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ON THE ACCORDANCE OF FRAME STABILITY WITH THE EFFECTIVE COLUMN LENGTH *

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The effective length concept has been widely accepted in the design of framed structures. An excessive accuracy is not to be expected from its origin apart from structural analysis, but there still remain certain cases where its estimation is far conservative beyond reality. The practice for a rectangular frame is set up differently depending upon whether braced or unbraced. Literally, the effective length has been designated as an axial strength of individual members. In this study, an inadequacy is found in those conventional practices, and an alternative manner is suggested to deal with the structural buckling of a rectangular frame.

Key Words : *rectangular frame, column unit, sway buckling, non-sway buckling*

1. INTRODUCTION

In our design of a framed structure, the stability as a whole is assured on the axial strengths of its consisting members. In such a scheme, the member strengths are not to be determined in themselves, but are required to take account of their interaction effects also. The *effective length method* is a well-known approach to the buckling resistance of each member after restrained with its adjacent members. In that method, an appreciation of trial members is immediately brought without structural analysis, but the strict basis on which those individual strengths are established seems not apparent so far. For instance, the effective length factor for compressed truss members is conventionally taken as unity, but, in a rigidly-connected truss, their actual values are supposed to be less than unity. So it still remains questionable to what extent the effective length concept has its validity.

The present study is confined on *rectangular rigid frames.* Their bucklings in vertical loading are characterized by the fact that columns are compressed enough into their unstable ranges to balance with the stable beam deformations. By the absence of axial forces, the beam restraints can be formulated for an assumed buckling mode. It is due to those restraints by its connected beams that the effective length of each compressed column is fairly estimated. In the AISC Manual,[1)] the two alignment charts for K-factors are such a result for braced and unbraced frames, separately. Recently in Ref. 3), it is found that a rectangular frame is exactly consisting of the structural units which stand for the respective column members stiffened with their adjoining members. Those *column units* have almost the same effective lengths as given in the AISC alignment charts.

In the field of design practice, as introduced in Ref. 2), there has been a certain assertion that the going effective lengths could have excessive margins. Since based on each compressed member subject to its neighboring restraints only, the effective length concept seems corresponding to the structural buckling in a uniform loading, rather than in a partial loading. In our design codes, however, the effective length is employed as an axial strength of members, with no regard to the whole loading condition. In this study, through numerical examples in different loading patterns, the *sway* and *non-sway* bucklings of rectangular frames are analyzed on their dependence on the respective effective column lengths.

*A main part of this paper has been presented at the JSCE Annual Conference,[4)] held in Kumamoto, 2001-10.

