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NON-ITERATIVE CAPACITY SPECTRUM METHOD BASED ON EQUIVALENT LINEARIZATION FOR ESTIMATING INELASTIC DEFORMATION DEMANDS OF BUILDINGS

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It is known that iterative procedures are always needed when the capacity spectrum methods based on the equivalent linear systems are employed to estimate the maximum deformation of existing structures. In addition to inefficiency, it has been shown that the existing method sometimes leads to the lack of convergence and accuracy. Besides, the problem of multiple solutions is encountered in many cases, and it is hard to decide which one is the best. To overcome these problems, this paper presents a non-iterative capacity spectrum method using a varied version of the equivalent linear methods for determining the maximum displacement demands of existing structures. A trustworthy single value will always be obtained by the procedure.

Key Words : *non-iterative capacity spectrum method, equivalent linear method, strength ratio, evaluation of existing structures*

1. INTRODUCTION

For evaluation and rehabilitation of the seismic capacity of the existing structures, one of the important steps is to estimate their maximum inelastic deformation demands under a given level of elastic design response spectrum or earthquake excitation. In order to avoid complex nonlinear time-history analysis, several simple evaluation methods have been proposed. Of these methods, the capacity spectrum method[1)] and the displacement coefficient method[2)] are the best-known. Both methods fall into the nonlinear static procedure. The capacity spectrum method included in the ATC-40 document[1)] is based on the equivalent linear method in which the maximum inelastic displacement of a structure is estimated by using an iterative method requiring analysis of a sequence of equivalent linear systems. Simultaneously, in the FEMA-273 document[2)], the displacement coefficient method is adopted in which the maximum inelastic deformation of a structure is estimated from the maximum linear elastic deformation of this structure by using a modifying factor C_1. In both methods, the maximum displacement demands in buildings are computed from the results of single-degree-of-freedom (SDOF) systems. Thus, estimation of the maximum displacement demands of the inelastic SDOF systems is a fundamental issue for the seismic design and evaluation of the multiple-degree-of-freedom (MDOF) structures.

The capacity spectrum method was originally developed as a rapid evaluation method for a pilot seismic risk project of the Puget Sound Naval Shipyard for the U.S. Navy in the early 1970s[3)]. During this period, it has been modified several times[4)]. In its implementation in the ATC-40, the maximum inelastic displacement of a structure is estimated by iteratively analyzing a sequence of equivalent linear systems from an assumed displacement until the computed displacement is equal or close enough to the assumed displacement. The equivalent linear systems implemented here have the following two characteristics: (a) a lateral stiffness equal to the secant stiffness at maximum displacement of structures, and (b) an equivalent viscous damping ratio greater than 5%. Both characteristics depend on the level of inelastic deformation of structures.

Recently, a modified capacity spectrum method also based on the equivalent linearization was proposed by Iwan and Guyader[5)]. This method uses the optimal equivalent stiffness (which locates between the initial stiffness and the secant stiffness of the nonlinear structures) instead of the secant stiffness for defining the equivalent linear systems. Although the capacity spectrum method is simple and straightforward, the following shortcomings exist. (a) Iterative procedures are always needed in order to obtain the maximum inelastic deformation demands of structures. (b) The peak deformation of inelastic systems is underestimated significantly for a wide range of periods when the Type A idealized hysteretic damping model is used[6]. (c) Procedure A in ATC-40 does not necessarily converge, and it does not converge to the correct values even if converged in many cases[6),7)]. (d) The situation of multiple solutions meets often[7)], i.e., different results may be obtained if different initial displacements are taken.

Even though the inelastic response spectrum can be used in capacity spectrum method to avoid iteration[6),8)–10)], this study focuses on the equivalent linear method still. The objective of this paper is to present a non-iterative and more accurate capacity spectrum method according to the equivalent linear systems for estimating the maximum inelastic deformations of the existing structures. The procedure not only can improve the efficiency of structural evaluation but also is no necessary to worry about the problems of unconvergence and multiple solutions because it is noniterative. A dependable single value can always be obtained. In order to verify the accuracy of the proposed procedure, mean ratios and standard errors of the approximate to exact maximum inelastic displacements are also carried out for a wide period range of elastic-perfect-plastic SDOF systems subjected to 72 earthquake ground motion records. In the end of the studies, estimates derived from the capacity spectrum method of ATC-40 based on the existing equivalent linear systems are compared with those derived from the non-iterative capacity spectrum method proposed by this paper to assess their accuracy.

