as shown in Fig. 7 (a).

Fig. 7 (b) shows the influence of the degree of blockade. In the case of $q_0 = 1800$, $\alpha = 0.5$, the beginning time of recommendation is about three times later than one of $\alpha = 1.0$.

The influence of position of accident on the recommendation time is shown in Fig. 7 (c). If the accident occurs within about 5 km downstream from the off-ramp, the enforcement discharge will be implemented without recommendation discharge.

Then the expiration of enforcement discharge will be issued at the time of $\theta_*$,

$$\theta_* = 2 \times (1+\sqrt{1}) \times \frac{12}{80} \times 60 - \frac{12}{80} \times 60$$

$$= 27 \quad \text{(minutes)},$$

in the case of $x_0 = 12$ km and $\alpha = 1.0$.

## 6. DECISION PROCEDURE

The decision procedure is done by the following steps.

1) Confirmation of accident condition:

The position of accident, $x_0$, and its occurence time, the degree of blockade on lanes, $\alpha$.

2) Confirmation of traffic condition before the accident:

Calculation of traffic density $k_0$ by the method of

(a) estimating from the time occupancy observed,

(b) estimating from a pre-determined flow-density curve through $q_0$,

(c) estimating from the difference of accumulated traffic volumes between two points,

and so on.

3) Selection of the correspoding traveling time along the alternative route, $T_*$.

4) Calculation of the beginning time of discharge recommendation.

The beginning time $\tau_*$ is calculated as follows:

(a) the values of $\tau_1$, $\tau_2$ and $\tau_3$ are determined by Eqs. (19), (20) and (21).

(b) for the determined value of $T_*$ and $\tau_2 \leqq \tau < \tau_3$, we have the value of $\tau_*$ so as to satisfy $T(\tau_*) = T_*$, from Eqs. (32), (33) and (30), that is,

$$\tau_* = \frac{1}{\sqrt{\alpha}+1-2p_0} \cdot \frac{1}{\sqrt{\alpha}-1+2p_0}$$

$$\times\left[\frac{1}{T_*}\left(T_*-\frac{L-x_0}{v_f}\right)^2-4p_0\frac{x_0}{v_f}\right]. \qquad \cdots\cdots(38)$$

(1) if $\tau_*>\tau_3$, the recommendation of discharge is not implemented but the enforcement of discharge will have been issued at time $\tau=\tau_3$,

(2) if $\tau_3>\tau_*\geqq\tau_2$, it should be recommended to discharge from the time of $\tau_*$,

(3) if $\tau_2>\tau_*$, $\tau_*$ is meaningless. Therefore, the following steps must be carried out.

(c) the value of $\tau_*$ in case of $\tau_1\leqq\tau<\tau_2$ is calculated from Eqs. (32), (33) and (22), namely,

$$\tau_*=\frac{p_0^2}{(\sqrt{\alpha}+1-2p_0)(\sqrt{\alpha}-1+2p_0)}\cdot\frac{1}{\theta_0}\left(\theta_0-\frac{1}{p_0}\cdot\frac{x_0}{v_f}\right)^2 \qquad \cdots\cdots(39)$$

where,

$$\theta_0=\frac{C^2-2v_fp_0x_0-\sqrt{C^2(C^2-4v_fp_0x_0)}}{2v_f^2p_0^2} \qquad \cdots\cdots(40)$$

$$C^2=[T_*v_f-(L-x_0)]^2/T_*. \qquad \cdots\cdots(41)$$

(1) if $\tau_1\leqq\tau_*<\tau_2$, the discharge recommendation will be begun at time $\tau_*$.

(2) if $\tau_*<\tau_1$, the car can move smoothly downstream. So any control does not be used.

5) When the congestion comes over the off-ramp upstream in spite of the discharge recommendation, it will be switched over to enforcement discharge.

6) The expiration of enforcement discharge is determined by Eq. (37).

## 7. CONCLUSION

We have proposed a criterion for recommendation of discharge by comparing the traveling time on the expressway with that on the alternative surface road. Moreover, we have also suggested that the recommendation should be changed to enforcement when the congestion reaches the upper off-ramp and afterwards the enforcement should be removed.

There may be some problems in applying this method to an actual control system. And so, the practical research should be developed successively.

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### NOTATIONS

$T$ : traveling time between ramps on expressway

$T_*$: traveling time between ramps on alternative surface road

$L$ : distance between ramps

$C$ : integral constant

$v$ : running speed

$v_f$ : free running speed

$v_0$ : running speed before the occurrence of accident

$v_1, v_2$ : running speed of upstream and downstream, respectively, after the occurrence of accident

$k$ : traffic density

$k_j$ : jam density

$k_0$ : traffic density before the occurrence of accident

$k_1, k_2$: traffic density of upstream and downstream, respectively, after the occurrence of accident

$p$ : relative density, $p=k/k_j$

$p_0$ : relative density before the occurrence of accident, $p_0=k_0/k_j$

$q$ : traffic volume

$q_c$ : usual traffic capacity

$q_0$ : traffic volume before the occurrence of accident

$q_1, q_2$ : traffic volume of upstream and downstream, respectively, after the occurrence of accident

$c_1$ : speed of shock wave propagating upstream

$c_2$ : speed of shock wave propagating downstream

$\alpha$ : the degree of blockade of lanes due to accident

$(\tau, x)$ : time-distance co-ordinates, whose origins are taken at the occurrence of accident and the position of upper off-ramp, and $\tau$ represents the clearance time of accident

$x_0$ : the position where the accident occurs

$\tau_1$ : the arrival time of the noticed car at upper off-ramp which catches up with congestion just on the point of obstacle

$\tau_2$ : the arrival time of the noticed car at upper off-ramp which catches up with congestion just on the point where the shock front is caught up with a starting wave propagating upstream

$\tau_3$ : the arrival time of the shock front at the upper off-ramp

$\tau_*$ : the beginning time of the recommendation of discharge

$(\theta, \xi)$ : time-distance co-ordinates, whose origins are taken at the cleared time and the position of obstacle, respectively

$\xi^1$ : the locus of starting wave

$\xi^2$ : the locus of shock front (the locus of the rear-end of queue) before $\theta=\theta_1$

$\xi^3$ : the locus of shock front (the locus of the read-end of queue) after $\theta=\theta_1$

$\xi^4$ : the locus of the noticed car

$(\theta_m, \xi_m)$: meeting point of the noticed car to the read-end of queue

$(\theta_0, \xi_0)$ : meeting point of the noticed car to the starting wave

$(\theta_1, \xi_1)$ : the position that starting wave catches up with shock front

$(\theta_2, \xi_2)$ : the most upstream point of shock front

$\theta_3$ : the time when congestion vanishes

$\theta_4$ : the time when congestion vanishes by enforced discharge control

$\theta_*$ : expiring time of the enforced discharge control

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