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FATIGUE ANALYSIS OF HIGHWAY BRIDGES

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ABSTRACT

The increased possibility of minor as well as major fatigue failure of highway bridges of relatively short span now being subjected to heavy trucks more frequently than ever before, appears to be a critical problem that calls for a basic re-evaluation of the fatigue performance of these bridges constructed under current and past design codes. For the purpose of establishing a framework of the study along this line, the present paper outlines, with a numerical example, a method of sensitivity analysis for the fatigue failure dealing with a system of a simple bridge and moving vehicles. Specifically, the sensitivity of the expected fatigue life to various parameters of the bridge and traffic flow characteristics such as failure criterion, traffic intensity, highway surface roughness, bridge damping, etc. is investigated.

1. INTRODUCTION

In recent years, an increasing number of heavy trucks are seen on the nation's highways. This obviously creates a variety of technical problems. In particular, the increased possibility of minor as well as major fatigue failure of highway bridges of relatively short span now being subjected to such heavy trucks more frequently than ever before, appears to be a critical problem that calls for a basic re-evaluation of the fatigue performance of these bridges constructed under current and past design codes.

For the purpose of establishing a framework of the study along this line, the present paper outlines, with a numerical example, a method of sensitivity analysis for the fatigue failure dealing with a system of a simple bridge and moving vehicles. Specifically, the sensitivity of the expected fatigue life to various parameters of the bridge and traffic flow characteristics is investigated.

It is acknowledged that encouragement and request to publish the result of this study as it stands, mixed with constructive criticisms on the technical details were offered by those who understood the problem and were kind enough to approve of this work as a pilot study. Partly in response to these requests and encouragement and partly in view of the authors' belief that the work as it is represents a meaningful fundamental study under a grant from the National Science Foundation and can serve as a useful guide to a more realistic and practical future study, the authors have decided to publish this paper with the discussion in Chapter 7 suggesting some of the more important subjects to be studied in the future.

2. DYNAMIC ANALYSIS OF BRIDGE/MOVING VEHICLE SYSTEM

A vibrational analysis of a system of a bridge and moving vehicles is outlined in this subsection. The analysis takes into account both the dynamics of moving vehicles and the impact forces resulting from random surface roughness and expansion joints of the bridge.

(1) Vibration of a Bridge and Moving Vehicles

Consider a system of a bridge and moving vehicles as shown in Fig. 1. For simplicity of analysis and brevity of numerical computation, the bridge considered consists of identical simple girders (see Fig. 2) of span l and uniform combined bending rigidity EI. A discussion will be given later as to the validity of the assumption that EI is uniform. Approximating the deflection $y(t, x)$ of the bridge at time t and location x as the sum of the modes, one obtains

$$y(t, x)=\sum_{k=1} q_k(t) \sin \frac{k\pi x}{l} \quad \cdots\cdots\cdots(1)$$

Note that each vehicle is idealized in terms of

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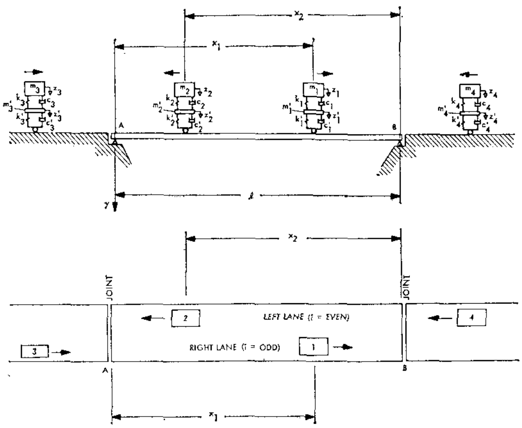

Fig. 1 Vehicle-bridge system

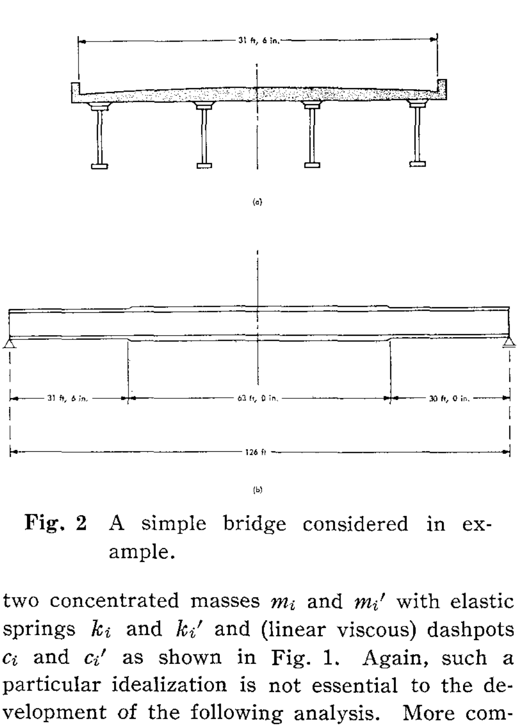

Fig. 2 A simple bridge considered in example.

two concentrated masses m_i and m_i' with elastic springs k_i and k_i' and (linear viscous) dashpots c_i and c_i' as shown in Fig. 1. Again, such a particular idealization is not essential to the development of the following analysis. More complex models of the vehicle, for example, a model with two axles, can be considered at the expense of more increased complexity of analysis and longer computer time.

The potential energy V, the kinetic energy T, and the Rayleigh dissipation function D of the system can be written as

$$V=\frac{EI}{2}\int_0^l\left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx+\sum_i\left\{\frac{1}{2}k_i(z_i-z_i')^2+\frac{1}{2}k_i'(z_i'-y_i)^2-(m_i+m_i')gy_i\right\} \quad \cdots\cdots (2)$$

$$T=\frac{1}{2}\frac{w}{g}\int_0^l \dot{y}^2dx+\sum_i\left\{\frac{1}{2}m_i\dot{z}_i^2+\frac{1}{2}m_i'\dot{z}_i'^2\right\} \quad \cdots\cdots (3)$$

$$D=\frac{1}{2}c''\int_0^l \dot{y}^2dx+\sum_i\left\{\frac{1}{2}c(\dot{z}_i-\dot{z}_i')^2+\frac{1}{2}c'(\dot{z}_i'-\dot{y}_i)^2\right\} \quad \cdots\cdots (4)$$

where the summation over i obviously covers only those vehicles on the bridge, w=weight of bridge per unit length, g=acceleration due to gravity, c''=coefficient of linear viscous damping for bridge, and y_i=deflection at the point of contact at $x=x_i$ between the bridge surface and the i-th vehicle;

$$y_i\equiv y_i(t,\, x_i)=\sum_i q_k(t)\sin\frac{k\pi x_i}{l}+y_0(x_i) \quad \cdots\cdots (5)$$

with $y_0(x)$ representing the surface roughness of the bridge. In the formulation above, it is assumed that the contact between the bridge and the vehicles is always maintained.

