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NUMERICAL ANALYSIS OF EXTERNALLY PRESTRESSED CONCRETE BEAMS CONSIDERING FRICTION AT DEVIATORS

Bui Khac DIEP[1], Hidetaka UMEHARA[2] and Tada-aki TANABE[3]

[1]Member of JSCE, Grad. student, Dept. of Env. Tech. & Urban Planning
Nagoya Inst. of Tech. (Gokiso-cho, Showa-ku, Nagoya, 466-8555,Japan)
[2]Member of JSCE, Ph. D., Professor, Dept. of Env. Tech. & Urban Planning
Nagoya Inst. of Tech. (Gokiso-cho, Showa-ku, Nagoya, 466-8555,Japan)
[3]Fellow of JSCE, Dr. Eng., Professor, Dept. of Civil Eng., Nagoya University
(Furo-cho, Chikusa-ku, Nagoya, 464-8603,Japan)

This paper presents a method of analysis for externally prestressed concrete beams based on the deformation compatibility of beams and friction at the deviators. A large number of beams tested with external cables are simulated to verify the reliability of the analytical method adopted. The characteristic responses such as load vs. deflection and load vs. increase of cable stress are presented and discussed. A parametric study is also performed to investigate the effect of friction at the deviators on the behavior of externally prestressed concrete beams in four cases, namely, free slip, slip with friction, partially fixed and perfectly fixed. The predicted results are then discussed with emphasis on the effects of friction.

Key Words: *external cable, deviator, cable strain, unbonded, friction*

1. INTRODUCTION

In recent years, the beams prestressed by means of external cables have attracted the engineer's attention. Especially, the use of external prestressing is gaining popularity in bridge constructions because of its simplicity and cost-effectiveness. A large number of bridges with monolithic or precast segmental block have been built in the United States, European countries and Japan by using the external prestressing technique. The external prestressing, moreover, is applied not only to new structures, but also to existing structures, which need to be repaired or strengthened. Although various advantages of external prestressing have been reported elsewhere[1)], some questions concerning the behavior of externally prestressed concrete beams at ultimate often arise in the design practice.

One of the major problems concerning the beams prestressed with external cables is in calculating the cable stress beyond the effective prestress. In the case of beams prestressed with bonded cables, since the cable strain is assumed to be the same as the concrete strain at the cable level, the calculation of cable strain under the applied load is a problem related only to a section of maximum moment, i.e., the increase of cable strain is a section-dependent. This is totally different in the case of beams prestressed with external cables. Since the cable is unbonded, the cable freely moves in the relative change of the beam deformation. Therefore, the cable strain is basically different from the concrete strain at every cross section, i.e., the cable strain cannot be determined from the local strain compatibility between the concrete and the cable. For the calculation of cable strain, it is necessary to formulate the global deformation compatibility of beam between the extreme ends. This makes the analysis of beam with external cables more complicated, and a proper modeling of the overall deformation of beam becomes necessary.

When behavior of externally prestressed concrete beams was investigated, many researchers attempted to calculate the increase of cable stress beyond the effective prestress either by using their formulations with some parameters involved for certain cases[2)~4)], or by using equations, which are provided in the codes for unbonded beams[5), 6)]. Since there is no crack under the service loads, the stress increase in the cable is extremely small, it can be negligible. As a result, the cable stress at ultimate could be computed by using the strain compatibility with the strain reduction coefficient[7)]. Whereas Lu and Zhang[8)] assumed that after cracking, the total elongation of a cable is equal to the total crack widths of concrete surrounding the prestressing cable. Some researchers[9)~11)] assumed that the strain variation in a cable is uniform over its entire length, and tried to calculate the cable strain by adopting the assumption, which states that the total elongation of a cable must be equal to the total elongation of concrete at the cable level. Since the prestressing force is transferred to the concrete beam through the deviator points and anchorage ends, the cable friction obviously exists at the deviator points, resulting in a different level of strain increase between the two successive cable segments. Due to the complicated calculation of cable strain, almost all analytical approaches, however, did not consider friction at the deviators because of its unknown extent. For the purpose of simplicity, some researchers consider only two extreme cases, namely, free slip and perfectly fixed at the deviator when the cable strain is being computed[12), 13)].

Although an extensive body of experimental studies has been conducted to understand the behavior of externally prestressed concrete beams, a method of prediction, which gives results in close agreement with experimental observations, is still in the research process. Nevertheless, the characteristic behavior of externally prestressed concrete beams at the ultimate state is a research topic, which has not yet been well understood in depth. The demand for a better understanding of experimental observations has been an analytical research need.

In this study, an analytical method based on the deformation compatibility of beam and friction at the deviators is firstly proposed to predict the entire response of externally prestressed concrete beams up to the ultimate state. The accuracy of the proposed method is then verified by comparing the predicted results with experimental observations, which are available in the literature. Finally, a parametric study is performed to investigate the effect of friction at the deviators on the behavior of externally prestressed concrete beams.

2. METHOD OF ANALYSIS

(1) Non-linear analysis algorithm

To obtain a whole deformed shape of beam, a finite element method is commonly used as one of the popular tools in the structural analysis. The conventional finite element method often approximates a deformed shape of beam element with interpolation functions such as a cubic polynomial function for transverse displacement and a linear function for longitudinal displacement. However, the analysis of unbonded beams in general or the analysis of externally prestressed concrete beams in particular necessitates an accurate evaluation of strain variation in the concrete since the compatibility requirement should be formulated with the values of concrete strain at the cable level.

In the previous study[14), 15)], a non-linear finite element program together with the displacement control method has been developed to obtain the entire behavior of externally prestressed concrete beam up to the ultimate state. The program uses a stepwise analysis and deformation control to trace the nonlinear response of prestressed concrete beams with external cables. This program is capable of accounting not only for the flexural deformation, but also for the shear deformation, friction at the deviators, and external cables with different configuration (straight or polygonal profile). In the analysis, the beam is represented by a set of beam elements connected together by nodes located at either end. Each node has three degrees of freedom, namely, horizontal displacement, vertical displacement and rotation. A cable stress equal to the effective stress after all losses, is taken as the initial value in the analysis. Cross section of the beam is divided into layers, in which each layer might have different materials, but its properties are assumed to be constant over the layer thickness. Based on the effective stress of cable, the concrete strain of each layer for every beam element is determined, and appears to take as the initial condition of beam. In this study, a single displacement controlled point, which could be arbitrarily chosen among the loading points, is applied in the analysis.