2. EQUIVALENT LINEAR METHODS

The equivalent linear method is the fundamental of the capacity spectrum method. It allows the maximum inelastic displacement demand of a structure to be estimated from the maximum linear elastic displacement demand of an equivalent linear system described with equivalent period (T_{eq}) and viscous damping ratio (ξ_{eq})[5),11)–16)]. In this study, the following equivalent linear method is used[16)].

$$T_{eq} = T_0 \sqrt{\frac{R}{1 + \alpha(R-1)}} \tag{1}$$

$$\xi_{eq} = \xi_0 + 0.263\left(1 - \frac{1}{\sqrt{R}}\right) + 0.05(1-R)e^{-10T_0} \tag{2}$$

in which T_0 and ξ_0 denote the initial elastic period and inherent viscous damping ratio of the nonlinear structures, respectively. α represents the strain hardening ratio. R is the strength ratio which is referred as

$$R = \frac{mS_a}{f_y} \tag{3}$$

where m is the mass of systems; f_y is the lateral yield strength of systems. This strength can be obtained by conducting the static nonlinear analysis (pushover analysis) of the systems; S_a is the ordinate of acceleration spectral. The numerator (mS_a) in Eq.(3) represents the lateral strength required to maintain the nonlinear system elastic.

It is known that the equivalent linear systems of the existing capacity spectrum method are defined by the ductility ratio (μ). However, differing from the existing method, the equivalent linear systems presented by Lin and Miranda[16)] are adopted here because both the equivalent period (T_{eq}) and viscous damping ratio (ξ_{eq}) in their equations are defined not by the ductility ratio but by the strength ratio (R). For existing structures, the value of R is known while that of μ is unknown. Hence, iterations are no need if Eqs.(1) and (2) are used in the capacity spectrum method.

Eq.(1) was derived based on the secant stiffness at maximum nonlinear deformation of systems in terms of the strength ratio. Eq.(2) was obtained from statistical analyses and nonlinear regression analyses of SDOF systems with different natural periods and strength ratios when subjected to 72 earthquake ground motions recorded on firm site. For detained information of the two equations, refer to Lin and Miranda[16)].

It should be noted that Lin and Miranda[16)] shows how Eqs.(1) and (2) are derived and the basis of them, while this paper presents the application of the two equations in design and evaluation practice including the use of damping reduction factors which is adopted for constructing the high-damped response spectra.

3. CONSTRUCTION OF HIGH-DAMPED RESPONSE SPECTRUM

Since the equivalent viscous damping ratio (ξ_{eq}) of the equivalent linear systems is usually greater than 5%, it is needed to construct the high-damped elastic responses spectrum during the capacity spectrum process. For seismic design and evaluation of structures, the typical manner is to get them from the 5%-damped elastic response spectrum through the damping reduction factor (B)[17)–20)]. In this study, the damping reduction factor proposed by Newmark and Hall[17)] will be used for plotting the high-damped re-

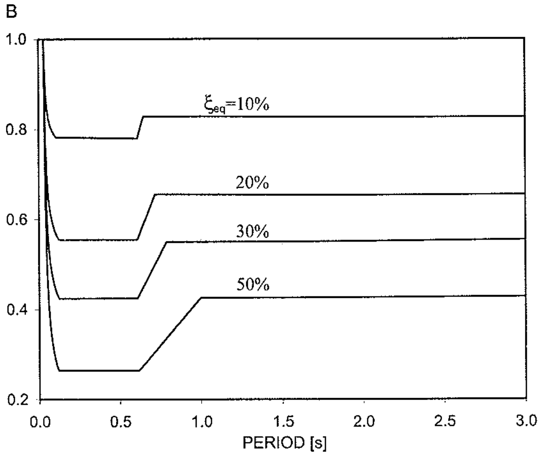

Fig.1 Damping reduction factors proposed by Newmark and Hall[17].