Applying the Lagrange equation of motion and considering up to the n-th mode of vibration, one can derive the following equations of motion:

$$\left.\begin{aligned}&\ddot{q}_k+2\beta_k p_k\dot{q}_k+p_k^2q_k-\sum_i\left[\frac{2gR_i}{R_i'}\{\mu_i'(z_i'-y_i)+\nu_i'(\dot{z}_j'-\dot{y}_i)+R'+1\}\sin\frac{k\pi x_i}{l}\right]=0\\&\qquad (k=1,\,2,\,\cdots,\,n)\\&\ddot{z}_i+g\{\mu_i(z_i'-z_i)+\nu_i(\dot{z}_i'-\dot{z}_i)\}=0\\&\ddot{z}_i'+R'g\{\mu_i(z_i'-z_i)+\nu_i(\dot{z}_i'-\dot{z}_i)\}+g\{\mu_i'(z_i'-y_i)+\nu_i(\dot{z}_i'-\dot{y}_i)\}=0\end{aligned}\right\} \quad \cdots\cdots (6)$$

$$p_k=\frac{k^2\pi^2}{l^2}\sqrt{\frac{EI}{w/g}}, \qquad \beta_k=\frac{c''}{2p_k(w/g)}$$

$$\mu_i=k_i/m_ig, \qquad \mu_i'=k_i'/m_i'g$$

$$\nu_i=c_i/m_ig, \qquad \nu_i'=c_i'/m_i'g$$

$$R_i'=m_i/m_i', \qquad R_i=m_ig/wl$$

$$x_i=v(t-t_i) \qquad (i=\text{odd})$$

$$x_i=l-v(t-t_i) \qquad (i=\text{even})$$

t_i=arrival time of the i-th vehicle

If at any time there are N vehicles ($i=j,\, j+1,\, \cdots,\, j+N-1$) moving on the bridge, then Eq. (6) represents a set of $n+2N$ simultaneous equations for the same number of unknowns $q_1,\, q_2,\, \cdots,\, q_n,\, z_j,\, z_{j+1},\, z_{j+1}',\, \cdots,\, z_{j+N-1}$. This is indeed a complex system that can only be solved by means of numerical integration where the computer program is to be divised so that as the time proceeds, the vehicle leaving (entering) the bridge at a certain time instant is eliminated from (added to) the system of equations (Eq. (6)) after that time instant.

(2) Impact Loadings Due to Random Surface Roughness and Expansion Joints

One of the special features considered in the present study is the effect of the surface roughness $y_0(x)$ on the system vibration and ultimately the (fatigue) life expectancy of the bridge. It is well known that $y_0(x)$ can be treated as a homogeneous random process. In fact, typical field measurements result[1] in the power spectral densities $S(\Omega)$ as shown in Fig. 3 (irregular solid and dashed curves). Curves (a), (b) and (c) are

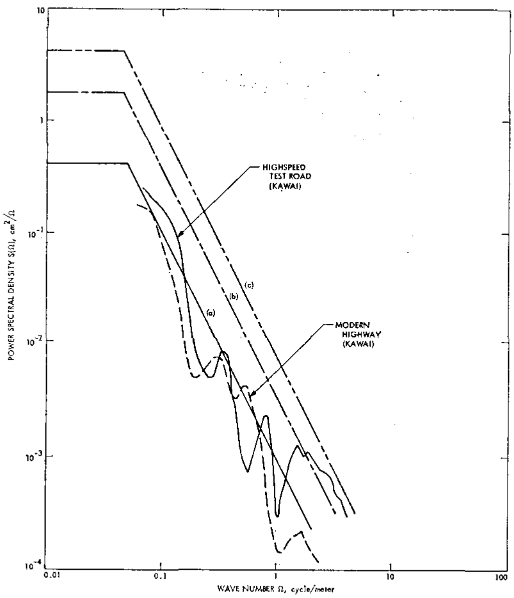

Fig. 3

idealized spectral densities constructed for the purpose of numerical simulation. On the assumption that the amplitude distribution of the surface roughness act on Gaussian distribution, the method developed in Ref. 2 is conveniently used for this purpose in which the surface roughness is simulated in the following form;

$$y_0(x)=\sigma\sqrt{\frac{2}{N}}\sum_{i=1}^{N}\cos(\Omega_i x+\varphi_i) \quad \cdots\cdots\cdots(7)$$

where

$$\sigma=\sqrt{\int_{-\infty}^{\infty}S(\Omega)d\Omega} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(8)$$

Ω_i = random variables identically distributed with density function $g(\Omega)\equiv S(\Omega)/\sigma^2$; independent of Ω_j $(i\neq j)$

φ_i = random variables identically distributed with uniform density between 0 and 2π; independent of φ_j $(i\neq j)$ and of Ω_j $(j=1, 2, \cdots, N)$

N = a large positive integer

For the purpose of generating realization of $y_0(x)$ on a digital computer, Eq. (7) is used with Ω_i and φ_i replaced by their respective realizations. This simply requires a generation of independent sequences of random variables; a routine task. The result of such digital generation of $y_0(x)$ is then employed in Eq. (6).

Another significant feature to be studied currently is the effect of impact produced by the expansion joint as the vehicle enters the bridge. Suppose that the maintenance or the construction of the joint is not ideal and leaves a rectangular indentation of depth 1.2 in. and length 2 in.. The effect of such a joint can be approximated in terms of an impact $A\delta(t)$ to be applied to the vehicle as it enters the bridge; for example, $A=0.04$ in.-sec if the vehicle moves with a speed of 40 mph.

The surface roughness and impact can be incorporated into the process of numerical integration of the system of equations of motion (Eq. (6)).

These generalization of the system of equations of motion (Eq. (6)) to a multiple-span bridge with any number of lanes is apparent. A general computer program subroutine is established, which is capable of generating and numerically solving a system of equations of motion. This is the first step toward the automatic rational design of highway bridges.