(2) Force equilibrium at a deviator

Fig.1 showed that F_i, F_{i+1} are tensile forces in the cable segments (i) and ($i+1$) at the deviator (i). Correspondingly, θ_i, θ_{i+1} are cable angles, respectively. Thus, the force equilibrium in the X direction can be expressed as:

$$F_i\cos\theta_i + (-1)^{k_i}\mu(F_i\sin\theta_i + F_{i+1}\sin\theta_{i+1}) = F_{i+1}\cos\theta_{i+1} \quad (1)$$

where coefficient k_i depends on the slipping direction, and has a value $k_i = 1$ if $F_i\cos\theta_i > F_{i+1}\cos\theta_{i+1}$ and $k_i = 2$ if $F_i\cos\theta_i < F_{i+1}\cos\theta_{i+1}$; μ is the friction coefficient at the deviator.

Eq.(1) can be rewritten in terms of the increments of tensile forces as:

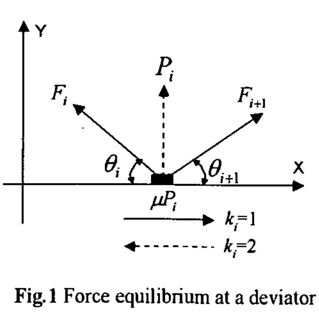


Fig.1 Force equilibrium at a deviator

$$\Delta F_i \cos\theta_i + (-1)^{k_i} \mu(\Delta F_i \sin\theta_i + \Delta F_{i+1} \sin\theta_{i+1}) = \Delta F_{i+1} \cos\theta_{i+1} \quad (2)$$

where ΔF_i, ΔF_{i+1} are the increments of tensile forces of cable at the both sides of the deviator.

Since the stress in an external cable usually remains below the elastic limit up to failure of the beam, it is possible to rewrite the force equilibrium at the deviator in terms of the increments of cable strain by dividing both sides of Eq.(2) by $E_{ps}A_{ps}$, the force equilibrium can be then expressed as:

$$\Delta\varepsilon_{si} \cos\theta_i + (-1)^{k_i} \mu(\Delta\varepsilon_{si} \sin\theta_i + \Delta\varepsilon_{si+1} \sin\theta_{i+1}) = \Delta\varepsilon_{si+1} \cos\theta_{i+1}$$

or

$$[\cos\theta_i + (-1)^{k_i} \mu\sin\theta_i]\Delta\varepsilon_{si} + [-\cos\theta_{i+1} + (-1)^{k_i} \mu\sin\theta_{i+1}]\Delta\varepsilon_{si+1} = 0 \quad (3)$$

where E_{ps} and A_{ps} are the elastic modulus and area of the cable; $\Delta\varepsilon_{si}$, $\Delta\varepsilon_{si+1}$ are the increments of cable strain at the both sides of the deviator, respectively.

(3) Strain variation in external cables

Since the deflection of the external cable does not follow the beam deflection, except at the deviator points as the beam is deformed, the strain in a cable totally differs from the strain in the concrete at the cable level. The strain induced in the concrete at the cable level varies according to the bending moment diagram, while the strain in an external cable is uniform over the length of a cable segment between two successive deviators or anchorage end. Since there is no strain compatibility between the cable and the concrete at every cross-section, the strain variation in a cable must be evaluated by taking into account the whole structure, rather than performing the calculation at each section, independently. An analytical model for externally prestressed concrete beams, therefore, should satisfy by the total compatibility requirement, i.e., the total elongation of a cable must be equal to the integrated value of concrete deformation at the cable level. This requirement is commonly adopted elsewhere[14), 15)], and is referred to as "deformation compatibility of beam" in this study. The mathematical expression of the deformation compatibility of beam is expressed as:

$$\sum_{i=1}^{n} l_i \Delta\varepsilon_{si} = \int_0^l \Delta\varepsilon_{cs} dx \quad (4)$$

where $\Delta\varepsilon_{si}$ is the increment of cable strain; l_i is the length of cable segment under consideration; $\Delta\varepsilon_{cs}$ is the increment of concrete strain at the cable level.

Combining Eq.(4) with the force equilibrium at the deviator, which is expressed in Eq.(3), one can analytically obtain the increment of cable strain of each segment at a particular loading stage, and it can be expressed as the following:

$$\begin{bmatrix} l_1 & l_2 & l_3 & \cdots & l_{n-1} & l_n \\ c_1+(-1)^{k_1}\mu s_1 & -c_2+(-1)^{k_1}\mu s_2 & 0 & \cdots & 0 & 0 \\ 0 & c_2+(-1)^{k_2}\mu s_2 & -c_3+(-1)^{k_2}\mu s_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -c_{n-1}+(-1)^{k_{n-2}}\mu s_{n-1} & 0 \\ 0 & 0 & 0 & \cdots & c_{n-1}+(-1)^{k_{n-1}}\mu s_{n-1} & -c_n+(-1)^{k_{n-1}}\mu s_n \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{s1} \\ \Delta\varepsilon_{s2} \\ \Delta\varepsilon_{s3} \\ \vdots \\ \vdots \\ \Delta\varepsilon_{sn-1} \\ \Delta\varepsilon_{sn} \end{Bmatrix} = \begin{bmatrix} \int_0^l \Delta\varepsilon_{cs} dx \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

or

$$[M]\{\Delta\varepsilon_s\} = [N]\{d\} \quad (5)$$

Finally, the increment of cable strain can be defined by using the inverse matrix operation as:

$$\{\Delta\varepsilon_s\} = [M]^{-1}[N]\{d\} = [C]\{d\} \quad (6)$$

where the letters c_i and s_i are denoted as cosine and sine of the cable angle, and the subscripts under these letters indicate the cable angle number; $\{d\}$ is the increment of nodal displacement vector.

