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INVITED PAPER

# INSPECTION, PREDICTION AND DECISION-MAKING IN INFRASTRUCTURE MANAGEMENT: FRAMEWORK, MODELS AND COMPUTATION

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Decision-making models provide highway agencies with a plan for optimal decisions about maintenance and repair activities. The objective of these models is to minimize the total expected cost of maintaining a system of facilities incurred by an agency and the users over a given planning horizon. Recent models take into account measurement error in the inspection process and optimize the inspection schedule. Other state-of-the-art models include uncertainty in performance forecasting. Our research develops a model that jointly determines when to inspect and what maintenance activity to perform, while taking into account both uncertainty in the measurements and feedback in the estimation of the deterioration rate. A computational implementation is performed in order to study empirically the relative significance of uncertainties in the deterioration rate and the state of the system.

## 1. INTRODUCTION

### (1) Infrastructure management

Infrastructure management is the process through which agencies collect and analyze data about infrastructure systems and make decisions on maintenance, repair, and reconstruction (MR & R) of facilities over a planning horizon. Bridge maintenance, road improvement, and highway rehabilitation are examples of MR & R activities.

In each period, usually every year, agencies face two decisions for each facility in an infrastructure network: to inspect or not, and which MR & R action to perform, if any. In this role, they are supported by Infrastructure Management Systems (IMS) that provide them with tools to help them in this three step process:

1. Data collection;
2. Performance forecasting; and
3. Decision-Making.

Agencies base their MR & R decisions on performance models that forecast the behavior of the infrastructure facility under the effect of MR & R actions and deterioration. Their objective is to minimize the expected cost related to the facilities' use and maintenance over the planning horizon. This problem has been extensively researched and is known in the literature as the Infrastructure Maintenance and Repair problem.

### (2) Scope

The focus of this paper is on decision-making at the facility level. The methodology that we develop can be used for each facility an agency is responsible for, as long as it does not face any budget constraint. In reality, agencies will operate under budget constraints and level-of-service constraints, in the context of a network of facilities.

The main motivation of this research is to examine the effect of relaxing the annual inspection constraint, in the case of infrastructure under model uncertainty. We develop a formulation that is built upon recent developments in the application of adaptive control (AC) schemes, i.e., decision-making algorithms that explicitly account for uncertainty in the performance models, see for example Durango and Madanat (2002)[5]. Furthermore, we relax the constraint of annual inspection that is typically imposed.

## 2. LITERATURE REVIEW

### (1) The event chain in infrastructure management

The columns in **Table 1** explain how the event chain, i.e., the management process, is captured in IMS. Each row corresponds to a different type of formulation. In the first phase, traffic, weather, and ageing contribute to facility deterioration which is represented as a stochastic process. MDP formulations use a single model to represent the physical deterioration process. On the other hand, AC formulations and the proposed model capture the uncertainty that exists in choosing a representation from a set of possible deterioration models. The approach is explained in **Section 2.(4)**. An agency observes facility condition at the start of each period. Condition assessment may or may not be error-free, depending on the inspection technology. Given the measured state of the system, an agency makes decisions concerning the actions to be taken at the start of the period. The decision rule in state-of-the-art IMS is to choose actions that minimize the total expected cost. The actions include MR & R and inspection activities.

**Table 1** Features of MR & R models

| Formulation | Deterioration Model | Condition assessment | Decisions considered |
|---|---|---|---|
| MDP | Certain | True state | MR & R |
| Joint MDP | Certain | True state | MR & R Inspection |
| Latent MDP | Certain | Measurement errors | MR & R Inspection |
| AC | Uncertain | True state | MR & R |
| Proposed Model | Uncertain | Measurement errors | MR & R Inspection |

A review of existing research shows that, so far, no model has been developed to simultaneously take into account uncertainty in the choice deterioration model and in the condition assessment process.

### (2) Performance forecasting: Markov Decision Process (MDP) formulations

MDP address the issue of uncertainty in facility deterioration forecasting. A finite set of states, $\mathcal{S}$, is used to represent facility condition, and the deterioration process is represented by transition probabilities:

$$\pi_{ij}^{t}(a) \equiv \Pr(X_{t+1}=j \mid X_t=i, A_t=1), \ \forall i, j, a, t \quad (1)$$

where:

$X_t$ : state of the facility at the start of $t$

$A_t$ : action applied during $t$

$i, j$ : elements of $\mathcal{S}$

$a$ : element of a (finite) set of available actions $\mathcal{A}$

The transition probabilities can be derived from empirical data. Several approaches to estimate the probabilities are reported in the literature. One that uses statistical estimation and time series data is found in Carnahan et al. (1987)[3] and Olsonen (1988)[15]. Another is proposed by Madanat (1991)[10] and is based on a performance model and the mathematical properties of Markov Chains. Madanat and Wan Ibrahim (1995)[12] describe how Poisson regression and, more generally, negative binomial regression can be used to estimate the probabilities. These methods are statistically sound and recognize the discrete representation of condition. Finally, Mishalani and Madanat (2002)[14] develop a stochastic duration-based method to estimate the probabilities, which specifically takes into account the effect of causal variables, and recognizes the correlation between successive identical states.