3. CONSTRUCTION OF DYNAMIC STRESS HISTOGRAMS

A method of calculating dynamic stresses in representative bridge members from the previous vibrational analysis is presented. In particular, a method of constructing tables or charts of stress histograms suitable for use as design guides is presented herein in detail.

After the system of equations of motion (Eq. (6)) is integrated numerically, the maximum stress (extreme fiber stress) $s_0(t, x)$ within a cross-section at x can be evaluated as

$$s_0(t,x)=-\frac{EI}{Z}\frac{\partial^2 y}{\partial x^2}=\frac{EI}{Z}\sum_{k=1}^{n}\left(\frac{k\pi}{l}\right)^2 q_k(t)\sin\frac{k\pi x}{l} \quad \cdots\cdots\cdots(9)$$

where Z=section modulus. Then, the total stress is obtained by adding to $s_0(t, x)$ the stress $s_d(x)$ due to dead load which can easily be evaluted

$$s(t, x)=s_0(t, x)+s_d(x) \quad \cdots\cdots\cdots(10)$$

The number of terms to be retained in Eq. (9) depends on the convergence of $k^2q_k(t)$ rather than $q_k(t)$ as k increases, which in turn depends on physical parameters involved in the system, such as natural frequencies of the vehicle. For a set of parameter values assumed herein, $k^2q_k(t)$ is completely negligible compared with $(k-1)^2 \cdot q_{k-1}(t)$ for $k \geqq 5$ and that the third term is of considerable magnitude since the third natural frequency (15.5 cps) of the bridge happens to be close to the higher natural frequency (17.0 cps) of the vehicle. Consequently, the first four terms are retained in Ref. 2 (n=4). Such a decision, however, has to be made carefully on a case-by-case basis.

In spite of its importance, damping for different modes is difficult to infer in the face of lack of experimental evidence. A preliminary study carried out indicates that there is not much difference in the bridge response when $\beta_1=\beta_2=\beta_3=\beta_4=5\%$ are used or when $\beta_1=5\%$, $\beta_2=7.5\%$, $\beta_3=10\%$, and $\beta_4=12.5\%$ are used. A more careful sensitivity study, however, is needed on this point.

From the result of the stress analysis just mentioned, one can derive a considerable amount of pertinent information; in particular, the statistical distribution of peak stress values. Consider, for example, a stress history at a certain cross-section x as schematically shown in Fig. 4. It

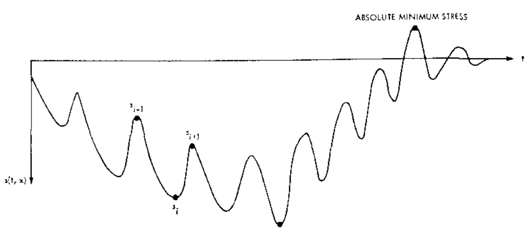

Fig. 4 A shematical stress history

is considered that the knowledge of the (statistical) distribution of peak stresses such as s_{i-1}, s_i and s_{i+1} is a signification to the fatigue analysis. Therefore, each set of successive local maximum and minimum stresses (s_i and s_{i+1}) or of successive local minimum and maximum stresses (s_{i-1} and s_i) is plotted as a point in the s_{min}-s_{max} plane. Dividing the plane into square cells of size Δs, and recording the number of points located within each cell, one obtains a table

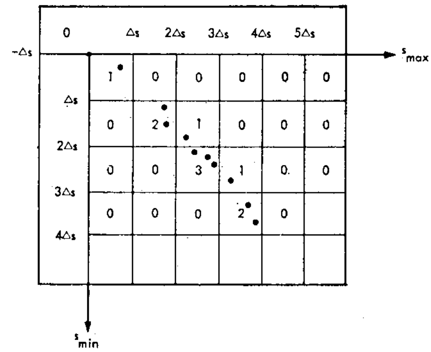

Fig. 5 A schematical stress histogram

such as that shown in Fig. 5. This table contains all of the information that is considered significant for the fatigue analysis and serves in lieu of a conventional stress peak histogram. If, for example, only the absolute maximum and minimum stresses are considered important, one can read these from the table; s_{max} corresponding to the cell located on the extreme right gives the absolute maximum stress whereas s_{min} associated with the cell located at the upper most position the absolute minimum stress (this can be negative). It goes without saying that all of these can be handled by a computer.

4. ANALYTICAL APPROACH TO LOAD ESTIMATION

(1) General Approach

One of the basic problems in the estimation of life expectancy of a bridge is the prediction of the traffic pattern, such as traffic volume, speed, weight, percentage of trucks, etc. for which a bridge is designed. One approach to this problem is a direct simulation of traffic flow pattern. Another approach is to formulate a statistical model and to construct a number of loading patterns of the traffic (being consistent with the observed traffic flow). The advantages of a simulation model lie in the fact that the result will probably reflect reality reasonably well, and it is relatively easy to perform such a simulation. It is, however, too expensive in terms of computer time, especially for the sensitivity studies of the life expectancy of a bridge with respect to the various important parameters involved. On the other hand, if an analytical model is constructed properly, a sensitivity analysis can be performed easily without involving extraordinary costs for computer time. This will become apparent in the next subsection where a sensitivity study is carried out on a typical example of a highway bridge, based on the analytical model proposed herein.

It is the purpose of this subsection to propose an analytical traffic model in such a way that the analysis results can be used for comparison and verification of the future results using the simulation model.

(2) Statistical Model and Traffic Loading Pattern

To formulate the analytical traffic model, the following assumptions are made essentially for the sake of simplicity in presentation and for the purpose of comparison with the result in Ref. 3 where the same assumptions were used.

1) The traffic consists of mainly these types of vehicles:
   a) Heavy trucks weighing 142 kips 1%
   b) Light trucks weighing 72 kips 20%
   c) Passenger cars 79%
2) The passenger cars can be disregarded in the dynamic and the fatigue analyses in view of their negligibly small weight.
3) One can compute the probability of exactly n trucks moving on the bridge (both lanes combined) from the traffic flow consisting only of trucks with the arrival rate equal to 21% of the original traffic flow.

