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BEHAVIOR OF REINFORCED CONCRETE BEAMS SUBJECTED TO BI-AXIAL SHEAR

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Thirteen reinforced concrete beams with square and rectangular sections were tested to investigate ultimate capacity under bi-axial shear loading and the ultimate bi-axial shear capacities of concrete and shear reinforcement are defined separately. The test results show that the ellipse formula underestimates bi-axial shear capacity of concrete and overestimates bi-axial shear capacity of shear reinforcement of specimens with rectangular section in which a model to calculate shear reinforcement capacity is formulated based on diagonal crack configuration. It is found that the estimations of bi-axial shear capacity of shear reinforcement of rectangular reinforced concrete beams give the values in the range of 0.88-1.27 of the tests.

Key Words : *bi-axial shear capacity; diagonal cracking; interaction relationship.*

1. INTRODUCTION

During the past decades, more accurate design method was required in order to reduce the construction cost of large structures, especially reinforced concrete ones. Generally speaking, the shear capacity of reinforced concrete members predicted by most of current design codes is quite conservative due to brittle failure mode. In the design of reinforced concrete beam under uni-axial shear loading, the calculation of shear capacity of concrete and shear reinforcement is clearly specified in the codes such as JSCE[1)], ACI[2)]. However, a reinforced concrete beam is occasionally subjected to bi-axial shear, for example spandrel beam. In this case, some codes such as JSCE provide the method to calculate shear capacity by using interaction curve, which some such as ACI do not provide. Hence more investigation on the bi-axial shear behavior is needed to verify or improve the present design code.

Concerning the past research on bi-axial shear of reinforced concrete member, the equation describing the effect of shear capacity reduction in one direction due to the presence of shear at the other orthogonal direction has been investigated through the test series of reinforced concrete member subjected to multi-directional loading, such as Ref.[3], as shown in **Fig.1**. In this test, a vertical constant axial load N was first applied at the top in the strengthening column zone. Then, a constant horizontal shear force was applied in X-direction (PX). While maintaining two forces above, increment horizontal displacement PY was induced until the column specimen loses capability to withstand additional load. It was found from the experiment that ultimate shear capacity in X-direction relates to shear capacity in Y-direction with ellipse formula.

For conventional design method of reinforced concrete member following the modified truss mechanism[1)], the uni-axial shear capacity is obtained by summing up the concrete contribution termed V_c with stirrups contribution termed V_s, i.e. $V_u = V_c + V_s$. The concrete contribution is calculated by using an available empirical equation. Stirrups contribution is obtained by taking equilibrium in vertical direction along the critical cracked plan, which is assumed to be inclined at 45-degree along member.

To estimate the bi-axial shear capacity, it is not easy to recognize the equilibrium along the diagonal cracked plane. JSCE puts forward the use of conventional design method with adopting the interaction ellipse formula. First, uni-axial shear capacity of x and y directions are calculated. Then the ultimate capacity of bi-axial shear is obtained by

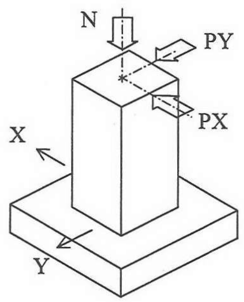


Fig.1 Bi-axial shear test of RC column by Yoshimura [3)]

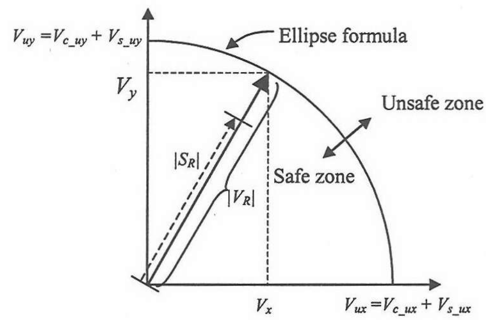


Fig.2 Ellipse formula for design of bi-axial shear capacity [2)].

ellipse formula between shear capacity in these two directions, as shown in **Fig.2**. If the magnitude of applied external shear $|S_R|$ is less than the capacity $|V_R|$ or fall inside the ellipse curve, the structure is safe from shear damage. But, if the magnitude is larger than the capacity or fall outside the curve, the structure is unsafe.

In spite of the convincing test results obtained by Yoshimura[3)], the scope of his test specimen is limited to square members. Up to the authors' knowledge, the bi-axial shear capacity of rectangular reinforced concrete member has never been investigated. In addition, it is more rational to evaluate stirrups contribution separately based on diagonal cracking similar to the case of uni-axial shear. Therefore, this study aims to propose the more elaborate design method of reinforced concrete beams with rectangular section based on the series of bi-axial shear test on reinforced concrete beams.

2. EXPERIMENTAL PROGRAM

(1) Experimental set-up

The test of simply-supported reinforced concrete beam under a concentrated load at midspan is performed in this study. The vertical force P is applied monotonically to specimen with one loading stub at midspan and two support stubs at both ends (**Fig.3**). These loading and support stubs are made of reinforced concrete, which are cast together with the specimens. The reinforcement inside the stubs is arranged to prevent bearing failure at the stubs. The applied vertical force is resolved into two forces acting in the direction of principal axes of a beam section. Hence, two shears are induced simultaneously. Ratio of shear loads in x and y directions can be changed in accordance with the inclination angle of the beam (β) between principal axes and line of load P. This experimental set-up requires only one hydraulic jack to apply bi-axial shear to the beam specimen, and hence the test procedure is quite simple but the results obtained are informative similar to the set-up by using multiple jacks in **Fig.1**.

