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REFINED TEMPERATURE MODELING IN LAKE YANAKA BY USE OF SECCHI DEPTH AND A PHYSICALLY-BASED EDDY DIFFUSIVITY

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In this study, a model for computing water temperature in the Lake Yanaka is established. The main feature of the model is the use of secchi disk depth to better estimate the heat budget. Secchi disk depth observation is incorporated with empirical relationship to calculate more accurately the solar radiation fraction coefficient and the extinction coefficient. Besides, the eddy diffusion coefficient, under the assumption of complete mixing, is calculated from the wind shear, density stratification, and inflow and outflow rate. Model results are compared with the observation data.

***Key Words**: Secchi disk depth, heat budget, stratification, extinction coefficient, eddy diffusion*

1. INTRODUCTION

The distribution of thermal energy in a lake has a significant influence on lake's operations and water quality. Water temperature is a dominant factor determining density, which, in turn, controls the placement of inflowing water, associated constituent load, and the outflow distribution. Water temperature also affects many biological and chemical rate processes.

(1) Study site

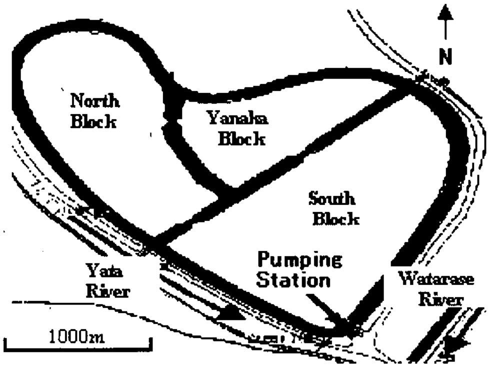

Fig.1 Layout of Lake Yanaka

The study site, Lake Yanaka (see **Fig.1**), is part of the Watarase retarding basin, north of Tokyo, described in detail elsewhere[4)]. Its main purposes are three folds: flood protection, drinking water supply to Tokyo area and maintenance of water supply for downstream rivers. This lake has a surface area of 4.5 ha, a maximum depth of 9 m, and a mean depth of 6 m. The lake is divided into three blocks by levees, named as south block, north block and Yanaka block, connected with gaps. In this lake, polymictic conditions prevail with mixing every few days or even daily most of a year.

Simulation of water temperature in lakes or reservoirs has relied heavily on the use of one-dimensional (vertical) modeling technique. A well-documented approach is the so-called eddy diffusivity model. There exists a large body of knowledge on the subject, various eddy diffusivity formulations have been proposed and evaluated[6)]. Particularly, many research works are only concerned with wind inducing mixing. Inflow and outflow inducing mixing are much neglected[3), 5)]. In the present study both mixing mechanisms are handled effectively in our modeling efforts. Addition to this physically based eddy diffusivity consideration, we improved the estimation of heat budget and temperature distribution for each layer by making use of secchi disk depth observation.

2. THE MODEL DEVELOPMENT

The mathematical structure of Water-Mud temperature model is based on a set of differential equations that express conservation of mass or energy in each horizontal layer. Their solutions provide temperature as functions of time and depth. The general formulation, from the basis of ***Eddy diffusivity Model*** approach[5)] is summarized as following equation.

$$\frac{\partial T}{\partial t}V + \frac{\partial}{\partial z}(A(z)w(z)T)\Delta z$$

$$= \frac{\partial}{\partial z}[A(z)(\alpha + K(z))\frac{\partial T}{\partial z}]\Delta z \qquad (1)$$

$$+ q_{IN}T_{IN} - q_{OUT}T$$

$$- \frac{1}{\rho C_p}\frac{\partial(Q_{NS}(z)A(z))}{\partial z}\Delta z$$

Where, $A(z)$ = cross section area at depth z , T = temperature, w = velocity in the z direction, t = time, z = depth, Δz = thickness of the slice, α = coefficient of molecular diffusion, $K(z)$ = coefficient of eddy diffusion at depth z , $Q_{NS}(z)$ = flux of solar radiation at depth z, q_{IN} = inflow rate, q_{OUT} = outflow rate and V = volume of the slice.