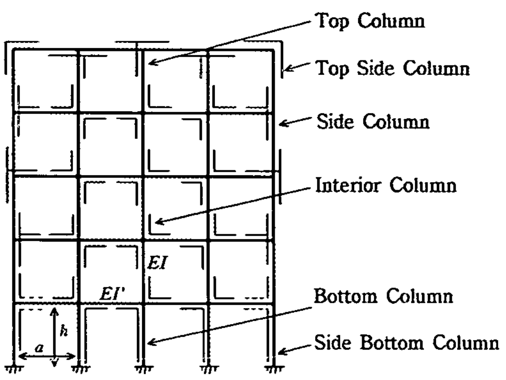

Fig. 1 Column units in a simple rectangular frame

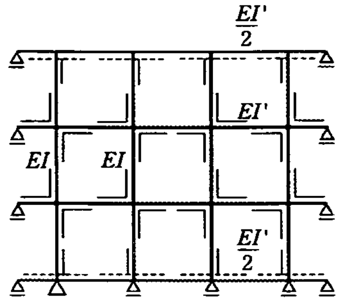

Fig. 2 A finite assembly of interior column units

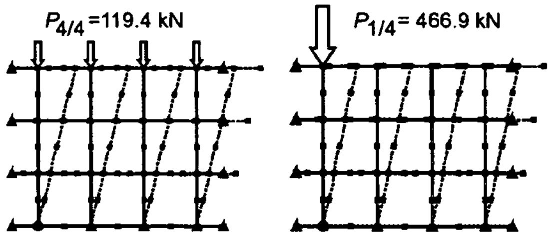

Fig. 3 Sidesway buckling

2. SWAY BUCKLING UPON AN IDEALIZED FRAME

It is possible for a rectangular frame to be decomposed into the *column units*,[3] instead of into the column and beam members. They are called "unit" for the following two: a usual frame is assembled from those units of different types; and, for each type, there exists a uniform frame consisting of the same units only.

As an example, the segmentation in a simple rectangular frame is shown in **Fig. 1**. In a frame of many panels, which is bounded with the side units and the top and bottom units, the buckling rigidity as a structure depends primarily on the interior units. A mere frame of the *interior-column units* is favorable for our essential argument, but their simple assembling results into an infinite frame. For the actual computation, let a subassembly of 3×3 panels, shown in **Fig. 2**, be considered: the side beams are cut at their mid points; and the bottom and uppermost beams are split off by a vertical plane into one half of the cross section.

We first argue the finite frame in its sidesway bucklings. Let the cut ends of the side beams be supported by horizontal rollers so that they are kept as inflection points. The notations are as follows: width a and height h of a panel; modulus of elasticity E; area A and moment of inertia I of the column cross-section; and area A' and moment of inertia I' of the beam cross-section (the lowermost and uppermost beams have $I'/2$ and $A'/2$). In the numerical examples below, the bucklings of a framed structure are determined in the *method of separation into rigid displacement and deformation.*[5] On the other hand, the critical axial forces in the respective column units are obtained from their buckling equations formulated on the *beam-column theory.*

In Ref. 3), the units' buckling equations are numerically solved in a variety of the bending stiffness ratio, and the critical values are shown in the effective length factor. The same values are here converted into the critical axial-force factor, which are shown in **Appendix–A**.

With $a = h = 500\,\text{cm}$, the columns and beams were once assumed to have a cross-section H250× 125: $I = I' = 255\,\text{cm}^4$, $A = A' = 32.68\,\text{cm}^2$ and $E = 206\,\text{GPa}$. But, the buckling equations of the column units are independent of the extension rigidity EA. To reduce the shortening effects in their comparison, the cross-section area is fictitiously magnified to $A = A' = 3268\,\text{cm}^2$.

Under uniform axial loading into the four columns, the frame buckling is determined at $P_{4/4} = 119.4\,\text{kN}$, which is almost equal to the critical axial force, $P_0 = 119.5\,\text{kN}$, from the buckling equation of the interior unit. In the one-point loading shown in **Fig. 3**, the sway buckling takes place at $P_{1/4} = 466.9\,\text{kN}$. In other cases of the equal point loads: $P_{3/4} = 159.0$ and $P_{2/4} = 237.4\,\text{kN}$. And, $P_{1/3} = 353.3\,\text{kN}$, from the three-column model. Concerning those bucklings of the four-column frame, to be noted, their respective axial forces are summed up to much the same value, namely $P_0 \times 4$. As stated in Ref. 2), since the joints on each story are swayed with the same magnitude, the buckling is confined to being a phenomenon between stories. Actually, as shown in the figure, the two buckling modes are

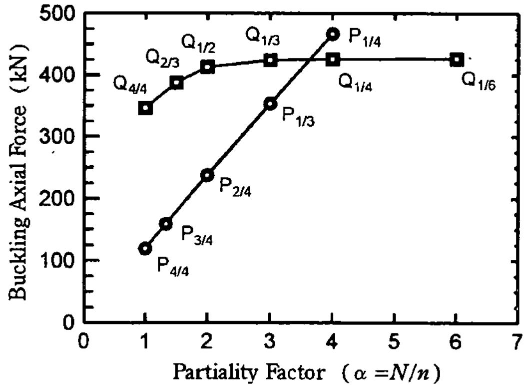

Fig. 4 Column axial force in the sway bucklings (P), and in the non-sway bucklings (Q)

quite similar, that is, are uniform over the consisting units. Also stated in Ref. 2) : "sidesway will not occur until the total frame load on a story reaches the sum of the potential individual column loads for the unbraced frame." This is really shown in the above examples. More specifically: the structural resistance is given by the sum of the *critical axial forces of column units* on a story.