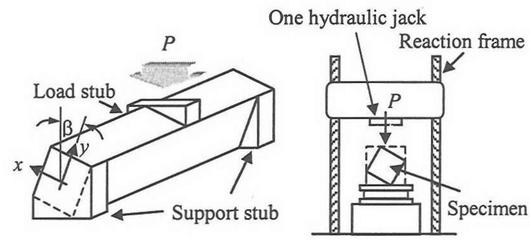


Fig. 3 Simplified bi-axial shear test

Table 1 General properties of test specimens

Series	Specimen	f_c' (MPa)	Stirrups	h/b (mm/mm)	Tilt angle (Degree)
I	B0_1	31	RB6@100mm	1	0
	B45_1	31	RB6@100mm	1	45
	B45W_1	31	-	1	45
II	B0_2	28	RB6@100mm	2.33	0
	B20_2	28	RB6@100mm	2.33	20
	B20W_2	28	-	2.33	20
	B45_2	28	RB6@100mm	2.33	45
	B45W_2	28	-	2.33	45
	B90_2	27.5	RB6@100mm	2.33	90
III	B0_3	32	RB6@100mm	2.25	0
	B25_3	32	RB6@100mm	2.25	25
	B45_3	32	RB6@100mm	2.25	45
	B90_3	32	RB6@100mm	2.25	90

(2) Test specimens

Three series of reinforced concrete beams were designed in order to investigate bi-axial shear capacity, especially the applicability of the ellipse formula specified in JSCE to rectangular section.

The series I contains three reinforced concrete beams having square section with the inclination angle β of 0 degree for B0_1 and 45 degrees for B45_1 and B45W_1. The designation which will be used herein for all the test specimens are that the number that follows B letter represents the degree of inclination. The next letter W, if any, is without shear reinforcement. Then, the number after the bottom dash line is named for series number. The series II contains 6 specimens and 4 specimens belong to series III. In total, there are 13 beams. Dimension of specimens of series I, II and III are described in **Fig.4**. Although, the inclination of specimens series II and III at 90 degrees likely changes behavior from beam to slab, Kani[4] has shown that the beam width effect is negligible.

Concrete compressive strength of beam specimens are 31, 27.5-28 and 32 MPa for series I, II, III, respectively. Deformed bar with diameter of 25 mm having yield strength of 440 MPa is used for longitudinal reinforcement. Two types of stirrups of all specimens are 6-mm and 9-mm diameter with yield strength of 370 MPa.

Adequate longitudinal reinforcements are provided in all specimens to prevent flexural failure. **Fig.4** shows reinforcement arrangement of specimen in the three test series. General properties of the test specimens are tabulated in **Table 1**. Tested area is on one side in which the reinforcement for resisting shear is weaker.

For the shear span-to-depth ratio (a/d) of the specimens subjected to vertical loads (β=0), the series I has a/d = 2.6, series II has a/d = 2.58 and series III has a/d = 2.68. These are the ratio classified in the range of the slender beam which should fail in diagonal tension mode.

(3) Instrumentation

The instruments were arranged to measure the applied vertical loads, vertical displacements at midspan and quarter span (i.e. mid-shear span), and strains in the extreme tension reinforcement and stirrups. Typically, all four legs of closed stirrups were attached with electrical strain gauges in every stirrups spaced at 200 mm. in shear span for series II and III.

3. TEST RESULTS

(1) Bi-axial shear capacity

Ultimate load capacities (V_u), concrete contribution (V_c) and shear reinforcement contribution (V_s) obtained from the test are shown