Assumptions 1) and 2) can be relaxed immediately while the relaxation of assumption 3) requires careful study. Under the assumptions mentioned above, the arrival rate λ of the truck traffic flow (without passenger cars) can be computed as $\lambda=0.21\lambda_0$ where λ_0 is the arrival rate of the true traffic flow (including passenger cars). Assuming the widely used gamma distribution for the time interval, T, between two successive arrival (e.g., Ref. 4), the density function $f_T(t)$ of T can be written as

$$f_T(t)=\frac{\mu}{(K-1)!}(\mu t)^{K-1}e^{-\mu t} \quad \cdots\cdots(11)$$

where

$$\mu=K\lambda \quad \cdots\cdots(12)$$

with K being a positive integer. Eq. (11) reduces to the exponential density when $K=1$. The probability $P(m)$ that exactly m cars will arrive in a time interval t_0 is then given by

$$P(m)=\frac{(\mu t_0)^m}{m!}e^{-\mu t_0} \qquad (K=1) \quad \cdots\cdots(13)$$

$$P(m)=\left\{\frac{(\mu t_0)^{2m}}{(2m)!}+\frac{(\mu t_0)^{2m+1}}{(2m+1)!}\right\}e^{-\mu t_0'} \qquad (K=2) \quad \cdots\cdots(14)$$

$$P(m)=\left\{\frac{(\mu t_0)^{3m}}{(3m)!}+\frac{(\mu t_0)^{3m+1}}{(3m+1)!}+\frac{(\mu t_0)^{3m+2}}{(3m+2)!}\right\}e^{-\mu t_0} \qquad (K=3) \quad \cdots\cdots(15)$$

and similar expressions for $K\geq 4$.

When $t_0=l/v$ (l=bridge span length, v=speed of traffic flows), the frequency interpretation of $P(m)$ is as follows: consider the traffic flow extending on the highway; take a section of this flow consisting of K (K=a positive integer) intervals of length l; and count the number $K(m)$ of intervals containing exactly m trucks at a certain time instant. Then, $P(m)=\lim_{K\to\infty} K(m)/K$. It is important to realize that this probability can also be interpreted as the portion of time one lane of the bridge is occupied by exactly m trucks. It then follows that the probability P_{ij} of exactly i trucks on the right lane and exactly j trucks on the left lane can be written as

$$P_{ij}=P_{ji}=P(i)P(j) \quad \cdots\cdots(16)$$

where it is evident that

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}P_{ij}=1\,, \qquad \sum_{i=0}^{\infty}P(i)=1$$

Table 1 shows those fifteen loading patterns that will be employed for the dynamic analysis in the next subsection. For example, the load pattern 11 represents a sequence of three trucks on the right lane and another sequence of two trucks on the left lane. The dynamic effect of this loading pattern is identical to that produced by the pattern in which three trucks are on the left lane and two on the right lane. Since the probabilities for both of these patterns are eqaul ($P_{32}=P_{23}$), the probability that the bridge will be subject to the vibration due to these patterns is defined as $P_{32}^*=2P_{32}$. Similar definitions can be given to P_{ij}^*; $P_{ij}^*=2P_{ij}$ ($i\neq j$) and $P_{ij}^*=P_{ij}$ ($i=j$). Obviously, however, trucks in the same lane generally have to maintain a certain minimum distance and therefore the bridge cannot accommodate more than a certain number of trucks. It is assumed that this distance is 30 ft. for all of the loading patterns (to be precise, this depends upon the speed of traffic flow), and each lane of the bridge can be occupied at most by four (4) trucks when the bridge span l is 126 ft. The probabilities P_{ij}^* with $i>4$ and $j>4$ are included in P_{44}^*.

At this point, it should be noted that there are a number of combinations of heavy and light trucks in a loading pattern. When exactly j trucks are moving on one lane of the bridge, the probability $P_{j'|j}$ that j' of them ($j'\leq j$) will be heavy trucks and j-j' will be light trucks, is given by

$$P_{j'|j} = {}_jC_{j'} q^{j'} (1-q)^{j-j'} \quad \cdots\cdots\cdots\cdots(17)$$

where q is the ratio of the number of heavy trucks to that of all trucks in the traffic considered. In the present case, the ratio is 1:21 as mentioned previously and hence $q=1/21$. Therefore, given that there are i trucks on the right lane and j trucks on the left lane, the (conditional) probability $P_{i'j'|ij}$ that i' of these i trucks on the right lane and at the same time j' of these j trucks on the left lane will be heavy ones is given by

$$P_{i'j'|ij} = P_{i'|i} P_{j'|j} \quad \cdots\cdots\cdots\cdots(18)$$

It is evident that

$$\sum_{i'=0}^{i} \sum_{j'=0}^{j} P_{i'|j'|ij} = 1 \quad \cdots\cdots\cdots\cdots(19)$$

and that

$$P^*_{m';ij} = P_{ij}{}^* \left(\sum_{i'} \sum_{j'} P_{i'j'|ij}\right) \qquad (m' \le i+j) \quad \cdots\cdots\cdots\cdots(20)$$

$$i'+j'=m'$$

indicates the probability that the bridge will be occupied by i trucks on one lane and j trucks on the other in which m' trucks (among $i+j$ trucks) are heavy ones. Again, it is evident that

$$\sum_{j=0}^{4} \sum_{i=0}^{4} \sum_{m'=0}^{i+j} P^*_{m';ij} = 1\,. \quad \cdots\cdots\cdots\cdots(21)$$

5. NUMERICAL EXAMPLE AND DISCUSSION

In this section, a numerical example of a two-way, two-lane highway bridge is presented to demonstrate the applicability and practicality of approaches proposed in the preceding sections. Many interesting features derived from the numerical example are discussed.

For the dynamic analysis, a bridge as shown in Fig. 1 having a fundamental frequency 1.725 cps and consisting of four main steel girders (each with an average moment of inertia 2.02×10^5 in.^{4}) is considered. Referring to Fig. 1, each truck has the following relationship among physical parameters: (1) $m_i=4.41$ (kips sec^2/ft)$=10\,m_i'$, (2) $\sqrt{k_i/m_i}=3.5$ cps and $\sqrt{k_i'/m_i'}=13.0$ cps; this results in 2.7 cps and 17.0 cps for the fundamental and the second frequencies of the truck, (3) $c_i=0.01(2\sqrt{m_i k_i})$ and $c_i'=0.05(2\sqrt{k_i' m_i'})$.