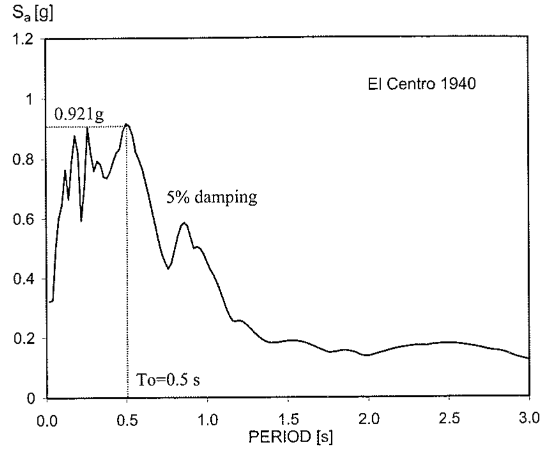

Fig.2 Design response spectrum for example.

sponse spectrum as.

$$B = \begin{cases} 1.514 - 0.321\ln(\xi_{eq}) & \text{for acceleration region} \\ 1.400 - 0.248\ln(\xi_{eq}) & \text{for velocity region} \\ 1.309 - 0.194\ln(\xi_{eq}) & \text{for displacement region} \end{cases} \quad (4)$$

Eq.(4) was derived from mean response spectrum. **Fig.1** shows a plot of this equation corresponding to different levels of viscous damping ratios.

4. NON-ITERATIVE EVALUATED PROCEDURE

The non-iterative capacity spectrum method based on the equivalent linearization proposed by Lin and Miranda[16] for estimating the maximum inelastic deformation demands (Δ_{ap}) of existing structures is described as a sequence of steps. Here, we will focus on the SDOF systems. However, the proposed procedure can easily be extended to MDOF systems according to the ATC-40 document.

1. Conduct the pushover analysis of the evaluated structure to obtain the diagram of force-deformation relationship between base shear and top displacement, commonly called as the pushover curve. Then, the yield force (f_y), yield displacement (Δ_y) and natural vibrating period ($T_0 = 2\pi\sqrt{m\Delta_y/f_y}$) of this structure can be evaluated from the curve. In addition, the ordinate of acceleration spectral (S_a) can be obtained from the 5%-damped elastic response (or design) spectrum according to the period (T_0).
2. Transfer the pushover curve and the 5%-damped elastic response spectrum to the capacity diagram and the 5%-damped elastic demand diagram. Both diagrams are in the A–D format (Acceleration–Displacement).
3. Compute the strength ratio (R) of this structure by Eq.(3) with known f_y and S_a.
4. Estimate the equivalent viscous damping ratio (ξ_{eq}) of the equivalent linear system for the non-linear structure based on Eq.(2).
5. Calculate the damping reduction factor (B) by Eq.(4) and then plot the elastic demand diagram for ξ_{eq} determined in Step 4.
6. Read-off the displacement Δ_{ap} where the demand diagram obtained from Step 5 intersects the capacity diagram. Then, Δ_{ap} is the maximum inelastic deformation demand of the structure.

5. EXAMPLE

The proposed non-iterative procedure is implemented to evaluate the earthquake-induced deformation demands of an elastic-perfect-plastic SDOF example system ($\alpha = 0$) with $T_0 = 0.5$ sec and $f_y = 0.1842w$, where w is the weight of the system. The design spectrum is shown in **Fig.2** which is the 5%-damped response spectrum of the 1940 El Centro earthquake with a peak ground acceleration of 0.34 g.