The Markovian assumption implies that the probability of a transition between any pair of states only depends on the condition at the start of the period and the action applied during the period. The assumption that deterioration is stationary/time-homogeneous implies that the transition probabilities are constant over time. Among other things, this means that deterioration is independent of facility age. In this case, $\pi_{ij}^{t}=\pi_{ij}, \forall t$.

The MDP for the problem of finding optimal

MR & R policies for infrastructure facilities is usually formulated as a dynamic program (DP). The objective value function is defined as the expected, discounted cost until the end of the horizon. The cost incurred during each period includes both the user costs and the cost of applying MR & R actions. The solution gives a set of actions for each period and every possible state of the facility.

The primary assumptions of this model are:

- The true state of a facility is observable;
- The evolution of the system depends only on the previous state and the last action (Markovian assumption); and
- Inspection are performed annually.

MDP formulations inherently fail to:

- Include the possibility of a flexible inspection schedule;
- Account for uncertainty in the inspection process; and
- Incorporate information about the physical deterioration process[1]

Those methods extend to the network maintenance problem fairly easily through a linear programming (LP) formulation. Several adjustments are necessary in the context of infrastructure network planning. The agency conducts centralized planning under budget constraints. The decision variables are the fractions of the network in each condition state to which given actions should be applied. Transitions are formulated as constraints of the LP, according to the Chapman-Kolmogorov equations. This mathematical programming implementation has been applied to the Arizona Pavement Management System (Golabi et al., 1982)[6].

The incorporation of joint decisions that includes inspection and MR & R activities is relatively straightforward. The DP objective function depends on three variables: time, state, time since last inspection was performed. This issue has been investigated by several researchers, such as Klein (1962)[9] and Mine and Kawai (1982)[13]. Nevertheless, these models and their variations fail to account for uncertainty in the measurement process.

In the remainder of this section, we discuss Latent MDP formulations and adaptive control formulations in more detail.

### (3) Uncertainty in the inspection process and unconstrained inspection decision-making: Latent MDP formulations

Research by Humplick (1992)[8] has shown that there are significant measurement errors in existing infrastructure inspection technologies. Measurement errors can lead to the selection of inappropriate actions when a policy specifies different actions for the true condition and the measured condition. A second limitation of traditional MDP formulations is the lack of systematic methodology for inspection decision-making. Traditional methods for inspection scheduling are ad hoc and subjective; see for example Shahin and Kohn (1981)[16].

The purpose of the LMDP is to account for uncertainty in the inspection process and allow MR & R decision making in each period-not only when an inspection is performed. This entails the violation of a basic assumption of the MDP methodology: perfect information about the state. Therefore, the technique of state augmentation for DP, described by Bertsekas (1995)[1], was used by Madanat and Ben-Akiva (1994)[11]. This technique consists in taking into account the fact that at any point in time, the decision-maker knows the history of the past actions taken and observed states. In the LMDP, the measured state is related to the true state of the facility by measurement probabilities.

Under the state augmentation technique, the state of the system in $t$ takes into account all the information available to the decision-maker, since the start of the planning horizon. This is summarized into an information set $I_t, \forall t$. This set includes at any point in time the information about previous actions and observations, i. e.,

$$I_t \equiv \{I_1, A_1, \cdots, \hat{X}_{t-1}, A_{t-1}, \hat{X}_t\} \qquad (2)$$

$I_1$ represents the initial information available at the start of the planning horizon, and $\hat{X}_t$ is the observed/measured condition at $t$. The set can also be defined recursively as $I_t = \{I_{t-1}, A_{t-1}, \hat{X}_t\}$. This shows how the evolution of the information set follows a Markovian process:

$$\Pr(I_t | I_1, A_1, \cdots, \hat{X}_{t-2}, A_{t-2}, \hat{X}_{t-1}) = \Pr(I_t | I_{t-1}) \qquad (3)$$

Formulating a dynamic program with a state-space that corresponds to the information set consti-

[1]In practice, agencies update their deterioration model by using the observed transitions. A failure to account for this updating in the DP formulation results in suboptimal policies.

tutes a natural extension of the framework described earlier. This is done by considering the conditional distribution of states for each information set. The probability that the facility is in state $X_t$ given $I_t$ is denoted $\Pr(X_t|I_t)$. The vector of probabilities for each state is denoted $P_t \lceil I_t$. This vector is referred to as the information set or as the sufficient statistic.