In the present study, to distribute the source terms (Q_{NS}) in above formulation, secchi disk depth observation is utilized for better estimation of heat budget of each water slice.

(1) Heat budget estimation

In contrast to upwelling and entrainment, heat cycles are relatively easily measured. They give total accounting of the gain and loss of heat by the system during a specified time period on the lake surface. To estimate the heat budget, the components of lake's heat cycle[1)] are modified in the following format:

$$Q_N = \lambda Q_{NS} + Q_L - Q_B - Q_E - Q_C - K(z)\frac{\partial^2 T}{\partial z^2}/_{Z=D} \qquad (2)$$

Where, Q_N = net storage rate of heat in lake's surface layer, Q_{NS} = net solar radiation, Q_L = atmospheric radiation, Q_B = back radiation, Q_E = evaporation loss, Q_C = sensible heat transfer, which is roughly equal to conduction, λ = surface layer absorption part, and rest is the gradient loss at the bottom of surface layer. In our model, surface layer depth (D) is controlled as 0.6m or slightly more. It depends mainly on current water level. And rest part of the water column is divided into several layers with constant thickness (Δz). In surface layer, we especially define a depth, Z^*, to estimate the λ accurately.

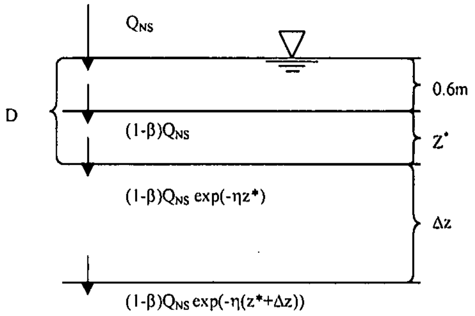

Fig.2 Solar radiation fraction in each layer

$$Z^* = D - 0.6 \qquad (3)$$

$$\lambda = \begin{cases} \beta & D = 0.6\text{m} \\ 1-(1-\beta)^*\exp\{-\eta Z^*\} & D > 0.6\text{m} \end{cases} \qquad (4)$$

Where, the fraction of solar radiation absorbed within 0.6m depth, β, and the extinction coefficient, η [9)], are estimated by using the following empirical equations.

$$\eta = 1.1 Z_S^{(-0.73)} \qquad (5)$$

$$\beta = 0.27\ln(\eta) + 0.61 \qquad (6)$$

Where, Z_S is the secchi disk depth. The following figure shows the details about the surface layer fraction of solar radiation.

The short and long wave radiation, water surface evaporation, back radiation, and sensible heat losses are calculated according to McCutcheon [8)].

(2) Water surface temperature calculation

From the heat budget the following equation is used to describe[8)] the water surface temperature calculation.

$$C_P \rho \frac{\partial T}{\partial t} = A \times Q_N / \Delta Vol \qquad (7)$$

Where, C_P = specific heat of water, ρ = density of water, A = surface area, ΔVol = volume for surface layer. The volume distribution for each layer is calculated from the following empirical equation obtained from the observational data.

Total Volume at water level Y.P

$$=\{-35332.829+4117.636YP\}*1000m^3 \qquad (8)$$

Where, *Y.P* is the water surface elevation from the datum. Due to the complexity of determining water surface temperature on hourly basis, an iterative procedure is followed.

(3) Vertical temperature distribution

Stratification is of crucial importance in determining water quality in lakes and it is therefore important to be able to predict temperature patterns in vertical direction. This prediction mainly depends on accurate eddy diffusivity formulation. To determine the vertical eddy diffusivity coefficient for each slice at each time interval the following steps are considered;

Step1: Dissipation per unit mass by wind shear mixing is calculated by the following equation.

$$DW = \frac{TKE_W}{\rho V_T \Delta t} \tag{9}$$

Where, V_T = Total volume of lake, TKE_W = available wind inducing turbulence kinetic energy for lake mixing. It is calculated by the following equation.

$$TKE_W = \int_{A_S} CW_{*} \tau \Delta t dA \tag{10}$$

Where, W_* = Square root of (τ/ρ_w), $\tau = \rho_a C_d W^2$ and C = empirical coefficient. It is a correction factor of available water surface for wind mixing. Here it is assumed as unity. Addition to this, ρ_a = density of air, ρ_w = density of water, W= wind speed and if $W <$ 15m/s, the drag coefficient, C_d = $0.0005W^{0.5}$ else $C_d = 0.0026$.

