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THEORETICAL ARGUMENT ON TURBULENT STRUCTURE OF GUSTY WIND

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ABSTRACT

For design purpose of long or tall structures such as suspension bridges or skyscrapers, knowledge on the transversal or vertical as well as longitudinal turbulent structure of wind is indispensable.

The cross spectrum, coherence and phase of turbulence in shear flow fields are estimated based on the theoretical arguments. They are shown to compare well with experiments on gusty winds.

1. INTRODUCTION

The investigation of turbulent flow dates back to the famous experiment by Reynolds in 1886. The complicated fine structures of turbulent real flows have been disclosed progressively through the statistical theory on isotropic turbulence by Taylor (1935-38) and the local isotropy hypothesis by Kolmogorov (1941) and so on (see Batchelor[1]). However, it is relatively recent that the advances in turbulence theory are taken into the engineering design.

In the first stage, the one-dimensional turbulent structure (spectrum and auto-correlation) was sufficient for design purpose. Davenport[4] obtained an empirical formula on gust spectrum. On the other hand, Hino[5],[7] derived a gust spectral formula based on theoretical consideration. With increase in scale of our construction activities, knowledge on the transversal or vertical as well as longitudinal turbulent structures has become indispensable for efficient design of long or tall structures such as suspension bridges and skyscrapers.

Owing to the difficulty in observation installation the measurements reported on this problem are relatively few until recently. However, stimulated by urgent needs of reasonable and reliable design of the long-span suspension bridge over the Seto inland sea, the detailed observation data have been collected and analyzed by Shiotani and Iwatani[9],[10]. On the other hand, the writer[6] have represented a note from a theoretical view point.

It is shown in this paper that the turbulent structures such as cross-spectrum, coherence and phase may be estimated from the theoretical consideration.

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2. THEORETICAL CONSIDERATION ON TURBULENT STRUCTURE OF GUSTY WIND

(1) Cross-spectrum Estimated from the Frozen Turbulence Hypothesis

Let ξ and η denote the spatial separation distance in the direction of mean wind and that perpendicular to it, respectively; and let τ be the time lag. Then, the two-dimensional spatial and temporal cross-correlation is defined by Eq. (1) as

$$R_{uu}(\xi, \eta; \tau)=\overline{u(x_0, y_0, t)u(x_0+\xi, y_0+\eta, t+\tau)} \quad \cdots\cdots(1)$$

where overbar denotes ensemble mean, or in the case of stationary homogeneous turbulence field, it is equivalent to the mean with respect to space (x_0, y_0) or time (Fig. 1).

The Fourier transform of the cross-correlation Eq. (1) with respect to τ gives the frequency cross-spectrum $\Gamma(\xi, \eta, \omega)$;

$$\Gamma(\xi, \eta, \omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} R_{uu}(\xi, \eta; \tau) \exp(-i\omega\tau)d\tau \quad \cdots\cdots(2)$$

On the other hand, the Taylor hypothesis on the so-called frozen turbulence states that $R_{uu}(\xi, \eta, \tau)$ may be approximated by

$$R_{uu}(\xi, \eta, \tau) = R_{uu}(\xi - U_c\tau, \eta, 0)$$
$$= R_{uu}\left(0, \eta, \tau - \frac{\xi}{U_c}\right) \quad \cdots\cdots(3)$$

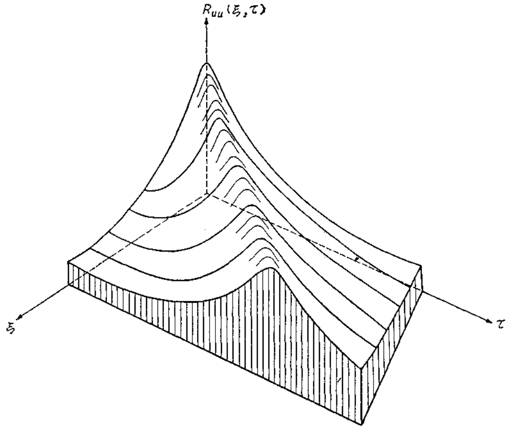

Fig. 1 Schematic graph of cross-correlation, $R_{uu}(\xi, \tau)$.