Under these assumptions, the measured state is now only probabilistically related to the true state. The distribution of the measurement relative to the true state is known and depends on the technology used. Although only the observed state is available, forecasting models are still based on the true state.

The model assumes the availability of the following measurement probabilities:

$$\epsilon^c_{jk} \equiv \Pr(\tilde{X}_t = k | X_t = j,\ C_t = c),\ \forall c, j, k, t \qquad (4)$$

where $C_t$ is the technology used to measure and $c$ is an element of the set of available technologies $C$.

Measurement probabilities can be derived empirically using measurement error models. Such models relate infrastructure performance to specified indicators. Commonly used indicators in the field of pavement management are the Pavement Condition Index (PCI) as detailed in Shahin and Kohn (1981)[16] and Present Serviceability Index in HRB (1962)[7]. In the remainder of this paper we will only consider the PCI scale. Madanat (1991)[10] derives a relationship between the true value of an indicator and its measured value using a given technology. It assumes normality in the error distribution and ignores bias in measurement error. Humplick (1992)[8] shows that biases can be statistically estimated, so they are neglected in this model, since they can be corrected for.

The DP formulation is implemented in the following manner: at the at the beginning of the planning horizon, the agency determines an initial information vector according to its beliefs about the state of the facility. The state vector is then updated forward in time, according to Bayes' Law, taking into account:

- The previous information vector;
- The forecasting model;
- The measurement made at the start of the period; and
- The measurement error model.

At each stage of the DP problem, the decision variables are now whether to inspect or not, and MR & R action to perform.

The DP solution is in the form of a policy, i.e., $\{A_1(I_1), \cdots, A_T(I_T)\}$, where $T$ is the length of the planning horizon, and $A_t(I_t) \in \mathcal{A} \times C$, $\forall t$.

LMDP models address the issue of uncertainty in the inspection process and allow flexible joint inspection and MR & R decision making, but:

- They do not allow for uncertainty in the the choice of deterioration models; and
- They fail to take into account possible feedback from observations to improve the performance models.

The following section describes AC formulations, which address both limitations. The LMDP formulation has been extended to the network level by Smilowitz and Madanat (2000)[17].

### (4) Uncertainty in performance forecasting: Adaptive Control formulations

The models described in the preceding sections rely on one deterioration model specified with a single set of transition probabilities for performance forecasting. Yet Carnahan (1988)[2] acknowledges that those models are subject to significant uncertainty particularly in the exogenous factors and are often updated in the course of operations by the agency. Hence the need to include inter temporal feedback and performance model updating in the original planning process.

Current research on the application of AC to the MR & R problem has focused on the characterization of performance forecasting by deterioration models, for example Durango and Madanat (2002)[5]. As the information about the physical deterioration model is uncertain, it can be described by a discrete probability mass function over a finite set of models. The mass function is denoted $\vec{Q}_t = \{Q^1_t, \cdots, Q^{|\mathcal{R}|}_t\}$, where $\mathcal{R}$ is a set of models that can be used/combined to represent the physical process. The elements of the vector represent the probability assigned to the event that deterioration is governed by each of the models, i.e., $Q^r_t = \Pr(Y = r | I_t)$. $Y$ is a random variable that represents the physical deterioration process.

Durango and Madanat (2002)[5] use state augmentation to account for the uncertainty in choosing a deterioration model. The augmented state at the beginning of each period is defined by the true state, $X_t$, and the beliefs about the deterioration model, summarized in each vector $\vec{Q}_t$. Two AC formulations were presented: *closed-loop* (CL) and *open-loop feedback* (OLF). The CL formulation includes Bayesian updating in the argument of the objective function, i.e, it captures the value in learning about deterioration. In contrast, the OLF formulation ignores future updating, i.e., it only exploits available knowledge about deterioration. Neither formulation allows for flexible inspection schedules or accounts for measurement errors.

A network level version of the AC MR & R formulation is presented in Durango (2002)[4]. A distinctive feature of this model compared to other network level formulations is the classification of the facilities into groups according to their deterioration model. This allows for the generation of a MR & R schedule according to the state and the deterioration class of each facility. The results show that convergence of the beliefs about the model for each group of facilities is achieved. this study has further demonstrated that the benefit from targeting the action to be applied jointly to the class of deterioration model and the state increases as budgets are tighter.

## 3. FORMULATION

### (1) Notation

The proposed formulation combines the Latent MDP with the adaptive control formulations. This requires adjustment of the information sets and of the belief vectors. This is done as follows:

$$I_t=I_{t-1}, A_{t-1}, C_t, \hat{X}_t\}, \forall t \quad (5)$$

The sets include the measured state and the technology used. The belief vectors must be updated to fit this information structure. They are denoted $\vec{Q}_t(C_t, \hat{X}_t, P_{t-1} \lceil I_{t-1}, A_{t-1})$.

