LIQUEFACTION PROCESS OF INHOMOGENEOUS SAND BED UNDER A SERIES OF OSCILLATING WATER PRESSURE

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This paper deals with the liquefaction process of the inhomogeneous sand bed under the oscillating water pressure. The inhomogeneous sand bed treated in this study has a smaller porosity in upper zone and has a larger porosity in the lower zone. In our experiment, it is formed after the loosely deposited sand bed has been once liquefied. The study is carried out experimentally and numerically, by means of the vertical one-dimensional model of sand bed. The applicability of the analytical model proposed in our past study to the inhomogeneous sand bed is verified.

Key Words: inhomogeneous sand bed, pore water pressure, effective stress, liquefaction, densification.

1. INTRODUCTION

Under an oscillating water pressure, the loosely deposited uniform sand bed (the porosity in the vertical direction is constant) will be liquefied to a certain depth and then the recovered sand layer resulted from the sedimentation will be densified. As a result, an inhomogeneous sand bed is formed. The porosity in the upper zone near the sand bed surface becomes smaller, while the porosity in the lower zone still remains the same value as the initial loose state.

The inhomogeneous sand bed may be liquefied once again to the deeper zone under the action of the oscillating water pressure with the amplitude larger than that having experienced before. This phenomenon is the object of our study in this study.

To study the liquefaction process of inhomogeneous sand bed is of a great deal of practical meaning to analyze the stability problem of both the natural sand bed and the reclaimed land. And then it is significant work to build the prediction method of the surface settlements of these sand beds. The liquefaction of inhomogeneous sand bed holds the characteristics similar to the liquefaction of homogeneous sand bed in the variation process of pore water pressure. But it is more complicated because of the inhomogeneity.

In the literature¹, the liquefaction process of a loosely deposited sand bed is classified into two stages of liquefaction.

The first stage of liquefaction is the sustained liquefaction in which the excess pore pressure includes both the oscillatory pore pressure and the residual pore pressure. This stage of liquefaction with the build-up and fall-down of residual pore pressure appears in the initial period of cyclic loading.

The second stage of liquefaction is the cyclic transient liquefaction with the damping of amplitude and the lag of phase in the pore water pressure, which follows the first stage of liquefaction. In this stage, the sand bed within some depth has become denser, and the excess pore pressure contains only oscillatory pore pressure.

Since 1978, many researches on the dynamic behavior of the sand bed considering soil-water-structure interaction have been developed in the
fields of coastal and geotechnical engineering \(^1\)–\(^{11}\). The mechanism of the second stage of liquefaction has been well known.

As to the first stage of liquefaction, the progressive liquefaction process of the loosely deposited sand bed under initial two cycles of oscillating water pressure was dealt in literature \(^{12}\) theoretically and experimentally, by using the vertically one-dimensional model. In that paper, the elasto-plastic model was proposed for the theoretical treatment and proved to be fairly good by the experiments. Since the analysis was focused on the very short initial time period of excitation, the sedimentation of sand in the liquefied zone is neglected.

In the literature \(^{13}\), the total process of both liquefaction and densification of the loosely deposited sand bed in the first stage of liquefaction, which has been once liquefied under a series of oscillating water pressure, was treated theoretically and experimentally by using the same model as that in literature \(^{12}\). The sedimentation of sand and the porosity variation of sedimentary sand layer were taken into account, and the concepts of both the recovery velocity and the liquefaction ratio were applied in that analysis.

In this paper, the experiments on the liquefaction process of both the loosely deposited sand bed and the inhomogeneous sand bed are introduced. Then, the liquefaction process of the inhomogeneous sand bed with different porosity in the vertical direction is analyzed by the vertically one-dimensional model mentioned in the literature \(^{13}\). The applicability of the analysis model proposed in the literature \(^{15}\) to the inhomogeneous sand bed with different porosity is investigated.

2. EXPERIMENTAL INVESTIGATION

(1) Apparatus and procedures of experiment

The vertical cylinder made of the transparent resin shown in Fig.1 was used as the vessel of sand column. It is 8.9cm in inner diameter and is 326cm high. The column was filled with loosely deposited sand to 316cm from its bottom, and the water depth above the sand surface was about 100cm.