1. $f_y = 0.1842w$ and $T_0 = 0.5$ sec. According to the 5%-damped elastic response spectrum of the El Centro earthquake (**Fig.2**), the ordinate of acceleration spectral (S_a) corresponding to $T_0 = 0.5$ sec is 0.921 g.
2. **Fig.3** shows the capacity diagram and the 5%-damped elastic demand diagram for this system.
3. The strength ratio (R) of this structure is

$$R = \frac{mS_a}{f_y} = \frac{m \times 0.921g}{0.1842w} = 5 \quad (3)$$

4. The viscous damping ratio (ξ_{eq}) of the equivalent linear system is calculated based on Eq.(2) as

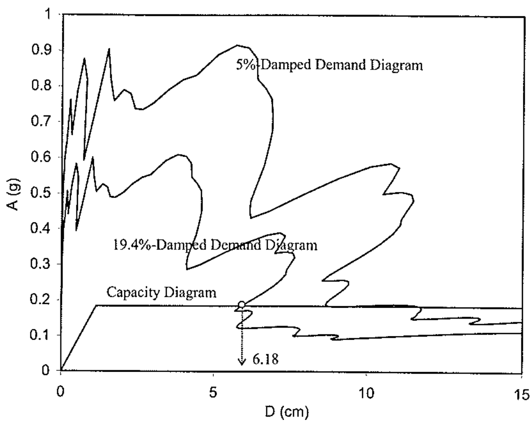

Fig.3 Non-iterative capacity spectrum method for example system subjected to the 1904 El Centro earthquake, where D = displacement and A = acceleration.

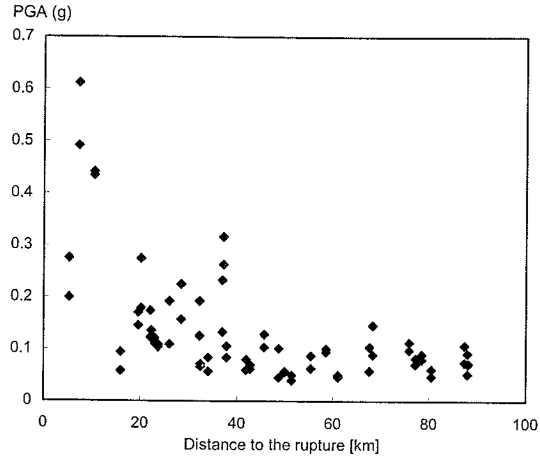

Fig.4 PGA vs. distance closest to fault rupture for considered earthquake ground motions of 72.

$$\xi_{eq} = 0.05 + 0.263\left(1 - \frac{1}{\sqrt{5}}\right) + 0.05(1-5)e^{-10\times0.5} = 19.4\% \quad (2)$$

5. From Eq.(4), the damping reduction factor corresponding to $\xi_{eq} = 19.4\%$ is as follows.

$$B = \begin{cases} 1.514 - 0.321\ln(19.4) = 0.562 & \text{acc. region} \\ 1.400 - 0.248\ln(19.4) = 0.665 & \text{vel. region} \\ 1.309 - 0.194\ln(19.4) = 0.734 & \text{disp. region} \end{cases} \quad (4)$$

The B multiplied by the 5%-damped elastic demand diagram is the 19.4% damped elastic demand diagram.

6. The 19.4%-damped elastic demand diagram intersects the capacity diagram at $D = 6.18$ cm (**Fig.3**). Hence, the maximum inelastic deformation demand (Δ_{ap}) of the structure under the 1940 El Centro earthquake is 6.18 cm.

From this example, it is clear that the maximum inelastic deformation demand of a structure can be determined easily and directly once the natural vibrating period (T_0) and yield force (f_y) of the structure are known.

6. ACCURACY OF THE PROPOSED PROCEDURE

In this section, a wide period range of elastic-perfect-plastic SDOF systems subjected to a number of earthquake records are carried out by the proposed procedure for estimating the (approximate) maximum inelastic displacements (Δ_{ap}). Then, these approximate values (Δ_{ap}) will be compared with their exact ones (Δ_{ex}) derived from the nonlinear time-history analysis to assess the accuracy of the presented procedure. The following term is defined for the purpose.

$$\gamma_{T_0,R} = \frac{1}{n}\sum_{i=1}^{n} \frac{\Delta_{ap}(T_{eq}, \xi_{eq})_i}{\Delta_{ex}(T_0, \xi_0, R)_i} \quad (5)$$

where $\gamma_{T_0,R}$ is the mean ratio of approximate $\Delta_{ap}(T_{eq}, \xi_{eq})$ to exact $\Delta_{ex}(T_0, \xi_0, R)$ maximum inelastic displacement for a given period of vibration (T_0) and strength ratio (R); n is the total number of earthquakes; i is the i-th earthquake. The proposed procedure will give the best estimation if $\gamma_{T_0,R}$ is equal to 1.0.