### (2) The decision-making framework

**Figure 1** provides the reader with a summary of the main decision making steps that are accounted for in the DP formulation that we present in this section. The first event at the beginning of each period is the usage of the facility, which implies a user cost that is a function of the true state of the facility. Then an inspection can be made according to the decision made in the previous period, which increases the information available to an agency. With this information, the agency can update its beliefs about the deterioration model and about the the state of the facility. Decision-making involves the choice of action to perform during period $t$, as well as whether to inspect or not at the beginning of $t+1$-note that period's $t+1$ inspection cost is accrued during $t$. Inspections reveal information about the current condition of a facility. In turn, this reveals information about the deterioration process, and about measurement errors associated with the technology. Weather, traffic and ageing produce changes in condition before the start of the next period.

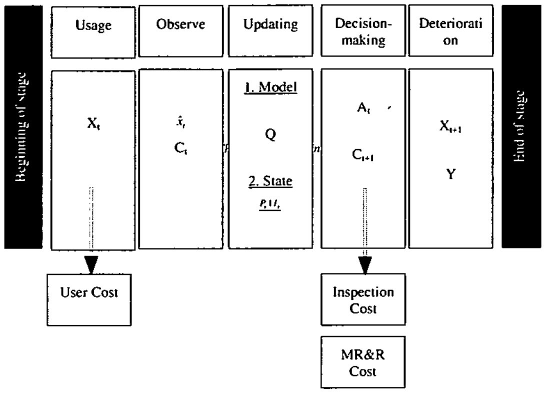


**Figure 1** The decision making framework

Prior to presenting the formulation we describe assumptions related to performance models, measurement errors, and costs.

### (3) Parameter specification

#### a) Transition probabilities

The transition probabilities are used to forecast the effect of each action on the true state of the facility. A set of transition probabilities is specified for each of the (stationary, Markovian) deterioration models. The transition probabilities depend on true state, the action selected and the deterioration model being used and are denoted:

$$\pi^r_{ij}(a) \equiv \Pr(X_{t+1}| Y=r, X_t=i, A_t), \forall r, i, j, a \quad (6)$$

#### b) Measurement error

The notation for representing measurement error is the same as the one presented for the Latent MDP formulation. The inspection decision is represented by a choice between two types of technology: one with the measurement precision associated with each inspection technology, and the other with a measurement error of infinite variance. The model can accommodate a set of different technologies. However, in the computational study presented in **Section 4** we reduce the choice to a binary decision: ($C_t=1$ for inspection and $C_t=0$ for no inspection). $C_t=0$ refers to a technology where for each state the probability of measuring any state is uniformly distributed. That is,

$$\epsilon_{jk}^{0}=\Pr(\hat{X}_t=k|X_t=j,\ C_t=0)=\frac{1}{|\mathcal{S}|},\ \forall j,\ k,\ t \quad (7)$$

where $|\mathcal{S}|$ denotes the cardinality of the set $\mathcal{S}$. This case, where every condition state is equally likely to be observed regardless of the true state, is shown to be equivalent to not inspecting in Madanat and Ben-Akiva (1994)[11]. The associated cost is set to zero.

**c) Cost**

We define $g(X_t, A_t, C_{t+1})$ as the generic cost incurred during period $t$ associated with activity $A_t$ on a facility in state $X_t$ and choosing to use inspection technology $C_{t+1}$ at the beginning of next period. We consider a cost structure that includes components for user costs, inspection costs, costs associated with MR & R activities, and a salvage cost at the end of the horizon. The salvage cost is denoted $s(i)$, $\forall i\in\mathcal{S}$.

### (4) Dynamic programming formulation

We consider a finite horizon formulation for the problem with a discount factor of $\alpha$. The length of the planning horizon is $T$. The formulation of the facility-level joint inspection and MR & R problem consists of the objective value function definition, the recursive formula, and a set of boundary conditions. The information available at the start of $t$ consists of $P_t\lceil I_t$ and $\vec{Q}_t$. The first vector summarizes the information about the current facility condition. The second vector captures the beliefs about deterioration, which in turn determine an agency's predictions about future condition. The formulation is presented below.