In the experiment, the grain size of sand (\(d_{50}\)) was 0.25mm (Toyoura sand), and its specific weight is 2.65.

The oscillating water pressure on the sand surface was generated with a generator of oscillating air pressure. The varying water pressure in the water and the pore water pressure in the sand bed were measured by the pressure transducers attached to the side of the cylinder. The measured points Pt.1 was in water and Pt.2–Pt.8 were 36cm, 46cm, 76cm, 126cm, 156cm, 196cm and 296cm from the initial surface of sand bed, respectively.

The experimental procedure is composed of the following two steps:

1. Formation of the inhomogeneous sand bed

Here, the inhomogeneous sand bed is formed by means of liquefaction of the homogeneous sand bed.

The dried sand was dropped freely into the water of the vessel to set up the homogeneous sand bed for the whole depth of sand layer. It took about 30 minutes after the finish of dropping to obtain the stationary state. The permeability coefficient of sand layer for the initial state was about 0.04 cm/sec. The porosity \(n_w\) at that condition was about 0.48.

The periodically oscillating air pressure was acted on the water surface for 10 minutes. The amplitude \(a_0\) was about 60cm in water head and the frequency \(f\) was 1.0 Hz. The sand bed liquefied and densified in some extent. The sand bed surface at the end of the above excitation subsided 6.6cm, and the thickness of sand bed became about 309cm. At this time, the positions of the measured points Pt.2–Pt.8 were 29cm, 39cm, 69cm, 119cm, 149cm, 189cm and 289cm from the surface of sand bed, respectively. The sand layer in the upper zone has
been densified and the sand layer in the lower zone under some depth was still in the loose state.

2 Liquefaction of the inhomogeneous sand bed
For the inhomogeneous sand bed resulted from the step①, the oscillating air pressure that the amplitude(\(a_0\)) was 113cm in water head and the frequency(\(f\)) was 1.0Hz was acted for 100 minutes continuously. Then, the excitation was stopped.

The water pressures were measured continuously for 10 minutes long from the beginning of the excitation with an interval of 0.02 second to each step. During this period, the subsidence of the sand bed surface was also observed.

The sand bed was divided into four parts of cylinder after the finish of excitation. The volume of each cylinder and the weight of the dried sand inside were measured. The mean porosity of sand layer in each cylinder was calculated by using the volume of each cylinder, the weight of the dried sand and the specific weight of sand. The obtained values were adopted as the porosity at the center of each cylinder.

Because the above experimental process was the continuous process, the porosity distribution of sand bed at the end of the step① could not be measured. Therefore, to get the porosity distribution of sand bed at the initial time of the step② needed in the next numerical analysis, the experiment for the same case was carried out once more until the finish of
step ① and then the experiment was stopped, and the porosity of sand bed were found by the method mentioned in the step ②.

(2) Experimental results

a) Variation of pore water pressure

The variations of pore water pressure measured in the sand bed for both the experiment step ① and the experiment step ② are shown in Fig.2. Moreover, the details of these pore pressure variations for the initial 20 seconds of each step are shown in Fig.3.

Fig.3  Variation of pore water pressure during the initial 20s (measured).

In Fig.2 and Fig.3, $\bar{h}$ represents the variation of pore water pressure in water head. It does not include the hydrostatic pressure.

It is understood from Fig.3 that the time until that the variations of pore water pressures at each measured points became nearly constant in the step ② is longer than one in the step ①. That is, in the step ②, the time until that the sand layer is liquefied at each measured point becomes longer. It means that the sand bed being densified in the upper zone needs longer time to become the liquefied state.
b) Variation of mixed-fluid depth

Under the action of oscillating water pressure on the surface of sand bed, the pore water pressure increases and effective stress decrease in sand bed. Under certain conditions, the effective stress within some depth in the sand bed becomes zero. This phenomenon is called as liquefaction of sand bed. The zone where effective stress is equal to zero is called as liquefaction zone. In the case of the loose sand bed (its porosity is about 0.48), the sand layer in the liquefaction zone is completely converted into a mixture of sand, water and small amount of pore air. It is similar to fluid in characteristics. Here, it is called as mixed-fluid. With respect to the dense sand bed (its porosity is about 0.40), the sand layer in the liquefaction zone is not converted into mixed-fluid.