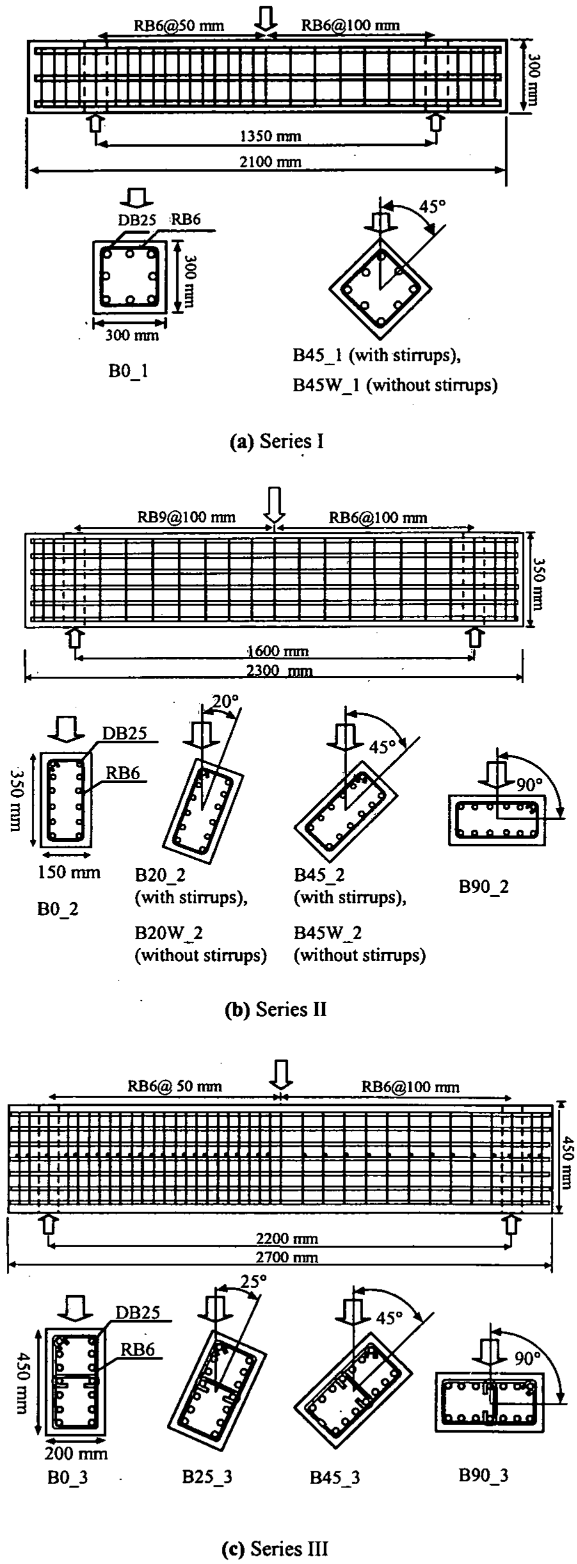


Fig.4 Dimension and reinforcement arrangement

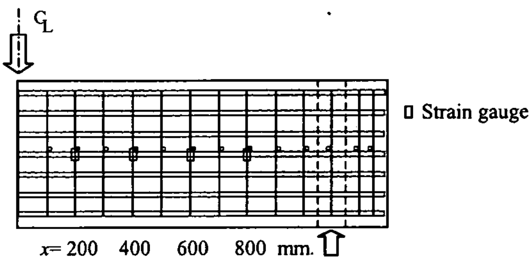
in **Table 2**. Failure mode of all beams with stirrups is diagonal tension after yielding of stirrups. Based on the fact that the slender beam which has moderate shear span-to-depth ratio will fail suddenly after appearing of diagonal crack, the concrete contribution is defined as the shear force corresponding to diagonal cracking load. In the test, it is recognized at the commencement of straining of stirrups[5)]. In the case of series I and II, in which the specimens without web reinforcement were tested, the average value between the results with and without web reinforcement is taken. The test results of specimen B45_3, **Fig.5** show the relationship between applied external force and strains of stirrups placed at x = 200, 400, 600, 800 mm distance from midspan (**Fig.5a**). Stirrups legs at the lower face, i.e. strain gauges number 2, 7, 12 and 17 (damaged), have been early strained by flexural cracking. Based on the cracking propagation, the diagonal crack has been observed first at the section near mid-shear span and extended forward to loading stub and backward to support stubs. Therefore, the applied load creating high stirrups strain at the other sections is larger than that observed at mid-shear span due to delayed occurrence of diagonal crack. Hence, strain measured at the section x = 600 mm which is close to the mid-shear span is used to define the diagonal cracking load. As can be seen in **Fig.5**, diagonal cracking load defined at section x = 600 mm is almost equal to that defined at section x = 400 mm and smaller than those defined at sections x = 200 mm and x = 800 mm.

Table 2 shows the comparison between the test results and calculation using the ellipse formula specified in JSCE. The plots of ultimate capacity obtained from the three test series and ellipse formula representing concrete contribution and ultimate state are shown in **Fig.6**. In the calculation of ultimate load (V_u) and shear resistance of concrete part (V_c), the bi-axial shear capacities are obtained from the ellipse formula, which are defined by uni-axial test results (V_{ux} when β=0 and V_{uy} when β=90), as shown in **Fig.6**. It can be seen from the comparison in **Table 2** and **Fig.6** that the calculations underestimate shear resistance of concrete part (V_c). The averaged concrete capacity of specimens B45_1 and B45W_1 from the test is 20 percent higher than that of the calculation. It can be explained by the enlargement of shearing area due to bi-axial shear leading to the increasing of interlocking of aggregate along the diagonal crack planes. In addition, by tilting the specimens, the effective depth may be increased from the case of B0_1 while the shear span is not changed, i.e. shear span-to-depth ratio is decreased. Similarly, the average of B20_2 and B20W_2 and that of B45_2

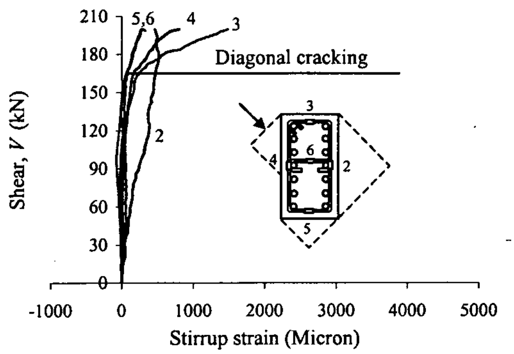
Table 2. Capacity of tested specimens

Series	Specimen	Experiment (kN)				Calculation (Ellipse formula) (kN)			Exp./Cal.		
		V_u	V_c	V_c (Avg.)	^1V_s	V_u	V_c	^1V_s	V_u	V_c	^1V_s
I	B0_1	215.9	118.4	118.4	97.5	-	-	-	-	-	-
	B45_1	233.5	137.3	142.2	96.1	215.9	118.4	97.5	1.08	1.20	0.99
	B45W_1	147.2	147.2		-	118.4	118.4	-	1.20	1.20	-
II	B0_2	164.7	80.2	80.2	84.5	-	-	-	-	-	-
	B20_2	156.5	108.2	108.1	48.4	138.7	75.6	63.1	1.13	1.43	0.77
	B20W_2	108.0	108.1		-	75.6	75.6	-	1.43	1.43	-
	B45_2	113.4	90.7	86.0	27.4	99.3	64.8	34.4	1.14	1.33	0.79
	B45W_2	81.3	81.3		-	64.8	64.8	-	1.33	1.33	-
	B90_2	77.6	55.8	55.8	21.8	-	-	-	-	-	-
III	B0_3	250.2	127.5	127.5	122.6	-	-	-	-	-	-
	B25_3	215.7	142.2	142.2	73.5	221.4	123.8	97.6	0.97	1.15	0.75
	B45_3	199.7	137.3	137.3	62.4	187.9	117.9	69.9	1.06	1.16	0.89
	B90_3	156.8	110.2	110.2	46.6	-	-	-	-	-	-
								Range	0.97-1.14[2] (-3)-(+14)	1.15-1.43 (+15)-(+43)	0.75-0.99 (-25)-(-1)