When the *partiality* of equal axial loads into n columns out of the total N is expressed by

$$\alpha = \frac{N}{n} \tag{1}$$

the buckling column load $P_{n/N}$ comes to be proportional to this factor α. The above $P_{4/4}$ to $P_{1/4}$ are actually plotted in **Fig. 4**.

3. NON-SWAY BUCKLING OF THE REPRESENTATIVE FRAME

To examine the non-sway bucklings, the preceding rollers at the side ends are now replaced by vertical guide supports (rotation fixed). By their restraints, the behavior of our frame is kept in mirror symmetry with respect to two vertical lines passing through the side ends.

For the same values of length and cross-section, the *braced* unit of interior column has a critical axial force of $Q_0 = 346.0$ kN, which is about three times as large as the *unbraced* P_0. The frame buckling under the uniform loading is determined at the same $Q_{4/4} = 346.0$ kN, with a uniform non-sway mode. When loaded into a single column shown in **Fig. 5**, the buckling value is raised to $Q_{1/4} = 425.7$, but the ratio to $Q_{4/4}$ is only 1.23 in the present case of $EI'h/EIa = 1$. In other paneled models: $Q_{2/3} = 387.4$; $Q_{1/2} = 412.9$; $Q_{1/3} = 424.2$; and $Q_{1/6} = 426.5$ kN.

In a braced frame, the joints are not allowed to have a translation beyond the member ex-

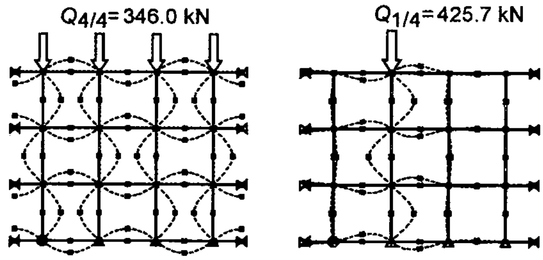

Fig. 5 Non-sway buckling

tensions. In the non-sway buckling, it is only through the joint rotations that the (unstable) deformations of compressed columns are transmitted to other restraining members. Such deformations are naturally absorbed in a short chain of members: in the above example of one-point loading, the two columns neighboring to the loaded one are somewhat involved in the buckling, via their connecting beams; but almost no deformations are seen beyond those two columns. Even under a partial loading, not such an extreme rise of the axial forces as in a sway-mode buckling is needed to yield the localized buckling.

The above $Q_{4/4}$ to $Q_{1/6}$ are also plotted in **Fig. 4**. The buckling column loads are almost constant for $\alpha \geq 3$. It thus appears that the partial-loading effects can be dealt with for $N = 3$. That is, even in a frame of more than three columns, the non-sway buckling under a partial loading is fairly predicted by examining the axial forces in every three adjacent columns. When the line from $Q_{4/4}$ to $Q_{1/3}$ in the figure is approximated by a polynomial, the cubic curve with vertex at $\alpha = 3$ is suitable. Noting that $Q_{3/3}$ is equivalent to the unit's critical Q_0, let the interpolation be expressed in a normalized form :

$$\bar{q}(\alpha) = \bar{q}_{\frac{1}{3}} - (\bar{q}_{\frac{1}{3}} - 1)\left(\frac{3-\alpha}{2}\right)^3 \quad \text{for } 1 \leq \alpha \leq 3 \tag{2}$$

where $\bar{q}(\alpha) = Q(\alpha)/Q_0$ and $\bar{q}_{1/3} = Q_{1/3}/Q_0$.