According to the vertical distribution of the variation of pore water pressure, the depth of the mixed-fluid was estimated and was shown in Fig.4. The subsidence of sand bed surface is also shown in the same figure. The curves \(t=0-600s\) show the experimental data with respect to step ① and the curves \(t=600-1200s\) show that with respect to step ②. The maximum of mixed-depth was about 130cm in the experiment step ① on the uniform sand bed and was about 180cm in the experiment step ② on the inhomogeneous sand bed. It is considered that the difference of the maximum depth of mixed-fluid zone is mainly caused by the difference of amplitude of oscillating water pressure. It is clear that the subsidence of sand bed surface becomes greater with the increase of mixed-fluid depth.

\[\text{Porosity } \phi \]

\[
\begin{array}{ccccccc}
0.37 & 0.39 & 0.41 & 0.43 & 0.45 & 0.47 & 0.49 \\
0 & 50 & 100 & 150 & 200 & 250 & 300 \\
\end{array}
\]

Fig.5 Porosity distribution.

The divided cylinder, so that the shapes of porosity curves are the histogram. It is known from Fig.5 that under the action of oscillating water pressure with the greater amplitude, the porosity of sand bed decreases wholly from top to bottom.

3. NUMERICAL ANALYSIS

(1) Governing equations

The dynamic behavior of the sand bed formed at the end of the step ① under oscillating water pressure is investigated by using the vertically, one-dimensional sand bed model.

Under the assumptions that sand bed is the multi-porous, elastic material, and both pore air and pore water are compressible, the system of governing equations for dynamic analysis has been
derived in the literature. It is rewritten in the following:
\[ \frac{1}{\alpha} \frac{\partial^2 \tilde{u}}{\partial y^2} = \rho_v g \frac{\partial \tilde{h}}{\partial y} \]
\[ \rho_w g \left( \beta n_w + \frac{n_x}{P} \right) \frac{\partial \tilde{h}}{\partial t} + \frac{\partial}{\partial t} \left( \frac{\partial \tilde{u}}{\partial y} \right) = k \frac{\partial^2 \tilde{h}}{\partial y^2} \]

where,
- \( \tilde{u} \) : displacement of sand layer (not including the initial displacement)
- \( \tilde{h} \) : variation of pore water pressure in water head (\( \tilde{h} = h - y - h_0 \))
- \( h \) : total pore water pressure in water head
- \( h_0 \) : still water depth on the sand bed surface
- \( \alpha \) : elastic compressibility of the skeleton of sand bed
- \( \beta \) : compressibility of water
- \( n_w \) : porosity occupied by water
- \( n_x \) : porosity occupied by air
- \( P \) : absolute pressure (\( P = P_0 + \rho_w g h \))
- \( P_0 \) : atmospheric pressure
- \( \rho_w \) : density of water
- \( g \) : acceleration due to gravity
- \( k \) : permeability coefficient of sand bed
- \( y \) : depth from the initial sand bed surface
- \( t \) : time

The system of equations (1) is taken as the governing equation in the following analysis. \( \tilde{u} \) and \( \tilde{h} \) in Eq.(1) are the unknown variables.

The following equation Eq.(2) has been derived from the equilibrium equation between the effective stress and the pore pressure in the sand bed.
\[ \sigma_y + \rho_v g h' = (\rho_s - \rho_w) g (y - y_{\text{L}}) (1 - n) \]  
(2)

where,
- \( \rho_s \) : density of water
- \( n \) : porosity (\( n = n_x + n_w \))
- \( y_{\text{L}} \) : the depth of mixed fluid (when the sand bed is loose (\( n = 0.48 \)), it is equal to the liquefaction depth \( y_0 \) at which effective stress is zero.)
- \( \sigma_y \) : effective stress (\( \sigma_y = \tilde{\sigma}_y + \sigma_{y0} \))
- \( \sigma_{y0} \) : part of effective stress caused by the weight of sand particles
- \( \tilde{\sigma}_y \) : effective stress increment induced by excess pore pressure \( h' \).
- \( h' \) : excess pore pressure represented by
\[ h' = \frac{1 - n}{\rho_s - \rho_w} \left( \frac{\rho_s - \rho_w}{\rho_w} y_{\text{L}} - h_0 \right) \]  
(3)

in which,
- \( h_0 \) : the oscillating water pressure on the sand bed surface in water head.