The SDOF systems considered here have the following characteristics. (a) period of vibration (T_0) between 0.1 and 3.0 sec with increments equal to 0.05 sec for $T_0 \leq 2.0$ sec and equal to 0.1 sec for $2.0 < T_0 \leq 3.0$ sec; (b) seven levels of the strength ratio (R = 1, 1.5, 2, 3, 4, 5 and 6); (c) an inherent damping ratio of 5% (ξ_0 = 5%). Furthermore, a total of 72 earthquake acceleration response histories recorded in the state of California of the U.S.A. from 9 different earthquakes are used in the following discussions. These ground motions are the same as those used by Lin and Miranda[16] and have the following characteristics: (a) recorded on firm sites; (b) recorded on free field stations or in the first floor of low-rise buildings with negligible soil-structure interaction effects; (c) recorded in earthquakes with surface wave magnitudes (M_s) between 5.8 and 7.5; and (d) records had the peak ground acceleration (PGA) greater than 45 cm/s^2. The relationships between the PGA and distance closest to fault rupture for these earthquake ground motions are shown in **Fig.4**. For detained information about these records, refer to Lin and Miranda[16].

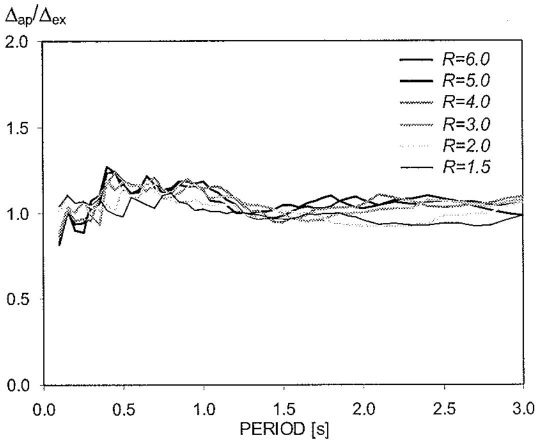

Fig.5 Mean approximate to exact maximum inelastic displacement ratio (Δ_{ap}/Δ_{ex}).

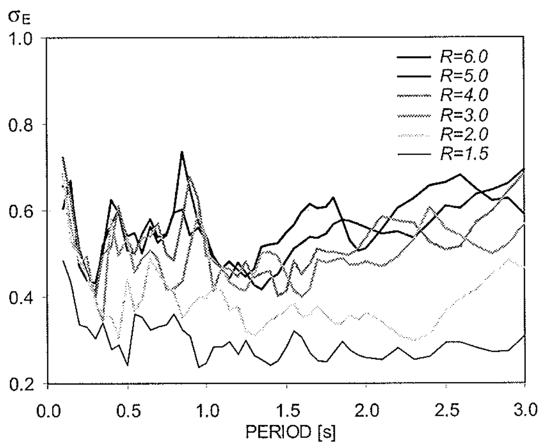

Fig.6 Standard error for Δ_{ap}/Δ_{ex}.

Fig.5 shows the mean approximate to exact maximum inelastic displacement ratios (Δ_{ap}/Δ_{ex}) derived from the 72 ground motions. For all levels of strength ratio (R), the proposed non-iterative procedure predicts the maximum inelastic displacement very well. **Fig.6** depicts the dispersion of Δ_{ap}/Δ_{ex} to the 72 earthquakes. It is defined by the standard error $\sigma_E(T_0, R)$ as shown in Eq.(6) for each T_0 and R. The standard error is practically the root-mean-square of the relative errors. This quantity can quantify the spread of the approximate maximum inelastic displacements around the exact counterparts. As the quality of the approximate inelastic displacement increases, the standard error approaches zero. Note that the quantity is different from the standard deviation, which quantifies the spread of the data around the mean (not around the exact value).

$$\sigma_E(T_0, R) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{\Delta_{ap}(T_{eq}, \xi_{eq})_i}{\Delta_{ex}(T_0, \xi_0, R)_i} - 1\right)^2} \quad (6)$$

The standard errors of **Fig.6** rise with increasing the level of strength ratios, but the increments decrease with increasing the level of strength ratios. They nearly stop rising when $R > 3.0$.