The uppermost $\bar{q}_{1/3}$ itself depends on the bending stiffness ratio, $\kappa = EI'h/EIa$. With different values of κ, the three-column model shown in **Fig. 6** is computed for $Q_{1/3}$ (Q_0 in parentheses). The resulting $\bar{q}_{1/3}$ are plotted in **Fig. 7**: the maximum rate is only 1.226 at $\kappa = 1$; and, on the logarithmic axis of κ, the descent for smaller κ is, roughly, similar to that for larger κ. For those $\bar{q}_{1/3}$, the corresponding $\bar{q}_{1/2}$ and $\bar{q}_{2/3}$ are estimated by Eq.(2), which are indicated by broken lines in the figure. Those ratios to Q_0 are much smaller than $P_{n/N}$ to P_0 in the sway-mode bucklings (see **Fig. 4**), and so might be disregarded as

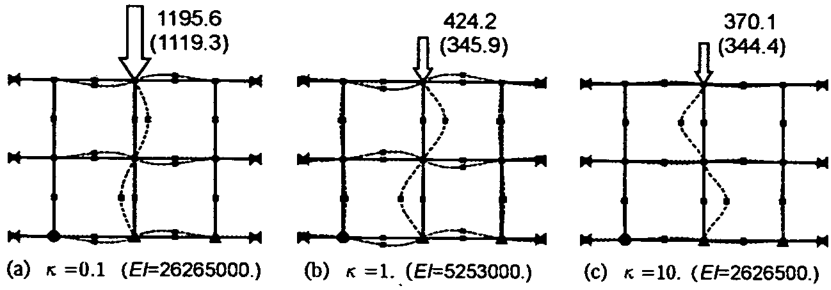

Fig. 6 Buckling mode in different bending stiffness ratios (kN, cm)

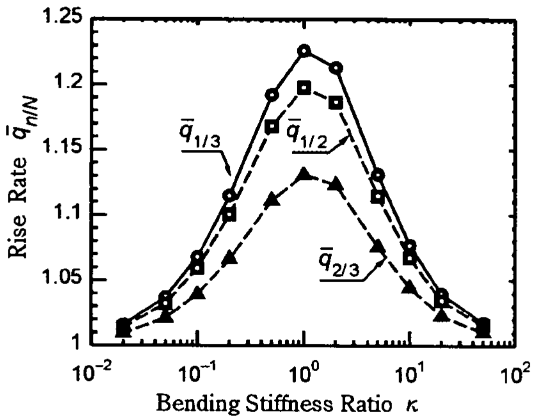

Fig. 7 $Q_{n/N}\,/\,Q_0$ in braced rectangular frames

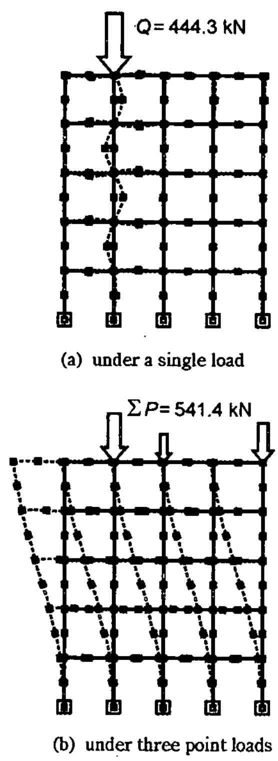

Fig. 8 Two bucklings of a simple rectangular frame

a practical treatment. But, if required, an improved estimation of the buckling axial forces is possible by the use of Eq.(2) and **Fig. 7**.

4. SIMPLE RECTANGULAR FRAME

The existing stability design of a rectangular frame is set up differently depending upon whether braced or unbraced. We here consider an unbraced simple rectangular frame shown in **Fig. 8**, consisting of the same members as assumed in **Sec. 2**. Illustrated with this example is the fact that an unbraced frame is not always withheld from the non-sway buckling.