The boundary conditions of sand bed at the upper surface of the un-liquefied sand layer (\( h' = 0 \)) and at the bottom of sand bed (impermeable, fixed) are as follows:
\[ \tilde{h} = \frac{(1 - n)(\rho_s - \rho_w)}{\rho_w} y_{\text{L}} + h_0 \quad \text{at} \quad y = y_{\text{L}} \]  
(4)
\[ \frac{\partial \tilde{h}}{\partial y} = 0, \quad \tilde{u} = 0 \quad \text{at} \quad y = D \]

At the beginning of excitation, the excess pore pressure and the liquefaction don't occur in the sand bed. Then, the initial condition is given in the following:
\[ y_{\text{L}} = 0, \quad \tilde{h} = 0, \quad \text{at} \quad t = 0 \]  
(5)

When the oscillating water pressure is exerted on the sand surface, the system of basic equations (1) applied to the un-liquefied sand layer are solved under boundary conditions (4) and initial condition (5) to find \( \tilde{h} \) and \( \tilde{u} \).

Finite element method is applied with respect to spatial differentiation, and finite difference method is applied with respect to time differentiation.

Excess pore pressures at nodes are calculated by Eq.(3) with the pore water pressure \( \tilde{h} \), and then the effective stress can be calculated by Eq.(2) with excess pore pressure \( h' \).

Under the variation of oscillating water pressure \( \Delta h_0 \), If the stress \( \sigma_y \) at some position deeper than the mixed fluid depth \( y_{\text{L}} \) becomes zero, the corresponding liquefaction depth \( y_0 \) can be also obtained from Eq.(2).

(2) The outline of the numerical analysis

The elasto-plastic analysis method given in literature was applied in the following analysis. That is, the elastic compressibility (\( \alpha \)) of the skeleton of sand bed in Eq.(1) is replaced by the plastic compressibility (\( \alpha_p \)), while the effective stress of sand layer exceeds the yield effective stress (\( \sigma_y \)). The yield effective stress (\( \sigma_y^* \)) is the normalized variable that the yield effective stress (\( \sigma_y \)) is divided by the submerged-weight of the sand layer above. This normalized yield effective stress was regard as the function of porosity. For we have not enough data, it was assumed as the next linear relation.
\[ \sigma_y^* = N - M \times n_w \]  
(6)

The values of parameters \( N, M \) were determined by test computation comparing analytical pore water pressure with the measured one.

The concept of liquefaction ratio introduced in the literature is an index to describe the relation between the mixed-fluid depth (\( y_{\text{L}} \)) and the
liquefaction depth ($\gamma_0$).

The recovery of sand layer due to sand sedimentation in the mixed-fluid zone is considered in this study and the permeability coefficient of sand layer is determined by our empirical formula with the porosity and the diameter of sand grain.

The points in the analysis are explained in the following.

**a) Liquefaction ratio**

In the literature\textsuperscript{13}, the next formula of $R_L$ was adopted for the case that the amplitude of oscillating water pressure is 80cm.

$$
\begin{align*}
R_L &= 0, \quad \text{for} \quad n_w \leq n_{wQ2} \\
R_L &= E \times n_w - F, \quad \text{for} \quad n_{wQ1} \leq n_w \geq n_{wQ2} \\
R_L &= 1.0, \quad \text{for} \quad n_w \geq n_{wQ1}
\end{align*}
$$

where,

$n_{wQ1} = 0.48$, $n_{wQ2} = 0.45$, $n_{wQ1}$ is the porosity at point Q1 and $n_{wQ2}$ is the porosity at point Q2 in Fig. 6.