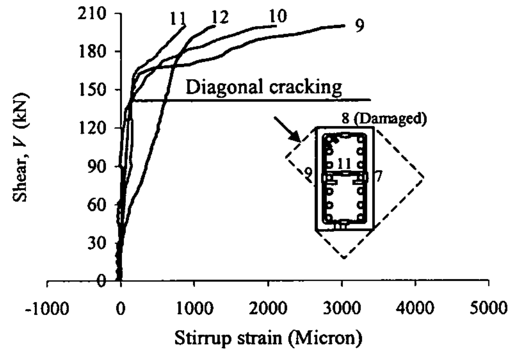
[1] is calculated by taking the difference between ultimate V_u and concrete part V_c (=V_u-V_c)
[2] considered only specimens with stirrups



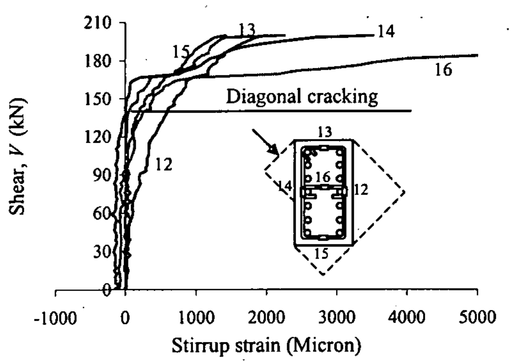
(a) Stirrups with attached strain gauges (see also at **Figs.(a)-(c)**)



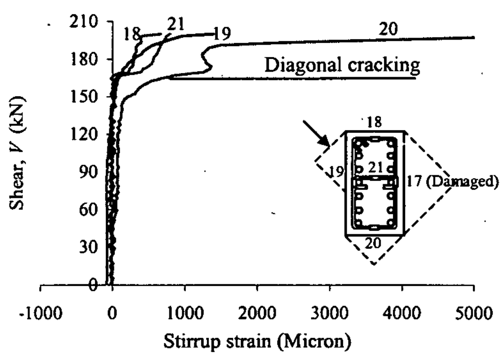
(b) Strains in stirrups at section x=200 mm from midspan



(c) Strains in stirrups at section x=400 mm from midspan



(d) Strains in stirrups at section x=600 mm from midspan



(e) Strains in stirrups at section x=800 mm from midspan

Fig.5 Applied load-stirrups strain relationship of B45_3

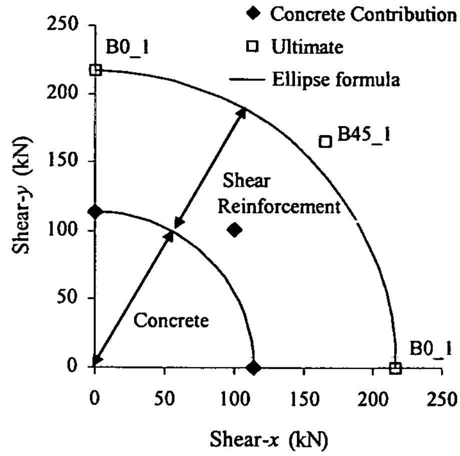
and B45W_2 from test results are 43 and 33 percent higher than the calculations. For series III, the calculations underestimate 15 and 16 percent in comparison with the test results of B25_3 and B45_3 respectively. From **Table 2**, comparison for the part of shear reinforcement capacity (V_s) indicates that the calculations using the ellipse formula are over-estimation for the case of specimens Series II and III (rectangular cross section). In other words, the calculations of shear reinforcement capacity of B20_2 and B45_2 are 23 and 21 percent higher, respectively. For Series III, the calculations are 25 and 11 percent higher. Regarding the ultimate load (V_u), i.e. the sum of concrete and shear reinforcement capacity, the present JSCE design practice using ellipse formula seems to be conservative. The ultimate loads from the test of reinforced concrete beams with shear reinforcement are ranged as (-3)-(+14) percent of calculated ones. The calculation using ellipse formula underestimates part of concrete in the range of (+15)-(+43) percent, overestimates part of shear reinforcement capacity in the range of (-1)-(-25) percent.

(2) Cracking behavior

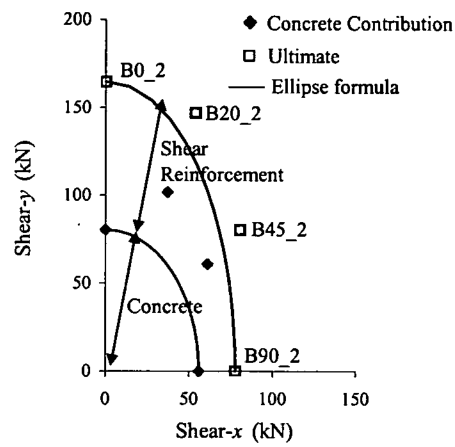
The cracks observed from the specimens in this study may be divided into two categories according to the dominant influences on their formation, i.e. flexure crack and diagonal shear crack. Flexural crack at or near midspan was observed first and spread vertically up to compression zone. With the well-known beam theory, the crack development is well corresponding to the direction of neutral surface. By increasing load, new flexural cracks were generated in shear span. Incorporated with shear stress, the flexural cracks developed in the direction inclined to member axis, especially when the shear force reached diagonal shear strength. The position and propagation of diagonal shear cracks are different among beams with different cross sectional shape and angle of inclination of specimens (β). The difference in cracking behavior reflects the evaluation of shear capacity especially, shear reinforcement capacity.