**a) Optimal Objective Value Function and Recursive Formula**

The optimal objective value function, $f_t(P_t\lceil I_t, \vec{Q}_t)$, $\forall P_t\lceil I_t, \vec{Q}_t, t$, represents the minimum expected, discounted cost until the end of the planning horizon given the information. It corresponds to:

$$\min_{A_t\in\mathcal{A},\, C_{t+1}\in C}\left[\sum_{i\in\mathcal{S}}\Pr(X_t=i|I_t)\begin{bmatrix} g(i,\ A_t,\ C_{t+1})+ \\ \alpha\cdot\sum_{r\in\mathcal{R}} Qo_t^r \\ \sum_{j\in\mathcal{S}}\pi_{ij}^r(At) \\ \sum_{k\in\mathcal{S}}\epsilon_{jk}^{C_{t+1}}\cdot \\ f_{t+1}(P_{t+1}\lceil I_{t+1},\ \vec{Q}_{t+1}) \end{bmatrix}\right] \quad (8)$$

In the recurrence relation, A represents the discount factor. The expression is helpful in understanding why the inspection decision for $t+1$ is made in $t$: the inspection in $t+1$ directly influences the information vector $P_{t+1}\lceil I_{t+1}$, which is used in the recursive computation of the objective function in period $t$ and the measurement probabilities $\epsilon_{jk}^{C_{t+1}}$.

**b) Boundary Conditions**

The boundary conditions for the problem are presented below. They are used to assign the salvage cost for the facility at the end of the planning horizon (start of period $T+1$).

$$f_{T+1}(P_{T+1}\lceil I_{T+1},\ \vec{Q}_{T+1})=\sum_{i\in\mathcal{S}}\Pr(X_{T+1}=i|I_{T+1})s(i),$$
$$\forall P_{T+1}\lceil I_{T+1},\ \vec{Q}_{T+1} \quad (9)$$

**c) Updating Beliefs about Deterioration**

Finally, we describe how the beliefs about deterioration, $\vec{Q}_t$, $\forall t$, and the information sets, $I_t$, $\forall t$, are updated in each period. The updates reflect how an agency's beliefs about deterioration and about the true condition change to account for periodic measurements of a facility's condition.

After a measurement at the start of $t$, the beliefs about deterioration are updated as follows:

$$\begin{aligned} Q_t^r &= \Pr(Y=r|I_t) \\ &= \Pr(Y=r|I_{t-1},\ A_{t-1},\ C_t,\ \hat{X}_t) \\ &= \frac{\Pr(\hat{X}_t|Y=r,\ I_{t-1},\ A_{t-1},\ C_t)\cdot\Pr(Y=r|I_{t-1},\ A_{t-1},\ C_t)}{\sum_{s\in\mathcal{R}}\Pr(\hat{X}_t|Y=s,\ I_{t-1},\ A_{t-1},\ C_t\cdot\Pr(Y=s|Y=s|I_{t-1},\ A_{t-1},\ C_t)} \end{aligned}$$

As $\Pr(Y=r|I_{t-1}, A_{t-1}, C_t)=\Pr(Y=r|I_{t-1})=Q_{t-1}^r$, we can write:

$$Q_t^r=\frac{\Pr(\hat{X}_t|Y=r,\ I_{t-1},\ A_{t-1},\ C_t)Q_{t-1}^r}{\sum_{s\in\mathcal{R}}\Pr(\hat{X}_t|Y=s,\ I_{t-1},\ A_{t-1},\ C_t)Q_{t-1}^s}$$

where $\Pr(\hat{X}_t=k|Y=r, I_{t-1}, A_{t-1}, C_t)$

$$=\sum_{j\in S}\Pr(\hat{X}_t=k|X_t=j,\ Y=r, I_{t-1}, A_{t-1}, C_t). \quad (10)$$

$$\Pr(X_t=j|Y=r, I_{t-1}, A_{t-1}, C_t)$$
$$=\sum_{j\in S}\epsilon_{jk}^{C_t}\cdot\sum_{i\in S}\Pr(X_t=j|Y=r, X_{t-1}=i, A_{t-1})\cdot\Pr(X_{t-1}=i|I_{t-1}) \quad (11)$$
$$=\sum_{j\in S}\epsilon_{jk}^{C_t}\sum_{i\in S}\pi_{ij}^{r}(A_{t-1})\Pr(X_{t-1}=i|I_{t-1})$$

Thus, given $\hat{X}_t=k$, $Q_t^r$ can be written as:

$$\frac{\sum_{j\in S}\epsilon_{jk}^{C_t}\sum_{i\in S}\pi_{ij}^{r}(A_{t-1})\Pr(X_{t-1}=i|I_{t-1})}{\sum_{s\in\mathfrak{R}}\sum j\in S\ \epsilon_{jk}^{C_t}\sum_{i\in S}\pi_{ij}^{s}(A_{t-1})\Pr(X_{t-1}=i|I_{t-1})} \quad (12)$$

Note that when the decision is not to inspect, the beliefs about deterioration are not updated. Indeed, no additional information is available to the decision-maker.