For the case analyzed in the following, in which the amplitude of oscillating water pressure is about 120cm, the relationship of liquefaction ratio $R_L$ with porosity is also assumed as follows (shown in Fig. 6).

$$
\begin{align*}
R_L &= 0, \quad \text{for} \quad n_w \leq n_{wQ3} \\
R_L &= E \times n_w - F, \quad \text{for} \quad n_{wQ1} \leq n_w \geq n_{wQ3} \\
R_L &= 1.0, \quad \text{for} \quad n_w \geq n_{wQ1}
\end{align*}
$$

where,

$E,F$: the coefficients relating to the sand bed conditions and the amplitude of oscillating water pressure. They should be determined according to the data of laboratory test.

$n_{wQ3} = 0.44$, it is the porosity at point Q3, the value of which was optimized by comparing the result of numerical analysis with one of laboratory test.

As for the inhomogeneous sand bed, the average porosity of sand layer in the zone between the lower boundary of mixed-fluid and the depth where the effective stress equals zero, was adopted as the porosity in Eq.(8) in the following analysis.

**b) Recovery velocity**

A recovery velocity is the upward velocity of interface between the mixed fluid and the sedimentary sand layer due to the sedimentation of sand in the liquefied zone.

In our experiment, the pore water pressures in water and at different depths of sand bed were continuously measured with pressure transducers, and were automatically saved into the large capacity of disk in the digital form. Then, the recovery time of sand layers at the different measure points can be understood from the graph of the recorded pore water pressure. The recovery velocity of sand layer was decided from the recovery time obtained above and the positions of the measurement points. It is considered that the recovery velocity is mainly related to $a_0$, $f$, $g$, $\gamma_s$ after the dimensional analysis on the above data. The empirical formula of recovery velocity $V_r$ was obtained as follows:

$$
V_r = C_{nw} \left[ 0.0015 \times \left( \frac{g}{a_1 f^2} \right) + 0.0081 \right] \times f \sqrt{a_1 \gamma_s}
$$

where,

$a_1$: amplitude of oscillating water pressure in water head

$\gamma_s$: the mixed fluid depth

$C_{nw}$: porosity factor, including the influence of porosity of mixed fluid. (here, its value was about 0.75)

**c) Densification of sand layer**

The porosity of sand layer in the densification zone was determined by the following formula in the following analysis (as shown in Fig. 7).

$$
n_w = A - B \times \ln(m)
$$

where,

$A,B$: the coefficients depended on the initial porosity of sand bed and the amplitude of the oscillating water pressure.
$m$ : cycle number of oscillating water pressure which acted on the sand layer of interest from the initial state of the loose sand bed.

In the following analysis, the initial porosity $n_{w0}$ was 0.486 and the amplitude of oscillating water pressure was 113 cm. Because we have not sufficient data, the coefficients $A$, $B$ were assumed by referring to the only porosity data measured at the end of step② in the above mentioned model test.

This $m$ is counted from the time when the sand layer of interest is liquefied first time. Because the method of count is different, equation (10) is different in form from Eq. (17) in literature. It

If the porosity of some sand layer is $n_{w1}$, the equivalent cycle number $m_1$ to $n_{w1}$ is found by the next equation.

$$m_1 = \exp \left( \frac{A - n_{w1}}{B} \right)$$

After the sand layer is acted by the cycle number $\Delta m$ of oscillating water pressure, the porosity of sand layer is determined with the following equation.

$$n_{w2} = A - B \times \ln(n_1 + \Delta m)$$

It is known from our experimental data that the densification zone of sand bed is within a depth, being about amplitude of oscillating water pressure, from the surface of sand bed on which oscillating water pressure is exerted. The densification zone in the following analysis was supposedly set as a zone from the surface or interface of sand layer to the depth being equal to a half of amplitude below it.

d) Permeability coefficient of sand layer

The permeability coefficient of sand bed varies with the variation of porosity in sand bed. On the Toyoura sand ($d_{50} = 0.25$ mm) used in our test, the permeability coefficient of sand bed was determined by the following formula obtained from our permeability coefficient test.

$$k = (0.064 \times n_w - 0.023) \sqrt{gd}$$

$$n_w = 0.42 \sim 0.48$$

(13)

where,

$k$ : permeability coefficient

$n_w$ : porosity in sand bed

$g$ : acceleration due to gravity (cm/s²)

$d$ : grain size of sand (here, $d = d_{50}$, cm)

(3) Results of numerical analysis

The above mentioned experiment step② is the object of our analysis with numerical method in the following. The initial porosity distribution in sand bed is shown in Fig. 8. For the numerical analysis, the measured data was transformed into the smooth curve.