Figure 6 indicates the result of the dynamic analysis under the (reference) loading pattern 15 (Table 1) consisting of 8 light trucks moving at a speed of 40 mph ($v=40$ mph) in both directions.

Table 1 Loading patterns

PATTERN CODE	PROBABILITY	NUMBER OF TRUCKS	RIGHT LANE (LEFT LANE)				LEFT LANE (RIGHT LANE)			
1	P^*_{00}	0								
2	P^*_{01}	1				↓				
3	P^*_{02}	2			↓	↓				
4	P^*_{03}	3		↓	↓	↓				
5	P^*_{04}	4	↓	↓	↓	↓				
6	P^*_{11}	2				↓	↓			
7	P^*_{12}	3			↓	↓	↓			
8	P^*_{13}	4		↓	↓	↓	↓			
9	P^*_{14}	5	↓	↓	↓	↓	↓			
10	P^*_{22}	4			↓	↓	↓	↓		
11	P^*_{23}	5		↓	↓	↓	↓	↓		
12	P^*_{24}	6	↓	↓	↓	↓	↓	↓		
13	P^*_{33}	6		↓	↓	↓	↓	↓	↓	
14	P^*_{34}	7	↓	↓	↓	↓	↓	↓	↓	
15	P^*_{44}	8	↓	↓	↓	↓	↓	↓	↓	↓

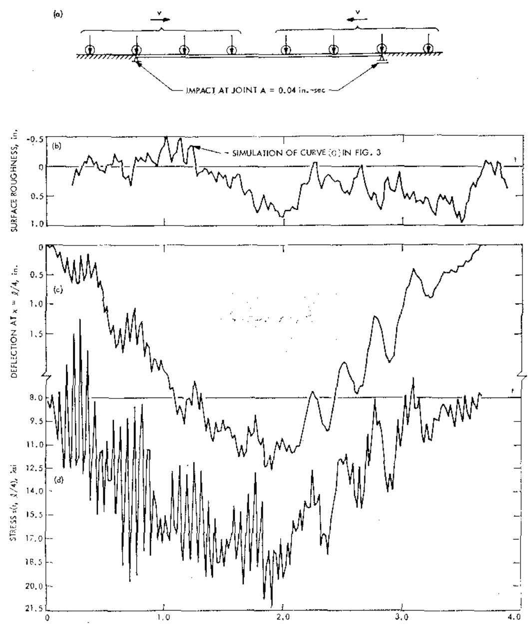

Fig. 6 Loading pattern 15

Figure 6 (b) shows the surface roughness corresponding to Curve (c) in Fig. 3; Fig. 6 (c) shows the deflection at $x=l/4$ where a change in the cross-sectional moment of inertia I exists as depicted in Fig. 2 (b) (for the first approximation, such a change in I has been disregarded in the dynamic analysis; see section 7 for details); and Fig. 6 (d) shows the stress $s(t, x)$ at $x=l/4$. Stresses are considered at $x=l/4$ since a stress concentration is likely to exist at this point where flanges of different sizes are supposedly welded. The parameter A representing the intensity of impact at the joints at the bridge supports is assumed to be 0.04 in.-sec. Table 2 shows the peak stress distribution obtained from this result. A similar result is also shown in Fig. 7 for comparison, where the surface roughness corresponds to curve (b) in Fig. 3 and $A=0$, although the same loading is considered. The

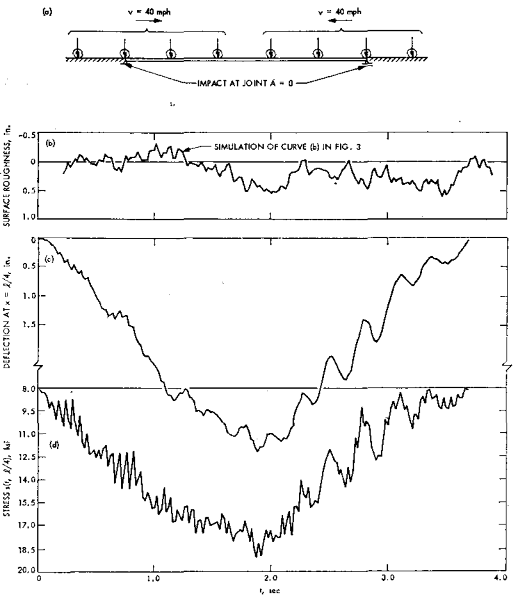

Fig. 7 Loading pattern 15

stress history is much less violent in this case reflecting the effects of a smoother surface (curve (b) rather than curve (c) in Fig. 3) and of no impact at the joints. Similar computations are carried out for other reference loading patterns resulting in corresponding tables of peak stress distribution.

In all these computations, a standard technique for step-by-step integration of a system of equations of motion is used with $\Delta t = 0.0001$ sec. Also, initial positions of the truck sequences in both directions are at some distance from the bridge (60 ft. away for the leading truck is sufficient) so that when the trucks reach the bridge they have already realistic vibrations.

The bridge itself is assumed to be at rest when the first truck in a loading pattern arrives. This implies a rather important hypothesis; there is enough time for bridge vibration due to a loading pattern to damp out before the next one arrives. Obviously, the probability P_{ij}^* is computed previously without accommodating such a hypothesis. In the present example, however, it is used for first approximation. The validity of this hypothesis should be reexamined not only from theoretical point of view but also on the basis of the result of more realistic future simulation study.

A preliminary study has supported the following assumptions:

1) The peak stress distribution for a particular loading pattern such as shown in Table 2 is relatively insensitive to the order in which heavy and light trucks arrive from both directions so long as the same number of heavy and light truchs are involved.
2) The peak stresses such as s_{i-1}, s_i, s_{i+1} in Fig. 4 due to a certain loading pattern consisting of a particular combination of heavy and light trucks can be obtained from those due to the same loading pat-

Table 2 Peak stress distribution (loading pattern 15; light trucks only)