Table 1 Critical axial force of the column units (kN)

unbraced	side column	mid column
top story	119.5	153.0
middle story	82.1	119.5
bottom story	126.7	155.1

braced	side column	mid column
top story	345.9	440.3
middle story	283.5	345.9
bottom story	482.6	529.1

The consisting column units are of six types: the critical axial forces in their sway modes, and in their non-sway modes, are given in **Table 1** (also shown in **Figs. A–1** and **A–2**). According to the result of **Sec. 2**, the story resistances to the sway-mode buckling are estimated as follows :

$$\text{top story}\ldots\ 119.5\times 2 + 153.0\times 3 = 698.0$$
$$\text{middle story}\ldots\ 82.1\times 2 + 119.5\times 3 = \underline{522.7}$$
$$\text{bottom story}\ldots\ 126.7\times 2 + 155.1\times 3 = 718.7$$

Consider the two loading conditions, shown in the figure. As for the non-sway buckling under a partial loading of $\alpha = 3$, multiplying the units' critical values by $\bar{q}_{1/3} = 1.23$, we have 541.6, 425.4 and 650.9 kN for the mid three columns, respectively.

Under the one-point loading, the frame buckling occurs in a non-sway mode; for, the smallest column strength 425.4 is approached prior to the story strength, 522.7 kN. In case of the three point loads which are applied in ratio 3 : 1 : 2, the buckling is in a sway mode: when the total frame load is increased up to 522.7, the maximum axial force is 261.4, much short of the column strength 345.9 kN. After the structural analysis, the actual buckling loads are determined at 444.3 and 541.4 kN, which are slightly larger than the predicted 425.4 and 522.7 kN, respectively. This is because the column units being considered are restrained by the upper and lower units having larger buckling rigidities.

5. SUMMARY

Instead of in the effective length, the expression in the axial-force factor is suitable for the stiffened columns to be argued on their interactions. Trivially, an unbraced frame does not exceed the braced correspondent in the buckling rigidity, but, at the same time, is not inhibited from the non-sway buckling. The two types of buckling are different in their arising mechanics: the non-sway buckling is resisted largely by the individual column units carrying the compressive loads; but the column units on a story are combined against the sway buckling under a general loading. In principle, the two capacities of a rectangular frame are estimated as follows :

$$[\textit{story resistance to the sway buckling}] = \left[\begin{array}{l}\textit{the sum of critical axial forces}\\ \textit{of the column units on a story}\end{array}\right]$$

$$[\textit{column resistance to the non-sway buckling}] = [\textit{critical axial force of the column unit}] \times [\textit{amplifying factor for the partial loading}]$$

By the alliance among the column units against the sway-mode buckling, it can happen under a localized loading such as $\alpha \geq 4$ (see **Fig. 4**) that the stability limit of an unbraced frame is determined by the non-sway buckling.