In series I, beam specimens were subjected to a load acting at zero and 45 degrees. Since condition of stress distribution and crack propagation about loading direction are symmetrical, the neutral axis is perpendicular to loading direction and cracking propagation observed on two opposite parts along the loading direction was similar to each other. For normal beam with 0 or 90 degrees of inclination, B0_1 also B0_2, B90_2 in Series II and B0_3, B90_3 in Series III, crack propagation is analogous to a beam typically failed by uni-axial shear observed by most of the past investigators. For B45_1, B45W_1, flexural crack extended all the bottom part. Failure was reached by the excessive extension of diagonal crack line from support to center loading point. **Fig.7** shows crack configuration on top and bottom faces in failure span of B45_1.

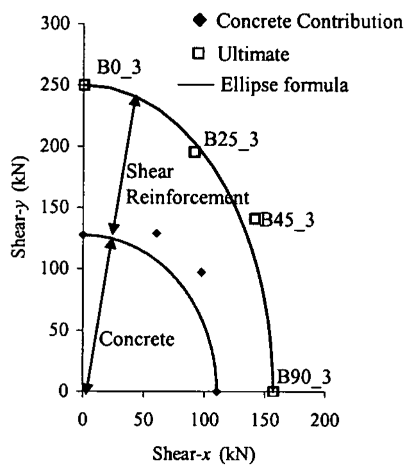
The cracking behavior of specimens explained previously concerns loading and cross sectional shape that simulates symmetrical condition about loading direction. The following explanations are made for specimens with rectangular cross sectional shape and complex loading direction. In other words, the geometrical dimension and stress distribution are no longer in symmetry. For the specimens having rectangular section in series II and III, stress distributions are more complicated than series I. Based on elastic beam theory, **Fig.8** explains the orientation of neutral axis of a tilted rectangular reinforced concrete beam subjected to vertical load. The orientation is governed by the second power of aspect ratio (h/b) and tilted angle of beam cross-section (β) or the direction of applied load. The direction of neutral axis is no longer perpendicular to



(a) Interaction diagram of specimen Series I



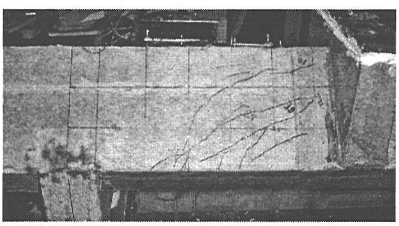
(b) Interaction diagram of specimen Series II



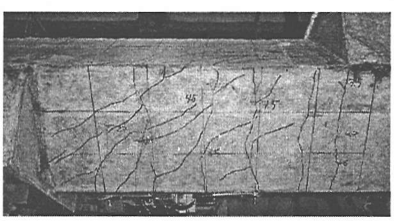
(c) Interaction diagram of specimen Series III

Fig.6 Bi-axial shear capacity and elliptical strength boundary loading direction as the previous case with square section.

For the beams tilted at 45 degrees, the calculation for direction of neutral surface of specimen B45_2 and B45_3 are respectively 79.6 and 78.8 degrees. As shown in **Fig.9** for B45_2, at the section near midspan, flexural cracks were opened fully on face C and the flexural crack tips were on face B and D. Hence, it indicates that the neutral axis is more or less parallel to the faces A and C. With the continuing increase of load, the new flexural cracks were observed in the shear span. First diagonal crack occurred on face B and D at section near mid shear span and extended to top fiber of face B and D at midspan. After that, the diagonal cracks became wider and propagated down to support stubs. When the applied load approached the ultimate one, splitting cracks were observed on face A taken place along the member axis. Due to the small compression area on face A, the cracks observed on this face were generated by highly applied compressive stress. The propagation of splitting cracks on face A also confirms the direction of neutral surface on cross section.

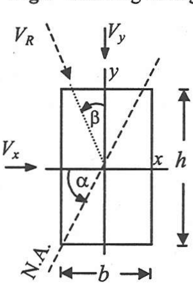


(a) Top face of failure span



(b) Bottom face of failure span

Fig.7 Cracking configuration of B45_1 of series I



$$I_x = \frac{1}{12} bh^3 \quad I_y = \frac{1}{12} hb^3$$

$$\alpha = \tan^{-1}\left(\frac{I_x}{I_y}\tan\beta\right)$$

$$= \tan^{-1}\left(\left(\frac{h}{b}\right)^2 \tan\beta\right)$$

Fig. 8 Direction of neutral axis

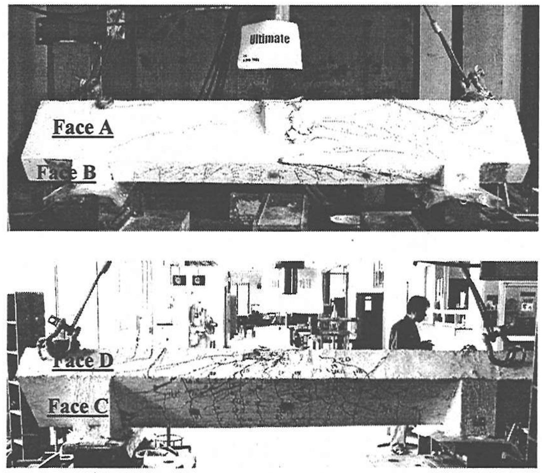
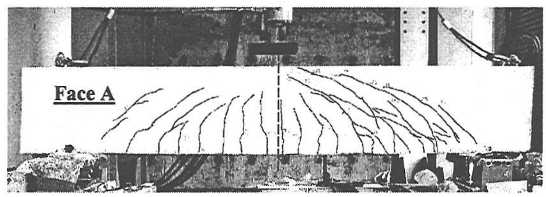
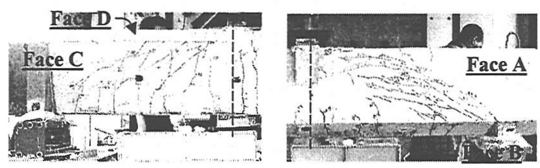