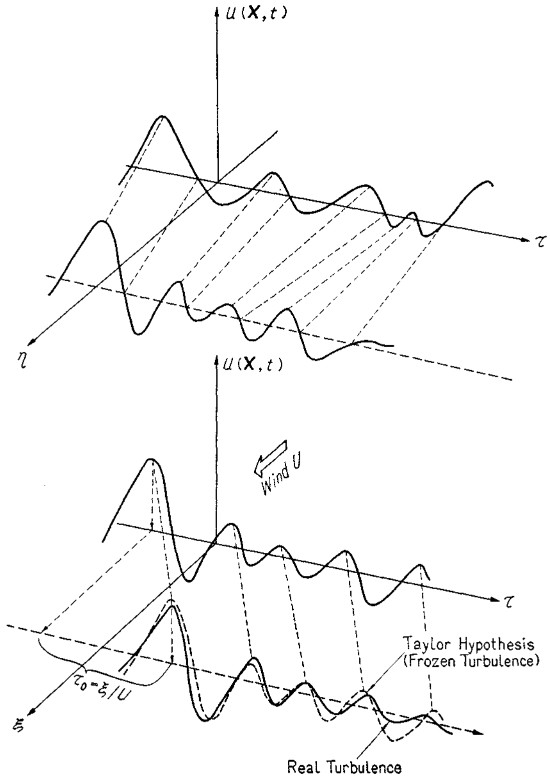

Fig. 2 Gradual deformation of turbulent fluctuation with separation distance η (in the lateral direction) and ξ (in the longitudinal direction).

where U_c represents the convection velocity (Fig. 2).

Although the Taylor hypothesis needs some modification for strong shear flow fields very near to a boundary, it may be applied with sufficient reliability to the case of structural design concerned. Substitution of Eq. (3) into Eq. (2) yields the frequency cross-spectrum of Eq. (4)

$$\Gamma(\xi, \eta; \omega) = \Gamma(0, \eta, \omega) \exp\left(\frac{i\omega\xi}{U_c}\right) \quad \cdots\cdots(4)$$

Experimentally, cross-correlation of fluctuating velocity components at two point separated by (ξ, η) is

$$\Gamma(\xi, \eta; \omega) = \Gamma(0, \eta, \omega) \cos\left(\frac{\omega\xi}{U_c}\right) \quad \cdots\cdots(4a)$$

This functional form, Eq. (4a), of the cross-spectrum is also given by Corcos[2] from experiments on wall pressure fluctuation.

The above equation represents that the cross-correlation for a certain frequency component ω varies with increase in the separation ξ sinusoidally without damping. This is a contradiction brought about by the assumption of frozen turbulent field. In reality, the cross-correlation will decrease slowly with increase in ξ and may

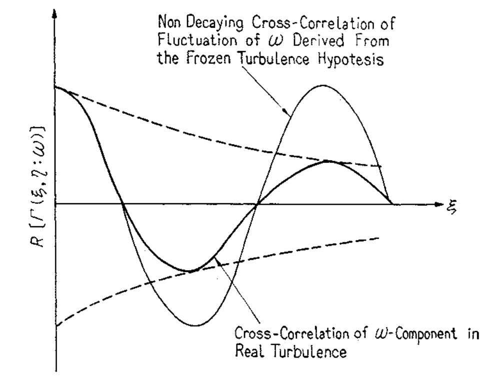

Fig. 3 Schematic representation of the real part of cross-spectrum (i.e. the cross-correlation of turbulent fluctuation with frequency ω) derived from the frozen turbulence hypothesis and the one of real turbulent flow.

be expressed multiplied by a damping function $A(\omega\xi/U_c)$ (Fig. 3),

$$\Gamma(\xi, \eta, \omega) = \Gamma(0, \eta, \omega) \exp\left(\frac{i\omega\xi}{U_c}\right) A\left(\frac{\omega|\xi|}{U_c}\right) \quad \cdots\cdots(5)$$

The inverse Fourier transform of the above equation leads to cross-correlation of velocity fluctuation at two points separated by (ξ, n)

$$R(\xi,\eta;\tau)=\int_{-\infty}^{\infty}\Gamma(\xi,\eta;\omega)\exp(i\omega\tau)d\omega$$
$$=\int_{-\infty}^{\infty}\Gamma(0,\eta;\omega)$$
$$\times\exp\left\{i\omega\left(\tau+\frac{\xi}{U_c}\right)\right\}A\left(\frac{\xi\omega}{U_c}\right)d\omega$$

In the above equations, the function Γ with $\eta=0$; i.e. $\Gamma(0,0,\omega)$ is the one-dimensional turbulent spectrum $P(\omega)$.