Since the elongation of a cable depends on the concrete strain at the cable level, the portion under the integral of Eq.(5) should be formulated in terms of the coordinate of the cable elements. Within one cable element, the coordinate of cable can be determined by the equation of a straight line as $y=px+q$. Therefore, the integral of Eq.(5) can be derived as following:

$$\int_0^{l_i} \Delta\varepsilon_{cs} dx = \int_0^{l_i} (u' - yv_b'')dx = \int_0^{l_i} [u' - (px+q)v_b'']dx$$

$$= \left[-1,\ -\frac{p}{Ts},\ q+\frac{6pK}{l_i Ts},\ 1,\ \frac{p}{Ts},\ -q-\frac{1}{Ts}\left(pl_i+\frac{6pK}{l_i}\right)\right]\{d\} \quad (7)$$

where $\Delta\varepsilon_{cs} = u' - yv_b''$; u and v_b is the horizontal displacement and the vertical displacement, respectively; y is the coordinate of the cable in the Y direction; $K = EI/GA$ is the stiffness ratio; EI and GA is the flexural stiffness and the shear stiffness, respectively; $Ts=1+12K/L^2$; L is the length of concrete element.

It can be seen from Eq.(5) that the strain variation in an external cable depends mainly on the overall deformation of the beam, friction at the deviators and cable angle. The change of the beam deformation under the applied load is in the relative change of cable elongation. The adequate evaluation of cable strain depends on the accuracy of the calculation of concrete strain at the cable level. Therefore, the concrete beam should be necessarily divided into a large number of short elements by using finite element method.

For precast segmental beams, since continuous reinforcement is not usually provided across the joints between segments, all the cracks are commonly concentrated at the joint locations as the applied load increases, thus inducing opening of the joints. In this study, a model for the joint opening is referred from the previous studies[16), 17)], in which the joint opening depends on the horizontal displacement and rotation of the joints.

It should be noted that the beams prestressed with internally unbonded cables do not exactly meet the proposed method of analysis since no deviator points are provided. The proposed method, however, can simulate the beams prestressed with unbonded cables by setting a large number of "fictitious deviator" along the beam and zero-friction at these deviators. To do that the behavior of beams prestressed with either external cables or unbonded cables can be investigated by the same method of analysis. Because the proposed method is member-analysis, the accuracy of the outcomes depends strongly on the initial condition of the beam.

(4) Material model

In the analysis, the stress-strain curve for the concrete in compression is assumed to be a parabolic ascending branch and a linear descending branch as shown in **Fig.2**. The stress-strain curve is expressed as follows:

$$f_c = f_c'\left[\frac{2\varepsilon_c}{\varepsilon_{co}} - \left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] \quad \text{for } \varepsilon_c < \varepsilon_{co} \qquad (8)$$

$$f_c = f_c' - m_1(\varepsilon_c - \varepsilon_{co}) \quad \text{for } \varepsilon_{co} < \varepsilon_c < \varepsilon_{cu} \qquad (9)$$

where f'_c is the compressive strength of concrete; $\varepsilon_{co}=0.002$ is the concrete strain at the peak stress; $\varepsilon_{cu}=0.0035$ is the concrete strain at ultimate; $m_1=0.8f'_c/(\varepsilon_{cu}-\varepsilon_{co})$ is the post peak slope controlling the descending branch of concrete. A bilinear curve for the stress-strain relationship of prestressing cable is also presented in **Fig.2**, in which ε_{py} and ε_{pu} has a value of 0.008 and 0.035, respectively.

3. VALIDITY OF PROPOSED METHOD

The accuracy of the proposed method is verified by comparing the predicted results with the experimental observations of externally prestressed concrete beams reported recently by Nishikawa, K., et al.[18)] and Zhang, Z., et al.[19)]

In the study by Nishikawa, K., et al.[18)], five beams in series of flexural test are taken to analyze. A layout scheme of beam, cross section, cable configuration, and loading arrangement are shown in **Fig.3**. A summary of the test variables is provided in **Table 1**. All the beams with a flanged section were cast-in-situ, except beam GS1 with

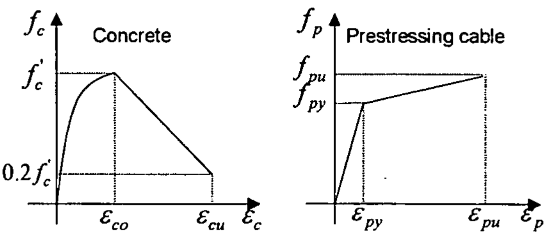


Fig.2 Stress-strain relationships

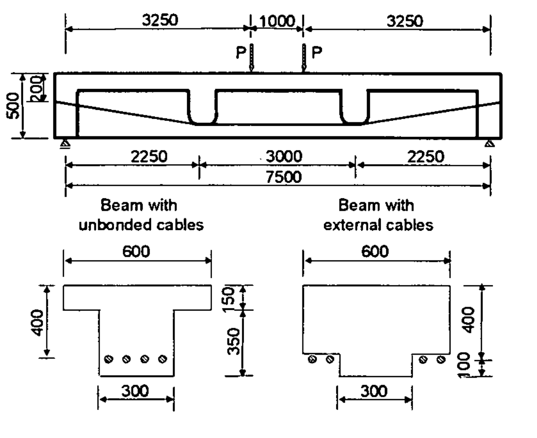


Fig.3 Layout scheme of beams with external cables (Test by Nishikawa, K., et al.)

Table 1 Test variables and materials

Beam No	Con-crete strength N/mm²	Cable strength f_{pu} N/mm²	Amount of reinfor-cement	Type of prest-ressing	Devia tor dis-tance, m
G1	55.7	1773	4-D10	Ext.	3.0
G4	51.3	1773	4-D10	Unbnd.	-
G5	51.2	1773	4-D10	Ext.	5.0
G6	50.5	1773	4-D16	Ext.	3.0
GS1	51.9	1773	4-D10	Ext.	3.0

Ext.=external; Unbnd.=unbonded

precast segmental block. The main test variables included the area of non-prestressed reinforcement, distance between the deviators, type of prestressing (internally unbonded cables or external cables), and type of casting method. Before loading test, all the beams were prestressed by four cables type 1S12.4 with 981.7 mm^2 per a cable. The effective stress in the cable was approximately 52% of the ultimate strength of cable. Two loading points were provided at the distance of 1000.0 mm and symmetrically located from the midspan section of the beam.