Step2: Mixing also results from inflows and outflows. This localized mixing is restricted to regions of flow. The turbulence kinetic energy (TKE_Q) generated by these flows is given by the following equation.

$$TKE_Q = 0.5 \rho q \Delta t (\frac{q}{l \Delta z})^2 \tag{11}$$

Where, q = longitudinal flow inside each layer and l = width of the layer. Dissipation per unit mass due to this mixing is given by the following equation.

$$DQ = \frac{TKE_Q}{\rho \Delta V \Delta t} \tag{12}$$

Step3: With the consideration of stability of the density stratification, both dissipation energies are finalized. The vertical eddy diffusivity coefficient for each layer is defined as the following format.

$$K = \Delta t^2 \{[\frac{C_1 \times DW}{1+Ri}] + [\frac{C_2 \times DQ}{1+(1/Fr)^2}]\} \tag{13}$$

Where, C_1 & C_2 are the calibration coefficients, Fr = densimetric Froude number and Ri = Local Richardson number for each layer.

(4) Sediment heat flux

A simple numerical method[2)] that simulates heat exchange of sediments with the water column is adopted here to account for the sediment impact.

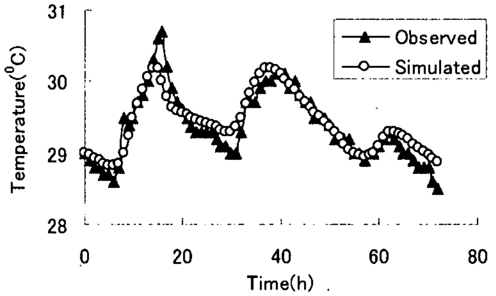

Fig.3 Surface water temperature for Lake Yanaka on 1-3 of August 1996

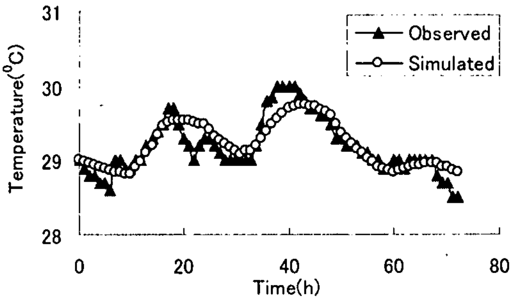

Fig.4 Bottom water temperature for Lake Yanaka on 1-3 of August 1996

(4) Simulation input data

In our calculation procedure, we select Δt as one hour. Calculations start from mid-night of the simulation day. Observed air temperature, humidity, wind speed, sunshine hour ratio, inflow rate and temperature, outflow rate, and air pressure are read in on hourly basis. Addition to these, we used secchi-disk depth data, initial water depth and temperature distribution were prescribed according to field data. These input data and values of all model constants used in this study were documented by Pathmathevan[7)].

3. RESULTS

For randomly selected days, the simulated results and field data from Tone River upstream work office are compared. The following figures show the outputs of the simulation:

Figure 3 shows the comparison of water temperature at 0.5m under the water surface. Meanwhile the Fig.4 shows the comparison at 1m above lake bottom. Unfortunately we have no more depth-wise observational data to compare the results at each depth level. Figure 5 shows the eddy diffusivity, at surface and bottom layer, and wind distribution. It is clear that they are well correlated and eddy diffusion doesn't change very much with water depth. During the simulation period, influences from inflow, outflow and density gradient on eddy diffusivity was very less. Because of the water depth during the simulation period was very low.

4. CONCLUSION

Introduction of secchi disk depth, for surface solar radiation fraction formulation, reduces the complexity of heat budget preparation and improves the agreement with the observed values. And the predicting results are further improved by using the physically based vertical eddy diffusion. The analysis and results clearly show the importance of these modifications. The modified simulation (simulated) results follow the trend with the observed values, while simulation (Sim-old), assuming an experimental constant value for eddy diffusivity, failed to catch the peak variation of observed values (**Fig.6**).