7. COMPARISON WITH EXISTING METHODS

In this section, the approximate maximum inelastic deformations (Δ_{ap}) derived from the proposed procedure is compared with the existing capacity spectrum method adopted in the ATC-40 document. Since it has been proved that the maximum deformation of inelastic systems is underestimated significantly for a wide range of periods when the Type A idealized hysteretic damping model is used[6], the following two damping models are implemented here for computing the equivalent viscous damping ratios (ξ_{eq}). The two models are used because they provide more accurate results than others[21].

a. Hysteretic damping model presented by Gulkan and Sozen[12]. According to the Takeda hysteretic model[22] and experimental shake table results of small-scale reinforced concrete frames, Gulkan and Sozen[12] developed the following empirical equation to compute the equivalent damping ratio.

$$\xi_{eq} = \xi_0 + 0.2\left(1 - \frac{1}{\sqrt{\mu}}\right) \quad (7)$$

b. Hysteretic damping model proposed by Kowalsky[15]. This method was also derived from Takeda hysteretic model[22] by including the effect of stiffness degrading and the energy dissipated in a vibration cycle of the inelastic system and of the equivalent linear system. For an unloading stiffness factor of 0.5, the equivalent viscous damping ratio is given by

$$\xi_{eq} = \xi_0 + \frac{1}{\pi}\left(1 - \frac{1-\alpha}{\sqrt{\mu}} - \alpha\sqrt{\mu}\right) \quad (8)$$

The following steps illustrate briefly the capacity spectrum method adopted in the ATC-40 for SDOF systems. As mentioned early, this is an iterative procedure.

1. Plot the pushover curve and the 5%-damped elastic response (or design) spectrum, both in the A–D format (Acceleration–Displacement) to obtain the capacity diagram and 5%-damped elastic demand diagram, respectively.
2. Assume the peak displacement demand D_i and determine the corresponding acceleration A_i for the capacity diagram.
3. Computed the ductility ratio $\mu = D_i/u_y$.

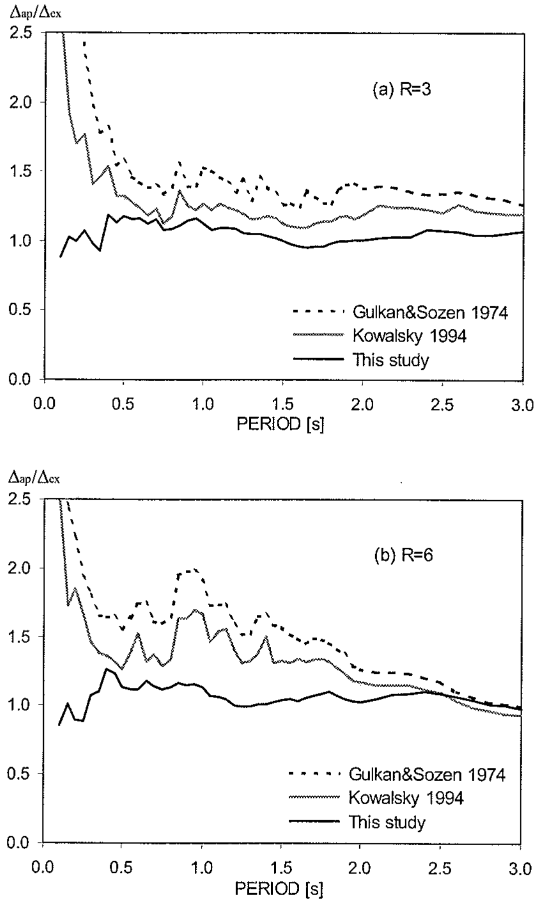

Fig.7 Mean approximate to exact displacement ratio (Δ_{ap}/Δ_{ex}) computed with various methods. (a) $R = 3$. (b) $R = 6$.