In the experimental program conducted by Zhang, Z., et al.[19], among ten tested beams, nine beams of them are taken to analyze. A layout scheme of beam, cross section, cable configuration, and loading arrangement are shown in **Fig.4**. A summary of the test variables is provided in **Table 2**. All the beams were simply supported with a flanged section, and were divided into two groups with different configuration of cable. In each group, the beams were designed with different amount of non-prestressed reinforcement in order to examine its effect on the behavior of the prestressed beams with external cables at ultimate. Six beams in the first group were prestressed by two cables with the straight profile at the depth of 350.0 mm from the top surface of the beam (beams with series A). Also the beams of the first group were subdivided into three pairs. In each pair, one beam was subjected to a single concentrated load at the midspan section, while the other was subjected to two loading points, symmetrically located at the distance of 2.0 m from each end of the beam. In the second group, three beams were prestressed by two cables with the polygonal profile at the depth of 375.0 mm, and two deviator points were provided at the distance of 2700.0 mm (beams with series B). The main test variables included the area of non-prestressed reinforcement, cable profile (straight or polygonal profile), and loading type (one or two loading points).

As mentioned earlier, there is a friction between the cable and the deviator, and the friction can be expressed in terms of the friction coefficient, μ as shown in Eq.(1). The real value of friction coefficient depends on many factors, and it can be only determined by the experimental investigations. The friction coefficients, however, are not easy to find in any of the available literature. For the analytical purposes, the friction coefficients at the deviators should be assumed to have a certain value, and they are assumed to be 0.2 for the beams tested by Nishikawa, and 0.15 for the beams tested by Zhang in this study. Although these values might not be true in the tested beams, they are, however, only adopted for the analytical purpose.

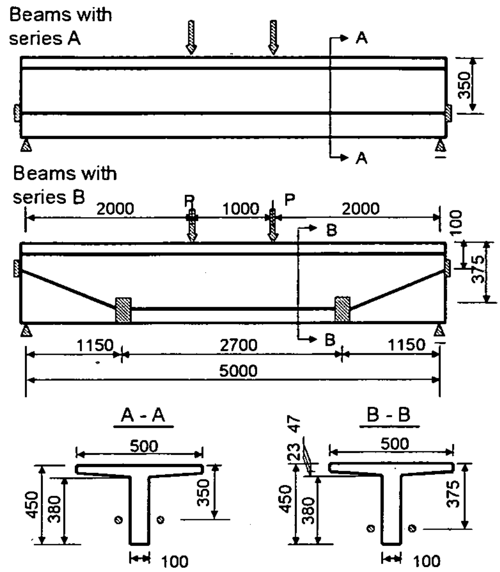


Fig.4 Layout scheme of beams with external cables (Test by Zhang, Z., et al.)

Table 2 Test variables and materials

Beam No	Concrete strength N/mm^2	Prestressing cable		Area of reinforce -ment mm^2	Loading type
		A_p mm^2	f_{pe} N/mm^2		
A1-1	52.3	981.7	322.9	157.1	One point
A1-2	52.3	981.7	325.9	157.1	Two points
A2-1	49.8	981.7	332.3	235.6	One point
A2-2	49.8	981.7	335.4	235.6	Two points
A3-1	52.6	981.7	331.3	358.1	One point
A3-2	52.6	981.7	326.8	358.1	Two points
B1-2	52.7	392.7	805.7	157.1	Two points
B2-2	52.7	392.7	843.4	201.1	Two points
B3-2	49.3	392.7	822.2	402.1	Two points

f_{pe} is the effective prestress at the prestressing stage

A comparison between the analytical predictions and the experimental results of the beams tested by Nishikawa in terms of load vs. deflection curves is presented in **Fig.5**. In order to show the test variables in the experimental program by Nishikawa, beam G1 is chosen as a reference beam, and the predicted results as well as the experimental observations of beam G1 are compared with results of the other beams. It can be seen from **Fig.5** that all the beams behave essentially the same before cracking regarding the use of different parameters such as amount of reinforcement, the distance between the deviators and the type of prestressing. However, the ultimate strength of beam G1 is comparatively much larger than that of segmental

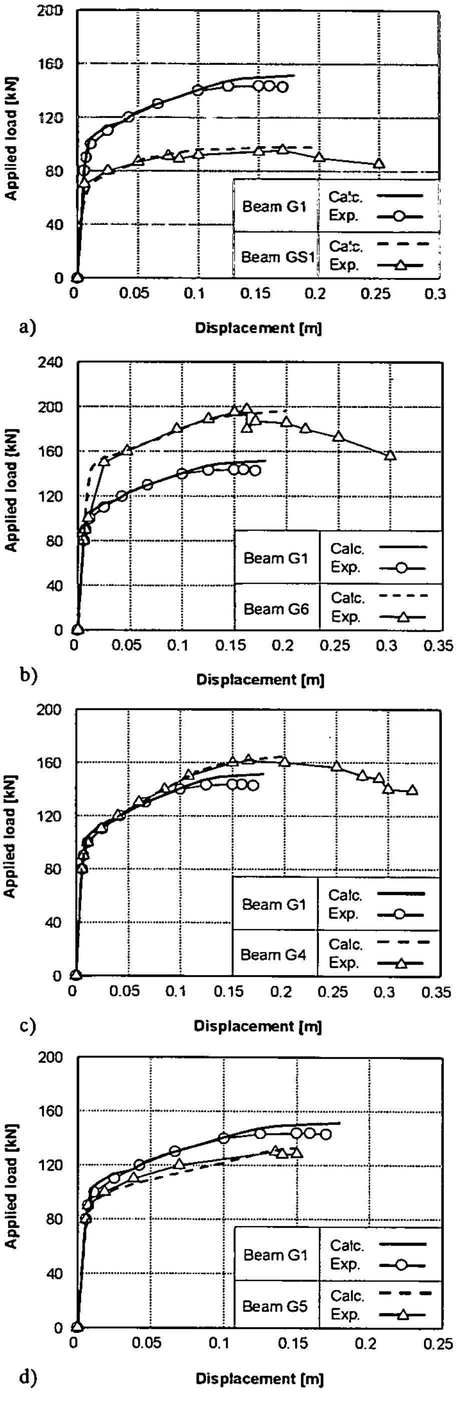


Fig.5 Comparison of calculated load-deflection response with experimental results

beam GS1 (see Fig 5a), and much smaller than that of beam G6 with additional amount of reinforcement (see **Fig.5b**). The difference of ultimate strength of the beams is obvious and is approximately 55% for the case of segmental beam