**d) Updating the beliefs about facility state**

After updating the beliefs about deterioration, a decision-maker revises the components of the sufficient statistic that represents facility condition as follows:

$$\Pr(X_t=j|I_t)$$
$$=\sum_{r\in\mathfrak{R}}\Pr(X_t=j|Y=r, I_t)\Pr(Y=r|I_t)$$
$$=\sum_{r\in\mathfrak{R}}Q_t^r\frac{\Pr(X_t=j|Y=r, I_{t-1}, A_{t-1}, C_t)\epsilon_{jk}^{C_t}}{\sum_{l\in S}\Pr(X_t=l|Y=r, I_{t-1}, A_{t-1}, C_t)\epsilon_{lk}^{C_t}}$$
$$=\cdots$$
$$=\sum_{r\in\mathfrak{R}}Q_t^r\cdot\frac{\sum_{i\in S}\pi_{ij}^{r}(A_{t-1})\Pr(X_{t-1}=i|I_{t-1})\epsilon_{jk}^{C_t}}{\sum l\in S\ \sum_{i\in S}\pi_{il}^{r}(A_{t-1})\Pr(X_{t-1}=i|I_{t-1})\epsilon_{lk}^{C_t}} \quad (13)$$

## 4. COMPUTATIONAL STUDY

### (1) Input Parameters

We present a computational study in the context of pavement management with a planning horizon of 15 years and a discount rate $\rho=5\%$, where $\alpha=1/(1+\rho)$. As in Carnahan et al. (1987)[3], we assume that pavement condition is represented by eight states, each representing 12.5 points on the PCI scale of 100. State 1 represents a failed pavement and state 8 represents a pavement in excellent condition. The agency can choose from the following MR & R actions: (1) do-nothing, (2) routine maintenance, (3) 1-in overlay, (4) 2-in overlay, (5) 4-in overlay, (6) 6-in overlay, and (7) reconstruction. The costs of applying MR & R actions and the user costs are taken from Carnahan et al. (1987)[3] and Durango and Madanat (2002)[5]. The salvage costs are used to enforce the restriction that the facility must provide adequate service until the end of the planning horizon. All costs are presented in **Table 2**.

We consider three deterioration models: (1) slow, (2) medium, and (3) fast. Each model being characterized by a set of seven transition probability matrices (one for each action). The models are taken from Durango and Madanat (2002)[5] and are such that:

- The effect of MR & R actions on transitions is assumed to follow a truncated normal distribution with the mean depending on the action and the model and the variance depending on the model;
- Actions are less effective in improving pavement condition under faster deterioration models; and
- Faster deterioration models have higher variance in forecasting.

The means and standard deviations of the effects of actions are presented in **Table 3**. The transition

**Table 2** Costs (\$/lane-yard)

| Pav. State | Maintenance & Repair Actions | | | | | | | User Costs |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | 0.00 | 6.90 | 19.90 | 21.81 | 25.61 | 29.42 | 25.97 | ∞ |
| 2 | 0.00 | 2.00 | 10.40 | 12.31 | 16.11 | 19.92 | 25.97 | 25.00 |
| 3 | 0.00 | 1.40 | 8.78 | 10.69 | 14.49 | 18.30 | 25.97 | 22.00 |
| 4 | 0.00 | 0.83 | 7.15 | 9.06 | 12.86 | 16.67 | 25.97 | 14.00 |
| 5 | 0.00 | 0.65 | 4.73 | 6.64 | 10.43 | 14.25 | 25.97 | 8.00 |
| 6 | 0.00 | 0.31 | 2.20 | 4.11 | 7.91 | 11.72 | 25.97 | 4.00 |
| 7 | 0.00 | 0.15 | 2.00 | 3.91 | 7.71 | 11.52 | 25.97 | 2.00 |
| 8 | 0.00 | 0.04 | 1.90 | 3.81 | 7.61 | 11.42 | 25.97 | 0.00 |

**Table 3** Means and standard deviations of action effects on change in pavement condition

| | Deterioration Model: | | |
|---|---|---|---|
| | Slow | Medium | Fast |
| Std. Dev. | 0.30 | 0.50 | 0.70 |
| Action | Mean Effects | | |
| 1 | −0.25 | −0.75 | −1.75 |
| 2 | 0.50 | 0.00 | −0.50 |
| 3 | 1.75 | 1.00 | 0.25 |
| 4 | 3.00 | 2.00 | 1.00 |
| 5 | 4.25 | 3.00 | 1.75 |
| 6 | 5.50 | 4.00 | 2.50 |
| 7 | 8.00 | 6.00 | 4.00 |

probabilities are presented in Durango (2002)[4].

If an inspection is performed, the agency is said to have "perfect state information". That is, we ignore measurement error in this study, i.e., $\epsilon^1_{kj}=\{1$ if $k=j$; 0 otherwise$\}$. This assumption was made to reduce the number of parameters and simplify the interpretation of the results. As in Madanat and Ben-Akiva (1994)[11], the inspection cost is assumed to be $0.065/lane-yard. Although this does not reflect the actual cost of the assumed error-free process, it is only used for comparison purposes.