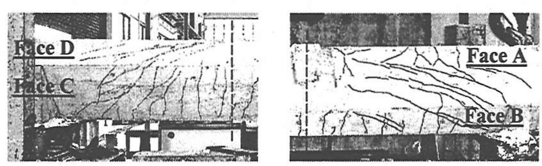
Fig.9 Crack configuration of B45_2 of series II



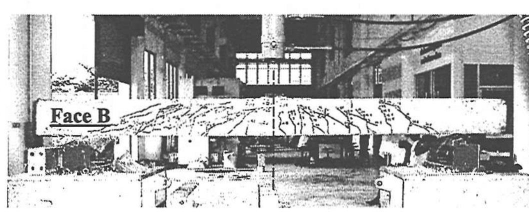
(a) B0_3



(b) B25_3



(c) B45_3



(d) B90_3

Fig.10 Crack configuration of series III

Fig.10 shows the effect of tilt angle on crack pattern of specimens series Ⅲ (failed span). The broken lines in **Fig.10** means beam center. At section near midspan, the effect of tilt angle on flexural cracking was observed. With the change of tilting angle from 0 to 90 degrees, the diagonal crack occurred at face A and C of specimen B0_3 and changed to occur at face B and D of specimen B0_90. However, the change of diagonal cracking face was faster than the change of tilt angle, and hence most diagonal cracks of specimen B45_3 appeared on face B and D.

(3) Diagonal crack plane

In order to calculate shear reinforcement capacity more accurately, diagonal crack configuration should be defined in three-dimensional view, i.e. not only the inclination with respect to member axis but also crack orientation across beam cross section. The occurrence of diagonal crack stems from the combination of flexural and shear stresses and its direction depends on the ratio of the two stresses. The investigation of orientation of diagonal crack plane across beam cross section requires the understanding of distribution of flexural and shear stress distribution over the cross section. If the plane section remains plane after deformation, axial flexural stress at any points of the cross section away from neutral surface with the same distance will be identical. Shear stress induced on the beam is related to the variation of flexural stress of two consecutive sections. Hence shear stress distributing over the cross section at the same distance away from neutral surface is also identical. Thus, the diagonal crack plane is likely parallel to the neutral surface. The experiment of bi-axial shear test of four beams with square section[6] indicated that the diagonal crack planes intersected with cut section straight normal to the direction of applied shear at the middle zone of the cut section and curved above and below the middle zone.

The inclined specimens in Series II were cut through the section at quarter shear span (section ①-① which is 200 mm from beam midspan) and at half of shear span (section ②-② which is 400 mm from beam midspan). **Fig.11** shows the configurations of diagonal crack on the cut sections, which seems to be parallel to the neutral axis with the angle α as can be calculated from **Fig.8**. The cut section ①-① of B20W_2, B20_2 and the cut section ②-② of B45_2 are rotated vertically to ease the observation of the development of diagonal crack plane.

4. SHEAR REINFORCEMENT CAPACITY

It is generally assumed that shear reinforcement does not help resisting externally applied shear force before diagonal cracking. It is withstood only by the concrete contribution. After diagonal cracking, stirrups reinforced across diagonal crack will obstruct the extension of the crack. Then, ultimate load is reached at yielding of the crossed stirrups. Hence, the shear reinforcement capacity depends on the amount of effective stirrups, which is calculated as shown in **Eq.(1)**.

$$V_s = nA_w f_w \tag{1}$$

where n, A_w and f_w are number of effective stirrups legs crossed by diagonal crack plane, cross sectional area and yield strength of stirrups steel respectively.

From **Eq.(1)**, cross sectional area and yield strength of stirrups steel are essentially the beam properties. Only amount of crossed stirrups legs are needed to be evaluated. The estimation of the number of stirrups requires the diagonal plane in three-dimensional view. The crack inclination along member axis and orientation with respect to cross section are needed. For the uni-axial shear loading, the number of effective shear reinforcement is defined by horizontal projection length to be divided by stirrups spacing, and then multiplied by two legs of stirrups crossing the diagonal crack (for general closed stirrups). Hence, shear reinforcement capacity of uni-axial shear applied in principal y direction is calculated as,

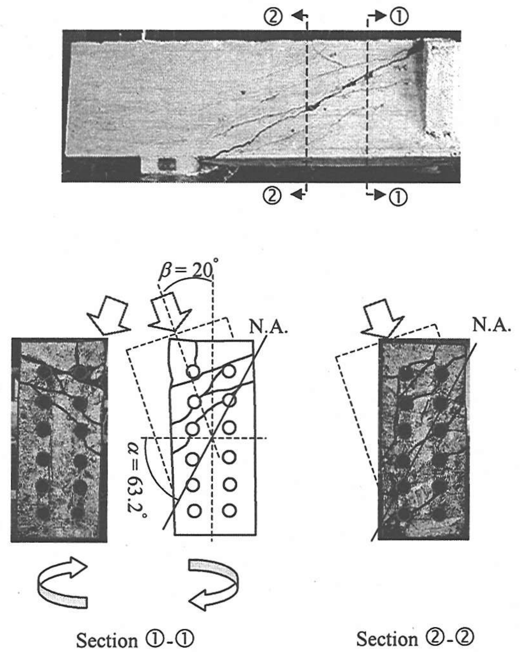
$$V_{s\_uy} = n_{wy} A_w f_{wy} = 2\left(h_{sy} \cot\theta / s\right) A_w f_{wy} \tag{2}$$

where h_{sy} is the length of stirrups in the y direction, s is stirrups spacing, θ is diagonal crack inclination with respect to member axis. f_{wy} (f_{wx}) is yield strength of stirrups arranged in y (x) direction.