APPENDIX–A. CRITICAL AXIAL LOAD OF COLUMN UNITS

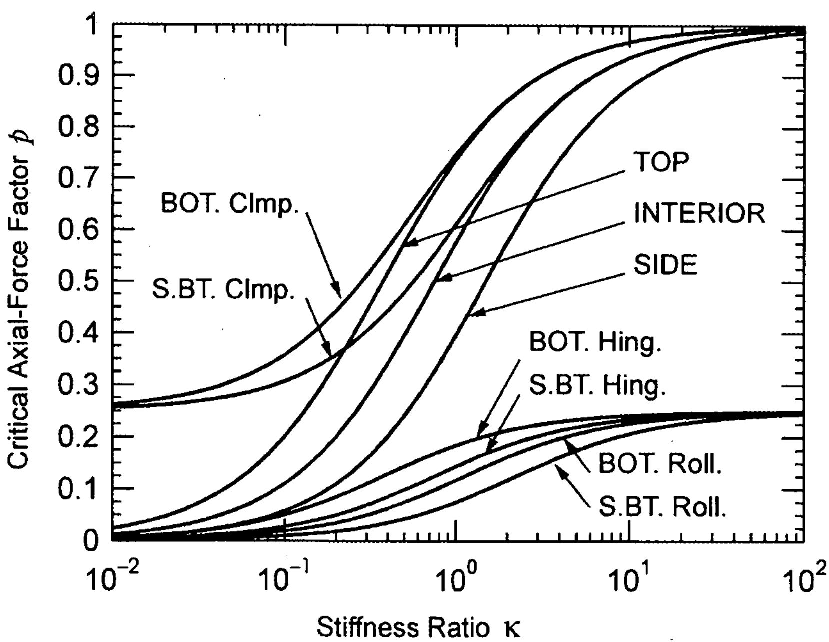

Fig. A–1 Critical axial force of unbraced column units ($P_0 = p\pi^2 EI / h^2$)

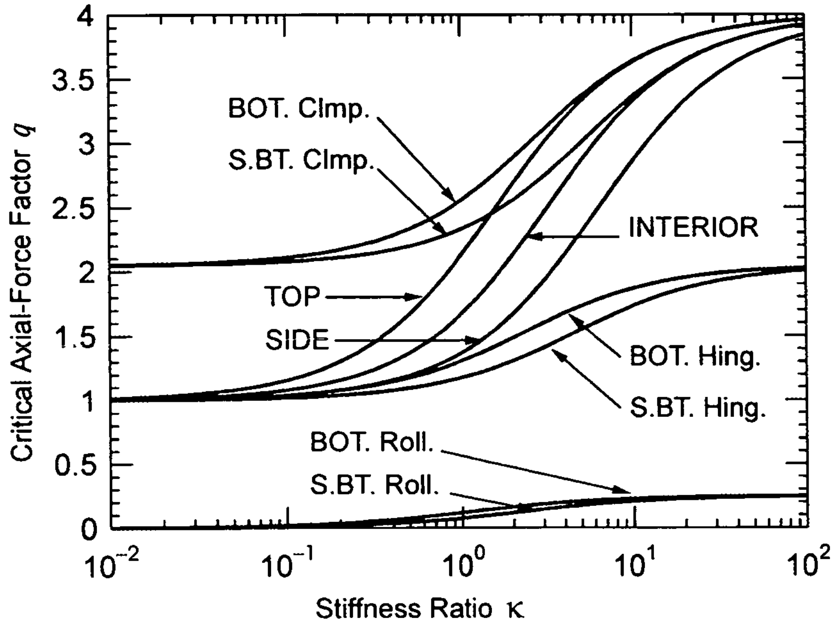

Fig. A–2 Critical axial force of braced column units ($Q_0 = q\pi^2 EI / h^2$)

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骨組の全体座屈と有効柱長の整合性について

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有効長による骨組構造の安定性評価は，全体構造解析を必要とせず，元来 高精度を期待する方法ではないが，非現実的に安全側に過ぎる値を与える場合があることが，これまでに指摘されている．本文では，矩形ラーメンに限って，その適用性を検討している．これまで，有効長は各柱の部材強度として用いられており，側拘束の有無によって最初から異なる値が設定されている．部分荷重などの一般的な鉛直載荷に対して，横振り座屈では，各層に含まれる柱すべてが抵抗部材となり，側拘束型座屈は，基本的には圧縮を受ける柱部材とそれに直結するはり部材（構造単位）のみによって抵抗されることを，構造解析を行いながら確認した．無側拘束であっても，極端な部分載荷では，横振り型よりも側拘束型座屈の方が先行して生じるなど，横振り座屈に対しては，柱部材毎の強度ではなく，層毎の抵抗値を設定することの必要性を示している．