For short-term analysis, multi-dimensional approach with hydrodynamic flow model consideration may give more accurate results. On the other hand for long-term simulation, this approach is more suitable by comparing simulation time and the importance of accuracy level. This model is also a useful tool for quick prediction of the lake response to particular management decisions. Anyhow it should be mentioned that further model refinement work is needed to minimize model errors, as an example, to catch sudden changes in temperature distribution.

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REFERENCES

1) Alexandder, J.H. and Goldman, C.R.: *LIMNOLOGY second edition*, McGraw-Hill international edition, 1994.
2) Benoit, G. and Hemond, H.F.: *Vertical eddy diffusion calculated by the flux gradient method: Significant of Sediment-water heat exchange,* Limnol. Oceanogr., 41(1), pp. 157-168, 1996.
3) Dennis, E.F. and Stefan, H.G.: *Thermal Predictions Using Integral Energy Model,* Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 106, No. HY1, January 1980.
4) Huang, G.W. and Tamai, N.: *An improved phosphorus budget model and its application to Lake Yanaka,* Environmental Hydraulics, Lee, Jayawardena & Wang (eds) Balkema, Rottedam, ISBN 90 5809 035 3, 1999.

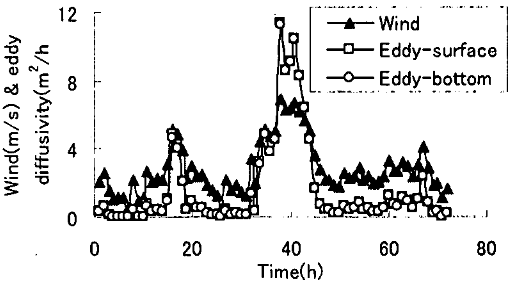

Fig.5 Wind magnitude and eddy diffusivity, at surface (Eddy-surface) and bottom (Eddy-bottom) layers, correlation for 1-3 of August 1996

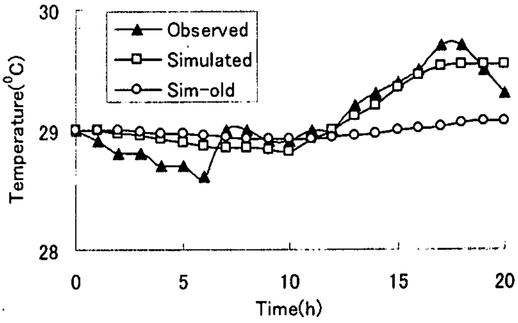

Fig.6 The bottom temperature distribution with constant (sim-old) and modified (simulated) eddy diffusivity on 1st of August 1996

5) James, A.: *An Introduction to Water Quality Modeling, 2nd Edition*, John Wiley & Sons Ltd, 1993.
6) Jassby, A. and Powell, T.: *Vertical pattern of eddy diffusion during stratification in Castle Lake, California,* Limnol. Oceanogr. 20: pp.530-543, 1975.
7) Pathmathevan, M.: *Thermal structure and dissolved oxygen dynamics in Lake Yanaka*, Master's Thesis, Dept. of Civil Engg., The University of Tokyo, 2000.
8) McCutcheon, S.C.: *Water Quality Modeling, Vol I*, CRC Press, Inc, Boca Raton, Florida, 1989.
9) Williams, D. T., Drummond, G. R., Ford, D. E. and Robey, D. L.: *Determination of Light Extinction Coefficients in Lakes and Reservoirs*, Proceeding of the Symposium on Surface Water Impoundments, American Society of Civil Engineers, Vol. 2, pp. 1329-1335, 1981.

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透明度および物理に基づいた渦動拡散係数を用いた谷中湖の水温計算

Mahadevan PATHMATHEVAN・黄　光偉・玉井　信行

湖沼では，水表面に到達した日射量の一部は薄い水表層に吸収され，残り部分が水中に透過して水に熱エネルギーを与える．谷中湖の水温モデルを構築する際，透明度（セッキ深度）の観測データを用いて水表層の熱吸収率及び水中の吸光係数を見積った．さらに，風，成層及び流入・流出の影響を考慮した渦動熱拡散係数を導入した．計算結果と観測データの比較によって，以上の改良の有効性が示された．