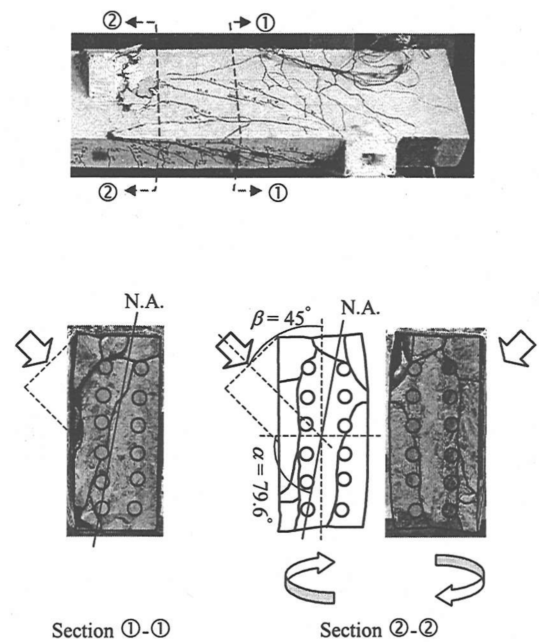
In fact, the diagonal crack propagates from the bottom tension zone to the top compression zone. The calculation in JSCE design code considers the tip of diagonal crack depth z, which is $d/1.15$, and d represents effective depth of the beam. **Eq.(2)** is rearranged replacing h_{sy} with z as,

$$V_{s\_uy} = n_{wy} A_w f_{wy} = 2\left(z \cot\theta / s\right) A_w f_{wy} \tag{3}$$

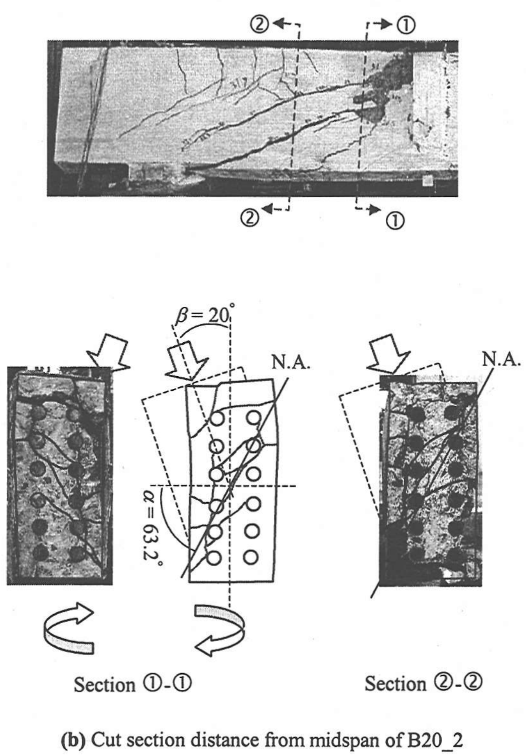
The calculation for shear reinforcement capacity in the case of reinforced concrete beam subjected to uni-axial shear is quite simple. However, when the member is subjected to bi-axial shear, equilibrium of external applied shear force and internal shear resistance provided from stirrups becomes more complex. The orientation of crack on the beam cross



(a) Cut section distance from midspan of B20W_2



(c) Cut section distance from midspan of B45_2



(b) Cut section distance from midspan of B20_2

Fig.11 Crack configuration of the cut sections

section is then required to estimate the shear resistance in x and y directions. **Fig.12** shows the calculation of number of effective stirrups of a member subjected to inclined load at β with respect to principal y axis. In the calculation, the diagonal crack plane is assumed to be parallel to neutral surface as observed from the present test, which is rotated to α degree with respect to principal x axis. Regarding the member axis, diagonal crack is assumed to propagate inclined from the bottom to the top at θ degree with respect to member axis. In Range 1 and Range 3, one stirrups leg placed in each x and y direction is active, respectively. In Range 2, the active shear reinforcement is two legs of stirrups reinforced in y direction. The number of stirrups in resisting shear in x and y directions, n_{wx} and n_{wy}, based on this diagonal crack plane are expressed in **Eq.(4a)** and **(4b)**.

$$n_{wx} = \frac{h_{sx}\sin\alpha\cot\theta}{s} + \frac{h_{sx}\sin\alpha\cot\theta}{s} = \frac{2h_{sx}\sin\alpha\cot\theta}{s} \quad (4a)$$

$$n_{wy} = \frac{h_{sx}\sin\alpha\cot\theta}{s} + 2\frac{(h_{sy}\cos\alpha - h_{sx}\sin\alpha)\cot\theta}{s} + \frac{h_{sx}\sin\alpha\cot\theta}{s} = \frac{2h_{sy}\cos\alpha\cot\theta}{s} \quad (4b)$$

Substituting the number of effective stirrups in **Eq.(1)** results in

$$V_{sx} = \frac{2h_{sx}\sin\alpha\cot\theta}{s}A_{wx}f_{wx} \tag{5a}$$

$$V_{sy} = \frac{2h_{sy}\cos\alpha\cot\theta}{s}A_{wy}f_{wy} \tag{5b}$$

Consideration of **Eqs.(2), (3)** and **(5)**, the equation for estimating shear reinforcement capacity in principal x and y directions can be appeared in a familiar manner as shown in **Eq.(6).**

$$V_{sx} = V_{s\_ux}\sin\alpha \tag{6a}$$

$$V_{sy} = V_{s\_uy}\cos\alpha \tag{6b}$$

Regarding the equilibrium requirement, the resultant shear reinforcement capacity is calculated as,