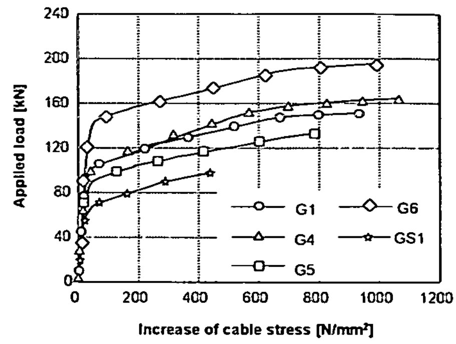


Fig.6 Calculated results in terms of load vs. increase of cable stress

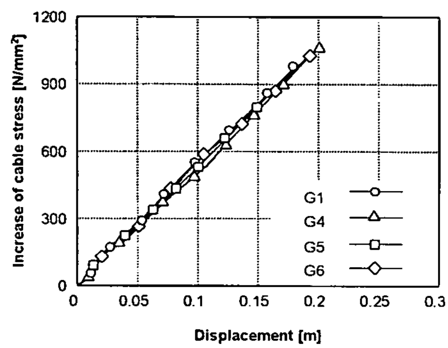
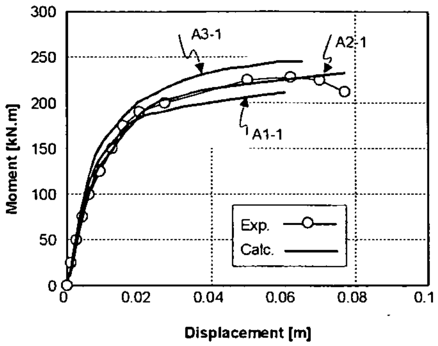


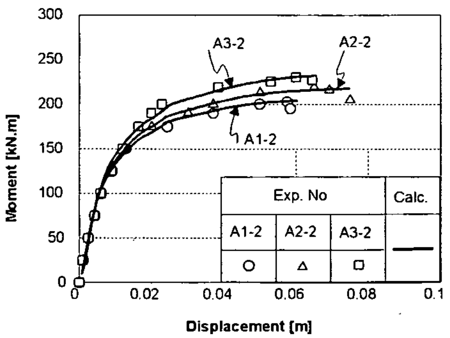
Fig.7 Calculated results in terms of increase of cable stress vs. deflection

GS1 and 30% for the case of beam G6. This is because the joint of the midspan section of segmental beam GS1 opens rapidly as the applied load increases, resulting in the reduction of stiffness of the beam, leading to premature failure at a low strength. The load-deflection curve of the precast segmental beam GS1 is also indicated by a rather plateau as shown in **Fig.5a**. This implies that the deflection of the beam increases rapidly with a little increase in the applied load. While beam G6 with additional amount of reinforcement is attributed to the resistance of the reinforcement itself as well as to its effect in distributing and limiting the cracks in the concrete, leading to give a higher strength to the beam.

Beam G1 is also compared with two other beams as shown in **Figs.5c, d**. It is apparently indicated that the distance between the deviators increases from 3.0 to 5.0 m, the ultimate strength of beam G5 reduces approximately 14% compared with beam G1 (see **Fig.5d**). Because of shortening of free length of cable, beam G1 with less free length of cable produces a greater stress variation in a cable than beam G5 with more free length of cable does (see **Fig.6**). This also leads to a higher strength of beam G1 as compared with beam G5. Since there

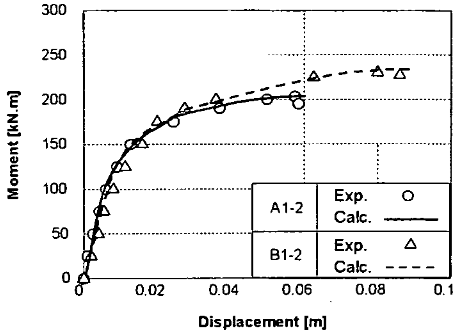


a) Beams with one loading points

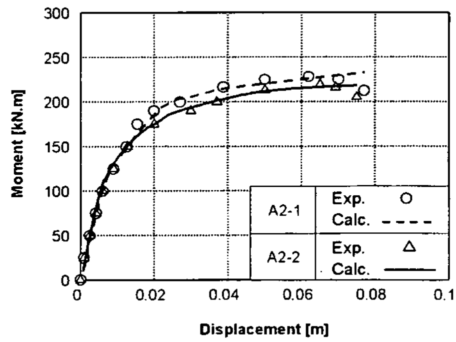


b) Beams with two loading points

Fig.8 Moment vs. deflection of beams with series A



a) Effect of cable configuration



b) Effect of loading pattern

Fig.10 Effect of cable configuration and loading pattern

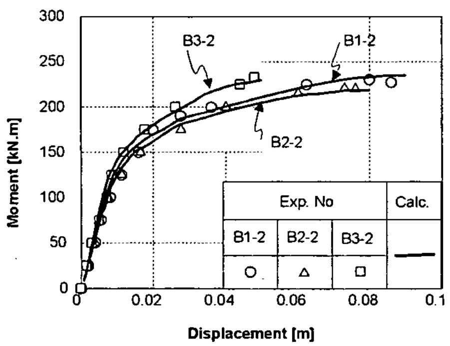


Fig.9 Moment vs. deflection of beams with series B

are no second-order effects in the case of unbonded beam G4, the ultimate strength of beam G4 increases approximately 9% as compared with beam G1, which is prestressed with external cables.

Fig.6 shows the calculated results of all the beams in terms of load vs. increase of cable stress responses. It can be seen from this figure that the curves of increase of cable stress exhibit essentially similar to the load-deflection relationships. This means that the stress in the external cables increases very little before cracking. However, it more rapidly increases after that, i.e., the major part of stress increase in a cable develops as the deflection of the beam becomes large. The segmental beam GS1 shows the smallest value of stress variation in the cable, while the monolithic beam G4 with unbonded cables indicates the biggest value of stress increase in series of tested beams. A comparison between the calculated results and the experimental data could not be made because the experimental data are not available in the literature.