![Fig. 8 Initial porosity distribution for analysis.](image)

Table 1 Parameters adopted in the numerical analysis.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{w0}$ (t=0)</td>
<td>0.486</td>
<td>--</td>
</tr>
<tr>
<td>$n_{w}$ (t=0, $h_1$=30 cm)</td>
<td>0.0012</td>
<td>--</td>
</tr>
<tr>
<td>$n_0$ (t=0)</td>
<td>0.486</td>
<td>--</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$6 \times 10^{-9}$</td>
<td>m²/N</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>$100 \times \alpha$</td>
<td>m²/N</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$4.3 \times 10^{-10}$</td>
<td>m²/N</td>
</tr>
<tr>
<td>$D$</td>
<td>309</td>
<td>cm</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>2.65</td>
<td>t/m³</td>
</tr>
<tr>
<td>$h_0$</td>
<td>100</td>
<td>cm</td>
</tr>
<tr>
<td>$\Delta t$</td>
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<td>s</td>
</tr>
<tr>
<td>$\Delta y$</td>
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<td>cm</td>
</tr>
<tr>
<td>$a_0$</td>
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<td>cm</td>
</tr>
<tr>
<td>$a_1$</td>
<td>113</td>
<td>cm</td>
</tr>
<tr>
<td>$f$</td>
<td>1.0</td>
<td>Hz</td>
</tr>
<tr>
<td>$A$</td>
<td>0.486</td>
<td>--</td>
</tr>
<tr>
<td>$B$</td>
<td>0.013</td>
<td>--</td>
</tr>
<tr>
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</tr>
<tr>
<td>$N$</td>
<td>6.2</td>
<td>--</td>
</tr>
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</table>

The parameters adopted in the numerical analysis are given in Table 1. $\Delta y$ in the table denotes the height of element in the analysis.

In the analysis, the yield effective stress, liquefaction ratio and porosity were decided by Eq. (6), Eq. (8) and Eq. (10) respectively. The recovery velocity of sedimentation sand layer and the permeability coefficient in the process of liquefaction were calculated by the formula given in Eq. (9) and Eq. (13) respectively. The permeability coefficient is the function of the porosity ($n_w$) of sand layer concerned.

In the analysis, the relation $\alpha_1 = 100 \times \alpha$ was adopted according to literature. For comparison, the case in which $\alpha_1$ is equal to $10 \times \alpha$ was also
Fig. 9  the measured pore water pressure and the analyzed pore water pressure.

Simulated. It is stood from the results that the difference of the maximum depth of mixed-fluid is about 5 cm in the above cases. A significant change was not seen.

The pore water pressures obtained by numerical analysis and by the experiment are shown in Fig. 9.

In Fig. 9, the variation of pore water pressure with both the progressive process of liquefaction and the recovery process of sand layer is displayed at different depths (28.8cm, 119.0cm, 148.8cm, 189.0cm, 288.9cm).

It is understood from Fig. 9 that the pore pressure obtained by numerical analysis can express the trend of pore pressure variation measured in the experiment comparatively well. However, the amount of damping of pore water pressure obtained from the numerical analysis is smaller than that from the experiment, especially in the latter part of the process.