$$V_{sR} = V_{sy}\cos\beta + V_{sx}\sin\beta \tag{7}$$

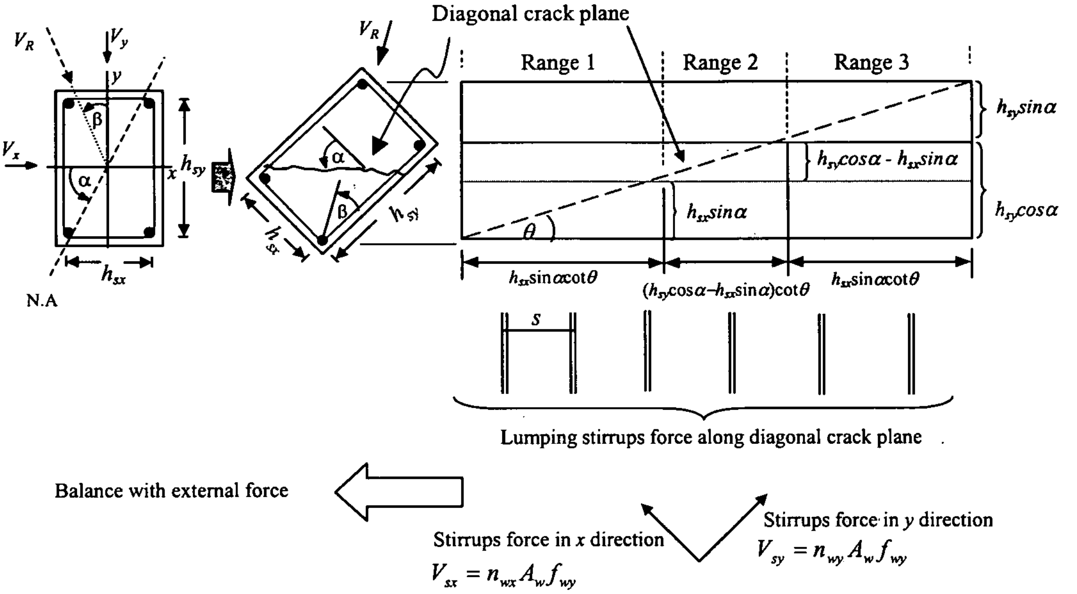


Fig.12 Effectiveness of shear reinforcement when diagonal crack is parallel to N.A.

Table 3 Comparison of bi-axial shear capacity of shear reinforcement

Series	Specimen	β (Degree)	α (Degree)	Exp. (kN)	Cal. (kN)			V_{s_exp}/V_{s_cal}
				V_{s_exp}	V_{s_ux}	V_{s_uy}	V_{s_cal} (V_{sR})	
I	B0_1	0	0	97.5	-	-	-	-
	B45_1	45	45	96.1	97.5	97.5	97.5	0.99
II	B0_2	0	0	84.5	-	-	-	-
	B20_2	20	63.2	48.3	21.8	84.5	42.4	1.14
	B45_2	45	79.6	22.7	21.8	84.5	25.9	0.88
	B90_2	90	90	21.8	-	-	-	-
III	B0_3	0	0	122.6	-	-	-	-
	B25_3	25	67.0	73.5	46.6	122.6	61.4	1.20
	B45_3	45	78.8	62.4	46.6	122.6	49.1	1.27
	B90_3	90	90	46.6	-	-	-	-
							Range	0.88-1.27

Table 3 shows the comparison between bi-axial shear reinforcement capacity from the test and the calculation by using the proposed method in **Eqs.(6)** and (7). In the calculation, uni-axial shear capacities of shear reinforcement in x and y direction, i.e. V_{s_ux} and V_{s_uy} , are taken from the corresponding uni-axial tests in each series. It can be seen that the calculation using the assumed diagonal crack configuration presented here predicts shear reinforcement capacity with the difference in the range of 0.88-1.27 percent.

For estimation of ultimate bi-axial shear capacity, superposition of concrete contribution obtained from ellipse formula and shear reinforcement contribution are calculated by using the presented model. It is noted that the estimation of concrete capacity needs ample evidence. Due to the scatter value of concrete contribution, as the past research of reinforced concrete beams subjected to uni-axial shear, statistical analysis may be further required. Regarding design for shear, the conservative design capacity of shear reinforcement is usually preferred to ensure ductility after diagonal cracking, as seen in the calculation of truss model using 45 degree of diagonal crack. It is considered that the proposed calculation method provides the safe value of shear reinforcement capacity adopting the 45-degree truss model in the calculation by substituting the 45 degrees into θ in **Eq.(5)**.

5. CONCLUDING REMARKS

Three series of reinforced concrete beams were tested under bi-axial shear by simple loading set-up requiring only one hydraulic jack. The main parameters in the present test are cross sectional shape and inclination angle of cross section. From the test results, ultimate capacities are discussed separately in terms of concrete contribution and shear reinforcement contribution.

The experimental results of bi-axial shear test show that the ellipse formula underestimates capacity of concrete part of reinforced concrete beams by about (+15)-(+43) percent. This may be due to that the increase of shearing area along the diagonal crack plane. In addition, the inclination may increase the effective depth while shear span remains constant, and hence shear span-to-depth ratio is decreased. For the contribution of shear reinforcement, the calculation using ellipse formula gives almost the same shear reinforcement contribution of the test specimens with square section. However, for the case of reinforced concrete beams with rectangular section, the calculations are about (-11)-(-25) percent over estimating capacity of shear reinforcement part. Eventually, ultimate capacities of specimens with shear reinforcement are about (-3)-(+14) percent of the calculated ones.

By observation of diagonal crack plane of the tested specimens in three dimensional view, it was found that diagonal crack plane intersects the cross section of the specimens by the line almost parallel to the neutral axis. Taking the direction of neutral surface into consideration and assuming the diagonal crack plane is parallel to the neutral surface, the calculation of shear reinforcement capacity gives the values in the range of 0.88-1.27 ((-12%)-(+27%)) of the experiments, fluctuating between the test results.

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