While a comparison between the curves of load vs. deflection and load vs. increase of cable stress for the individual beam is made, it is interesting to note that these two curves are very similar in shape, indicating the close relationship between the deflection and the stress increase in the external cables. The close relationship is also indicated by approximately linear relationship in terms of stress variation in the cable vs. deflection response as shown in **Fig.7**. The linear relationship between the midspan deflection and the stress increase in the cables confirms the findings from the experimental observations, which have been reported elsewhere[20)~22)]. Consequently, the deformation compatibility of beam as mentioned earlier is verified to be suitable for the analysis of prestressed beams with external cables.

Figs.8, 9 show a comparison between the analytical predictions and the experimental results of the beams tested by Zhang in terms of moment vs. deflection curves. It can be seen in these figures that since the stiffness of the beams prior to cracking remains the same, all the beams exhibit essentially identical before cracking, indicating no significant effect resulted in using the different area of non-prestressed reinforcement. Since the first crack of the beam with a smaller amount of non-prestressed reinforcement occurs a little earlier than the companion beams with a larger amount of reinforcement do, the beam with a smaller amount of non-prestressed reinforcement, therefore, produces a lesser moment at ultimate. After cracking, the beam with a smaller amount of non-prestressed reinforcement exhibits rather ductile, and fails by initial yielding of non-prestressed reinforcement, sequentially, collapses totally by crushing of concrete in the compression region. On the other hand, the beam with a higher value of non-prestressed reinforcement exhibits rather stiff, resulting in a higher strength at ultimate. The analytical results reproduce the experimental data with remarkably good agreement.

Fig.10 shows the effect of cable configuration and loading pattern for two pairs of beams (A1-2 vs. B1-2, and A2-1 vs. A2-2). It can be seen from **Fig.10a** that since the beam deflection and the accompanying reduction in cable eccentricity are small in the elastic range, the moment-deflection responses behave similarly in this stage, indicating insignificant effect of cable configuration on the moment-deflection response. As the applied load increases, the beam deflection becomes large. As a result, the reduction in cable eccentricity of the beam without deviator becomes more pronouncedly, leading to lower strength of the beam as compared to the beam with deviator.

Beam subjected to a single concentrated load at the midspan section exhibits a higher strength as compared with beam subjected to two loading points as shown in **Fig.10b**. Similar experimental observations for the beams prestressed with external cables have been reported elsewhere[23].

4. PARAMETRIC STUDY

The effect of friction is performed on a simply supported beam with a box section, which was tested at the Research Center for Experiments and Studies on Construction and Public Work (CEBTP) in France[24), 25)]. The dimensions of the beam, span length and loading arrangement are shown in **Fig.11**, and material properties are shown in **Table 3**. Two deviators were provided at the distance of 3.0 m from each other, and symmetrically located from the midspan section. The beam is analyzed by considering four different cases, namely: 1) free slip; 2) slip with friction coefficient of 0.17; 3) partially fixed; 4) perfectly fixed. For the case of cables being free slip, the friction coefficient is equal to zero, whereas for the case of cables being perfectly fixed, the friction coefficient should have a value, which is big enough to restrain any movement of a cable at the deviators. In this case the value of friction coefficient is assumed to be equal to 2.0, which is referred from Garcia-Vargas's model[25)]. For the case of partially fixed, the friction coefficient is assumed to be 1.0, which has an intermediate value between the cases of slip with friction and perfectly fixed in order to examine the extent of fixity at the deviators.

Fig.12 plots the predicted characteristics of the load-deflection response for four cases and also the results obtained from the experimental observations. It can be seen from this figure that the deflection responses behave essentially in the same manner as in the experimental observations until the decompression stage regardless of friction. This is because the beam deflection is very small, which induces a small tensile force in each cable segment, leading to an extremely small unbalanced force at a deviator. As a result, the cable slip generally cannot occur at this stage. That is the friction at the deviators does have an insignificant effect on the deflection response until the decompression stage.

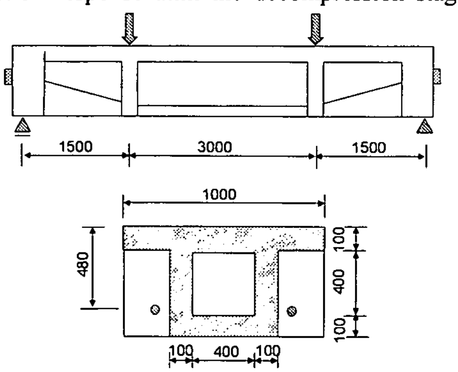
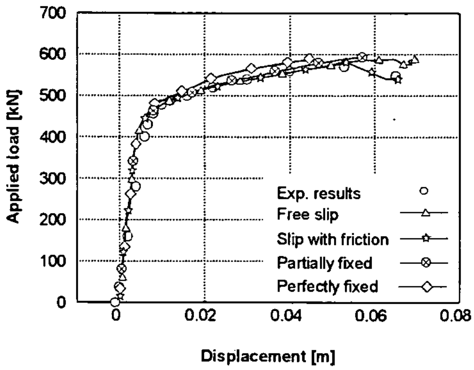


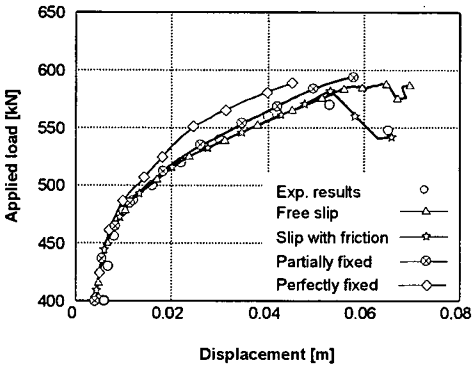
Fig.11 Layout scheme of beam tested by CEBTP

Table 3 Material properties (N/mm^2)

Concrete		Prestressing cable		
f'_c	E_c	f_{py}	f_{pu}	E_{ps}
41.0	3.8x10^4	1570	1860	1.95x10^5