S_{max}^i (IN ksi) / S_{min}^i (IN ksi)	8.0	8.75	9.5	10.25	11.0	11.75	12.5	13.25	14.0	14.75	15.5	16.25	17.0	17.75	18.5	20.0	20.75	21.5	22.75
4.25	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
5.0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
5.75	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
6.5	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
7.25	2	1	1	1	1	0	0	1	0	0	2	0	0	0	1	1	0	0	0
8.0	0	5	8	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0
8.75	0	0	1	0	1	0	0	0	0	0	1	0	2	0	1	0	0	0	0
9.5	0	0	1	0	3	0	0	0	3	1	2	0	0	0	0	0	0	0	0
10.25	0	0	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0
11.0	0	0	0	0	0	2	0	1	0	0	0	0	0	1	0	0	0	0	0
11.75	0	0	0	0	0	0	0	1	0	2	1	0	1	0	4	1	0	0	0
12.5	0	0	0	0	0	0	0	0	2	1	1	1	3	0	4	0	0	0	0
13.25	0	0	0	0	0	0	0	0	0	1	0	0	1	0	2	1	0	1	0
14.0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	2	0	0	0
14.75	0	0	0	0	0	0	0	0	0	0	0	6	1	2	0	1	0	0	0
15.5	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0
16.25	0	0	0	0	0	0	0	0	0	0	0	0	1	3	1	4	0	3	0
17.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17.75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19.25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20.75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TOTAL	2	8	12	2	6	4	0	4	8	8	10	8	12	8	16	12	0	4	0
GRAND TOTAL 124 (UPS AND DOWNS)																			

tern consisting only of light trucks (the reference pattern). This is done by simply multiplying the peak stresses due to the reference pattern by a factor that is equal to the ratio of the total weight of trucks involved in the particular combination to that in the reference pattern. If, for example, Fig. 5 represents the peak stress distribution due to a reference loading pattern consisting of m light trucks, one can produce the peak stress distribution due to the same loading pattern consisting, however, of m' heavy trucks by translating each and every point in the diagram (representing a set of local minimum and maximum stresses), say, $s_{\min}=s_{i-1}$ and $s_{\max}=s_i$ into a position $s_{\min}=\alpha s_{i-1}$ and $s_{\max}=\alpha s_i$ where

$$\alpha=\{m'W'+(m-m')W''\}/(mW'') \qquad \cdots\cdots(22)$$

with W' and W'' being the weights of heavy and light trucks (142 kips and 72 kips) respectively. In this fashion, the tables of the peak stress distribution associated with all combinations of heavy and light trucks within a particular loading pattern can be obtained from that associated with the reference pattern by means of simple computation thus resulting in a drastic saving in computer time. Therefore, the dynamic analysis is needed only for the 15 tables of peak stress distributions corresponding to these 15 reference patterns. On the basis of these distributions, one can then construct the tables of stress peak distribution for other combinations of heavy and light trucks. For the loading pattern 2, there are 2 such combinations; the truck is heavy or light, and for the loading pattern 3, there are 3 combinations; zero or one or two heavy trucks in the loading pattern, and so forth. In all, there are 75 combinations and therefore 75 tables of peak stress distribution, although obviously the table corresponding to loading pattern 1 consists only of zeros. These tables are combined into a master table taking the probabilities $P^*_{m';ij}$ (in Eq. (20)) into account. For example, the probability $P^*_{0;44}$ to be assigned to Table 2 associated with the reference loading pattern 15 indicates the relative frequency of this particular distribution of peak stresses to happen. Therefore, when Table 2 is compiled into the master table, it is done in such a way that all the numbers in the table are multiplied by $P^*_{0;44}$ and then placed into the corresponding cell of the master table. For example, $2P^*_{0;44}$ will be placed into the cell associated with $s_{\min}=7.25$ ksi and $s_{\max}=8.0$ ksi in the master table. In general, if $n^{(k,l)}_{m';ij}$ denotes the number in the cell located at the k-th row and the l-th column of the table of peak stress distribution of the loading pattern with i trucks on one lane and j trucks on the other, the cell at the same location in the master table has a number $n^*(k,l)$

$$n^*(k,l)=\sum_{j=0}^{4}\sum_{i=0}^{4}\sum_{m'=0}^{i+j} n^{(k,l)}_{m';ij}P^*_{m';ij} \qquad \cdots\cdots(23)$$

Though rather obvious, it is pointed out because of computational significance that the most time-consuming part (in terms of computer time) of the study, i.e., the dynamic analysis for the 15 reference loading patterns, does not have to be repeated when the assessment of probabilities of occurrence of these patterns (such as P_{ij}^*, $P^*_{i'j';ij}$, etc.) are modified. These modifications will change $n^*(k,l)$ only through Eq. (23). In Table 2, actually, the slight amplitude of dynamic stress are overbooked.

6. FATIGUE ANALYSIS; LIFE EXPECTANCY OF A BRIDGE; AND SENSITIVITY STUDY

This subsection presents a study of fatigue analysis and the prediction of the expected life of a bridge using the stress histograms obtained in the preceding subsection. The same numerical example presented previously is carried out to demonstrate that the results of the preceeding approaches can best be applied to the prediction of life expectancy of a bridge. Based on the proposed analytical approach, a sensitivity study is performed herein to evaluate the effects of changes of important parameters, such as traffic volume, traffic pattern, surface roughness, and damping of the bridge and vehicles, etc., on the ultimate life expectancy of a bridge. Very interesting results are presented and important items of future study are indicated.

(1) Fatigue Analysis and Life Expectancy of a Bridge

An obvious immediate difficulty in fatigue analysis is to make a choice from existing fatigue failure hypotheses applicable to the type of random stress history as exemplified by Fig. 6 (d) and Fig. 7 (d). It appears that the Palmgren-Miner rule of fatigue damage accumulation is a logical choice consistent with the various simplification and assumptions introduced in the preceding discussion. The rule simply states that (1) if a structural component survives N cycles of constant stress range between s_{min} and s_{max}, the (fatigue) damage $\Delta(s_{min}, s_{max})$ produced by a half cycle of such stress range, whether from s_{min} to s_{max} or from s_{max} to s_{min}, is equal to

$$\Delta(s_{min}, s_{max})=\frac{1}{2N} \quad \cdots\cdots(24)$$

and (2) the structural component fails when the amount of such damage reaches unity.

In a majority of cases, the rule has been applied to the fatigue failure under stress histories of variable stress range with a constant mean stress. However, the rule is now used for the present situation where the mean stress is also variable.