### (2) Expected cost

**Figures** 2 and 3 compare the expected costs when the physical process corresponds to the slow or fast model, respectively. The initial information set in these cases is set such that: $P_1 \lceil I_1=(0, 0.1, 0.1, 0.2, 0.4, 0.2, 0, 0)$. For the case of "slow" initial beliefs, we set the belief vector to $\vec{Q}_1=(0.8, 0.1, 0.1)$. That is, a probability of 0.8 is assigned to the event that the physical process is governed by the slow model, 0.1 to the medium, and 0.1 to the fast model. Similarly, for "fast" initial beliefs we set the vector such that: $\vec{Q}_1=(0.1, 0.1, 0.8)$. We also consider an initial belief vector that corresponds to a case of high model uncertainty. This case is labeled "no" which stands for the non-informative initial beliefs $\vec{Q}_1=(0.3, 0.4, 0.3)$. In our study we set the initial pavement condition to state 5.

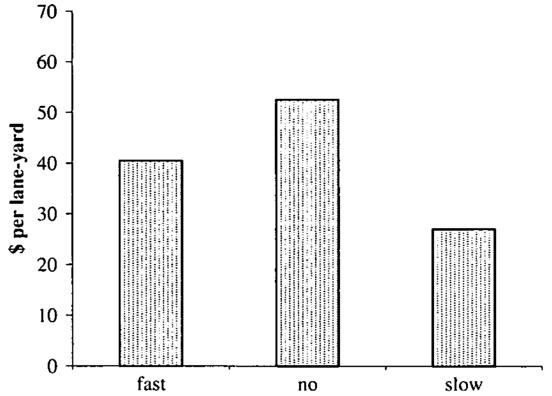


**Figure 2** Expected costs for slow deterioration

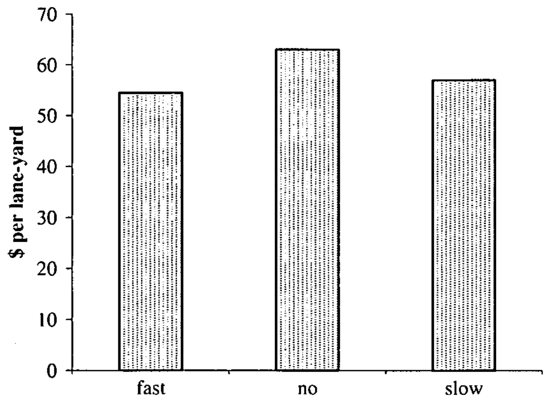


**Figure 3** Expected costs for fast deterioration

As expected, in both instances-whether the deterioration is slow or fast, when the initial beliefs "match" the physical process, the expected cost is the lowest. The expected costs are higher in **Figure** 3 than in **Figure** 2 because it is costlier to maintain a pavement that deteriorates faster. An interesting result is that the non-informative initial beliefs are the worst in both instances: it seems counter-intuitive to have lower expected cost when the initial beliefs about the model are incorrect. Similar qualitative observations can be found in Durango and Madanat (2002)[5].

An explanation can be found in **Figure 4**, which presents the result of a simulation performed according to the optimal policy given by our formulation. As above, the initial beliefs about the state are $P_1 \lceil I_1=(0, 0.1, 0.1, 0.2, 0.4, 0.2, 0, 0)$. The true initial state is assume to be 5. The physical deterioration process is assumed to be governed by the fast model. The beliefs assigned to the fast model in each period, $Q_t^3=\Pr(Y=3|I_t)$, are averaged over 1,000 simulation runs. We plot the trajectory of the average over the planning horizon.

The average $Q_t^3$ converges much faster when the initial beliefs are "slow", i.e., wrong, than when they are non-informative. Therefore, actions taken in

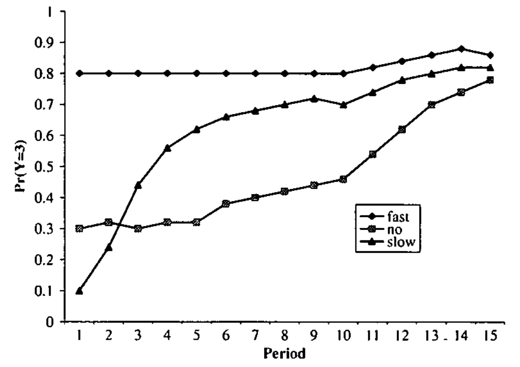


**Figure 4** Probability assigned to fast model

the non-informative case cannot be as close to optimality as those taken when the initial beliefs are wrong. Hence the higher expected cost when the initial beliefs about deterioration are more spread out.