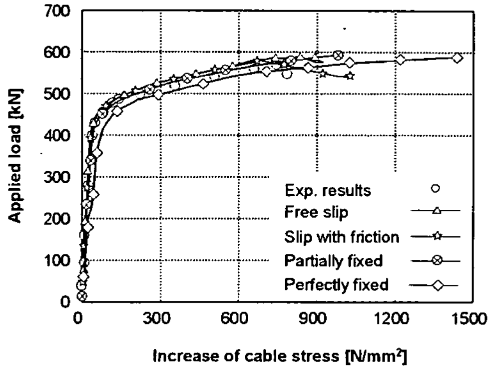


a) Entire responses

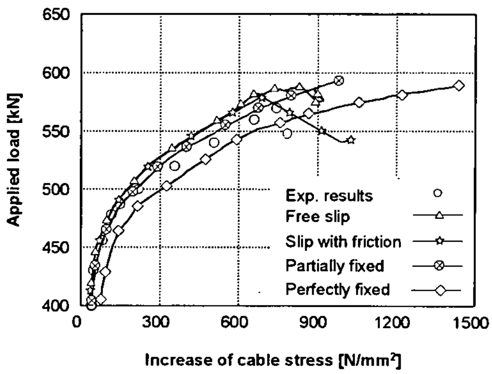


b) Responses after the decompression

Fig.12 Effect of friction at the deviator on the load-displacement responses



a) Entire responses



b) Responses after the decompression

Fig.13 Effect of friction at the deviators on the load-increase of cable stress responses

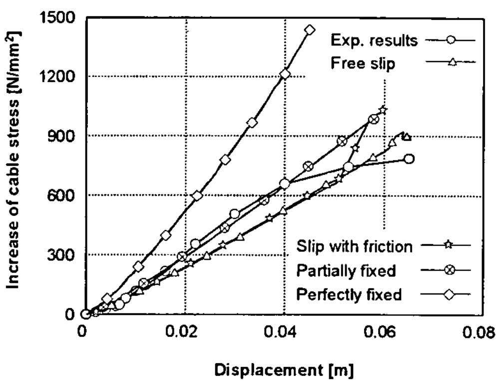


Fig.14 Increase of cable stress vs. deflection

After the decompression, the deflection response of beams with consideration of free slip and slip with friction is more or less identical to the experimental results, whereas for the case of perfectly fixed, the prediction overestimates the strength of the beam at ultimate. The reason for this can be explained that since the cables are assumed to be a perfectly fixed at the deviators, the stress increase in each segment is independent from that of the others. As the applied load increases, the deflection of midspan and the accompanying concrete strain at the cable level between the deviator points becomes large, resulting in a great increase of cable stress of middle segment (see **Fig.13**). A greater stress variation in the middle segment of a cable induces a higher load carrying capacity, resulting in overestimating prediction of ultimate strength of the beam.

Fig.13 presents the results of stress increase in the external cables. It is apparently seen that the increase of cable stress exceeds the yielding strength for the cases of partially fixed and perfectly fixed, and remains in the elastic range for the cases of free slip and slip with friction. Although a small discrepancy has been observed in the predicted results for the cases with free slip and slip with friction, the same rate of stress increase, however, is approximately found until the ultimate state, and very similar to the experimental observations.

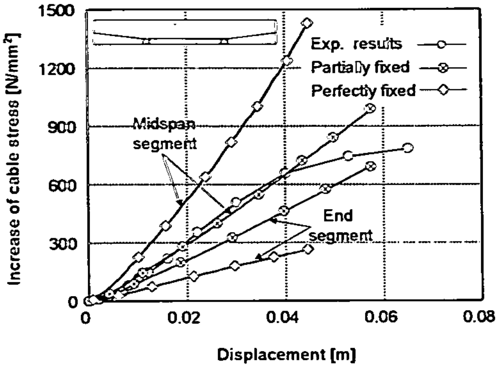
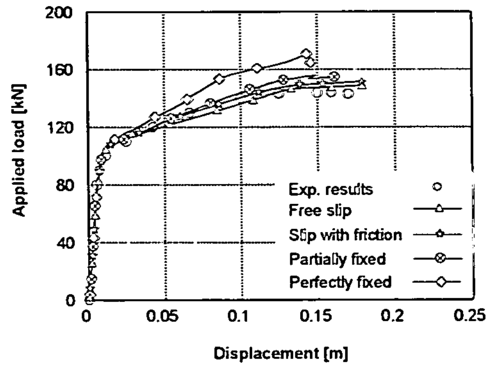


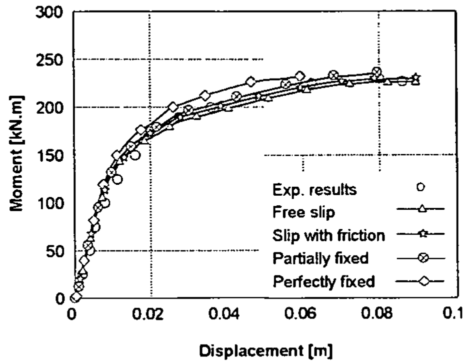
Fig.15 Comparison between the cases of partially fixed and perfectly fixed

Table 4 Comparison between the experimental observations and the calculated results

Case of study	Ultimate load kN	Ultimate deflection mm	Increase of cable stress N/mm²
Free slip	586.2	58.1	741.7
Slip with friction	580.6	54.0	679.4
Partially fixed	594.0	58.0	995.5
Perfectly fixed	589.9	45.4	1455.0
Exp. observations	570.0	53.0	745.0



a) Beam G1 tested by Nishikawa



b) Beam B1-2 tested by Zhang

Fig.16 Evaluation of the friction effect on behavior of beams prestressed with external cables

A fairly linear relationship between the increase of cable stress and the beam deflection is also observed as shown in **Fig.14**. This indicates that the stress increase in a cable is almost proportional to the midspan deflection until the crushing strain reaches in the concrete. However, the rate of stress increase in the case of cable being perfectly fixed is quite different from the other cases. It is also seen from this figure that the rate of stress increase is reduced from the deflection of 40.0 mm as observed in the experiment. This is because the rate of stress increase in the external cables is smaller than the rate of increase in the beam deflection as the applied load increases from this point. However, the rate of stress increase observed by the predictions does not change except the case of cable being slip with friction. This may be indicated in the calculated results for the ultimate load capacity, which are a little higher than that of the experimental observations (see **Table 4**). It is also found from the results of the case of slip with friction that the concrete strain at the critical section suddenly jumps as the applied load reaches the peak load. As the crushing strain reaches in the concrete at the compression region, the applied load is sharply reduced, accompanying the beam deflection increases significantly as shown in **Fig.12**. This causes the change in the rate of stress increase as shown in the curve of the increase of cable stress vs. deflection (see **Fig.14**). Because the deflection of beam increases noticeably after the crushing of concrete, the linear relationship, therefore, is terminated as shown obviously for the case of slip with friction.