One of the S-N relationships under constant stress range between s_{min} and s_{max} that can be employed conveniently in the present study is is proposed by Fisher and Viest[5].

$$\log N=A+B(s_{max}-s_{min})+Cs_{min}\pm E \quad \cdots\cdots(25)$$

where N=fatigue life and is taken to be infinite if Eq. (25) produces N larger than 10^6, A, B and C=material constants, and E=a quantity to indicate the bound due to experimental error and other factors. A set recommended values for these parameters are

$$A=6.827, \quad B=-0.0620, \quad C=-0.0056$$

$$E=\pm 0.180$$

Equation (25) combined with the Palmgren-Miner rule is extremely useful for the type of stress history considered here. For example, referring to Fig. 4, the incremental damage created by that part of stress history between s_{i-1} and s_i is $1/(2N_{i-1,i'})$, where $N_{i-1,i}$ is the fatigue life obtained from Eq. (25). In general, consider a stress range between $s_{max}=s_l$ and $s_{min}=s_k$ represented by the cell located at k-th row and l-th column in the table of peak stress distribution. The damage Δ_{kl} created by an application of half cycle of this stress range is

$$\Delta_{kl}=\frac{1}{2N_{kl}} \quad \cdots\cdots(27)$$

where N_{kl} is the fatigue life under the repeated stress cycle between s_k and s_l.

As an example, consider traffic in both directions such that a steady flow of M vehicles/hour at a speed of 40 mph=60 ft/sec is observed night and day. Then, the arrival rate of the truck is

$$\lambda=0.21\times\lambda_0=0.21\times\frac{M}{60\times 60} \quad (\text{sec}^{-1}) \quad \cdots\cdots(28)$$

and t_0 to be used in Eqs. (13)-(15) is

$$t_0=l/v=126/60=2.1 \quad (\text{sec}) \quad \cdots\cdots(29)$$

The average total number N_0 of times the bridge is subjected to loading patterns (all patterns combined) per day is

$$N_0=24\times 60\times 60/2.1=41143 \quad \cdots\cdots(30)$$

One can easily check the consistency of these quantities; for example,

$N_0\lambda t_0$=average total number of loading patterns/day/lane × expected number of trucks in a loading pattern

$$=\frac{24\times 60\times 60}{2.1}\times 0.21\times\frac{M}{60\times 60}\times 2.1 \quad \cdots\cdots(31)$$

$=0.21\times 24M$

=total number of trucks/day/lane

Combining Eq. (27) with the master table of peak stress distribution, the average damage per year Δ can be written as

$$\Delta=\sum_k\sum_l 365N_0n^*(k,l)\Delta_{kl} \quad \cdots\cdots(32)$$

from which it follows that the average life Y (in years) or the life expectancy of the bridge is

$$Y=1/\Delta \quad \cdots\cdots(33)$$

This computation is currently under way. Completed, however, are different set of computations based on a different S-N relationship

$$N=cS^{-b} \quad \cdots\cdots(34)$$

where N=fatigue life, S=stress amplitude. Eq. (34) is for the stress history with $s_{min}=0$ and $s_{max}=2S>0$ and employed in Ref. 3 with $b=4.18$ and $c=0.24\times 10^{12}$. Because of the fact that the value of s_{min} is fixed, the method of life estimation described in conjunction with Eq. (24) cannot be applied. Therefore, the estimation of life expectancy is made in this case in such a way that the damage due to a half cycle of $s_{max}=s_l$ and $s_{min}=s_k$ is replaced by the damage due to a half cycle of $s_{max}=s_l$ and $s_{min}=0$. This is equivalent to using Eq. (32) with $n^*(k,l)$ replaced by $\sum_k n^*(k,l)$ (similar to the total line in Tables 1, 2) and Δ_{kl} by Δ_{0l}, and obviously will produce a

Table 3 Life expectancy (Y) in years

M (No. of vehicles/hr/lane)		34		170		340		S-N Curve
K (Eqs. (13), (14))		1	2	1	2	1	2	
A	Condition 1	3.5×10^1	4.3×10^3	7.0×10^0	1.7×10^2	3.5×10^0	4.3×10^1	Eq. (34)
	Condition 2	7.5×10^1	9.5×10^3	1.4×10^1	3.7×10^2	6.0×10^0	9.0×10^1	
	Condition 3	1.0×10^2	1.3×10^4	2.0×10^1	4.9×10^2	1.0×10^1	1.2×10^2	
	Condition 4	2.6×10^2	3.1×10^4	5.0×10^1	1.2×10^3	2.5×10^1	3.0×10^2	
B	Condition 1	5.0×10^1	6.5×10^3	1.0×10^1	2.5×10^2	4.5×10^0	6.5×10^1	Eq. (34)
	Condition 2	2.4×10^2	3.0×10^4	4.5×10^1	1.2×10^3	2.0×10^6	3.0×10^2	
	Condition 3	6.0×10^2	7.3×10^4	1.1×10^2	3.0×10^3	5.0×10^1	6.5×10^2	
	Condition 4	2.3×10^3	3.0×10^5	4.2×10^2	1.1×10^4	2.0×10^2	3.0×10^3	
C	Condition 1	8.0×10^1	1.0×10^4	2.6×10^1	4.0×10^2	8.1×10^0	1.0×10^2	Eq. (24)
	Condition 2	1.7×10^2	2.1×10^4	3.5×10^1	8.0×10^2	1.3×10^1	2.2×10^2	
	Condition 3	1.7×10^6	2.5×10^{10}	6.8×10^4	4.1×10^7	1.8×10^4	3.0×10^6	
	Condition 4	over 10^{10}	over 10^{10}	over 10^{10}	over 10^{10}	over 10^{10}	over 10^{10}	

Condition			1	2	3	4
Impact A (in.-sec.)			0.04	0.04	0.04	0
Surface roughness (Fig. 3)			curve (c)	curve (c)	curve (b)	curve (b)
Damping	Bridge (β_i)		0.05	0.05	0.05	0.05
	Trades (Fig. 1)	$c/(2\sqrt{mk})$	0	0.01	0.01	0.01
		$c'/(2\sqrt{m'k'})$	0	0.05	0.05	0.05

conservative result. The result of computation for Y (in years) based on the S-N curve used in Ref. 3 is given in A in Table 3 under a variety of different conditions.

If the same computations are carried out using the absolute maximum stress for a particular loading pattern as s_{max} and disregarding all other stress fluctuations, one obtains the life expectancies as shown in B in Table 3. This is obviously a non-conservative result. The result $Y=$ 17 000 years indicated in Ref. 3 is basically estimated in this fashion, however, taking into consideration neither the dynamic interaction between trucks and the bridge, the surface roughness, the imact due to joints, nor the damping capacity of trucks. Since the effect of all these are significant as can easily be detected from Table 3, the result such as obtained in Ref. 3 should, to say the least, be treated with extreme care.