(2) Cross-spectrum, Coherence and Phase

Generally speaking, the cross-correlation is not symmetric with respect to $\tau=0$. As a result, the cross-spectrum of it defined by the complex Fourier transform is not a real valued function but has a non-zero imaginary part. That is,

$$\Gamma(\xi,\eta;\omega)=S(\xi,\eta;\omega)+iQ(\xi,\eta;\omega)$$

where the symbols S and Q denote the real and the imaginary parts of the cross-spectrum $\Gamma(\xi,\eta;\omega)$ and they are nominated as the co-spectrum and the quadrature spectrum, respectively.

Usually, cross-spectrum is conveniently expressed through the coherence function and the phase defined by

$$\mathrm{Coh}(\xi,\eta;\omega)=\frac{|\Gamma(\xi,\eta;\omega)|}{\Gamma(0,0,\omega)} \quad\cdots\cdots(6)$$

$$\Phi(\xi,\eta;\omega)=\tan^{-1}\left\{\frac{Q(\xi,\eta;\omega)}{S(\xi,\eta;\omega)}\right\} \quad\cdots\cdots(7)$$

where the spectrum at point (x_0, y_0) and that at point $(x_0+\xi, y_0+\eta)$ are assumed to be identical and given by $\Gamma(0,0;\omega)=P(\omega)$.

As a result, the cross-spectrum is rewritten as

$$\Gamma(\xi,\eta;\omega)=R[\Gamma(\xi,\eta;\omega)]+I[\Gamma(\xi,\eta;\omega)]$$
$$=\Gamma(0,0,\omega)\,\mathrm{Coh}(\xi,\eta;\omega)$$
$$\times\exp[i\Phi(\xi,\eta;\omega)] \quad\cdots\cdots(8)$$

As for the gust spectrum $P(\omega)=\Gamma(0,0;\omega)$, the author has recently proposed a formula as a function of height from the ground and the exponent of power law velocity distribution (Hino (1964, 1971)).

If we compare Eq. (8) with Eq. (5) and if it is reminded that coherence $\mathrm{Coh}(\xi,\eta;\omega)$ is real valued, the functional forms of coherence and phase are determined as follows (Fig. 4);

$$\mathrm{Coh}(\xi,0;\omega)=A\left(\frac{\omega|\xi|}{U_c}\right) \quad\cdots\cdots(9)$$

$$\Phi(\xi,0;\omega)=\frac{\omega\xi}{U_c} \quad\cdots\cdots(10)$$

Since $A(\omega|\xi|/U_c)$ should be unity for $\omega\xi/U_c=0$ and decrease uniformly with argument, it is most reasonable to assume that

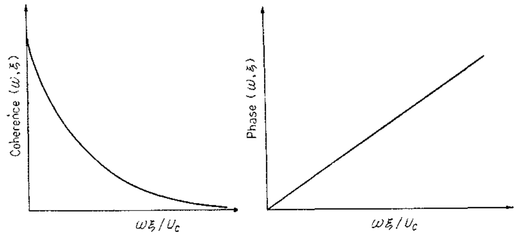

Fig. 4 Functional form of coherence and phase as functions of $\omega\xi/U_c$ (theoretically estimated).

$$A\left(\frac{\omega|\xi|}{U_c}\right)=\exp\left(-\frac{k_\xi\omega|\xi|}{2\pi U_c}\right) \quad\cdots\cdots(11)$$

More generally, coherence and phase may be given, refering to the above equations, by

$$\mathrm{Coh}(\xi,\eta;\omega)=\exp\left(-\frac{k_\xi|\xi|+k_\eta|\eta|}{2\pi U_c}\omega\right) \quad\cdots\cdots(12)$$

$$\Phi(\xi,\eta;\omega)=\left(\frac{m_\xi\xi+m_\eta\eta}{2\pi U_c}\right)\omega \quad\cdots\cdots(13)$$

Therefore, the cross-spectrum is expressed

$$\Gamma(\xi,\eta;\omega)=\Gamma(0,0:\omega)\,\mathrm{Coh}(\xi,\eta;\omega)$$
$$\times\exp[i\Phi(\xi,\eta;\omega)] \quad\cdots\cdots(14)$$

And, the cross-correlation is obtained by Fourier transform of the above equation,

$$R(\xi,\eta;\tau)=\int_{-\infty}^{\infty}\Gamma(0,0;\omega)\,\mathrm{Coh}(\xi,\eta,\omega)$$
$$\times\exp(i\Phi)\exp(i\omega\tau)d\omega$$

$$R(\xi,\eta;0)=\int_{-\infty}^{\infty}\Gamma(0,0;\omega)\,\mathrm{Coh}(\xi,\eta,\omega)$$
$$\times\exp(i\Phi)d\omega$$

Comparing Eqs. (10) and (13), m_ξ may be estimated to be $m_\xi=2\pi$. While, m_η will be zero for transversal direction in horizontal plane perpendicular to the average wind direction. However, it is not zero in the vertical direction.