It was considered that the discrepancy mentioned above was caused by the assumed values of compressibility coefficient and the yielding effective stress. But, According to the sensitivity analysis of parameters in the following section (4), it is clear that the changes of these parameters have very little influence on the damping of pore water pressure. At present, the treatment of porosity $n_s$ which was assumed as uniform vertically, was considered as one of important reasons. That is, considering that a part of pore air in the sand bed fled out in the liquefied state at step 7, the porosity $n_s$ in sand bed became smaller. Therefore, the smaller porosity $n_s$ in the whole vertical direction of sand bed was taken in this case. However, in reality, it is possible that porosity $n_s$ became smaller in the shallow sand layer and still keeps original value in the deep sand layer.
If so, the damping of pore water pressure in deeper sand layer became smaller than one measured in the experiment. The consideration needs to be verified in the future work.

For the comparison between the analyzed value and the measured value of pore water pressure during the initial stage, the enlarged figures of these pore water pressures during the initial 20 seconds are also shown in Fig.10.

It can be seen in Fig.10 that there are some extent of discrepancies between the analyzed pore pressure and the measured one. From Fig.10, we know that the liquefaction of sand bed in the analysis is faster than one in experiment. The mixed-fluid zone is determined by the liquefaction ratio, and the liquefaction ratio depends on the porosity \(n_w\) in liquefaction zone. The sand layer near the surface of sand bed is denser than that in deep zone, which is not easy to liquefy in the actual experiment. The liquefaction process comes about in the shallow zone, and then gradually develops toward the deep zone. In analysis, the average porosity of the liquefaction zone was taken as the porosity value to determine value of liquefaction ratio.
It is possible that the treatment makes the liquefaction in analysis faster than that in experiment in shallow sand layer. This is the theme of study in the future.

Fig. 11 describes the variation processes of the mixed-fluid depth and the settlement of sand bed surface obtained by the analysis and by the experiment.

It is known from Fig. 11 that the variation of the mixed fluid depth analyzed is nearly in agreement with the one obtained by the measured pore pressure, and the analyzed settlement of sand bed surface approximates to the measured one.

Fig. 12 shows the initial porosity and the analytical value of porosity at the 598th cycle. The pore water pressure was measured for only 598 cycles, which was applied to the simulation of porosity. The trend of porosity distribution curves analyzed basically is consistent with the measured ones, though there are some discrepancies in the deep position.

The histograms in Fig. 12 are the porosity values measured in our experiment. Because the lengths of test cylinders on the lower position in our experiment are rather long (210 cm, see Fig. 1), the detail data of porosity distribution are not obtained.

(4) Sensitivity analysis of parameters

In order to investigate the effects of parameters (E, F, N, M, \( \alpha \)) on both the damping of pore water pressure and the maximum depth of mixed-fluid in sand bed, the cases in Table 2 were simulated with numerical analysis method. Case BS-1 is the basic case, EF-1, EF-2 are for investigation of liquefaction ratio, NM-1, NM-2 are for yielding effective stress ratio and CC-1, CC-2 are for compressibility coefficient.

It is understood from results of analysis that effects of these parameters are not distinct on the damping of pore water pressure. The effects on the maximum depth of mixed-fluid are shown in Table 2. That the influences of these parameters are small have been recognized.

4. CONCLUSIONS

In this paper, the liquefaction process of the inhomogeneous sand bed under the oscillating water pressure were investigated by using the vertical one-dimensional model, and the analytical solutions were verified by the experiment. The conclusions can be drawn as follows:

It is clear that the solutions of the numerical analysis fairly well reproduced the liquefaction processes of the inhomogeneous sand bed. That is, the liquefaction process with the build-up of pore water pressure and the densification process of sand
layer with the dissipation of excess pore pressure during the liquefaction were explained quantitatively.

However, there are some discrepancies between the results of the analysis and the experiment in detail portion. The amount of damping of pore water pressure obtained by the numerical analysis is smaller than that by the experiment. It is shown that this phenomenon is not caused by the assumed values adopted for the liquefaction ratio, the yield effective stress and compressibility of sand bed. That the uniform porosity \( n_u \) adopted in analysis maybe influences the damping of pore water pressure is considered. It is necessary to make it clear from now on.

Moreover, through observation on the data of pore water pressure measured in experiment, it is understood that the damping of amplitude exists in the mixed fluid zone in experiment. But, it is not included in the analytical model. The modeling of this phenomenon is a theme in the future study.

**REFERENCES**


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