The same type of computation that lead to the result in B is performed using the S-N curve given in Eq. (24). The result is shown in C in Table 3. In this case, however, not only the absolute maximum stress but also the absolute minimum stress due to each loading pattern have been taken into consideration (this is possible because Eq. (24) can take arbitrary s_{min}) although other stress fluctuations are all disregarded. The comparison between the results in B and C gives some indication of sensitivity of the life expectancy to the S-N curve.

(2) Sensitivity Study Based on Proposed Analytical Approach

The results of Table 3 clearly indicates the amount of sensitivity of life expectancy of highway bridges on various important parameters. For example, the effect of impact (compare conditions 3 and 4) is such that the life expectancy is at least doubled without such an impact. This suggests that a careful workmanship in joint construction is worthwhile. The existence of damping in the truck (compare conditions 1 and

2) does increase the life expectancy again more than twice. Hence, information should be gathered on the damping characteristics of the truck. The surface roughness plays an important role (compare conditions 2 and 3) the extent of which ranges from 50% increase in case A to more than 100% increase in B using the same *S-N* curve (Eq. (34)) by decreasing the surface roughness from Curve (c) to Curve (b) of Figs. 1-3 (a reduction of 60% in term of the power spectral density). The increase in the life is much more in case C. This shows that the surface finish of bridge pavement should also be carefully executed.

Table 3 further indicates that most significant factors are (1) the characteristics of traffic flow such as traffic volume and the distribution of the interval of successive arrivals, etc., and (2) the choice of the *S-N* curve as well as of the failure Hypothesis.

7. DISCUSSION

(1) Traffic Flow

The intensity of the traffic flow is one of the significant characteristics and is usually specified in terms of the statistical distribution of time interval between two successive arrivals. Eq. (11) is a possible analytical form of such distribution often used in the literature. The inverse of the expected value of this distribution is the parameter known as the mean arrival rate. Another important characteristic of the traffic is the statistical distribution of vehicle weight. A special feature of this distribution is the fact that its density function is usually not unimodal. It often takes a bimodal shape (or a trimodal shape—indicating separate weight concentrations for passenger cars and trucks (or for passenger cars, light and heavy trucks). The combination of these intensity and weight characteristics can define the traffic flow to be light, medium and heavy. However, a more systematic field study appears to be needed on the traffic flow including the weight distribution so that a more representative traffic model can be devised for a type of studies presented here. In fact, the stress analysis of a bridge under a continuous simulation of traffic flow (determined by such a field study) of sufficient length, rather than under loading patterns applied separately as done in the present paper, will be more realistic to a specific bridge.

(2) Nonuniform Cross-section of Girders

Variability of the bending rigidity and the weight per unit length of the bridge can be taken into consideration in the following ways.

(a) The most straightforward method is to directly solve a set of differential equations describing the system of bridge and vehicles. Since the beam equation is a partial differential equation, such direct method of solution requires a use of the finite difference scheme not only with respect to the time variable but also with respect to the space variable. Evidently, this is expected to provide a most rigorous solution at the expense of a much larger amount of computer time than other approximate methods to be described below.

(b) The deflection of the bridge is expanded into a sum of the sine functions. Since in this case the bending rigidity and the weight per unit length are functions of the space variable, the sine functions are not the modes of bridge vibration. Therefore, a set of (ordinary) differential equations for generalized coordinates $q_i(t)$ resulting from the energy formulation will be coupled. It is pointed out, however, that the integration of such simultaneous differential equations presents no computational difficulty. In fact, if one considers only first a few modes, this problem is much less involved than any typical problem of the dynamic finite element analysis.

(c) The deflection is expanded into a sum of the exact mode shapes. Since then the mode shapes are exact, the resulting differential equations for generalized coordinates are uncoupled and the integration can proceed exactly in the same way as in the case of uniform bridge. If, however, the exact modes are not given in analytical form, the stress has to be evaluated by differentiating the deflection "twice numerically" with respect to the space variable. Obviously, this presents some computational difficulties.

(d) For a first but possibly reasonable approximation as is done in the present study, the variable cross-section is replaced by a uniform cross-section using the average values of the bending rigidity and the weight per unit length. It is desirable to check the validity of this approach in a future study.

(3) Mechanical Model of Vehicles

Vehicles can be idealized as a concentrated mass or masses supported by two or more axles. Such a model will produce not only the vertical motion but also the rotation (pitching) of the mass. This increase in the number of degrees of freedom of motion will neither be a major analytical nor computational hazard.

(4) Lateral Stress Distribution

A rigorous treatment of the lateral stress distribution would give rise to a considerable analytical and computational difficulty since it would involve the effect of the diaphragm system and the slab on the vibration.

It has been observed, however, in an experimental study (p. 55, Ref. 6) that "In general, the shape of the static and dynamic stringer stress distributions was essentially the same." Therefore, the dynamic lateral stress distribution may be considered the same as the static lateral stress distribution for a first approximation. Although it is possible to use those ratios of stress distribution observed in Ref. 6, a literature survey should be made for an additional amount of information as to the lateral stress distribution, static or dynamic, experimental or analytical.

(5) Modeling Errors and Model Analysis

Dealing with any structure as complex and as large as a bridge, one cannot completely avoid modeling errors. The same reference[6] provides valuable data which help assessing such errors. In this connection, it is anticipated that the stress history as a result of the modal analysis may not agree with that observed in a field study. At the same time, however, it is expected that the modal analysis will provide a reasonable estimation of the dynamic stress histogram as shown in Fig. 5. This will be particularly true if field data are available for a proper adjustment of analytical data.

(6) Surface Roughness

A more rigorous treatment of the effect of surface roughness should consider the fact that the contact between each wheel of a vehicle and the surface is made over a certain area rather than at a point. This will probably eliminate the effect of those components of roughness with larger wave numbers.

(7) Other Factors

Other factors that are to be investigated more closely in the future study are: (a) damping characteristics of trucks and bridges, (b) natural frequencies of trucks and bridges, (c) fatigue failure hypothesis, (d) impact characteristics of joints, (e) effect of stress concentration in structural details and (f) standard deviation of life expectancy.

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