Fig.15 shows a comparison between the cases of perfectly fixed and partially fixed in terms of the increase of cable stress vs. deflection curves. It can be seen from this figure that since the external cables are being perfectly fixed at the deviators as in the case of perfectly fixed, the stress increase in the midspan segment and the end segment is totally different. While for the case of the cables being partially fixed at the deviators, the difference of the stress increase in the midspan segment and the end segment is lesser as compared to the case of perfectly fixed. This indicates that some cable slip might occur at the deviator points, resulting in transfer of cable stress from the midspan segment to the end segment. This phenomenon is agreed well with the experimental observations, which have been conducted by Fujioka, A., et al.[26]

It is also found from the calculated results that the ultimate load of the beam with consideration of partially fixed at the deviators does not increase much as compared with the cases of free slip and slip with friction (see **Fig.12** and **Table 4**). However, the stress increase in the external cables is much higher as the comparisons have been made. This is because the strain variation in the external cables depends not only on the overall deformation of the beam, but also on the free length of a cable between two successive deviators, i.e., it depends on a ratio of L_d/L (the distance between the deviators per the total span length). For the beam tested by CEBTP, this ratio of L_d/L is equal to 0.5, which seems to be considerably large. In this case the extent of fixity of cable at the deviators has a significant effect on the stress increase in the external cables rather than on the ultimate strength of the beam. It is believed that when the ratio of L_d/L is rather small, both the ultimate strength and the stress increase in the cables are significantly increased due to the extent of fixity of cable at the deviators. The improvement due to the fixity of cable is also verified by the experimental observations for two pairs of beams with the different ratio of L_d/L, which have been reported elsewhere[26].

The results at the ultimate stage for the beams under the different bondage of cable at the deviators are presented in **Table 4**. It should be, generally, noted that friction at the deviators reduces the ultimate deflection and increases the stress in the prestressing cables. However, it is found from the analysis that the results of the case of slip with friction show somewhat contrary to the other cases. The reason for that might be the strain jump, which is happened in the concrete at the critical section as explained early. Note that the calculated results in terms of load vs. deflection and load vs. increase of cable stress curves have been observed somehow similar for the both cases of free slip and slip with friction.

It is also found from the predicted results that beam with partially fixed condition shows a higher ultimate load but a lower increase of cable stress as compared with beam having perfectly fixed condition. This is rather contrary to the previous findings that beam having a higher cable stress should also have a higher ultimate load capacity in general. The reasons for this can be explained that since the cables are perfectly fixed at the deviators as in the case of perfectly fixed, the cable stress usually reaches the yielding strength at the lower level of the applied load as compared with the case of partially fixed. As a result, the ultimate load capacity of the beam in the case of perfectly fixed is a little smaller than that obtained from the case of partially fixed. Moreover, the value of friction coefficient adopted for the case of perfectly fixed in this study is not exactly known for the real condition. This reason might also lead to overestimate the stress increase in the external cables. For the others cases of this study, the predicted results are agreed well with the findings from the previous studies.

The effect of friction is also investigated on the beams tested by Nishikawa, K., et al.[18] and Zhang, Z., et al.[19]. The calculated results are plotted in **Fig.16**. It is apparently shown that the friction at the deviators have some influence on the load-deflection curves of a prestressed concrete beam with external cables. Although a small difference between the cases of free slip and slip with friction has been observed, the experimental results, however, fit more closely with the assumption of slip with friction. The same effect of friction at the deviators is also found as in the case of the beam presented in **Fig.12**. Similar predictions of the friction effect on the behavior of the beams with external cables have been reported elsewhere[27), 28)]. It should be noted that since no any means to prevent the movement of a cable at the deviator points are generally provided, the assumption of either free slip or slip with friction seems to be more realistic rather than the assumption of perfectly fixed in the numerical analysis.

5. SUMMARY AND CONCLUSIONS

A non-linear analysis using a finite element algorithm together with the deformation compatibility of beam is performed to predict the entire response of the beams prestressed with external cables up to the ultimate loading stage. The accuracy of the proposed method is verified by comparing the predicted results with the experimental observations. The predicted results in terms of load vs. deflection and load vs. increase of cable stress curves are in reasonably close agreement with the experimental data. The close agreement between the experimental data and the predicted results apparently indicates a validity and potential of the proposed method for the analysis of beams prestressed with external cables. The proposed method is generally suitable for the investigation of all kinds of beam prestressed with external cables such as simply supported or multiple spans continuous beams with or without deviators. It should be noted that the proposed method might be best for the purpose of research rather than for the design practice.

There is a close relationship between the two curves of load vs. deflection and load vs. increase of cable stress. This relationship is also verified by the fairly linear response between the midspan deflection and the increase of cable stress for the individual beam.

The appropriate amount of non-prestressed reinforcement should be provided for externally prestressed concrete beams to improve the ultimate strength of the beams. The externally prestressed concrete beams with the adequate addition of bonded non-prestressed reinforcement exhibit like the flexural members after cracking rather than the shallow tied arch members.

In consideration of friction at the deviators, the cables with free slip and slip with friction produce more or less equalized stress increase at all loading stage, and very similar to the experimental observations. While the cables with consideration of partially fixed and perfectly fixed at the deviators overestimate the stress increase as well as ultimate strength of the beam.

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デビエーターでの摩擦を考慮した外ケーブル方式PC桁の解析に関する研究

Bui Khac DIEP・梅原秀哲・田辺忠顕

本論文は, 変形の適合条件に基づき, デビエーターでの摩擦を考慮した外ケーブルプレストレスコンクリートはりの解析手法を示したものである. 外ケーブルを有する数多くのコンクリートはりの実験を数値解析し, 荷重－変位関係, 荷重－外ケーブル応力関係等について実験値と比較, 考察を行うことにより, 本解析手法の信頼性を検証した. また, デビエーターでの摩擦が, 外ケーブルプレストレスコンクリートはりの挙動に与える影響を探るために, デビエーターでのケーブルの挙動について, 摩擦が無く滑る場合, 摩擦を伴い滑る場合, 部分的に固定する場合, 完全に固定する場合の 4 つのケースについて感度解析を行った. 解析値と実験値を比較することにより, 摩擦がはりの挙動に及ぼす影響を考察した.