The mean scale of eddies with frequency ω in either direction may be defined by the integration of the coherence function;

$$l_\xi(\omega)=\int_0^{\infty}\exp\left(-\frac{k_\xi\omega\xi}{2\pi U_c}\right)d\xi=\frac{2\pi U_c}{k_\xi\omega} \quad\cdots\cdots(16)$$

$$l_\eta(\omega)=\int_0^{\infty}\exp\left(-\frac{k_\eta\omega\eta}{2\pi U_c}\right)d\eta=\frac{2\pi U_c}{k_\eta\omega} \quad\cdots\cdots(17)$$

Then, the ratio of eddy scales in the mean wind direction and in the perpendicular (vertical or lateral) direction to it is given independently of the frequency as

$$l_\xi(\omega):l_\eta(\omega)=1:\frac{k_\xi}{k_\eta} \quad\cdots\cdots(18)$$

3. COMPARISON WITH EXPERIMENTS ON GUSTY WINDS

Based on data of field observation on gusty wind, Cramer[3] (1959) and Shiotani[10] (1969) determined the functional form of the coherence function and the phase as follows,

$$\mathrm{Coh}\,(0, \eta; \omega)=\exp\left(-\frac{k_\eta|\eta|}{2\pi U}\omega\right) \quad \cdots\cdots(19)$$

$$\Phi(0, \eta; \omega)=\frac{m_\eta \omega \eta}{2\pi U} \quad \cdots\cdots(20)$$

These expressions of Eqs. (19) and (20) are exactly identical with the estimated functions with $\xi=0$. The values of k_η and m_η ranged as

$k_\eta=6-8.5$

$m_\eta=5$

On the other hand, from the cross-correlation measurement of pressure fluctuations on smooth walls, the coefficient in the longitudinal direction k_ξ is obtained by Corcos[2]

$k_\xi=0.7$

As a consequence, the scale ratio of eddies of frequency ω is determined

$l_\xi(\omega):l_\eta(\omega)=1:(0.12-0.08)$

Inoue[8] (1952) has obtained the ratios of maximum scales L of the so-called "turbulon" from the assumption of equality of dissipation rates in every direction $\varepsilon=(\bar{u}_\xi^2)^{3/2}/L_\xi=(\bar{u}_\eta^2)^{3/2}/L_\eta=(\bar{u}_\zeta^2)^{3/2}/L_\zeta$ and the measured turbulent intensities, as follows

$$L_\xi:L_\eta:L_\zeta=(\bar{u}_\xi^2)^{3/2}:(\bar{u}_\eta^2)^{3/2}:(\bar{u}_\zeta^2)^{3/2}$$
$$=1:0.15:0.04$$

The maximum scale of "turbulon" by Inoue is approximately equal to the integral of $l(\omega)$ for the whole frequency range of ω. Although there is some difference in the difinition of l and L, the ratios of scales in the ξ and η direction compare relatively well.

4. CONCLUSION

Recent day construction activities of large scale structures urgently need the detailed knowledge on spatial as well as temporal two- or three-dimensional structure of gusty winds. Based on the theoretical analysis, functional forms of the cross-spectrum $\Gamma(\xi, \eta; \omega)$, the coherence $\mathrm{Coh}\,(\xi, \eta; \omega)$ and the phase $\Phi(\xi, \eta; \omega)$ of gusty winds are estimated. They are represented as

$$\Gamma(\xi, \eta; \omega)=\Gamma(0, 0, \omega)\,\mathrm{Coh}\,(\xi, \eta; \omega) \times \exp\,[i\Phi(\xi, \eta; \omega)]$$

where

$$\mathrm{Coh}\,(\xi, \eta, \omega)=\exp\left(-\frac{k_\xi|\xi|+k_\eta|\eta|}{2\pi U_c}\omega\right)$$

$$\Phi(\xi, \eta; \omega)=\left(\frac{m_\xi\xi+m_\eta\eta}{2\pi U_c}\right)\omega$$

It is shown that they agree well with experimental data.

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