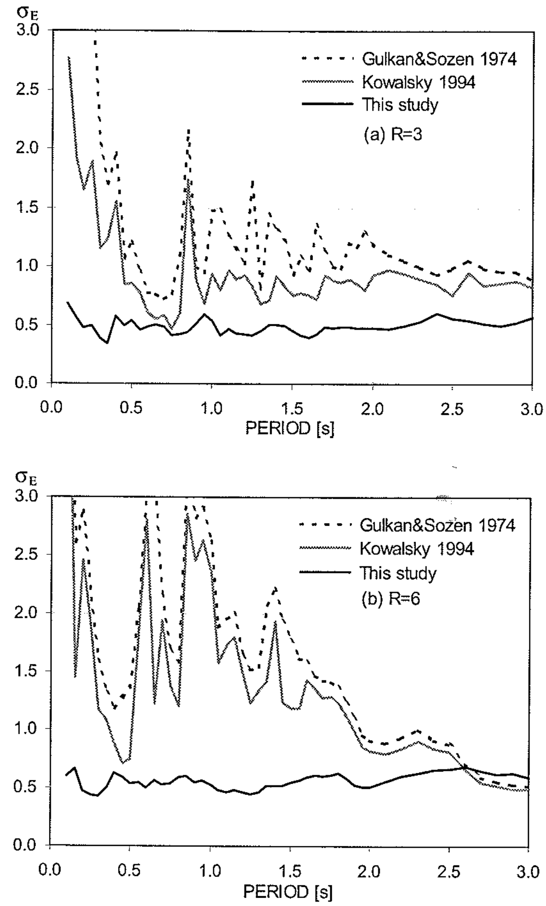

Fig.8 Standard error for Δ_{ap}/Δ_{ex} computed with various methods. (a) $R = 3$. (b) $R = 6$.

4. Calculate the equivalent damping ratio (ξ_{eq}) by Eq.(7) or (8).
5. Plot the elastic demand diagram for ξ_{eq} determined in Step 4.
6. Read-off the displacement D_j where the demand diagram obtained from Step 5 intersects the capacity diagram.
7. Check for convergence. If $(D_j - D_i) \div D_j \leq$ tolerance, the maximum inelastic displacement demand $\Delta_{ap} = D_j$. Otherwise, set $D_i = D_j$ (or another estimated value) and repeat Steps 3–7 until convergence occurred.

Fig.7 shows a comparison of the mean approximate $\Delta_{ap}(T_{eq}, \xi_{eq})$ to exact $\Delta_{ex}(T_0, \xi_0, R)$ maximum inelastic displacement (i.e., Eq.(5)) derived from various methods for $R = 3$ and 6 to the 72 earthquakes. It can be seen that the proposed procedure always leads to the minimum displacement errors no matter what strength ratio R and natural period T_0 of systems are. In addition, the method is especially useful than the others for systems having short natural period of vibration. During this period range, the approximate maximum inelastic displacements are obviously overestimated for the ATC-40 capacity spectrum method using either the Gulkan and Sozen or the Kowalsky hysteretic damping model. Among these curves, the hysteretic damping model suggested by Gulkan and Sozen generally gives the maximum errors.

Fig.8 presents the standard errors (Eq.(6)) derived from various methods for $R = 3$ and 6 to the 72 earthquakes. The conclusions are similar to those obtained from the mean ratios of approximate to exact maximum inelastic displacement. The proposed procedure gives minimum dispersion not only for systems with mid and long periods but also for those with short periods. On the contrary, the standard errors computed from the other two methods are relatively high particularly for systems with short periods.

8. CONCLUSIONS

A non-iterative procedure for capacity spectrum method based on equivalent linear systems has been proposed in this paper to estimate the maximum inelastic deformation demands of existing structures. In addition to significantly simplify the procedure of structural evaluation, it also provides more accurate results than the current capacity spectrum method. The proposed procedure does not have the problem of unconvergence and multiple values since it is non-iterative. The key point of non-iteration is to represent the equivalent period and viscous damping ratio of the equivalent linear systems with a function of the strength ratio rather than the ductility ratio because the former parameter (strength ratio) is always known in the existing structures.

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