The faster convergence of the beliefs in the wrong case compared to the non-informative case can be explained qualitatively by the contrast between the observations and the expectations. This contrast is augmented by the action taken in both cases: when the initial beliefs are wrong, i.e., "slow", the MR & R actions taken will be mild compared to the non-informative case. Therefore, worse states are more likely to be observed. Such unexpected outcomes provide feedback that leads to drastic and prompt revision of the beliefs in the wrong case. Quantitatively, this contrast is expressed by the weight of the "slow" model in the denominator of the Bayesian updating formula being equal to zero.

### (3) Value of information

A primary objective of this research is to understand the relative role of uncertainties in condition assessment and in choosing a deterioration model. In order to investigate the effect of uncertainty in the initial set of beliefs $\vec{Q}_1$ and $P_1 \lceil I_1$, we conducted a case study where $P_1 \lceil I_1 = (0, 0, 0, 0, 1, 0, 0, 0)$ and $P_1 \lceil I_1 = (0, 0.1, 0.1, 0.2, 0.4, 0.2, 0, 0)$. The initial uncertainty about the model can be high, in other words $\vec{Q}_1 = (0.3, 0.4, 0.3)$, or low. We only considered the effect of decreasing uncertainty about the model in the correct direction, i.e., $\vec{Q}_1 = (0.8, 0.1, 0.1)$ when the physical process is governed by the slow model, and $\vec{Q}_1 = (0.1, 0.1, 0.8)$ when it was fast.

**Figure 5** and **Figure 6** present the expected costs for the slow and fast models, respectively. The Δs represent the expected benefit of moving in the direction of the arrows. The double-lined arrows indicate the largest expected benefit in each case, if we start in the upper right corner of the figures, i.e., when both initial uncertainties are high.

We observe that a reduction in initial uncertainty always results in a decrease in the expected cost. If the physical model is fast, there is more value in first decreasing the uncertainty about the state as can be seen in **Figure 6**. This is because the beliefs about the model converge faster when the physical process is fast.

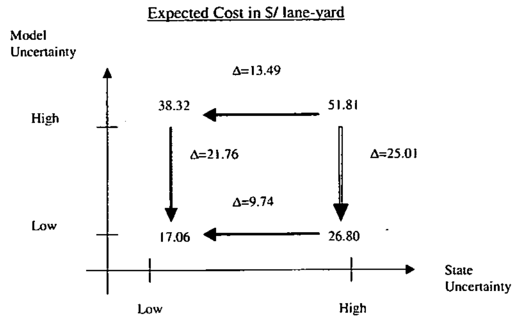


**Figure 5** Relationship between expected cost and sources of initial uncertainty: Physical process = slow

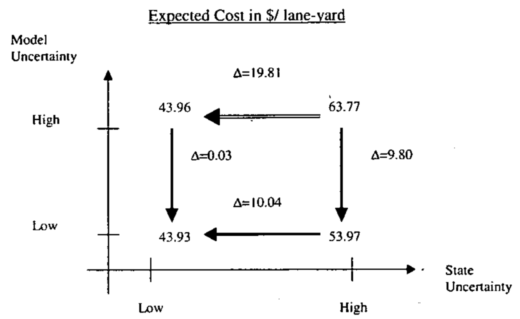


**Figure 6** Relationship between expected cost and sources of initial uncertainty: Physical process = fast

Assuming that the agency is initially in a situation where it has high uncertainty about both the model and the state, **Figure 5** shows that when the actual model is slow, reducing first uncertainty about the model brings more value, whereas **Figure 6** recommends a reduction in state uncertainty as the first step. If we consider that the physical process is hidden from the agency a cautious (maximin) strategy vis-a-vis the value of information consists of reducing the uncertainty about the state first.

In practice, agencies have fairly advanced measurement technologies, whereas they rarely have accurate and precise sets of performance forecasting models. It can be concluded from the above figures that the incorporation of improved performance forecasting models, that reduces the uncertainty about deterioration, in the planning process always provides value.

## 5. CONCLUSIONS

This paper has presented an inspection, MR & R model that explicitly allows for a flexible inspection schedule, takes into account measurement error, and includes feedback from measurements to improve the characterization of the deterioration process.

The results show that the least expected cost is observed when the initial beliefs about the deterioration model are correct, which is an intuitive result. Yet, the case of non-informative initial beliefs leads to the highest expected cost. The computational study clarified the role of the convergence of the beliefs about deterioration. We therefore recommend that an agency should not initialize the implementation phase with the probability of all the models being equal. Even a wrong initialization would lead to smaller expected cost.

Finally, as the proposed model accounts for both state and performance model uncertainties, it was possible to determine the relative value of decreasing each source of uncertainty. Results showed that, if an agency is assumed to have high variance in both initial beliefs about the model and the state, a cautious recommendation is to decrease uncertainty about the state first (by adopting a better inspection technology).

The scope of this research was intentionally limited to the facility level MR & R problem. An immediate extension is to see how the problem translates to the network level with budget constraints. The issue of implementation and "real-time" control can also be investigated in this context.

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